Radiative gluon effects in massive quark-antiquark production in collisions of positrons and polarized electrons at the Z^0 resonance

John B. Stav and Haakon A. Olsen
Institute of Physics, University of Trondheim, Institute of Physics, N-7055 Dragvoll, Norway
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Cross sections and asymmetries for massive quark-antiquark production in electron-positron collisions at the Z^0 resonance are calculated, for the case of longitudinally polarized electrons. The polarization of Z^0, P_{Z^0}, for the initial electron polarization P_- is composed of P_- and the "natural" polarization $P_{Z^0}(0) = -2av/(v^2+a^2)$, and the composition rule for polarization is given. The cross section differential in the quark emission polar angle θ is obtained in terms of form factors which are given as functions of the quark mass m_f in exact form. Convenient expansions to order m_f^2 are found and compared to previously published results. For the asymmetries the Z^0 polarization plays an important role. It is shown that for the forward-backward quark asymmetry $A_{\rm FB}(P_-)$ and the forward-backward left-right asymmetry $A_{{\rm FB},LR}(P_-)$ the quark dependence is the same including radiative corrections, with the relations $A_{\rm FB}(P_-) : A_{\rm FB}(0) : A_{{\rm FB},LR}(P_-) = P_{Z^0}(P_-) : P_{Z^0}(0) : P_-$.

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I. INTRODUCTION

The advance of the techniques of obtaining and using high-energy spin-polarized electron beams [1] has proved to be an important step for new possibilities for obtaining more accurate knowledge of electroweak interactions and for testing the standard model [2].

The theory of gluon bremsstrahlung from massless quarks and antiquarks produced in annihilation of high-energy unpolarized [3] and spin polarized [4] electron-positron collisions was initiated a long time ago. It was shown that beam polarizations affect cross sections and asymmetries in distinct ways, and that new information may be obtained with the use of polarized electron and positron beams. Radiative gluon effects give radiative QCD corrections which are observed as modifications of quark-antiquark cross sections and asymmetries. Calculations for massless quarks [5] show that polarization effects modify the theoretical results considerably.

At this point it should be remarked that radiative corrections are in general mass dependent through logarithmic dependence on mass divided by energy, m/E. Radiative corrections for zero mass fermions have therefore no meaning, with one important exception: the Kinoshita-Lee-Nauenberg theorem [6,7] guarantees that radiative corrections are mass independent, i.e., have no mass singularities, for total cross section sections, i.e., when all final states have been summed over. Actually the theorem can be relaxed in one respect: for the case at hand only the collinear gluon states are needed in the final-state sum. The cross section may still be a function of the quark emission angle, as discussed in [5].

The mass corrections are then important and may in general be of the form of sums of powers of m/E multiplied with powers of $\ln(m/E)$ for high energies, always in a way which is in accordance with our relaxed Kinoshita-Lee-Nauenberg theorm: the cross-section differential in a quark emission angle and summed over collinear gluon states should have no mass singularities, which means that logarithmic terms are always multiplied with m/E at least to the first power.

The first calculation of gluon bremsstrahlung from massive quarks for unpolarized beams was made by Grunberg, Ng, and Tye [8] neglecting Z^0 exchange, and by Jersák, Laermann, and Zerwas [9] who included Z^0 exchange. Recent calculations of cross sections and asymmetries for unpolarized electrons and positrons are given by Djouadi [10], Djouadi *et al.* [11], and Arbuzov, Bardin, and Leike [12]. We will compare our discussion and results with their findings. A recent paper by us [13] on gluon bremsstrahlung constitutes the basis for the present paper. Related QED processes are μ-$\bar{\mu}$ creation processes for massive μ particles with emission of photons in collisions of electrons and positrons [14].

We present in this paper a calculation of the cross section and asymmetries for the production of massive quark-antiquark pairs in collision of polarized electrons and unpolarized positrons, including radiative QCD corrections to first order in α_s. We consider specifically the cross section at the Z^0 pole, neglecting the contribution from photon exchange and interference terms.

In addition to the obvious calculational simplification obtained with this approximation, we believe that useful physical insight is gained, in particular regarding the mechanism of spin polarization and polarization transfer. The Z^0 boson, a spin-1 Proca particle, is produced with a polarization. This polarization is composed of the "natural" Z^0 polarization $P_{Z^0} = -2av/(v^2+a^2)$, due to the parity-violating electron axial vector coupling a and the electron polarization P_- transferred to Z^0. The produced Z^0 decays, transferring its polarization information to the final state $q\bar{q}g$, again through a parity-violating coupling, the quark axial vector coupling a_f. The polarization is also transferred to the gluon as circular polarization as shown in [15], and of course also to the quark and antiquark which is the polarization transfer, inverse to the initial Z^0 production process.

