Study of the color string configuration in a multiparton system

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Using the recursive formulation of matrix elements for $e^+e^- \to m(q\bar{q}) + ng$, we obtain the color effective Hamiltonian of the process, which makes it possible to develop the method proposed in our previous work on analyzing the color string structure for $m = n = 1$ and $m = 0, n = 3$ systems to more general multiparton ones. The method for calculating the probability of the color configuration in a multiparton system is given. The ratio of the singlet string identified by PQCD is found to decrease rapidly as the gluon number grows.

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I. INTRODUCTION

The multiparton state predicted by perturbative quantum chromodynamics (PQCD) is extensively verified by multijet phenomena observed in high energy collisions. The current theoretical treatment describing these processes involves two main procedures: One is to determine the kinematic state of the system; the other is to use a certain phenomenological model to hadronize [1,2]. For example, when the Lund model is applied to the process $e^+e^- \to m(q\bar{q}) + ng$, first, one might use the parton shower model (PSM) [2] or the color dipole model (CDM) [3] to give the kinematic state of this multiparton system and then separately assign the color string configuration (or the color dipole chain) stretched between partons [4]; next, one applies the string fragmentation model (SFM) to each substring or dipole in the same way as the $q\bar{q}$ singlet [2,5]. This color string structure of the multiparton system is regarded not only as a consequence of parton evolution, but also as the interface to connect hadronization models. Therefore it can be thought of as the bridge between perturbative and nonperturbative phases.

In the Lund model, the color string configuration for the multiparton system is an extension of the neutral color flow model for the simplest system $q\bar{q}g$. In the neutral color flow model, the gluon is always treated as a composite color object consisting of one color and one anticolor. For the system $q\bar{q}g$, i.e., $m = n = 1$, the neutral color flow surely provides a chain of single substrings (dipoles) in the subsystems q-g and g-$\bar{q}$ at the first approximation, and so the two substrings (dipoles) fragment into hadrons in the same way as a $q\bar{q}$ singlet. Lund then extrapolated the above picture to the multiparton system [2,3]. But this kind of color string structure based on neutral color flow implies a nonet gluon rather than an octet, and the neutral color flow is not just the singlet string and hence the confining attractive field for PQCD [9]. Lund assumes that some topological effects of non-PQCD in a multiparton system may lead to this string picture [6]. The Lund string model which implements this color string structure has achieved great success by providing a fair description for various reactions [1,7,8]. The momentum configuration is obtained via PQCD in the popular Lund model, while its color string structure is assigned by the neutral color flow model rather than by PQCD. It is worthwhile to find out in PQCD limits what the color configuration is like and how it differs from the color string picture inspired by the neutral color flow model. In our previous paper, we defined by PQCD some attractive string configuration among partons, say, **3**, **3***, and particularly the singlet string **1** in terms of which the color string structure is very close to that used by Lund for the $q\bar{q}g$ system [9]. Enlightened by the idea of our previous work, in this paper, according to PQCD, we present a more general approach to calculate the probability of a color string configuration for the process $e^+e^- \to m(q\bar{q}) + ng$ and to study their dependence on the strong coupling constant α_s and the gluon number n.

The conventional matrix element method to calculate the process $e^+e^- \to m(q\bar{q}) + ng$ by PQCD encounters great difficulties as the number of partons increases as a result of the vast number of Feynman diagrams involved and particularly the complexity of the loop structures. Recently, two authors have developed a kind of recursive method to compute the matrix elements which naturally include all Feynman diagrams at the tree level. This method makes it convenient to deal with the multiparton processes recursively [10]. In Sec. II, we derive the color effective Hamiltonian H_c from the recursive formulation of matrix elements as the basis to study the color string structure of the multiparton system. Section III discusses the physical meaning of the color string and the completeness set of SU$_c$(3) singlets. Section IV gives a general approach to calculate the probability of the color string configuration, particularly that of singlet string.

The ratio of singlet strings is found to decrease drastically with the increasing of gluon number. In the last section, we make the conclusive remarks on the results.

II. COLOR EFFECTIVE HAMILTONIAN H_c

The recursive formulation of the matrix element recently developed is an exact approach at the tree level. It has the great merit of a simple form of the color part which is easy to separate for further study [10]. This section presents the operator form of color freedom in order to find out the color effective Hamiltonian from the recursive amplitude for $e^+e^- \to m(q\bar{q}) + ng$ and some of its basic properties.

A. Operator form of color freedom

The color triplet **3** for a quark and the anticolor one **3*** for an antiquark are written as

$$|\Psi_{i_u}\rangle = (|R\rangle, |Y\rangle, |B\rangle), |\Psi^{i_u}\rangle = (|\overline{R}\rangle, |\overline{Y}\rangle, |\overline{B}\rangle) \ , \quad (2.1)$$

where i_u is the color index for quark u; $(|R\rangle, |Y\rangle, |B\rangle)$ are the three color charges, and $(|\overline{R}\rangle, |\overline{Y}\rangle, |\overline{B}\rangle)$ are the three anticolor charges. For convenience of expression hereafter we work in the particle number representation and introduce color creation and destruction operators as follows:

$$\begin{aligned} \Psi_{i_u}^\dagger &= (R^\dagger, Y^\dagger, B^\dagger) \ , \\ \Psi^{i_u\dagger} &= (\overline{R}^\dagger, \overline{Y}^\dagger, \overline{B}^\dagger) \ , \\ \Psi_{i_u} &= [(\Psi_{i_u}^\dagger)^T]^* = (R, Y, B) \ , \\ \Psi^{i_u} &= [(\Psi^{i_u\dagger})^T]^* = (\overline{R}, \overline{Y}, \overline{B}) \ , \end{aligned} \quad (2.2)$$

where T and the asterisk denote transposition and the complex conjugation manipulation, respectively. Obviously, we have

$$\begin{aligned} |\Psi_{i_u}\rangle = \Psi_{i_u}^\dagger|0\rangle \ , \quad |\Psi^{i_u}\rangle = \Psi^{i_u\dagger}|0\rangle, \\ \Psi^{i_u}|0\rangle = \Psi^{i_u}|0\rangle = 0 \ , \end{aligned} \quad (2.3)$$

where $|0\rangle$ is the color vacuum state. For the color creation and destruction operators of quarks u and v, define these commutation relations as

$$\begin{aligned} [\Psi_{i_u}, \Psi_{j_v}^\dagger] = [\Psi^{i_u}, \Psi^{j_v\dagger}] = \delta_{i_u,j_v}\delta_{u,v} \ , \\ \text{other commutators} = 0 \ . \end{aligned} \quad (2.4)$$

