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Z-peak subtracted representation of four-fermion processes at future e^+e^- colliders

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We propose a representation of four-fermion processes at one loop, at variable c.m. energy, in which the theoretical input contains certain quantities measured on top of the Z resonance at CERN LEP 1 and SLC, rather than the more familiar input parameter G_μ. This choice allows the calculation of the "residual" one-loop expressions in a way that exhibits interesting properties for cases of new physics, as shown with two specific examples of models of technicolor-type and of models with anomalous triple gauge couplings.

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I. INTRODUCTION

Whenever a search of virtual effects (characteristic of some theoretical model to be tested at a certain level) is performed in a high-precision measurement, two theoretical assumptions are normally implicitly considered as trivial necessary conditions in order that the program may become successful. The first one is that the values of those parameters to be considered as an input in the theoretical formulas must be known with a "suitable" accuracy, which means in practice that the possible error that affects them can be considered as negligibly small with respect to the experimental one of the high-precision test which is proposed. The second request is that the theoretical calculation of the virtual effect for a certain model may be actually performed in a clean reasonable way, without introducing too many extra *ad hoc* assumptions that would induce an unpleasant loss of generality, or of reliability, of the calculation.

A priori, it would appear natural to consider these two assumptions as totally unrelated and not mutually interacting. In this sense, choosing the "best" input parameters would simply mean selecting those that are known with the maximum accuracy. Once this selection is made, the calculation of the relevant virtual effects proceeds, meeting or avoiding computational difficulties that are, in a sense, intrinsic to the specific model.

To produce an illustrative example of this (vague) statement, consider the case of the calculation of possible technicolor effects on electroweak observables subject to very accurate measurements. Choosing as convenient qualities the Altarelli-Barbieri $\varepsilon_1, \varepsilon_3$ parameters [1], one normally proceeds by first writing a theoretical expression for Z leptonic observables which contains as input quantities the "canonical" best set, i.e., $\alpha \equiv \alpha_{\rm QED}(0), G_\mu$, derived from the muon lifetime, and M_Z. Then the calculation of technicolor effects proceeds in the way first illustrated by Peskin and Takeuchi [2]. In particular, one sees that the effect on ε_3, or on the original Peskin-Takeuchi parameter S, can be calculated in a "clean" and reasonable way by resorting to unsubtracted dispersion relations, without great loss of generality of the considered model. On the contrary, the calculation for ε_1 (or T) is much more delicate and model dependent in this case, involving the quantity where custodial symmetry is broken from fermion masses. Here the (still ambiguous) fine details of the model become dominant, and in conclusion it appears difficult to derive from the measured value of this parameter sufficiently general indications on technicolor [2].

In the considered example, certain quantities measured on the Z peak were involved. In particular, the leptonic width of the Z, Γ_l, and the effective angle $s^2_{\rm eff}(M^2_Z)$ measured at the Z-peak mass [3] were the two relevant observables whose theoretical expression was written in terms of α, G_μ, M_Z (the "input" set) and ε_1, ε_3 (the "test" parameters). With this choice, the results of measurement of $\Gamma_l, s^2_{\rm eff}(M^2_Z)$ provide values of $\varepsilon_{1,3}$, and thus allow test of a number of models "beyond" the minimal standard model (MSM) from the analysis of their possible virtual effects on four-fermion processes at total center-of-mass energy $\sqrt{q^2} = M_Z$.

Four-fermion process at $\sqrt{q^2} > M_Z$ will be measured either in the very near future at the CERN e^+e^- collider LEP 2, or long term at a more powerful new linear collider (NLC). Although the main aim of these machines will undoubtedly be that of direct production of as-yet undiscovered particles, the calculation of virtual effects will still be a very important activity either to investigate the fine details of possible new models, or to try again, in the less exciting case of no direct discovery, to identify small deviations from the MSM predictions for suitably chosen "test" parameters. In this case, one might imagine generalizing the previous parameterization already used on top of the Z resonance, and in principle the choice of α, G_μ, M_Z as input parameters would appear *a priori* the most convenient. In fact, one might imagine trading some of these parameters with other, new quantities measured, e.g., on Z resonance, typically, say, Γ_l. But at first sight this would seem not very convenient since, for example, the relative error on $G_\mu(\sim 2\times 10^{-4})$ is still sufficiently smaller than that on $\Gamma_l(\sim 2\times 10^{-3})$, and therefore one would feel that G_μ is, in any case, a better parameter than Γ_l for a theoretical description of such four-fermion processes at $\sqrt{q^2} > M_Z$.

The main goal of this paper is that of showing that the previous feeling is not always correct. To be more precise, we shall demonstrate that, if a calculation of virtual effects of popular existing models "*beyond*" the MSM has to be performed, the replacement of G_μ by Γ_l [and, also, the introduction of $s^2_{\text{eff}}(M_Z^2)$ as an input parameter] would be extremely useful for computational purposes. This is due to the fact that, as we shall explicitly show, the choice of the input parameters and the theoretical features of the related "test" parameters are now strictly correlated. In particular, a proper choice of the input set (the one that we propose in this paper) allows the "reabsorption" of quantities that would systematically introduce, in the test parameters, the most heavily model-dependent features of the models to be tested.