We obtain in Sec. II the annihilation cross section for the polarized electron and unpolarized positron to Z^0. The Z^0 polarization is obtained and the composition rule for obtaining the Z^0 polarization from the natural Z^0 polarization and the transferred electron polarization is found. In Sec. III we calculate for massless quarks the decay probability of the polarized Z^0 to a quark-antiquark state, the two jet process, and to the three jet quark-antiquark-gluon state. The cross section for the complete process $e^+e^- \to q\bar{q}g$ is obtained by multiplication of the Z^0-production cross section with the Z^0 decay probability, summing over the Z^0 intermediate state spins. The $q\bar{q}$ decay probability with gluon radiative corrections is obtained after elimination of the infrared singularity in the usual way, and the decay probability is written in terms of infrared finite factors. In Sec. IV quark mass effects on the form factors are obtained, and the formulas for the form factors are given in Secs. V and VI. We discuss and give results for various asymmetries in Sec. VII, including QCD radiative corrections and quark mass corrections.

II. Z^0 PRODUCTION

The cross section for producing Z^0 in electron-positron collisions is given by

$$d^3\sigma = \frac{2\pi}{v_{\rm rel}}|M|^2\delta^4(p_+ + p_- - p_{Z^0})d^3p_{Z^0} \; , \qquad (2.1)$$

where $v_{\rm rel}$ is the relative electron-positron velocity and the matrix element is, in standard notation,

$$M = -\frac{ie}{2\sin 2\theta_W}\frac{\bar{v}}{\sqrt{2E_+}}\gamma^\mu(v - a\gamma_5)\frac{u}{\sqrt{2E_-}}\frac{e_\mu}{\sqrt{2E_Z}} \; . \qquad (2.2)$$

Here e_μ is the Z^0 field polarization vector having three components as Z^0 is a massive Proca-field particle.

With the electron polarization P_- in the positive helicity sense, with the z axis along $\mathbf{p}_-$, and the positron unpolarized, averaging over positron polarizations, we find, for massless electrons and positrons,

$$\begin{aligned}\frac{1}{2}\sum_{\rm spins}|M|^2 &= \frac{\pi}{8M_Z^3}\frac{\alpha}{\sin^2 2\theta_W}{\rm Tr}\, \not{p}_+\gamma^\mu \not{p}_-\gamma^\nu(1 + P_-\gamma_5)(v^2 + a^2 - 2av\gamma_5)e_\nu e_\mu^* \\ &= \frac{\pi}{4M_Z}\frac{\alpha}{\sin^2 2\theta_W}[v^2 + a^2 - 2avP_- - s_z\{2av - (v^2 + a^2)P_-\}] \; , \end{aligned} \qquad (2.3)$$

with $s_z = i(\mathbf{e}\times\mathbf{e}^*)$ [16], the Z^0 helicity, the z component of the Z^0 spin. It should be noted that the e_z component does not contribute to the order E^2, since $\not{p}_+ \not{e}_z \not{p}_- \not{e}_z = m_e^2$. The physical cross section for specifications as given, with $v_{\rm rel} = 2$,

$$\sigma = \frac{\pi}{2}\sum_{\rm spins}|M|^2\delta(2E_+ - E_z) \; , \qquad (2.4)$$

is obtained when replacing the δ function representing a stable Z^0, by

$$\frac{1}{2\pi}\int dt\, e^{it(2E_+ - E_z) - \Gamma|t|} = \frac{1}{\pi\Gamma} \; ,$$

for $2E_+ - E_Z = 0$, representing the decaying Z^0, with Γ the total line width of Z^0.

The cross section for producing polarized Z^0 is then

$$\begin{aligned}\sigma(s_z) = \frac{\pi}{4}\frac{\alpha}{\sin^2 2\theta_W}\frac{1}{M_Z\Gamma}[v^2 + a^2 - 2avP_- \\ -s_z\{2av - (v^2 + a^2)P_-\}] \; . \end{aligned} \qquad (2.5)$$

The polarization of Z^0 is

$$\begin{aligned}P_{Z^0}(P_-) &= \frac{\sigma(s_z) - \sigma(-s_z)}{\sigma(s_z) + \sigma(-s_z)} \\ &= \frac{-2av + (v^2 + a^2)P_-}{v^2 + a^2 - 2avP_-} \\ &= \frac{P_{Z^0}(0) + P_-}{1 + P_{Z^0}(0)P_-} \; , \end{aligned} \qquad (2.6)$$

where we have introduced the Z^0 polarization for unpolarized electrons, $P_- = 0$, the effect of parity violation,

$$P_{Z^0}(0) = -\frac{2av}{v^2 + a^2} \; . \qquad (2.7)$$

$P_{Z^0}(P_-)$ as a function of P_- is given in Fig. 1.