From the above formula, we have

$$\begin{aligned} \langle\Psi_{i_u}|\Psi_{j_v}\rangle = \langle\Psi^{i_u}|\Psi^{j_v}\rangle = \delta_{i_u,j_v}\delta_{u,v} \ , \\ |\Psi_{i_u}\Psi_{j_v}\rangle \equiv |\Psi_{i_u}\rangle|\Psi_{j_v}\rangle = |\Psi_{j_v}\rangle|\Psi_{i_u}\rangle \equiv |\Psi_{j_v}\Psi_{i_u}\rangle \ , \\ \langle\text{color}|\text{anticolor}\rangle = 0 \ . \end{aligned} \quad (2.5)$$

We may define the ascending and the descending operators $I_\pm$, $U_\pm$, and $V_\pm$, for I, U, and V spin, respectively,

$$\begin{aligned} I_+ &= R^\dagger Y + \overline{Y}^\dagger \overline{R} \ , \\ I_- &= Y^\dagger R + \overline{R}^\dagger \overline{Y} \ , \end{aligned} \quad (2.6)$$

and they satisfy

$$[I_+, I_-] = 2I_3 \ , \quad (2.7)$$

where I_3 is the component of I spin along the third axis. We can use $I_\pm$ to ascend or descend one $\mathrm{SU}_c(3)$ multiplet to another along the I_3 direction. We have, for example,

$$I_+|Y\rangle = |R\rangle \ , \quad I_+|\overline{R}\rangle - |\overline{Y}\rangle \ , \ \text{etc.} \ , \quad (2.8)$$

for **3** and **3*** multiplets. So do $U_\pm$, $V_\pm$.

For the $\mathrm{SU}_c(3)$ octet of gluon colors, we define their creation and annihilation operators as

$$\begin{aligned} A_u^{1\dagger} &= (R^\dagger\overline{Y}^\dagger + Y^\dagger\overline{R}^\dagger)_u/\sqrt{2} \ , \\ A_u^{2\dagger} &= (Y^\dagger\overline{R}^\dagger - R^\dagger\overline{Y}^\dagger)_u/\sqrt{2}i \ , \\ A_u^{3\dagger} &= (R^\dagger\overline{R}^\dagger - Y^\dagger\overline{Y}^\dagger)_u/\sqrt{2} \ , \\ A_u^{4\dagger} &= (R^\dagger\overline{B}^\dagger + B^\dagger\overline{R}^\dagger)_u/\sqrt{2} \ , \\ A_u^{5\dagger} &= (B^\dagger\overline{R}^\dagger - R^\dagger\overline{B}^\dagger)_u/\sqrt{2}i \ , \\ A_u^{6\dagger} &= (Y^\dagger\overline{B}^\dagger + B^\dagger\overline{Y}^\dagger)_u/\sqrt{2} \ , \\ A_u^{7\dagger} &= (B^\dagger\overline{Y}^\dagger - Y^\dagger\overline{B}^\dagger)_u/\sqrt{2}i \ , \\ A_u^{8\dagger} &= (R^\dagger\overline{R}^\dagger + Y^\dagger\overline{Y}^\dagger - 2B^\dagger\overline{B}^\dagger)_u/\sqrt{2} \ , \\ |A^{a_u}\rangle &= A_u^{a_u\dagger}|0\rangle \ , \\ A^{a_u} &\equiv [(A^{a_u\dagger})^T]^* \ , \end{aligned} \quad (2.9)$$

where $i = \sqrt{-1}$ and $a_u = 1, 2, \ldots, 8$ in (2.9). They satisfy the relations

$$\begin{aligned} \langle A_u^{a_u}|A_v^{a_v}\rangle = \langle 0|A_u^{a_u}A_v^{a_v\dagger}|0\rangle = \delta_{a_u,a_v}\delta_{a,v} \ , \\ [A_u^{a_u}, A_v^{a_v}] = [A_u^{a_u\dagger}, A_v^{a_v\dagger}] = 0 \ . \end{aligned} \quad (2.10)$$

B. Color effective Hamiltonian H_c for the process $e^+e^- \to m(q\bar{q}) + ng$

We derive in this subsection the color effective Hamiltonian H_c from the invariant amplitude given by Ref. [10]. First consider the case $m = 1$, i.e., $e^+e^- \to q\bar{q} + ng$. According to Ref. [10], the invariant amplitude for the process in which a quark pair in color states $|\Psi_i\rangle$ and $|\Psi^j\rangle$ and n gluons in $|A_u^{a_u}\rangle$ ($a_u = 1,2,\ldots,8$; for $u = 1,2,\ldots,n$) are produced can be written as

$$M_{ij}^{a_1\cdots a_n} = \sum_{\rho=P(1,\ldots,n)} (T^{a_1}\cdots T^{a_n})_{ij}^{\rho} D^{\rho} , \qquad (2.11)$$

where $T^{a_u} = \lambda^{a_u}/2$ for $a_u = 1,2,\ldots,8$, λ^{a_u} is the Gell-Mann matrix for SU(3), the summation is over all permutations of $(1,2,\ldots,n)$, $D^{\rho} \equiv D(q,\bar{q},g_{P(1)},g_{P(2)},\cdots,g_{P(n)})$ is the function of the momenta of partons where momentum indices are suppressed, and ρ denotes a certain permutation of $(1,2,\ldots,n)$; $(T^{a_{P(1)}}\cdots T^{a_{P(n)}})_{ij}^{\rho}$ is the ith row and jth column element of the matrix $(T^{a_{P(1)}}\cdots T^{a_{P(n)}})$, where the denotion $\rho = P(1,2,\ldots,n)$ has been used.