Technically, this paper is organized as follows. Section II contains a brief description of the method, for the particularly simple case of final leptonic states. Section III is devoted to the two particularly illustrative examples of "technicolor-type" models and of models with anomalous triple gauge boson couplings. Section IV contains a final discussion and our conclusions.

II. THE METHOD

A. The unsubtracted representation

We consider the process of electron-positron annihilation into a charged fermion-antifermion couple at c.m. energy $\sqrt{q^2} > M_Z$. Although this is by no means essential, we shall first consider the case where the final fermions are massless leptons (not electrons). The generalizations to final, possible massive, quarks are straightforward but only slightly more involved, and will be given in a different paper.

The starting point of our analysis is the theoretical expression of the invariant scattering amplitude *at one loop* (the realistic limit of perturbation expansions for the considered electroweak processes). Ae this level, we shall find it rather convenient to use the decomposition

$$\begin{aligned}A_{el}^{(1)}(q^2,\theta) = \Bigg\{ &\frac{i}{q^2} v_\mu^{(\gamma)} v^{(\gamma)\mu}\left[1-\tilde{F}_\gamma(q^2,\theta)\right] \\ &+\frac{i}{q^2-M_{0Z}^2} v_\mu^{(Z)} v^{(Z)\mu}\left[1-\frac{\tilde{A}_Z(q^2,\theta)}{q^2-M_{0Z}^2}\right] \\ &-\frac{2i}{q^2-M_{0Z}^2} v_\mu^{(\gamma)} v^{(Z)\mu}\left[\tilde{F}_{\gamma Z}(q^2,\theta)\right]\Bigg\} \\ &+A_{el}^{(1)(\mathrm{QED})}. \end{aligned} \tag{1}$$

A few words of comment on Eq. (1) are now appropriate. We have introduced the following "generalized" bare vertices:

$$v_\mu^{(\gamma)} \equiv |e_0|\langle l_2|J_\mu^{(\gamma)}(0)|l_1\rangle, \tag{2}$$

$$v_\mu^{(Z)} \equiv \frac{|e_0|}{s_0 c_0}\langle l_2|J_\mu^{(Z)}(0)|l_1\rangle \tag{3}$$

with $J_\mu^{(\gamma),(Z)}$ defined in the conventional way: i.e.,

$$J_\mu^{(\gamma)} = \sum_i Q_i \bar{\psi}_i \gamma_\mu \psi_i, \tag{4}$$

$$J_\mu^{(Z)} = \sum_i \frac{1}{2}\bar{\psi}_i[\gamma_\mu g_{Vi,0} - \gamma_\mu\gamma_5 g_{Ai,0}]\psi_i. \tag{5}$$

($g_{Ai,0} \equiv I_{3L,i}$ and $g_{Vi,0} = I_{3L,i} - 2Q_i s_0^2$.)

The decomposition of $A^{(1)}$ given here is "along" the three possible independent Lorentz structures that may arise at one loop for massless final leptons, that might be indicated as $(\gamma\gamma)$, (zz), and $\gamma z)$, respectively. Since $A^{(1)}$ is automatically gauge independent, the same property must obviously be true for the multiplicative coefficients of the three independent structures. These are made by certain combinations of transverse self-energies, generalized vertices (i.e., with external fermion self-energies already included), and boxes (tadpoles are already included in the calculation). Denoting the transverse self-energies as

$$A_i(q^2) \equiv A_i(0) + q^2 F_i(q^2) \tag{6}$$

($i=\gamma, z, \gamma z$), with $A_\gamma(0) = A_{\gamma z}(0) = 0$ (which can always be achieved by properly reabsorbing a vertex term [4]), one finds that the three independent coefficients assume the form

$$\tilde{F}_\gamma(q^2,\theta) = F_\gamma(q^2) - 2(\Gamma_\mu^{(\gamma)}, v_\mu^{(\gamma)}) - A_{el,\gamma\gamma}^{(1)(B)}(q^2,\theta), \tag{7}$$

$$\frac{\tilde{A}_Z(q^2,\theta)}{q^2-M_{0Z}^2} = \frac{A_Z(q^2)}{q^2-M_{0Z}^2} - 2(\Gamma_\mu^{(Z)}, v_\mu^{(Z)}) - A_{el,ZZ}^{(1)(B)}(q^2,\theta), \tag{8}$$

$$\begin{aligned}\tilde{F}_{\gamma Z}(q^2,\theta) = F_{\gamma Z}(q^2) &- (\Gamma_\mu^{(Z)}, v_\mu^{(\gamma)}) \\ &-\frac{q^2-M_{0Z}^2}{q^2}(\Gamma_\mu^{(\gamma)}, v_\mu^{(Z)}) \\ &-(q^2-M_{0Z}^2) A_{el,\gamma Z}^{(1)(B)}(q^2,\theta). \end{aligned} \tag{9}$$

The meaning of the parentheses (Γ_μ, v_μ) is the following. We have defined the "generalized" weak vertex contribution, e.g., of Fig. 1, as

$$A_{el}^{(1)(\gamma\gamma',V)} \equiv \frac{i}{q^2} v_\mu^{(\gamma)} \Gamma^{(\gamma)\mu}, \tag{10}$$

where $v_\mu^{(\gamma)}$ is defined by Eq. (2).