It is interesting to note that for a pure vector coupling, $a = 0$, similar to QED, the electron polarization is directly transferred to Z^0:

$$P_{Z^0}(P_-) = P_- \; .$$

For a pure electron helicity state $P_- = \pm 1$, again the polarization is directly transferred to Z^0 as expected:

$$P_{Z^0}(\pm 1) = \pm 1 \; .$$

As is seen from Eq. (2.6), for small values of $P_{Z^0}(0)$ and P_-, such that $|P_{Z^0}(0)\cdot P_-| \ll 1$, the polarizations add directly:

$$P_{Z^0}(P_-) \simeq P_{Z^0}(0) + P_- \; . \qquad (2.8)$$

For larger values of polarizations, the composition of polarizations is more complicated. When we define the hyperbolic angles χ by

$$\begin{aligned}P_{Z^0}(P_-) &= \tanh\chi_{Z^0}(P_-) \; , \\ P_{Z^0}(0) &= \tanh\chi_{Z^0}(0) \; , \\ P_- &= \tanh\chi_{P_-} \; , \end{aligned} \qquad (2.9)$$

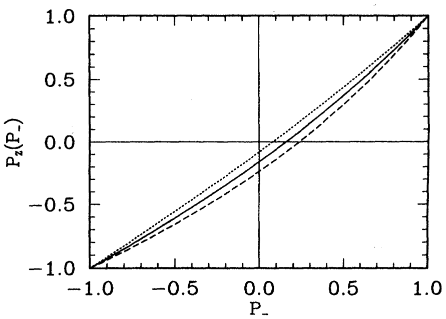

FIG. 1. Z^0 polarization as a function of the electron polarization P_-. Solid line: $\sin^2 2\theta_W = 0.23$; dashed line: $\sin^2 2\theta_W = 0.22$; and dotted line: $\sin^2 2\theta_W = 0.24$.

we find the addition rule

$$\chi_{Z^0}(P_-) = \chi_{Z^0}(0) + \chi_{P_-} \; . \tag{2.10}$$

III. Z^0 DECAY

The decay of polarized Z^0 to quark and antiquark of flavor f is obtained from the decay probability

$$d^6 w_f = \frac{1}{(2\pi)^2} \sum_{\text{spins,colors}} |M_f|^2 \times \delta^4(q + \bar{q} - p_{Z^0}) d^3 q d^3 \bar{q} \; , \tag{3.1}$$

where

$$M_f = -\frac{ie}{2 \sin 2\theta_W} \frac{v_{\bar{q}}}{\sqrt{2E_{\bar{q}}}} \gamma^\mu (v_f - a_f \gamma_5) \frac{\bar{u}_q}{\sqrt{2E_q}} \frac{e_\mu}{\sqrt{2E_Z}} \; . \tag{3.2}$$

Summation over q, $\bar{q}$ spins, and colors in Eq. (3.1) gives

$$\sum_{\text{spins,colors}} |M_f|^2 = \frac{3\pi}{2M_Z} \frac{\alpha}{\sin^2 2\theta_W} \{(v_f^2 + a_f^2)(1 + \cos^2\theta) - 4 s_z a_f v_f \cos\theta\} \; , \tag{3.3}$$

and the decay probability of a Z^0 boson with polarization s_z is

$$\frac{d^2 w_f^{q\bar{q}}(s_z)}{d\Omega} = \frac{3M_Z}{16\pi} \frac{\alpha}{\sin^2 2\theta_W} \{(v_f^2 + a_f^2)(1 + \cos^2\theta) - 4 s_z a_f v_f \cos\theta\} \; . \tag{3.4}$$

The physical decay probability of Z^0 produced in electron-positron collisions is obtained as a superposition of probabilities,

$$\frac{d^2 w_f^{q\bar{q}}(P_{Z^0})}{d\Omega} = \frac{1}{2}(1 + P_{Z^0}) \frac{d^2 w_f(1)}{d\Omega} + \frac{1}{2}(1 - P_{Z^0}) \frac{d^2 w_f(-1)}{d\Omega} \; ,$$

which amounts to replacing s_z in Eq. (3.4) by $P_{Z^0}(P_-)$, given by Eq. (2.6):

$$\frac{d^2 w_f^{q\bar{q}}[P_{Z^0}(P_-)]}{d\Omega} = \frac{3M_Z}{16\pi} \frac{\alpha}{\sin^2 2\theta_w} \{(v_s^2 + a_f^2)(1 + \cos^2\theta) - 4 P_{Z^0}(P_-) a_f v_f \cos\theta\} \; . \tag{3.5}$$

The differential cross section for the process $e^+e^- \to q\bar{q}$ is obtained by multiplying the decay rate $\{d^2 w_f^{q\bar{q}}[P_{Z^0}(P_-)]/d\Omega\}/\Gamma$ with the magnitude of the cross section, Eq. (2.5),

$$\frac{d^2\sigma_f}{d\Omega} = \frac{3}{4} \left(\frac{\alpha}{4 \sin^2 2\theta_W} \right)^2 \frac{1}{\Gamma^2} (v^2 + a^2 - 2avP_-) \times \{(v_f^2 + a_f^2)(1 + \cos^2\theta) - 4 P_{Z^0}(P_-) a_f v_f \cos\theta\} \; . \tag{3.6}$$

Written in this form, the Z^0 polarization is displayed explicitly, which may be useful for understanding the physical effect of electron polarization on the cross section and on polarization effects.