Now we find H_c for the initial color state $|0\rangle$ and the final one $|f\rangle = |\Psi_{i'}\Psi^{j'}A_1^{a_1'}\cdots A_n^{a_n'}\rangle$ for a parton system $q\bar{q}+ng$ which satisfies

$$\langle f|H_c|0\rangle = \langle \Psi_{i'}\Psi^{j'}A_1^{a_n'}\cdots A_n^{a_n'}|H_c|0\rangle = M_{i'j'}^{a_1'\cdots a_n'} . \qquad (2.12)$$

The following H_c satisfies the above relation:

$$H_c = \sum_{\rho} (T^{a_1}\cdots T^{a_n})_{ij}^{\rho} \Psi_i^{\dagger}\Psi^{j\dagger}A_1^{a_1\dagger}\cdots A_n^{a_n\dagger}$$
$$= \sum_{\rho} (1/\sqrt{2})^n \,\mathrm{Tr}(Q^{\dagger}G_1^{\dagger}\cdots G_n^{\dagger})^{\rho} D^{\rho} , \qquad (2.13)$$

where the repetition of two subscripts represents summing (we use this convention unless explicitly specified) and $(Q^{\dagger})_i^j \equiv \Psi^{j\dagger}\Psi_i^{\dagger}$ is the reducible nonet tensor composed of a color creation operator for a quark and an anticolor one for an antiquark. In (2.13),

$$G_u = (1/\sqrt{2})\lambda^{a_u}A_u^{a_u\dagger}$$
$$= (1/\sqrt{2})\begin{bmatrix} A^{3\dagger}+A^{8\dagger}/\sqrt{3} & A^{1\dagger}-iA^{2\dagger} & A^{4\dagger}-iA^{5\dagger} \\ A^{1\dagger}+iA^{2\dagger} & -A^{3\dagger}+A^{8\dagger}/\sqrt{3} & A^{6\dagger}-iA^{7\dagger} \\ A^{4\dagger}+iA^{5\dagger} & A^{6\dagger}+iA^{7\dagger} & -2A^{8\dagger}/\sqrt{3} \end{bmatrix}_u$$
$$= G_u'^{\dagger} - S_u^{\dagger}E/3 = \Psi_u^{i\dagger}\Psi_{uj}^{\dagger} - \Psi_u^{i\dagger}\Psi_{ui}^{\dagger}E/3 \qquad (2.14)$$

is the octet tensor operator for gluon color and $A_u^{a_u\dagger}$ is given in (2.9); E is a unit matrix.

It can be seen in (2.13) that $H_c^{\dagger} \neq H_c$. Since the effective Hamiltonian is another expression of an S matrix, it is not necessarily Hermitian.

The validity of H_c can be verified by the following calculation of the matrix element for the process $e^+e^- \to m(q\bar{q}) + ng$. For the color initial state $|0\rangle$ and the final one $|f\rangle = |\Psi_{i'}\Psi^{j'}A_1^{a_1'}\cdots A_n^{a_n'}\rangle$, we sum over the color indices i' and j' of the quark and antiquark and those $a_1', a_2' \ldots, a_n'$ of n gluons; then, we obtain

$$\sum_f |\langle f|H_c|0\rangle|^2 = \langle 0|H_c^{+}H|0\rangle = M_{ij}^{a_1\cdots a_n}(M_{ij}^{a_1\cdots a_n})^* , \qquad (2.15)$$

where we have used (2.4) and (2.5) and (2.9) and (2.10). We can see from (2.15) that the calculation of ordinary matrix elements through H_c returns to the original form.

In the same way, one can show that the color effective Hamiltonian H_c for $e^+e^- \to q_1\bar{q}_1q_2\bar{q}_2 + ng$ can be written as

$$H_c = \sum_{\rho=P(1,\ldots,n)} \sum_{k+l=n} \sum_{i,j} (1/\sqrt{2})^n D_{ij,kl}^{\rho} \,\mathrm{Tr}(Q_1^{\dagger}G_1^{\dagger}G_2^{\dagger}\cdots G_i^{\dagger}T^{x}G_{i+1}^{\dagger}\cdots G_k^{\dagger})\mathrm{Tr}(Q_2^{\dagger}G_{k+1}^{\dagger}G_{k+2}^{\dagger}\cdots G_{k+j}^{\dagger}T^{x}G_{k+j+1}^{\dagger}\cdots G_{k+l}^{\dagger}), \qquad (2.16)$$

where $D_{ij,kl}^{\rho}$ is a function of the momenta of partons and i and j the position indices of the inner gluon line which connects two $q\bar{q}$ pairs; k and l are numbers of gluons emitted by $q_1\bar{q}_1$ and $q_2\bar{q}_2$; x is the color index of the virtual gluon (see Fig. 1).

For the general case with m pairs of $q\bar{q}$ and n gluons in the final state, we can also write their Hamiltonian similarly.

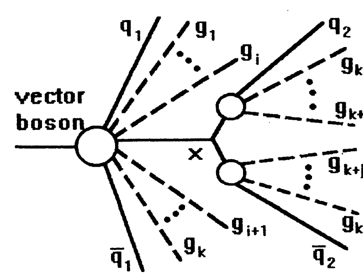

FIG. 1. Feynman diagram for process $e^+e^- \to$ vector boson $\to q_1\bar{q}_1q_2\bar{q}_2g_1g_2\cdots g_{k+1}$. The solid lines stand for quarks and the dashed lines for gluons. The circles stand for all structures at the tree level.

C. Matrix element of H_c

The matrix element of H_c between the initial and final states, $\langle f|H_c|0\rangle$, describes the invariant amplitude of the transition from the color vacuum state to a color configuration of a multiparton system. In this subsection, we list several properties of the matrix element for further use.

(a) Since H_c is a $SU_c(3)$ scalar and $\langle f|H_c|0\rangle$ obeys $SU_c(3)$ invariance, the physical color state $|f\rangle$ also must be a color singlet or scalar. This property excludes any color neutral states as physical ones unless they are singlets as well. The $SU_c(3)$ invariance plays an essential role in our exact PQCD analysis of the color string structure for a multiparton system.

(b) Let $|A_u^{a_u}\rangle$ be the color state of a gluon g_u and $|S_u\rangle$ be the singlet composed of its color charges; we have $\langle S_u|A_u^{a_u}\rangle = 0$; so if $|S_u\rangle \in |f\rangle$, then $\langle f|H_c|0\rangle = 0$. This property imposes that color charges from the same gluon must combine with those from different partons into a physical color singlet state. Most of present models satisfy this requirement automatically.