The one-loop generalized weak vertex initiated by a final γ will always be decomposable onto the two "orthogonal" directions $v_\mu^{(\gamma)}, v_\mu^{(Z)}$ with certain c-number coefficients, and in this sense we shall write

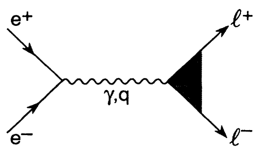

FIG. 1. Schematization of a one-loop effect with final photon "generalized" vertex, following the approach of [4].

$$\Gamma_\mu^{(\gamma)} \equiv (\Gamma_\mu^{(\gamma)}, v_\mu^{(\gamma)})v_\mu^{(\gamma)} + (\Gamma_\mu^{(\gamma)}, v_\mu^{(Z)}v_\mu^{Z}. \tag{11}$$

Analogous decompositions will be obtainable for the other (initial γ, initial, and final Z) weak vertices. Thus, one sees that the diagram of Fig. 1 contributes at one loop both to the $(\gamma\gamma)$ and the (γZ) Lorentz structures, and similar properties are valid for the other vertices. This is a known feature of the vertex component of the one-loop amplitude that has already been stressed, e.g., in a previous paper by Degrassi and Sirlin [4], to whose philosophy we shall stick to here. In fact, not to generate unnecessary confusion, we have tried to retain the same definitions as in [4] so that our vertices $\Gamma_\mu^{(\gamma)}, \Gamma_\mu^{(Z)}$ are exactly the quantities Γ_γ^μ, Γ_Z^μ defined by Eqs. (24) and (25) of that paper, in which a full discussion of the various contributions, including their gauge-dependent parts, was also given.

In a perfectly analogous way, one can decompose the fraction of $A^{(1)}$ coming from "genuine weak" (i.e., WW and ZZ) boxes $\equiv A^{(1)(b)}$ onto the *three* independent Lorentz structures of this process [5]. This decomposition is known and available in the literature [6], and we shall not give explicit expressions here.

Note that in Eq. (1) we still have *bare* masses and couplings everywhere. Note also that we have left out and explicitly denoted as $A^{(1)(\mathrm{QED})}$ the part of $A^{(1)}$ that is not "genuinely" weak. This consists of "classical" QED "radiation" diagrams, plus QED vertices and $\gamma\gamma$ *and* γZ boxes, that are already gauge invariant and must be treated separately and considered, at any q^2 value, a "known" contribution to the various structures to be evaluated numerically by some appropriate numerical, apparatus-dependent program [6].

To verify the gauge independence of the three combinations defined by Eqs. (7)–(9) is straightforward and particularly easy if one follows the Degrassi-Sirlin approach [4], as we did in this paper. This is an important check, particularly when calculations of extra effects will have to be performed in models of new physics that will introduce an extra explicit gauge dependence (for instance, models with anomalous triple gauge couplings).

Having illustrated (we hope in a clear and self-consistent way) our starting equation (1), we can now proceed with the derivation of our method. The next and immediate step is that of realizing that, at the pure one-loop level (i.e., throwing away systematically terms that are formally of a higher order in the perturbative expansion), Eq. (1) can be rewritten in a remarkably simple way, i.e., as

$$\begin{aligned} A_{el}^{(1)}(q^2\theta) = \Bigg\{ & \frac{i}{q^2} v_\mu^{(\gamma)} v^{(\gamma)\mu}\left[1-\tilde{F}_\gamma(q^2,\theta)\right] \\ & + \frac{i}{q^2-M_{0Z}^2} v_\mu^{(1)(Z)} v^{(1)(Z)\mu}\left[1-\frac{\tilde{A}_Z(q^2,\theta)}{q^2-M_{0Z}^2}\right] \\ & + \text{"QED"} \Bigg\}, \end{aligned} \tag{12}$$

where the definition of $v_\mu^{(1)(Z)}$ is formally identical to that of the corresponding bare quantity $v_\mu^{(Z)}$ [Eqs. (3) and (5)] but with the formal replacement

$$g_{Vl,0} \Rightarrow g_{Vl}^{(1)} \equiv g_{Vl,0} + 2s_1c_1\tilde{F}_{\gamma Z}(q^2,\theta) \tag{13}$$

(no change on the contrary for the axial coupling $g_{Al,0}$), where

$$s_1^2c_1^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}, \quad s_1^2 = 1-c_1^2 \simeq 0.212. \tag{14}$$

Equation (12) concludes the first part of this section. It can be viewed as a "normal" representation for the scattering amplitude at one loop that contains bare quantities and certain "coefficients" $\tilde{F}_\gamma$, $\tilde{A}_Z$, $\tilde{F}_{\gamma Z}$. From a technical point of view, it has the nice feature that the "genuine" electroweak component has, formally, the same Lorentz structure as at the tree level, with a number of precisely given replacements. This allows us to write immediately the one-loop expressions of the "electroweak" component of all independent observables of $e^+e^- \to l^+l^-$ process, once the corresponding tree-level formulas are given. But when doing that, one must also replace all the bare quantities with corresponding physical ones, making sure that all infinities cancel separately in the various structures. It is at this stage that the choice of a specific input set becomes relevant. This will be discussed in detail in the second part of this section.