It is useful to note that the cross section Eq. (3.6) may alternatively be found by summation over Z^0 helicity:

$$\frac{d^2\sigma_f}{d\Omega} = \sum_{s_z = \pm 1} \frac{\sigma(s_z)}{\Gamma} \frac{d^2 w_f(s_z)}{d\Omega} = \frac{3}{4} \left(\frac{\alpha}{4 \sin^2 2\theta_W} \right)^2 \frac{1}{\Gamma^2} \{(v^2 + a^2 - 2avP_-)(v_f^2 + a_f^2)(1 + \cos^2\theta) + [2av - (v^2 + a^2)P_-] 4 v_f a_f \cos\theta\} \; . \tag{3.7}$$

As described above only the helicity components $s_z = \pm 1$ contribute to the cross section.

The virtual gluon corrections are obtained by multiplication with the factor

$$1 + \frac{4\alpha_s}{3\pi} F_V(\bar{m}_f, \bar{\mu}) \; , \tag{3.8}$$

where $[4\alpha_s/(3\pi)] F_V(\bar{m}_f, \bar{\mu})$ is the well-known Schwinger correction modified by the color factor 4/3, with $\bar{m}_f = m_f/E$ and $\bar{\mu} = \mu/E$, the scaled gluon mass parameter,

$$F_V(\bar{m}_f, \bar{\mu}) = \left(2 \ln \frac{2}{\bar{m}_f} - 1 \right) \left(\frac{3}{2} - 2 \ln \frac{m_f}{\bar{\mu}} \right) - 2 \ln^2 \frac{\bar{m}_f}{2} - \frac{1}{2} + \frac{2\pi^2}{3} \; . \tag{3.9}$$

This is true in the case of massless quarks also for the axial vector coupling as demonstrated in [5].

The decay of the polarized Z^0 to quark and antiquark with emission of a gluon is obtained from [5]. The decay probability is for massless quarks:

$$\frac{d^2 w_f^{q\bar{q}g}(s_z)}{d\Omega} = \frac{\alpha_s}{\pi}\frac{M_Z}{4\pi}\frac{\alpha}{\sin^2 2\theta_W} \times\{(v_f^2+a_f^2)[\Psi_1(\bar{m}_f,\bar{\mu})(1+\cos^2\theta) +\Psi_2(\bar{m}_f)(1-3\cos^2\theta)] -s_z 4a_f v_f \Psi_3(\bar{m}_f,\bar{\mu})\cos\theta\}\ , \quad (3.10)$$

with

$$\Psi_1(\bar{m}_f,\bar{\mu}) = \int dx\, d\bar{x}\left\{\frac{x^2}{(1-x)(1-\bar{x})} - \frac{\bar{m}_f^2}{2}\frac{1}{(1-x)^2}\right\},$$
$$\Psi_2(\bar{m}_f) = \int dx\, d\bar{x}\left(\frac{x+\bar{x}-1}{x^2}\right)\ , \quad (3.11)$$
$$\Psi_3(\bar{m}_f,\bar{\mu}) = \Psi_1(\bar{m}_f,\bar{\mu}) - \int dx\, d\bar{x}\left(\frac{\bar{x}}{x}\right)\ .$$

We define the form factors F_i by

$$F_1 = \Psi_1(\bar{m}_f,\bar{\mu}) + F_V(\bar{m}_f,\bar{\mu})\ ,$$
$$F_2 = \Psi_2(\bar{m}_f)\ , \quad (3.12)$$
$$F_3 = \Psi_3(\bar{m}_f,\bar{\mu}) + F_V(\bar{m}_f,\bar{\mu})\ ,$$

which isolates the infrared term $F_V(\bar{m}_f,\bar{\mu})$, given by Eq. (3.9), in $\Psi_1(\bar{m}_f,\bar{\mu})$ and $\Psi_3(\bar{m}_f,\bar{\mu})$ which are infrared divergent.

The two-jet decay probability of Z^0 produced by polarized electrons and unpolarized positrons including first-order radiative corrections is then given by

$$\frac{d^2 w_f^{q\bar{q}}}{d\Omega} + \frac{d^2 w_f^{q\bar{q}g}}{d\Omega} = \frac{3M_Z}{16\pi}\frac{\alpha}{\sin^2 2\theta_W}\left\{(v_f^2+a_f^2)\left[\left(1+\frac{4\alpha_s}{3\pi}F_1\right)(1+\cos^2\theta) + \frac{4\alpha_s}{3\pi}F_2(1-3\cos^2\theta)\right] -4P_{Z^0}(P_-)a_f v_f\left(1+\frac{4\alpha_s}{3\pi}F_3\right)\cos\theta\right\}\ , \quad (3.13)$$

with $P_{Z^0}(P_-)$ given by Eq. (2.6) and, from [5],

$$F_1 = \frac{3}{4},\quad F_2 = \frac{1}{2},\quad F_3 = 0\ , \quad (3.14)$$

for massless quarks.