(c) The color configuration of the multiparton system is composed of color charges of m pairs of $q\overline{q}$ and those of n gluons. It belongs to the color space

$$3_{q_1} \otimes 3^*_{\overline{q}_1} \otimes \cdots \otimes 3_{q_m} \otimes 3^*_{\overline{q}_m} \otimes 3_1 \otimes 3^*_1 \otimes \cdots \otimes 3_n \otimes 3^*_n \ , \tag{2.17}$$

where 3_{q_u} and $3^*_{\overline{q}_u}$ $(u = 1, \ldots, m)$ are the color space for the uth quark pair, while 3_v and 3^*_v $(v = 1, \ldots, n)$ are the color and anticolor space for gluon v. The system states

$$|\Psi^{i_1}_{\overline{q}_1}\Psi_{q_1 j_1}\cdots\Psi^{i_m}_{\overline{q}_m}\Psi_{q_m j_m}\Psi^{k_1}_1\Psi_{1k_1}\cdots\Psi^{k_n}_n\Psi_{nk_n}\rangle \tag{2.18}$$

build the completeness bases for the color space (2.17), where $i_u, j_u = 1,2,3,\ k_v, l_v = 1,2,3$. There exist many ways to reduce the color space (2.17), while corresponding to each reduction there is one set of orthogonal singlet spaces whose bases make up a singlet completeness set. If a color singlet set is denoted by $|f_k\rangle$, $k = 1, 2, \ldots$, then

$$|f_k\rangle\langle f_k| = 1 \ , \quad \langle f_k|f_{k'}\rangle = \delta_{kk'} \ , \tag{2.19}$$

and

$$\begin{aligned}\sum_k |\langle f_k|H_c|0\rangle|^2 &= \langle 0|H_c|f_k\rangle\langle f_k|H_c|0\rangle \\ &= |M(q_1, \overline{q}_1, \ldots, q_m, \overline{q}_m, g_1, \ldots, g_n)|^2 \ .\end{aligned} \tag{2.20}$$

This property implies that the total sum of the cross sections over system singlets in a completeness set equals the total cross section σ_0: i.e., $\sum_k \sigma_k = \sigma_0$. This is the result of unitarity. Obviously, (2.19) and (2.20) also hold for another singlet set $|f_{k'}\rangle$. The two completeness sets differ by a unitary transformation.

III. COLOR STRING CONFIGURATION AND COMPLETENESS SET OF COLOR SINGLETS

It is necessary to determine the color string configuration before one uses fragmentation models to a multiparton system. In Sec. II C, we have shown that the physical color states of a multiparton system must satisfy the following conditions: (a) They are $SU_c(3)$ invariants or scalars; (b) they satisfy $\langle f|H_c|0\rangle \neq 0$. According to property (c) in Sec. II C, there is a multitude of color singlet sets which correspond to various reduction ways of the color space. In the Appendix, we take two systems $q\overline{q}g_1$ and $q\overline{q}g_1g_2$ as examples to illustrate how the color space is reduced and how system singlets are built. As one can see, each system singlet belongs to its own completeness set which is related to a specific method of reduction. Before we focus on the highly interesting topic of how the color charges of partons form a color string we shall recall the meaning of the color string and what the Lund model does at first. As is well known, the color string usually means a color-confining field, i.e., a linear confinement potential, between partons. Gustafson studied the properties of a confining force field among partons a decade ago [6]. He supposed that the confinement effects should lead to a subdivision of the full system into color singlet subsystems, and those singlet subsystems should connect with each other by some orderings. He could not prove such a picture from PQCD, and so he suggested that the topological properties of QCD which are not exhibited in perturbative theory play a fundamental role in specifying the above color string configuration. To determine it before these properties are clarified, the Lund model assumes that the attractive force field (color string) stretches between the color charge carried by one parton and its anticolor one by the other [6]. So in the Lund string model the color structure inspired by neutral color flow is only a model [5] and is based on the nonet gluon picture. The determination of momenta and the assignment of a color string configuration for the multiparton system are inconsistent, since the former is given by PQCD while the latter is not.

We know that the color interaction of a quark or gluon system can be written as [9,11]

$$V_{\text{int}} \propto V_0 \sum_{j=k}^{\beta} F_j(1)F_k(2) = (V_0/2)[C(1+2) - C(1) - C(2)] \ , \tag{3.1}$$

where V_0 is a positive constant, $F_j(u)$ is the generator of the $SU_c(3)$ multiplet for parton u, β is the number of generators, and $C(u)$ is the Casimir operator for a parton u given by

$$C(u) = \sum_{j=1}^{\beta}[F_j(u)]^2 \ , \quad u = 1, 2 \ . \tag{3.2}$$

$C(1+2)$ is the Casimir operator for the composite system of partons 1 and 2 and is written as

$$C(1+2) = \sum_{j=1}^{\beta}[F_j(1) + F_j(2)]^2 \ , \quad u = 1, 2 \ . \tag{3.3}$$

In our previous work, we have pointed out that the interaction between a color triplet and an anticolor one is attractive, i.e., $V_{\text{int}} < 0$, only if they combine into a singlet. For two color triplets (two anticolor triplets), an attractive interaction occurs when they form a multiplet $\mathbf{3}^*$ ($\mathbf{3}$). We can also apply (3.1) to the system of two octets. A short calculation shows that an attractive force field exists between them only when their composite system is a singlet or octet. We call this kind of attractive color force field the color string through which a composite color system in certain $\mathrm{SU}_c(3)$ multiplets binds together.

Take $e^+e^- \to q\bar{q}g_1$ as an example. The color of q and the anticolor of g_1 form the singlet $|1_{1q}\rangle$, while the color of g_1 and the anticolor of $\bar{q}$ make another one $|1_{\bar{q}1}\rangle$; these two color-anticolor pairs are both bound to be singlets by the attractive color force, i.e., color string obviously. The color-color from q-g_1 and anticolor-anticolor from $\bar{q}$-g_1 build the color string states $|3^*_{q1}\rangle$ and $|3_{\bar{q}1}\rangle$, respectively. They can never exist alone because they carry the color charge. They have to combine into a singlet to be free of color [see Fig. 2(a)]. This kind of system singlet is also formed by a color string. The simplest color string configuration with local properties is $|1_{1q}1_{\bar{q}1}\rangle$, which is the first system singlet in (A9). Thereafter we call this kind of system singlet the singlet string for short. In this completeness set, there is another system singlet $|8_{1q}\times 8_{\bar{q}1}\rangle$ whose form resembles that of a glueball, which is also bound by a string, as we have shown above [see Fig. 2(b)]. In summary, the attractive color force fields in all the above system singlets can be described as color strings. They can all exist independently as confining states because they are free of color. It can be shown that this statement is also true for a general multiparton system.