B. The Z-peak subtracted representation

To illustrate with a particularly simple example the philosophy of our approach, we consider the case of the pure photonic contribution at one loop to the electron-muon cross section $\sigma_\mu(q^2)$. To obtain this term is trivial (from our previous discussion), once the corresponding tree-level expression is known. In fact, the expressions of the various observables at tree level have already been explicitly given in a previous paper [7], where a preliminary presentation of our method (that did not take into account the complete set of one-loop virtual effects) was given, and we shall not rewrite them here. The term that we want to consider thus becomes

$$\sigma_\mu^{(1)(\gamma\gamma)}(q^2) = \left(\frac{4}{3}\pi q^2\right)\left(\frac{\alpha_0}{q^2}[1-\tilde{F}_\gamma(q^2)]\right)^2, \tag{15}$$

where $\tilde{F}_\gamma(q^2)$ is obtained by integrating over the c.m. angle θ in the differential cross section the combination of self-energy, vertices, and boxes that belonged to the

($\gamma\gamma$) structure in Eq. (12). In fact, Eq. (15) is usually written in a much more convenient form by resorting to the familiar definition of physical charge as the residue of the photon pole:

$$\alpha \equiv \alpha(0) \equiv \alpha_0(1 - \tilde{F}_{\gamma(0)}) \equiv \alpha_0 - \Delta\alpha \tag{16}$$

(where the bare quantity α_0 is $e_0^2/4\pi$), which enables us to write

$$\sigma_\mu^{(1)(\gamma\gamma)}(q^2) = \left(\frac{4}{3}\pi q^2\right)\left[\left(\frac{\alpha}{q^2}\right)^2 [1 + 2\tilde{\Delta}\alpha(q^2)]\right], \tag{17}$$

where

$$\tilde{\Delta}\alpha(q^2) \equiv \mathrm{Re}[\tilde{F}_\gamma(0) - \tilde{F}_\gamma(q^2)]. \tag{18}$$

Thus, replacing the bare charge with the photon "pole residue" leaves us with a "photon pole-subtracted" parameter to be calculated in the theoretical expression. As is well known, this (trivial) fact already has a great importance at a rather elementary level, since, for instance, its self-energy *hadronic* contribution can always be calculated via an unsubtracted dispersion relation, i.e., without introducing extra model-dependent assumptions [8].

We want to show that a remarkably analogous picture can be obtained for the remaining contributions to the various observables of the process, the "photon pole" being naturally replaced by the "Z peak." With this aim, we first consider the pure Z contribution to the muon cross section. At one loop, this reads

$$\sigma_\mu^{(1)(ZZ)}(q^2) = \left(\frac{4}{3}\pi q^2\right)\Bigg| \frac{\sqrt{2}}{4\pi} g_{Al,0}^2\{1 + [1 - 4\tilde{s}_l^2(q^2)]^2\} \times \frac{G_\mu M_Z^2}{q^2 - M_Z^2 + iM_Z\Gamma_Z(q^2)}\left(1 + \frac{\delta G_\mu}{G_\mu} + \mathrm{Re}\frac{\tilde{A}_Z(0)}{M_Z^2} - \tilde{I}_Z(q^2)\right)\Bigg|^2. \tag{19}$$

Here $\tilde{s}_l^2(q^2)$ is the result of the $\cos\theta$ integration in the differential cross section of the combination

$$\tilde{s}^2(q^2,\theta) \equiv s_1^2\left(1 + \tilde{\Delta}\kappa^1(q^2,\theta)\right) \tag{20}$$

with

$$\tilde{\Delta}\kappa^1(q^2,\theta) = \frac{c_1}{s_1}\tilde{F}_{\gamma Z}(q^2,\theta) + \frac{c_1^2}{c_1^2 - s_1^2}\left(\frac{\Delta_\alpha}{\alpha} - \frac{\Delta G_\mu}{G_\mu} - \frac{\Delta M_Z^2}{M_Z^2}\right) \tag{21}$$

and $\tilde{I}_Z(q^2)$ is the result of the analogous operation on the quantity

$$\tilde{I}_Z(q^2,\theta) = \frac{q^2}{q^2 - M_Z^2}\mathrm{Re}[\tilde{F}_Z(q^2,\theta) - \tilde{F}_Z(M_Z^2,\theta)] \tag{22}$$

while Γ_Z is the conventionally defined q^2-dependent Z width that will disappear in practice for the relevant values $q^2 \gg M_Z^2$:

$$M_Z\Gamma_Z = \frac{\mathrm{Im}\tilde{A}_Z}{1 + \mathrm{Re}\dot{\tilde{A}}_Z}. \tag{23}$$

Equation (19) provides a representation of the pure-Z contribution to σ_μ that contains the input parameters α (implicitly in the one-loop terms), M_Z and G_μ, plus the q^2-dependent combinations defined by Eqs. (20)–(22). One can use it and calculate the "test parameters" both in the MSM and in models of physics beyond it. An alternative possibility is provided by the observation that, at the Z-peak squared energy $q^2 = M_Z^2$, the following properties are exactly verified:

$$\frac{\delta G_\mu}{G_\mu} + \frac{\mathrm{Re}\tilde{A}_Z(0)}{M_Z^2} - \tilde{I}_Z(M_Z^2) \equiv \varepsilon_1, \tag{24}$$

where ε_1 is the original Altarelli-Barbieri parameter [1], and

$$\tilde{s}_l^2(M_Z^2) \equiv s_{\mathrm{eff}}^2(M_Z^2), \tag{25}$$

where $s_{\mathrm{eff}}^2(M_Z^2)$ is the quantity measured on the peak of the Z resonance in the conventional definition adopted by the various LEP1 groups [3].