IV. QUARK MASS EFFECTS

Finite quark masses will change the $q\bar{q}$ and $q\bar{q}g$ decay probabilities. The inclusion of mass terms in the projection operators in the spin sums of Eq. (3.3) gives the $q\bar{q}$ decay probability [13] instead of Eq. (3.5):

$$\frac{d^2 w_f^{q\bar{q}}[P_{Z^0}(P_-)]}{d\Omega} = \frac{3M_Z}{16\pi}\frac{\alpha}{\sin^2 2\theta_W}\beta\left\{\left[(v_f^2+a_f^2)\left(1+\frac{\bar{m}_f^2}{2}\right) - \frac{3}{2}a_f^2\bar{m}_f^2\right](1+\cos^2\theta) +\frac{1}{2}v_f^2\bar{m}_f^2(1-3\cos^2\theta) - 4P_{Z^0}(P_-)a_f v_f\beta\cos\theta\right\}\ , \quad (4.1)$$

where $\beta^2 = 1-\bar{m}_f^2$.

The virtual gluon corrections are obtained in a similar way as in Eq. (3.8) except that the additional term $F'_V(\bar{m}_f,\bar{\mu})$, appears. Equation (4.1) including virtual gluon corrections then becomes

$$\frac{d^2 w_f^{q\bar{q}}[P_{Z^0}(P_-)]}{d\Omega} = \frac{3M_Z}{16\pi}\frac{\alpha}{\sin^2 2\theta_W}\beta\left(\left[(v_f^2+a_f^2)\left(1+\frac{\bar{m}_f^2}{2}\right)\left(1+\frac{4\alpha_s}{3\pi}\left\{F_V(\beta,\bar{\mu}) + \frac{6}{2+\bar{m}_f^2}\mathrm{Re}F'_V(\beta)\right\}\right) -\frac{3}{2}a_f^2\bar{m}_f^2\left(1+\frac{4\alpha_s}{3\pi}\left\{F_V(\beta,\bar{\mu}) + \frac{2(5-2\bar{m}_f^2)}{3\bar{m}_f^2}\mathrm{Re}F'_V(\beta)\right\}\right)\right](1+\cos^2\theta) +\frac{1}{2}v_f^2\bar{m}_f^2\left(1+\frac{4\alpha_s}{3\pi}\left\{F_V(\beta,\bar{\mu}) + \frac{2}{\bar{m}_f^2}\mathrm{Re}F'_V(\beta)\right\}\right)(1-3\cos^2\theta) -4P_{Z^0}(P_-)a_f v_f\beta\left(1+\frac{4\alpha_s}{3\pi}F_V(\beta,\bar{\mu})\right)\cos\theta\right)\ , \quad (4.2)$$

where now the exact formulas for $F_V(\beta,\bar{\mu})$ and $F'_V(\beta)$ are given by

$$F_V(\beta,\bar\mu)=\left(1+\frac{1+\beta^2}{2\beta}\ln\frac{1-\beta}{1+\beta}\right)\ln\frac{1-\beta^2}{\bar\mu^2}-\frac{1}{\beta}\left(1+\beta^2-\frac{1}{2}\right)\ln\frac{1-\beta}{1+\beta}-2$$
$$+\frac{1+\beta^2}{2\beta}\left[-\frac{1}{2}\ln^2\frac{1-\beta}{1+\beta}+2\ln\frac{1-\beta}{1+\beta}\ln\frac{2\beta}{1+\beta}+2L_2\left(\frac{1-\beta}{1+\beta}\right)+\frac{2\pi^2}{3}\right],$$
$$F_V'(\beta)=\frac{1-\beta^2}{4\beta}\left(\ln\frac{1-\beta}{1+\beta}+i\pi\right). \qquad (4.3)$$

It should be noted that in the present calculation at the Z^0 pole, where photon and interference contributions have been neglected, only the real part of $F_v'(\beta)$ contributes. In exact calculations [17] the imaginary part of $F_V'(\beta)$ is taken into account. At the Z^0 pole, however, the contribution is extremely small, of the order $\bar m_f^2(\Gamma_Z/M_Z)$.