We have seen that for a multiparton system there are many singlet completeness sets which correspond to various methods of reduction. The physical choice depends on the dynamical knowledge of the system. This situation is similar to that in atomic physics: One needs an interactive Hamiltonian to select the physical coupling **JJ** or **LS**. But we have no such knowledge to determine the physical completeness set for a multiparton system. So the choices are rather arbitrary and vary with different models. The system singlets in the set $\{|8_Q\times 8_1\rangle\}$ for $q\bar{q}g_1$ and set (A24) for $q\bar{q}g_1g_2$ are similar to the glueball discussed in Ref. [11] where it is treated as a hadronization unit. This is equivalent to choosing these sets to be the physical ones. In the Lund model, each string segment or color dipole corresponds to a color singlet;

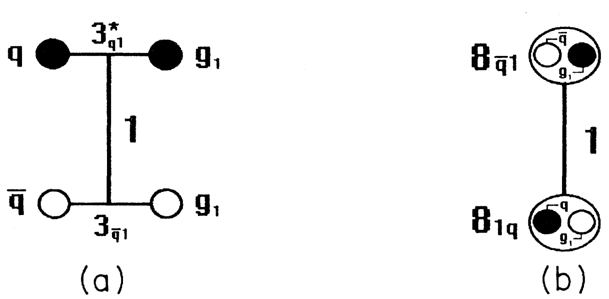

FIG. 2. (a) System singlet $\mathbf{3}_{\bar{q}1}\times\mathbf{3}^*_{q1}$ for system $q\bar{q}g_1$ which is formed by two triplets. (b) System singlet $8_{1q}\times 8_{\bar{q}1}$ for system $q\bar{q}g_1$ which is formed by two octets. Solid circles are color charges and open circles are anticolor charges.

then, the whole string or the dipole chain corresponds to the singlet strings in the completeness sets (A9) and (A26), while the other system singlets in these sets have no correspondence. In Ref. [9], we have shown that the probability of the singlet string $|1_{1q}1_{\bar{q}1}\rangle$ for the $q\bar{q}g_1$ system is 90%, which is rather close to the assumption of the Lund model. To contrast our consequences with the Lund string picture in the next section, we shall concentrate our attention on those completeness sets which contain singlet strings.

IV. CROSS SECTION OF COLOR STRING CONFIGURATIONS FOR MULTIPARTON SYSTEMS

By means of H_c, we can calculate the cross sections and, hence, the probabilities of any singlet string configurations for the process $e^+e^- \to m(q\bar{q}) + ng$. Let us take the case $m = 1$ for example and consider the three lowest order processes: i.e., $n = 1, 2, 3$. The probabilities of singlet strings in their completeness sets of the three cases are given and their dependences on the gluon number n and the strong coupling constants α_s are discussed.

The matrix element square of the process $e^+e^- \to q\bar{q}g_1$ is given by

$$|M(q,\bar{q},g_1)|^2 = M^{a_1}_{ij}(M^{a_1}_{ij})^* = 4|D|^2 \ , \qquad (4.1)$$

where the summation over color indices i, j, a_1 of $q, \bar{q}, g_1$ has been done; D is a function of the momenta of three partons. The cross section is

$$\sigma_0 \equiv \sigma_{\rm tree}(e^+e^- \to q\bar{q}g_1) = 4\int |D|^2 d\Omega \ , \qquad (4.2)$$

where Ω is phase space. According to (2.13), the color effective Hamiltonian of this case is given by

$$H_c = (1/\sqrt{2})\,\mathrm{Tr}(Q^\dagger G_1^\dagger)D \ . \qquad (4.3)$$

Using H_c in (2.13), we can give the matrix elements of any color singlets for the system $q\bar{q}g_1$. Here we calculate the cross section or the probability of the singlet string configuration in its completeness set. For the singlet string state $|f\rangle = \frac{1}{3}|1_{\bar{q}1}1_{1q}\rangle$ ($\frac{1}{3}$ is the normalization factor), we have

$$\langle f|H_c|0\rangle = \tfrac{1}{3}\langle 1_{\bar{q}1}1_{1q}|H_c|0\rangle = [8/(3\sqrt{2})]D \ . \qquad (4.4)$$

The cross section is

$$\sigma(1_{\bar{q}1}1_{1q}) = \tfrac{8}{9}\sigma_0 \ , \qquad (4.5)$$

so that the probability of the singlet string state is

$$P_1 \equiv \sigma(1_{\bar{q}1}1_{1q})/\sigma_0 = \tfrac{8}{9} \ . \qquad (4.6)$$

In this completeness set, there is another system singlet

$$|f\rangle = (1/\sqrt{8})\,\mathrm{Tr}(O^\dagger_{1q}O^\dagger_{\bar{q}1})|0\rangle = (1/\sqrt{8})|8_{1q}\times 8_{\bar{q}1}\rangle \ ,$$

which is orthogonal to the singlet string; then, we have

$$|\langle f|H_c|0\rangle|^2 = \tfrac{4}{9}|D|^2 \ . \qquad (4.7)$$

The probability is

$$P_8 \equiv \sigma(8_{1q} \times 8_{\bar{q}1})/\sigma_0 = |\langle f|H_c|0\rangle|^2/|M(q,\bar{q},g_1)^2 = \tfrac{1}{9} \ . \tag{4.8}$$

Though there always exist neutral color flows in (q,g_1) and $(\bar{q},g_1)$, the color singlet string is not always formed and its probability is $\frac{8}{9}$, near but not unity. The other $\frac{1}{9}$ of the probability is allocated to another system singlet $|8_{1q}\times 8_{\bar{q}1}\rangle$, which resembles a glueball in color structure. $|8_{1q}\times 8_{\bar{q}1}\rangle$ is built by four color charges and is regarded as a collective singlet. If this singlet is neglected, the parton system $q\bar{q}g_1$ can be treated as two singlet substrings as the SFM assumes. So the PQCD analysis for $q\bar{q}g_1$ is approximately consistent with the model. Note that P_1 and P_8 are independent of the strong coupling constant α_s.

The matrix element for the process $e^+e^- \to q\bar{q}g_1g_2$ is given by

$$M_{ij}^{a_1a_2} = (T^{a_1}T^{a_2})_{ij}D^{(12)} + (T^{a_2}T^{a_1})_{ij}D^{(21)} \ . \tag{4.9}$$

The square of the matrix element and the cross section are given by

$$|M(q,\bar{q},g_1,g_2)|^2 = \tfrac{16}{3}|D^{(12)}|^2 + \tfrac{16}{3}|D^{(21)}|^2 - \tfrac{4}{3}\,\mathrm{Re}(D^{(12)}D^{(21)*}) \ , \tag{4.10}$$

$$\sigma_0 \equiv \sigma_{\mathrm{tree}}(e^+e^- \to q\bar{q}g_1g_2) = \int |M(q,\bar{q},g_1,g_2)|^2 d\Omega \ , \tag{4.11}$$

where a summation over the color indices i,j,a_1,a_2 has been done.