If one now remembers the exact definition of the leptonic Z width [9],

$$\Gamma_l \equiv \frac{G_\mu M_Z^3}{6\pi\sqrt{2}}[1 + \varepsilon_1]g_{Al,0}^2\{1 + [1 - 4s_{\mathrm{eff}}^2(M_Z^2)]^2\}, \tag{26}$$

one easily realizes that, by simply properly "subtracting" in Eq. (19) the combinations $\tilde{I}_Z(M_Z^2)$ and $\tilde{s}_l^2(M_Z^2)$ calculated at the Z peak, one can rewrite the same one-loop expression in the form

$$\sigma_\mu^{(1)(ZZ)}(q^2) = \left(\frac{4}{3}\pi q^2\right)\left[\frac{3\Gamma_l}{M_Z}\right]^2 \frac{1}{[(q^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2]}\left[1 - 2R(q^2) - \frac{16(1 - 4s_1^2)c_1 s_1 V(q^2)}{\{1 + [1 - 4\tilde{s}_l^2(M_Z^2)]^2\}}\right], \tag{27}$$

where in the combinations

$$R(q^2) = \tilde{I}_Z(q^2) - \tilde{I}_Z(M_Z^2), \tag{28}$$

$$V(q^2) = \mathrm{Re}[\tilde{F}_{\gamma Z}(q^2) - \tilde{F}_{\gamma Z}(M_Z^2)], \tag{29}$$

a "subtraction" at the Z peak has been performed.

By comparing the two perfectly equivalent representations Eqs. (19) and (27), one sees that in the second one G_μ has been "traded" for Γ_l. The consequence of this operation is that the related "test" parameters have become the subtracted quantities $R(q^2), V(q^2)$. The essential feature of this exchange is that R, V no longer contain the q^2-independent quantities that enter into the parentheses of Eqs. (18) and (21). This will be the main point when calculations *for models of new physics* will have to be performed.

The operation that we have described can be repeated in other observables. In practice, only one new situation is met in the calculation of the final τ polarization at one loop (or, alternatively, of the longitudinal polarization asymmetry for leptons). Here the relevant expressions at one loop would also be *proportional* to the quantity

$$\tilde{v}_l(q^2) \equiv 1 - 4\tilde{s}_l^2(q^2). \tag{30}$$

This can be written at one loop as

$$\tilde{v}_l(q^2) = \tilde{v}_l(M_Z^2)\left[1 - \frac{4s_1c_1}{\tilde{v}_l(M_Z^2)}V(q^2)\right], \tag{31}$$

i.e., again in terms of the subtracted parameter $V(q^2)$ and of a quantity measured on top of the Z resonance, more precisely a certain linear function of $s_{\text{eff}}^2(M_Z^2)$. No other independent input on "test" parameters are required to describe the set of leptonic processes.

It can be useful at this point to give an approximate expression, where only the relevant terms have been retained, for the three independent leptonic observables: i.e., the muon cross section, the forward-backward muon asymmetry, and the final τ polarization asymmetry. In the "Z-peak subtracted" representation they read

$$\sigma_\mu^{(1)}(q^2) \simeq \left(\frac{4}{3}\pi q^2\right)\left\{\left[\frac{\alpha}{q^2}\right]^2[1 + 2\tilde{\Delta}_\alpha(q^2)] + \frac{1}{[(q^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2]}\left[\frac{3\Gamma_l}{M_Z}\right]^2\left[1 - 2R(q^2) - \frac{16(1 - 4s_1^2)c_1s_1V(q^2)}{[1 + \tilde{v}_l^2(M_Z^2)]}\right]\right\}, \tag{32}$$

$$A_{FB,\mu}^{(1)}(q^2) \simeq \frac{3}{4}\left(\frac{4\pi q^2}{3\sigma_\mu^{(1)}(q^2)}\right)\left\{\frac{2\alpha}{q^2}\left[\frac{3\Gamma_l}{M_Z}\right]\frac{q^2 - M_Z^2}{(q^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\frac{1}{1 + \tilde{v}_l^2(M_Z^2)}[1 + \tilde{\Delta}_\alpha(q^2) - R(q^2)]\right\}, \tag{33}$$

$$A_\tau^{(1)}(q^2) \simeq \left(\frac{4\pi q^2}{3\sigma_\mu^{(1)}(q^2)}\right)\left\{\left[\frac{3\Gamma_l}{M_Z}\right]\frac{A(M_Z^2)}{(q^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\left[\alpha\frac{q^2 - M_Z^2}{q^2} + \left(\frac{3\Gamma_l}{M_Z}\right)\right]\left[1 - \frac{8s_1c_1}{A(M_Z^2)}V(q^2)\right]\right\}, \tag{34}$$

where

$$A(M_Z^2) \equiv \frac{2\tilde{v}_l(M_Z^2)}{1 + \tilde{v}_l^2(M_Z^2)}. \tag{35}$$

Equations (32)–(34) are the main result of our paper. They provide the "Z-peak-subtracted" representation of four lepton processes that, we believe, turns out to be particularly convenient if one wants to calculate the effects of *models of new physics*. They contain α, M_Z, and two more quantities directly measured at the Z peak: i.e., Γ_l and $\tilde{v}_l(M_Z^2)$ [or $A(M)_Z^2)$]. The latter can be considered, in the zero-width approximation, as the "residues" of the Z propagator and of the γ-Z self-energy. In strict analogy with the photon case, the "residual" coefficients are differences of functions, "subtracted" at the Z peak. We still have to show that these coefficients are particularly convenient for an evaluation in models beyond the MSM. This will be done in Sec. III for two specific and particularly illustrative cases.