From [13] we find, in the same way for $q\bar q g$ decays, with the notation as in Eq. (3.11),

$$\frac{d^2w_f^{q\bar q g}[P_{Z^0}(P_-)]}{d\Omega}=\frac{\alpha_s}{\pi}\frac{M_Z}{4\pi}\frac{\alpha}{\sin^2 2\theta_W}$$
$$\times\{(v_f^2+a_f^2)[\Psi_1(\bar m_f,\bar\mu)(1+\cos^2\theta)+\Psi_2(\bar m_f,\bar\mu)(1-3\cos^2\theta)]$$
$$-2a_f^2[\Psi_4(\bar m_f,\bar\mu)(1+\cos^2\theta)+\Psi_5(\bar m_f,\bar\mu)(1-3\cos^2\theta)]$$
$$-4P_{Z^0}(P_-)a_fv_f\Psi_3(\bar m_f,\bar\mu)\cos\theta\}, \qquad (4.4)$$

where the extra terms Ψ_4 and Ψ_5 are terms of order $\bar m_f^2$ and higher. The Ψ functions are found [13] to be given by

$$\Psi_1(\bar m_f,\bar\mu)=\int dx\,d\bar x\left\{\frac{x^2-\bar m_f^2(1-x)-\frac{\bar m_f^4}{4}}{(1-x)(1-\bar x)}-\frac{\bar m_f^2}{2}\left(1+\frac{\bar m_f^2}{2}\right)\frac{1}{(1-x)^2}\right\},$$
$$\Psi_2(\bar m_f,\bar\mu)=\frac{1}{4}\int dx\,d\bar x\left\{\frac{(\bar x^2-\bar m_f^2)\sin^2\vartheta}{(1-x)(1-\bar x)}\left(1-\frac{\bar m_f^2}{2}\frac{1-x}{1-\bar x}\right)+\bar m_f^2\left[\frac{2(1-x_g)-\bar m_f^2}{(1-x)(1-\bar x)}-\frac{\bar m_f^2}{(1-x)^2}\right]\right\},$$
$$\Psi_3(\bar m_f,\bar\mu)=\frac{1}{2}\int\frac{dx\,d\bar x}{(1-x)(1-\bar x)}\left\{\left(x-\frac{\bar m_f^2}{2}\frac{x_g}{1-x}\right)(x^2-\bar m_f^2)\right.$$
$$\left.-\left(\bar x-\frac{\bar m_f^2}{2}\frac{x_g}{1-\bar x}\right)[2(1-x)(1-\bar x)-x\bar x+\bar m_f^2]\right\}\frac{1}{\sqrt{x^2-\bar m_f^2}}. \qquad (4.5)$$
$$\Psi_4(\bar m_f,\bar\mu)=\frac{3}{8}\bar m_f^2\int dx\,d\bar x\left\{\frac{2-\bar m_f^2-2x_g-\frac{x_g^2}{3}}{(1-x)(1-\bar x)}-\frac{\bar m_f^2}{(1-x)^2}\right\},$$
$$\Psi_5(\bar m_f,\bar\mu)=\frac{\bar m_f^2}{8}\int dx\,d\bar x\left\{\frac{(\bar x^2-\bar m_f^2)\sin^2\vartheta}{(1-x)(1-\bar x)}+\left[\frac{2-\bar m_f^2-2x_g-x_g^2}{(1-x)(1-\bar x)}-\frac{\bar m_f^2}{(1-x)^2}\right]\right\}.$$

with $x\beta_x\bar x\beta_{\bar x}\cos\vartheta=-x\bar x-2(1-x-\bar x)-\bar m_f^2$.

We may define the form factors F_1–F_5 similarly to as in Eq. (3.12):

$$\Psi_1(\bar m_f,\bar\mu)=\beta\left(1+\frac{\bar m_f^2}{2}\right)\left[F_1(\bar m_f)-F_V(\beta,\bar\mu)-\frac{6}{2+\bar m_f^2}\mathrm{Re}F_V'(\beta)\right],$$
$$\Psi_2(\bar m_f,\bar\mu)=\beta\left[F_2(\bar m_f)-\frac{\bar m_f^2}{2}F_V(\beta,\bar\mu)-\mathrm{Re}F_V'(\beta)\right],$$
$$\Psi_3(\bar m_f,\bar\mu)=(1-\bar m_f^2)[F_3(\bar m_f)-F_V(\beta,\bar\mu)], \qquad (4.6)$$
$$\Psi_4(\bar m_f,\bar\mu)=\beta\frac{3\bar m_f^2}{4}\left[F_4(\bar m_f)-F_V(\beta,\bar\mu)-\frac{2(5-2\bar m_f^2)}{3\bar m_f^2}\mathrm{Re}F_V'(\beta)\right],$$
$$\Psi_5(\bar m_f,\bar\mu)=\beta\frac{\bar m_f^2}{4}\left[F_5(\bar m_f)-F_V(\beta,\bar\mu)-\frac{2}{\bar m_f^2}\mathrm{Re}F_V'(\beta)\right].$$