Similarly, the color effective Hamiltonian is

$$H_c = \tfrac{1}{2}\,\mathrm{Tr}(Q^\dagger G_1^\dagger G_2^\dagger)D^{(12)} + \tfrac{1}{2}\,\mathrm{Tr}(Q^\dagger G_2^\dagger G_1^\dagger)D^{(21)} \ . \tag{4.12}$$

As mentioned above, we choose (A26) as our completeness singlet set whose normalized form is

$$\begin{aligned}\{|f_k\rangle, k=1,2,\ldots,6\} \\ \equiv \{(3\sqrt{3})^{-1}|1_{2q}1_{12}1_{\bar{q}1}\rangle, (2\sqrt{6})^{-1}|1_{2q}(8_{12}\times 8_{\bar{q}1})\rangle \ , \\ (2\sqrt{6})^{-1}|1_{12}(8_{2q}\times 8_{\bar{q}_1})\rangle, (2\sqrt{6})^{-1}|1_{\bar{q}1}(8_{2q}\times 8_{12})\rangle \ , \\ (\tfrac{3}{80})^{1/2}|8_{\bar{q}1}\times\{8_{12},8_{2q}\}\rangle, 48^{-1/2}|8_{\bar{q}1}\times[8_{12},8_{2q}]\rangle\} \ ,\end{aligned} \tag{4.13}$$

where

$$|\{8_{12},8_{2q}\}\rangle = |(O^i_{(12)j}O^j_{(2q)k}) - \tfrac{2}{3}\delta^i_k\,\mathrm{Tr}(O_{12}O_{2q})\rangle \ , \tag{4.14}$$

$$|[8_{12},8_{2q}]\rangle = |(O^i_{(12)j}O^j_{(2q)k} - O^i_{(2q)j}O^j_{(12)k}\rangle \ . \tag{4.15}$$

From (4.12) and (4.13), we derive

$$\begin{aligned}\langle f_1|H_c|0\rangle &= [32/(9\sqrt{3})]D^{(12)} - [4/(9\sqrt{3})]D^{(21)} \ , \\ \langle f_2|H_c|0\rangle &= -[16/(9\sqrt{6})]D^{(12)} + [2/(9\sqrt{6})]D^{(21)} \ , \\ \langle f_3|H_c|0\rangle &= -[16/(9\sqrt{6})]D^{(12)} + [2/(9\sqrt{6})]D^{(21)} \ , \\ \langle f_4|H_c|0\rangle &= [2(9\sqrt{6})]D^{(12)} + [20/(9\sqrt{6})]D^{(21)} \ , \\ \langle f_5|H_c|0\rangle &= (\sqrt{15}/27)D^{(12)} + (10\sqrt{15}/27)D^{(21)} \ , \\ \langle f_6|H_c|0\rangle &= [1/(3\sqrt{3})]D^{(12)} - [8/(3\sqrt{3})]D^{(21)} \ .\end{aligned} \tag{4.16}$$

From (4.16), we can verify the unitary relation (2.20):

$$\sum_{k=1}^{6} |\langle f_k|H_c|0\rangle|^2 = |M(q,\bar{q},g_1,g_2)|^2 \ . \tag{4.17}$$

We can neglect the interference term $\mathrm{Re}(D^{(12)}D^{(21)*})$ (in fact it is very small compared to other terms) and obtain the following probability of the singlet string without carrying out the integral:

$$P_1 = \int |\langle f_1|H_c|0\rangle|^2 d\Omega/\sigma_0 \approx 40\% \ . \tag{4.18}$$

Note that this probability is also independent of α_s, the same as $q\bar{q}g_1$, but it is much lower than $\frac{8}{9}$. In the same way, we derive the probability of the singlet string for $q\bar{q}g_1g_2g_3$, which is only 11.9% and much lower than $q\bar{q}g_1g_2$. We conclude that the probability for the singlet string configuration decreases drastically as the gluon number n increases. This effect results from the fact that the methods of reduction and hence the number of system singlets grow fast as the gluon number becomes larger, while, on the other hand, in each completeness set unitarity requires that the sum of probabilities of all system singlets must be unity, so that the percentage of each system singlet becomes lower and lower.

V. CONCLUSIONS AND DISCUSSION

The color string configuration for the multiparton system is the interface between parton states and hadronization models in the present treatment for high energy reactions and can be thought of as a bridge linking PQCD and non-PQCD phases. In the popular Lund model, the momentum configuration is obtained via PQCD, while its color string structure is assigned by the neutral color flow model rather than by PQCD analysis. It is worthwhile to find out in PQCD limits what the color configuration is like and how it differs from the color string picture inspired by the neutral color flow model.

With the help of the recursive forms of the amplitudes for $e^+e^- \to m(q\bar{q}) + ng$, we have developed a systematic analysis and derived a more general approach to calculate the probability of a color string configuration for the multiparton system $m(q\bar{q})+ng$. Although for the $m=n=1$ case the probability of a singlet string $|1_{1q}1_{\bar{q}1}\rangle$ which physically corresponds to the dipole chain in the CDM is 90%, for the cases $m=1$, $n=2$ and $n=3$, the probabilities of singlet strings $|1_{2q}1_{12}1_{\bar{q}1}\rangle$ and $|1_{3q}1_{12}1_{23}1_{\bar{q}1}\rangle$ decrease to 40% and 12%, respectively. The other color configurations in the completeness set are orthogonal to the singlet string and resemble a glueball in color structure which is not involved in the Lund model. There has been no hadronization model so far to deal with these two types of system singlets consistently.

It can be seen that the percentage of the singlet string in its own completeness set has no relevance to α_s and decreases rapidly as the gluon number grows, while those of other color configurations which are orthogonal to the singlet string in the same set become larger. This phenomenon seems to indicate the greater importance of collective effects because these color singlets contain four or more color charges. We should note that there are many ways of constructing singlets of the parton system, which make up various completeness sets. With an increase of the gluon number, more ways of building system singlets and hence more types of color string configurations emerge. We do not have enough knowledge to derive a physical set on the PQCD level so far. In addition to this, there are many problems to be solved in the future, such as how non-PQCD or soft gluon interference affect the color string configuration and whether these effects make the original color structure deviate from the dipole chain. So the determination of the color string structure for a multiparton system is still an open question. It is notable that in the recent work by Sjöstrand and Khoze [5] part of the above problems in the process $e^+e^- \to W^+W^- \to$hadrons has been studied from the string fragmentation scenario. We are trying to study similar problems along the line of this work.