III. APPLICATIONS

A. Models of "technicolor type"

The example of models of "technicolor type," i.e., with some vector resonance strongly coupled to the known gauge bosons, has already been discussed in great detail in [7]. This is particularly illustrative of the advantages of our representation. In fact, had one used a conventional parametrization of the type shown in Eq. (19), the contribution of such models, that by definition cannot be treated perturbatively, would be hard to estimate. In particular, the q^2-independent terms in the various brackets would contain the custodial-symmetry-violating self-energy component called $\Delta\rho(0)$ and other transverse self-energies not obeying any unsubtracted dispersion relation. By reabsorbing all such terms in the Z-peak observables, one is left with differences of quantities, whose self-energy components (the only ones that this model affects) do satisfy an unsubtracted dispersion relation (one may say that the relative subtraction constant is provided by Z-peak measurements) and may therefore be estimated in a "reasonable" way, i.e, one that is independent of several of the "fine" details of such models.

A full discussion of this case has already been given in [7]. In order to make this paper as self-contained as possible, we sketch here very briefly the main points and technical features of the relevant calculation.

We shall assume that a couple of vector and axial-vector resonances, to be generically called V and A, with unknown (but larger than $\sqrt{q^2}$) masses and unknown but "reasonable" widths, exist and that these particles are strongly coupled to the conventionally defined vector and axial components of the transverse self-energies, exactly like a ρ and an A_1 in the corresponding QCD case, with strengths F_V and F_A of typical strong interaction size. We shall also assume that there are no appreciable effects from ω-like resonances coupled to the "hypercharge" component. With standard isospin decomposition of the resonant contribution to the imaginary parts of our $\tilde{\Delta}\alpha(q^2)$, $R(q^2)$, $V(q^2)$ quantities, one is then led to the "effective" representations

$$\tilde{\Delta}\alpha(q^2) = \frac{\alpha q^2}{3\pi}\mathcal{P}\int_0^\infty \frac{ds\, R_{VV}(s)}{(s-q^2)s}, \tag{36}$$

$$R(q^2) = -\frac{\alpha(q^2-M_Z^2)}{3\pi}\frac{(1-2s_1^2)^2}{4s_1^2c_1^2}\mathcal{P}\int_0^\infty \frac{sds}{(s-q^2)(s-M_Z^2)}\left(R_{VV}(s)+\frac{R_{AA}(s)}{(1-2s_1^2)^2}\right), \tag{37}$$

$$V(q^2) = -\frac{\alpha(q^2-M_Z^2)}{3\pi}\frac{1-2s_1^2}{2s_1c_1}\mathcal{P}\int_0^\infty \frac{ds\, R_{VV}(s)}{(s-q^2)(s-M_Z^2)}. \tag{38}$$

To fix the normalization of our search, we remind the reader that the quantity originally called S in [2] was given by the expression

$$S = \frac{1}{3\pi}\int \frac{ds}{s}[R_{VV}(s) - R_{AA}(s)]. \tag{39}$$

In order to show the main features of our approach, we shall proceed to an illustration using the following oversimplified representation of the two resonances:

$$R_i = 12\pi^2 F_i^2 \delta(s-m_i^2). \tag{40}$$

Our investigation now proceeds in two steps. First, we assume as we did in [7] the validity of the two Weinberg sum rules [11] (but only fully exploited the consequences of the second one),

$$\int ds[R_{VV}(s) - R_{AA}(s)] = 12\pi^2 F_\pi^2 \tag{41}$$

[only the positivity of Eq. (41) has been exploited, since the value of F_π^2 is strongly model dependent],

$$\int dss[R_{VV}(s) - R_{AA}(s)] = 0, \tag{42}$$

and we made use of the experimental constraint [10] on the parameter S:

$$-1 \lesssim S \lesssim 0.5(2\sigma). \tag{43}$$

Figures 2 and 3 show the discovery limits for the masses of a couple of vector and axial resonances for $\sqrt{q^2} = 190$ and 500 GeV, respectively, using realistic experimental accuracies as fully discussed in [7,11]. For a more complete discussion we refer to previous references [8,12]. One sees that discovery limits ("almost" suitable at 500 GeV) would be rather poor at LEP2, unless for some reason a techniresonance of much smaller mass than in the canonical schemes [13] did exist [14].

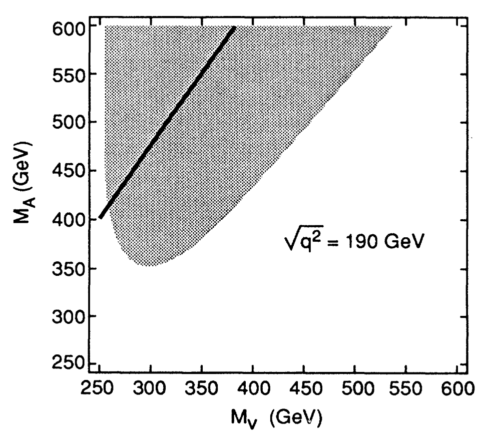

FIG. 2. Discovery limits in the (M_V, M_A) plane for a couple of vector and axial-vector strong resonances assuming the validity of the two Weinberg sum rules and using the experimental constraint on S, for $\sqrt{q^2} = 190$ GeV. The line corresponds to the indicative QCD value $M_A = 1.6M_V$, analogous to the A_1 and ρ case.