With these definitions the virtual corrections cancel and we are left with the $q\bar q$ decay probability with radiative corrections similar to Eq. (3.13), where now also the mass corrections are taken into account:

$$\frac{d^2w_f}{d\Omega}[P_{Z^0}(P_-)] = \frac{d^2w_f^{q\bar{q}}[P_{Z^0}(P_-)]}{d\Omega} + \frac{d^2w_f^{q\bar{q}g}[P_{Z^0}(P_-)]}{d\Omega}$$
$$= \frac{3M_Z}{16\pi}\frac{\alpha}{\sin^2 2\theta_W}\beta$$
$$\times\left\{\left[(v_f^2+a_f^2)\left(1+\frac{\bar{m}_f^2}{2}\right)\left(1+\frac{4\alpha_s}{3\pi}F_1(\bar{m}_f)\right)-\frac{3}{2}a_f^2\bar{m}_f^2\left(1+\frac{4\alpha_s}{3\pi}F_4(\bar{m}_f)\right)\right](1+\cos^2\theta)\right.$$
$$+\left[(v_f^2+a_f^2)\left(\frac{\bar{m}_f^2}{2}+\frac{4\alpha_s}{3\pi}F_2(\bar{m}_f)\right)-a_f^2\frac{\bar{m}_f^2}{2}\left(1+\frac{4\alpha_s}{3\pi}F_5(\bar{m}_f)\right)\right](1-3\cos^2\theta)$$
$$\left.-4P_{Z^0}(P_-)a_f v_f\beta\left(1+\frac{4\alpha_s}{3\pi}F_3(\bar{m}_f)\right)\cos\theta\right\}. \tag{4.7}$$

V. FORM FACTORS

The form factors, except $F_3(\bar{m}_f)$, may be calculated analytically. In fact the vector coupling form factor $F(\bar{m}_f)$ was calculated by Schwinger [18]. We write it in the form

$$F_1(\bar{m}_f) = \frac{3}{4}\left(\frac{2+3\bar{m}_f^2}{2+\bar{m}_f^2}\right)+\frac{1}{\beta}\left[\frac{11}{8}(2-\bar{m}_f^2)+\frac{(1-\bar{m}_f^2)^2}{2(2+\bar{m}_f^2)}\right]\ln\frac{1+\beta}{1-\beta}+G(\bar{m}_f)\ , \tag{5.1}$$

with

$$G(\bar{m}_f) = -4\ \ln\beta + 6\ \ln\frac{1+\beta}{2} - 3\ \ln\frac{1+\beta}{1-\beta} + \frac{2-\bar{m}_f^2}{\beta}\left[\frac{\pi^2}{6}+\ln\frac{1+\beta}{2}\ln\frac{1+\beta}{1-\beta}+2L_2\left(\frac{1-\beta}{1+\beta}\right)+L_2(\beta^2)\right.$$
$$\left.-4L_2(\beta)+2L_2\left(\frac{1+\beta}{2}\right)-2L_2\left(\frac{1-\beta}{2}\right)\right].$$

We find further

$$F_2(\bar{m}_f) = \frac{1}{2\beta}\left[\beta(1-3\bar{m}_f^2)+\frac{\bar{m}_f^2}{2}(3-\bar{m}_f^2)\ln\frac{1+\beta}{1-\beta}+\left\{\frac{\bar{m}_f}{2}(1-\bar{m}_f^2-2\bar{m}_f)(1+\bar{m}_f)H(\bar{m}_f)+(\bar{m}_f\Leftrightarrow-\bar{m}_f)\right\}\right]$$
$$+\bar{m}_f^2+\frac{\bar{m}_f^2}{2}\left\{\frac{1}{\beta}(2-\bar{m}_f^2)\ln\frac{1+\beta}{1-\beta}+G(\bar{m}_f)\right\}\ , \tag{5.2}$$

$$F_4(\bar{m}_f) = \frac{9}{4}-\frac{\bar{m}_f^2}{8}+\frac{1}{2\beta}\left(3-\bar{m}_f^2-\frac{\bar{m}_f^4}{8}\right)\ln\frac{1+\beta}{1-\beta}+G(\bar{m}_f)\ , \tag{5.3}$$

$$F_5(\bar{m}_f) = \frac{3}{2}-\frac{\bar{m}_f^2}{4}-\frac{\bar{m}_f^2}{8\beta}(4+\bar{m}_f^2)\ln\frac{1+\beta}{1-\beta}+\left\{\frac{\bar{m}_f}{2\beta}(1+\bar{m}_f)H(\bar{m}_f)+(\bar{m}_f\Leftrightarrow-\bar{m}_f)\right\}+F_4(\bar{m}_f)\ , \tag{5.4}$$