APPENDIX: SYSTEM SINGLETS FOR PARTON SYSTEM $q\bar{q}g_1$ AND $q\bar{q}g_1g_2$

For the system $q\bar{q}g_1$, each color state belongs to the color space

$$3_{q_1}\otimes 3^*_{\bar{q}_1}\otimes 3_1\otimes 3^*_1 \ , \tag{A1}$$

whose singlet subspaces correspond to its various reduction types. For the reduction type (a),

$$(3_{q_1}\otimes 3^*_{\bar{q}_1})\otimes(3_1\otimes 3^*_1)=(S_Q\oplus 8_Q)\otimes(S_1\oplus 8_1) \ , \tag{A2}$$

where S_Q and 8_Q are the singlet and octet spaces composed of (color, anticolor) from the quark pair; S_1 and 8_1 are those composed of (color, anticolor) from the gluon. Their bases are given by

$$|S_Q\rangle=|\Psi^i_{\bar{q}}\Psi_{qi}\rangle \ , \quad |8_Q\rangle=|\Psi^i_{\bar{q}}\Psi_{qj}\rangle-\tfrac{1}{3}\delta^i_j|S_Q\rangle\equiv|O^i_{Qj}\rangle \ , \tag{A3}$$

$$|S_1\rangle=|\Psi^i_1\Psi_{1i}\rangle, |8_1\rangle=|\Psi^i_1\Psi_{1j}\rangle-\tfrac{1}{3}\delta^i_j|S_1\rangle\equiv|G^i_{1j}\rangle \ . \tag{A4}$$

Only $8_Q\otimes 8_1$ can be reduced to the physical singlet space, and its basis $|8_Q\times 8_1\rangle$ is written as

$$|8_Q\times 8_1\rangle\equiv|\mathrm{Tr}(O_QG_1)\rangle=|O^i_{Qj}G^j_{1i}\rangle \ . \tag{A5}$$

Since $\langle S_QS_1|H_c|0\rangle=0$, S_QS_1 is unphysical according to property (b) in Sec. II C. Corresponding to this reduction type is the system singlet set $\{|8_Q\times 8_1\rangle\}$ where there is only one element. From (2.19) and (2.20), we have

$$|C_8\langle 8_Q\times 8_1|H_c|0\rangle|^2=|M(q_1,\bar{q}_1,g_1)|^2 \ , \tag{A6}$$

where C_8 is the normalization factor of the state $|8_Q\times 8_1\rangle$, i.e.,

$$|C_8|^2\langle 8_Q\times 8_1|8_Q\times 8_1\rangle=1 \ . \tag{A7}$$

For the reduction type (b),

$$(3_q\otimes 3^*_1)\otimes(3_1\otimes 3^*_{\bar{q}})=(1_{1q}1_{\bar{q}1})\oplus(8_{1q}\otimes 8_{\bar{q}1})\oplus\text{ other spaces} \ , \tag{A8}$$

where 1_{1q} and 8_{1q} are the singlet and octet made up of (anticolor, color) from the gluon and the quark, while $1_{\bar{q}1}$ and $8_{\bar{q}1}$ are those from the antiquark and gluon. Their corresponding singlet completeness set is

$$\{|1_{1q}1_{\bar{q}1}\rangle, |8_{1q}\times 8_{\bar{q}1}\rangle\} \ , \tag{A9}$$

and we have

$$\langle 1_{1q}1_{\bar{q}1}|8_{1q}\times 8_{\bar{q}1}\rangle=0 \ , \tag{A10}$$

$$|C_1|^2\langle 1_{1q}1_{\bar{q}1}|1_{1q}1_{\bar{q}1}\rangle=1 \ , \tag{A11}$$

$$|C_8|^2\langle 8_{1q}\times 8_{\bar{q}1}|8_{1q}\times 8_{\bar{q}1}\rangle=1 \ , \tag{A12}$$

$$|C_1|^2|\langle 1_{1q}1_{\bar{q}1}|H_c|0\rangle|^2+|C_8|^2|\langle 8_{1q}\times 8_{\bar{q}1}|H_c|0\rangle|^2=|M(q,\bar{q},g_1)|^2 \ ; \tag{A13}$$

here,

$$|1_{1q}\rangle\equiv|\Psi^i_1\Psi_{qi}\rangle, |8_{1q}\rangle\equiv|\Psi^i_1\Psi_{qj}\rangle-\tfrac{1}{3}\delta^i_j|1_{1q}\rangle\equiv|O^i_{1qj}\rangle \ , \tag{A14}$$
$$|1_{\bar{q}1}\rangle\equiv|\Psi^i_{\bar{q}}\Psi_{1i}\rangle, |8_{\bar{q}1}\rangle\equiv|\Psi^i_{\bar{q}}\Psi_{1j}\rangle-\tfrac{1}{3}\delta^i_j|1_{\bar{q}1}\rangle\equiv|O^i_{\bar{q}1j}\rangle \ ,$$

where C_1 and C_8 are the normalization factors of $|1_{1q}1_{\bar{q}1}\rangle$ and $|8_{1q}\times 8_{\bar{q}1}\rangle$, respectively.

For the reduction type (c),

$$(3_q\otimes 3_1)\otimes(3^*_{\bar{q}}\otimes 3^*_1)=(3^*_{q1}\otimes 3_{\bar{q}1})\oplus(6_{q1}\otimes 6^*_{\bar{q}1})\oplus\text{ other spaces} \ , \tag{A15}$$

where 6_{q1} and 3^*_{q1} are symmetric and antisymmetric spaces made up of (color, color) from the quark and the gluon, while $6^*_{\bar{q}1}$ and $3_{\bar{q}1}$ are those composed of (anticolor, anticolor) from the antiquark and gluon. The singlet set is written as

$$\{|3_{\bar{q}1}\times 3^*_{q1}\rangle, |6_{q1}\times 6^*_{\bar{q}1}\rangle\} \ , \tag{A16}$$

and we have

$$\langle 3_{\bar{q}1} \times 3^*_{q1} | 6_{q1} \times 6^*_{\bar{q}1} \rangle = 0 \ , \tag{A17}$$

$$|C_3|^2 \langle 3_{\bar{q}1} \times 3^*_{q1} | 3_{\bar{q}1} \times 3^*_{q1} \rangle = 1 \ , \tag{A18}$$

$$|C_6|^2 \langle 6_{q1} \times 6^*_{\bar{q}1} | 6_{q1} \times 6^*_{\bar{q}1} \rangle = 1 \ , \tag{A19}$$