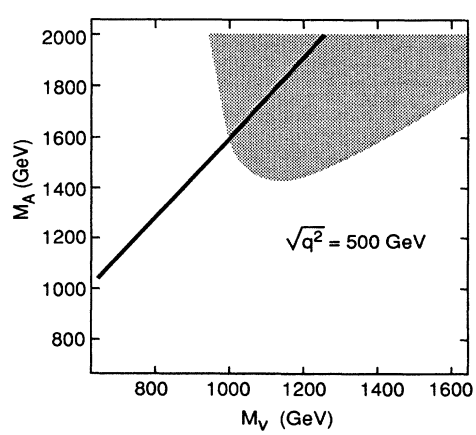

FIG. 3. Same as Fig. 1 for $\sqrt{q^2}$=500 GeV.

B. Models with anomalous triple gauge couplings

As a second, and also a particularly illustrative, example we choose that of models with anomalous triple gauge boson couplings [15]. In such models, contributions generally arise both to self-energies and to vertices, and the knowledge of the proper gauge-invariant combinations that make up $\tilde{\Delta}_\alpha$, R, and V is therefore essential. Here we shall choose the case of a general, dimension-six, fully $SU(2)_L \times U(1)_Y$ symmetric effective Lagrangian that conserves both C and CP, and is realized in a linear way, i.e., with the standard Higgs doublet, recently illustrated by Hagiwara, Ishihara, Szalapski, and Zeppenfeld [16], whose notations we shall keep. In such a model, four of the independent operators contribute at tree level and five different ("blind") ones contribute at one loop the various vertices and boxes, making a total of nine arbitrary parameters (plus the unknown Higgs boson mass and the scale Λ, usually assumed to be $\sim$ 1 TeV). To derive bounds or information on this model from a small number of experiments clearly represents a nontrivial task [17].

To visualize the (positive) influence of our representation in this case, we consider first the contribution of this model to the pure photonic contribution $\tilde{\Delta}_\alpha(q^2)$. This is in fact already computed in [16], and reads [we denote by (A) the anomalous contributions to the various combinations]

$$\tilde{\Delta}_\alpha^{(A)}(q^2) = -8\pi\alpha\frac{q^2}{\Lambda^2}[f_{DW}^r + f_{DB}^r], \tag{44}$$

where $f_{DW,B}^r$ are the renormalized expressions of the tree-level q^2-independent parameters $f_{DW,B}$ that appear in the effective Lagrangian. We see therefore that one begins with two parameters in the single observable pure photon combination.

When we move to the pure Z combination, we may try to use the "unsubtracted" representation of Eq. (19). We find in this case the following result for the term in the large parentheses:

$$\left[\left(\frac{\delta G_\mu}{G_\mu} + \mathrm{Re}\frac{\tilde{A}_Z(0)}{M_Z^2}\right) - \tilde{I}_Z(q^2)\right]^{(A)}$$

$$= \left[\left(\frac{-2M_W^2}{g^2\Lambda^2}f_{\phi,1}^r\right) - 8\pi\alpha\frac{q^2}{\Lambda^2}\left(\frac{c_1^2}{s_1^2}f_{DW}^r + \frac{s_1^2}{c_1^2}f_{DB}^r\right)\right]. \tag{45}$$

One sees that the "unsubtracted" coefficient contains a third renormalized *and q^2-independent* parameter, coming from the term

$$\left(\frac{\delta G_\mu}{G_\mu} + \frac{\tilde{A}_Z(0)}{M_Z^2}\right),$$

and a (different) combination of the same two parameters that enter the expression of $\tilde{\Delta}_\alpha$, Eq. (37). A similar result is obtained if one calculates the effect on $\tilde{s}_l^2(q^2)$: one finds a q^2-independent contribution that contains a certain combination of $f_{\phi,1}^4$ *and* of the fourth renormalized parameter of the model f_{BW}^r, and another different combination of $f_{DW,B}^r$:

$$\tilde{s}_l^{2(A)}(q^2) \simeq f(f_{\phi,1}^r, f_{BW}^r) + \alpha\frac{a^2}{\Lambda^2}g(f_{DW,B}), \tag{46}$$

where f, g are two linear functions of $f_{\phi,1}$, f_{BW}^r, and $f_{DW,B}^r$, whose explicit expression can be easily calculated. The result of this approach is that, retaining the unsubtracted representation, four different renormalized parameters of the model would enter into the theoretical expression of the three available observables. Note that adding extra realistic hadronic observables would introduce other parameters, e.g., related to the $Zb\bar{b}$ vertex.

This situation changes drastically if we use the subtracted representation. In this case all the q^2-independent parameters (that are, incidentally, the most heavily model-dependent, as shown in [16]) are automatically reabsorbed in the definition of Γ_l *and* $s_{\mathrm{eff}}^2(M_Z^2)$, and one is left with *two* independent parameters entering (three) different combinations. More precisely, one has now the following expressions of R, V:

$$R^{(A)}(q^2) = 8\pi\alpha\frac{q^2 - M_Z^2}{\Lambda^2}\left[\frac{c_1^2}{s_1^2}f_{DW}^r + \frac{s_q^2}{c_1^2}f_{DB}^r\right], \tag{47}$$

$$V^{(A)}(q^2) = 8\pi\alpha\frac{q^2 - M_Z^2}{\Lambda^2}\left[\frac{c_1}{s_1}f_{DW}^r - \frac{s_1}{c_1}f_{DB}^r\right]. \tag{48}$$

From Eqs. (37), (40), and (41), one can easily calculate the anomalous effect on the various leptonic observables. To give a more quantitative estimate, we have written here the approximate effects, only considering leading terms in each case. These are precisely (*relative* effects are shown)

$$\frac{\delta\sigma_\mu^{(A)}}{\sigma_\mu} \simeq -16\pi\alpha\frac{q^2}{\Lambda^2}[f_{DW}^r + f_{DB}^r], \tag{49}$$

$$\frac{\delta A_{FB,\mu}^{(A)}}{A_{FB,\mu}} \simeq 8\pi\alpha\frac{q^2}{\Lambda^2}\left[f_{DW}^r\left(1 - \frac{q^2 - M_Z^2}{q^2}\frac{c_1^2}{s_1^2}\right)\right.$$
$$\left. + f_{DB}^r\left(1 - \frac{q^2 - M_Z^2}{q^2}\frac{s_1^2}{c_1^2}\right)\right], \tag{50}$$

$$\frac{\delta A_\tau^{(A)}}{A_\tau} \simeq -120\pi\alpha\frac{q^2 - M_Z^2}{\Lambda^2}\left[\frac{c_1^2}{s_1^2}f_{DW}^r - f_{DB}^r\right]. \tag{51}$$

We assume $\Lambda \simeq 1$ TeV. From the (qualitative) bounds given in [16], one sees that both f_{DW}^r and f_{DB}^r are still allowed to be of order (1). Then, for $q^2 = 4M_Z^2$ (close to the realistic LEP2 energy), we see that the relative effects could be of a few percent both in σ_μ and in $A_{FB,\mu}$, and of a few "ten percent" in A_τ, which would lead in all cases to potentially visible effects (or clean bounds).

The conclusion of this second example is, we believe, positive. We have shown that by using our Z-peak-subtracted representation four-fermion processes can become an interesting way of studying, in a clean way, already at LEP2 energies, the effects of anomalous gauge couplings. Note that the couplings that are involved are quite different from the (blind) ones that would enter WW production, of which the four-fermion process would therefore represent a possibly interesting complementary alternative (a much more detailed discussion on this point will in fact be given in a separate paper).

An essential feature to be discussed at this point is that of whether the use of this representation, that practically corresponds to the replacement of G_μ by Γ_l [and $\tilde{v}(m_Z^2)$], does not introduce dangerous "theoretical" uncertainties coming from the experimental error on Γ_l, $\tilde{v}$. This will be discussed in Sec. IV.

IV. "UNCERTAINTIES" IN THE REPRESENTATION

To quantify the practical consequences of the "trading" of G_μ with Γ_l, $\tilde{v}(M_Z^2)$ in the theoretical expressions, the simplest thing to consider the expressions Eqs. (32)–(34) that represent the bulk of our paper. From the latest experimental analyses [18], we know that the experimental precision on Γ_l, $\tilde{v}$ is

$$\frac{\delta\Gamma_l^{(\mathrm{expt})}}{\Gamma_l} \simeq 2\times 10^{-3}, \tag{52}$$

$$\frac{\delta\tilde{v}(M_Z^2)}{\tilde{v}(M_Z^2)} \simeq 2\times 10^{-2}. \tag{53}$$

These relative precisions are certainly worse than that on G_μ ($\sim 2\times 10^{-4}$). However, they have to parametrize observables whose relative experimental precision will be of the percent (and not per mille) level. In particular, the error on Γ_l [Eq. (45)] does not affect σ_μ (largely dominated by the photon term) and introduces a few per mille error in $A_{FB,\mu}$ (i.e., roughly ten times smaller than the, optimistic, anticipated experimental error [19]). The error on $\tilde{v}$, Eq. (46) [in fact, on the equivalent quantity A, Eq. (35)] is also, qualitatively, much smaller than that which one could expect (10% or more) in a (possible) measurement of A_τ, that we consider here only as a potentially interesting observable (a rigorous experimental discussion on this subject is at the moment missing). Thus, in any case, the new input does not generate sizable "theoretical" uncertainties [note that the relative errors Eqs. (45) and (46) will certainly decrease in the course of the final LEP1, and SLAC Linear Collider (SLC), runs].

We are now in a position to draw some conclusions. We believe to have shown that the use of a Z-peak-subtracted representation of four-fermion processes allows the study of the effects of some models of new physics on realistic observables in a remarkably simple and clean way, without introducing dangerous theoretical uncertainties. We feel therefore that it might be worth generalizing our approach, both to the study of effects of other types of models [typically supersymmetric (SUSY) models or models with one extra Z] and to the study of other processes (e.g., final hadronic states *or* charged currents). Work in these directions is by now in progress.

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