$$|C_3|^2 |\langle 3_{\bar{q}1} \times 3^*_{q1} | H_c | 0 \rangle|^2 + |C_6|^2 |\langle 6_{q1} \times 6^*_{\bar{q}1} | H_c | 0 \rangle|^2 = |M(q, \bar{q}, g_1)|^2 \ , \tag{A20}$$

where C_3 and C_6 are the normalization factors of $|3_{\bar{q}1} \times 3^*_{q1}\rangle$ and $|6_{q1} \times 6^*_{\bar{q}1}\rangle$, respectively, and

$$\begin{aligned} |(3_{\bar{q}1})_i\rangle &\equiv \varepsilon_{ijk} |\Psi^j_{\bar{q}} \Psi^k_1\rangle \ , \\ |(3^*_{q1})^i\rangle &\equiv \varepsilon^{ijk} |\Psi_{qj} \Psi_{1k}\rangle \ , \\ |(6_{q1})_{\{ij\}}\rangle &\equiv |\psi_{qi} \Psi_{1j} + \Psi_{qj} \Psi_{1i}\rangle \ , \\ |(6^*_{\bar{q}1})^{\{ij\}}\rangle &\equiv |\Psi^i_{\bar{q}} \Psi^j_1 + \Psi^j_{\bar{q}} \Psi^i_1\rangle \ . \end{aligned} \tag{A21}$$

In (A21), ε_{ijk} and ε^{ijk} are defined by $\varepsilon_{ijk} = \varepsilon^{ijk} = 1$ if (ijk) is an even permutation of (123); $\varepsilon_{ijk} = \varepsilon^{ijk} = -1$ if (ijk) is an odd permutation; $\varepsilon_{ijk} = \varepsilon^{ijk} = 0$ for other cases. In the second line of (A21), $\{ij\}$ is the symmetric symbol and we have $\{12\} = \{21\}$, etc. $|3_{\bar{q}1} \times 3^*_{q1}\rangle$ and $|6_{q1} \times 6^*_{\bar{q}1}\rangle$ are defined by

$$|3_{\bar{q}1} \times 3^*_{q1}\rangle = \varepsilon_{ijk} \varepsilon^{ij'k'} |\Psi^j_{\bar{q}} \Psi^k_1\rangle |\Psi_{qj'} \Psi_{1k'}\rangle \ ,$$

$$|6_{q1} \times 6^*_{\bar{q}1}\rangle = |\Psi_{qi} \Psi_{1j} + \Psi_{qj} \Psi_{1i}\rangle |\Psi^i_{\bar{q}} \Psi^j_1 + \Psi^j_{\bar{q}} \Psi^i_1\rangle \ . \tag{A22}$$

Similarly, we can give system singlet sets for the parton system $q\bar{q}g_1g_2$ which correspond to four ordinary reduction types.

For the reduction type (a),

$$(3_q \otimes 3^*_{\bar{q}}) \otimes (3_1 \otimes 3^*_1) \otimes (3_2 \otimes 3^*_2) = (S_Q \oplus 8_Q) \otimes (S_1 \oplus 8_1) \otimes (S_2 \oplus 8_2) \ . \tag{A23}$$

The system singlet set is

$$\{|S_Q(8_1 \times 8_2)\rangle, |\mathrm{Tr}(O_Q[G_1, G_2])\rangle, |\mathrm{Tr}(O_Q\{G_1, G_2\})\rangle\} \ . \tag{A24}$$

The first system singlet is composed of the singlet $|S_Q\rangle$ and $|8_1 \times 8_2\rangle$, which is formed by the subtraction of two gluon octet tensors. The second and third consists of two subtracted octets; one is the octet state $|O_Q\rangle$ of the quark pair, and the others are the symmetric and antisymmetric states $|[G_1, G_2]\rangle$ and $|\{G_1, G_2\}\rangle$ of two gluons.

For the reduction type (b),

$$(3_q \otimes 3^*_2) \otimes (3_2 \otimes 3^*_1) \otimes (3_1 \otimes 3^*_{\bar{q}}) = (1_{2q} \oplus 8_{2q}) \otimes (1_{12} \oplus 8_{12}) \otimes (1_{\bar{q}1} \oplus 8_{\bar{q}1}) \ . \tag{A25}$$

The system singlet set is

$$\begin{aligned} \{&|1_{2q} 1_{12} 1_{\bar{q}1}\rangle, |1_{2q}(8_{12} \times 8_{\bar{q}1})\rangle \ , \\ &|1_{12}(8_{2q} \times 8_{\bar{q}1})\rangle, |1_{\bar{q}1}(8_{2q} \times 8_{12})\rangle \ , \\ &|8_{\bar{q}1} \times \{8_{12}, 8_{2q}\}\rangle, |8_{\bar{q}1} \times [8_{12}, 8_{2q}]\rangle\} \ . \end{aligned} \tag{A26}$$

The reduction type (c) is obtained by the interchange of gluons 1 and 2, i.e., $1 \Leftrightarrow 2$, in (A25) and so does the singlet completeness set in (A26).

For the reduction type (d),

$$(3_q \otimes 3_1 \otimes 3_2) \otimes (3^*_{\bar{q}} \otimes 3^*_1 \otimes 3^*_2) = (1_{q12} \oplus 8_+ \oplus 8_- \oplus 10_{q12}) \otimes (1^*_{\bar{q}12} \oplus 8^*_+ \oplus 8^*_- \oplus 10^*_{\bar{q}12}) \ . \tag{A27}$$

The system singlet set is

$$\begin{aligned} \{&|1_{q12} 1^*_{\bar{q}12}\rangle, |8_+ \times 8^*_+\rangle, |8_+ \times 8^*_-\rangle \ , \\ &|8_- \times 8^*_+\rangle, |8_- \times 8^*_-\rangle, |10_{q12} \times 10^*_{\bar{q}12}\rangle\} \ , \end{aligned} \tag{A28}$$

where 8_+ and 8_- are mixed symmetric and mixed antisymmetric octets from three color charges and 8^*_+ and 8^*_- are those from three anticolor ones; 1_{q12} and $1^*_{\bar{q}12}$ are antisymmetric singlets from three color and three anticolor charges, respectively, while 10_{q12} and $10^*_{\bar{q}12}$ are decuplets which are symmetric states of three color and three anticolor charges, respectively; $|10_{q12} \times 10^*_{\bar{q}12}\rangle$ is the singlet formed by the subtraction of two decuplets.

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