with

$$H(\bar{m}_f) = \ln\frac{2}{2+\bar{m}_f}\ln\frac{1+\beta}{1-\beta}+\left[L_2\left(\frac{1-\beta}{2}\right)+L_2\left(\frac{1+\beta}{2+\bar{m}_f}\right)+L_2\left(-\sqrt{(1-\beta)/(1+\beta)}\right)-(\beta\Leftrightarrow-\beta)\right]$$
$$+\left[L_2\left(\frac{1-\sqrt{1-\bar{m}_f}}{1+\sqrt{1+\bar{m}_f}}\right)-L_2\left(\sqrt{1+\bar{m}_f}\frac{1-\sqrt{1-\bar{m}_f}}{1+\sqrt{1+\bar{m}_f}}\right)-L_2\left(\frac{1-\sqrt{1-\bar{m}_f}}{1-\sqrt{1+\bar{m}_f}}\right)\right.$$
$$\left.+L_2\left(-\sqrt{1+\bar{m}_f}\frac{1-\sqrt{1-\bar{m}_f}}{1-\sqrt{1+\bar{m}_f}}\right)-(\sqrt{1-\bar{m}_f}\Leftrightarrow-\sqrt{1-\bar{m}_f})\right]. \tag{5.5}$$

The form factor $F_3(\bar{m}_f)$ is obtained by numerical integration; only for an expansion in powers of $\bar{m}_f$ given in Sec. VI is the result simple. The form factors F_i, $i = 1$–5, are given in Fig. 2.

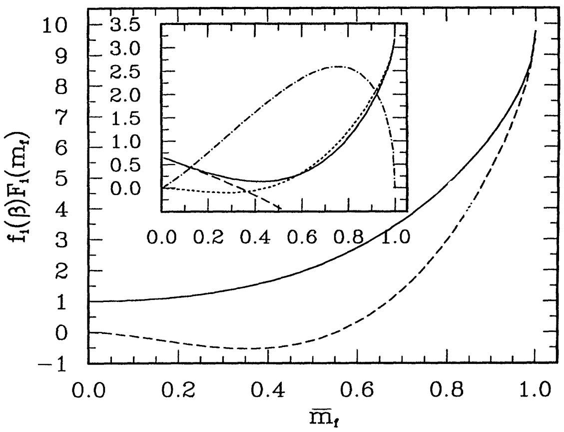

FIG. 2. The form factors $F_i(\bar{m}_f)$. Large figure: $\frac{2}{3}\beta(3-\beta^2)F_1(\bar{m}_f)$ (solid line); $2\beta(1-\beta^2)F_4(\bar{m}_f)$ (dashed line). Small figure: $\frac{4}{3}\beta F_2(\bar{m}_f)$ (solid line), expansion for small $\bar{m}_f$ (dashed line); $\frac{4}{3}\beta^2 F_3(\bar{m}_f)$ (dash-dotted line); $\frac{2}{3}\beta(1-\beta^2)F_5(\bar{m}_f)$ (dotted line).

VI. FORM FACTORS EXPANDED IN POWERS OF $\bar{M}_F$

We can obtain the expansions of the form factors to order $\bar{m}_f^2$ from the analytical expressions given in the previous section. For the case of $F_3(\bar{m}_f)$ we show in the Appendix how the expansion is obtained. The reason why we give this calculation explicitly is that the results of Djouadi *et al.* and Arbuzov, Bardin, and Leike [12] differ and also differ from our result.

From the formula (5.1)–(5.4) and from Eq. (A3) of the Appendix, we find, to order $\bar{m}_f^2$,

$$\frac{4\alpha_2}{3\pi}F_1(\bar{m}_f) = \frac{\alpha_s}{\pi}(1+3\bar{m}_f^2) \,, \tag{6.1}$$

$$\frac{4\alpha_s}{3\pi}F_2(\bar{m}_f) = \frac{\alpha_s}{\pi}\left[\frac{2}{3} - \frac{\pi^2}{6}\bar{m}_f - \bar{m}_f^2\left(\frac{1}{3}+\frac{\pi^2}{9}+\ln\frac{\bar{m}_f}{2} + \frac{1}{3}\ln^2\frac{\bar{m}_f}{2}\right)\right] , \tag{6.2}$$

$$\frac{4\alpha_s}{3\pi}F_3(\bar{m}_f) = \frac{\alpha_s}{\pi}\left[\frac{8}{3}\bar{m}_f + \bar{m}_f^2\left(\frac{7}{3}+\frac{\pi^2}{18}-\frac{2}{3}\ln\frac{\bar{m}_f}{2} + \frac{1}{3}\ln^2\frac{\bar{m}_f}{2}\right)\right] , \tag{6.3}$$

$$\frac{4\alpha_s}{3\pi}F_4(\bar{m}_f) = \frac{\alpha_s}{\pi}\left(3+4\ln\frac{\bar{m}_f}{2}\right) , \tag{6.4}$$

$$\frac{4\alpha_s}{3\pi}F_5(\bar{m}_f) = \frac{\alpha_s}{\pi}\left(5+4\ln\frac{\bar{m}_f}{2}\right) . \tag{6.5}$$

Here $F_1(\bar{m}_f)$, $F_4(\bar{m}_f)$, and $F_5(\bar{m}_f)$ agree with the results of Djouadi *et al.* [11] and Arbuzov, Bardin, and Leike [12]. Our result for $F_2(\bar{m}_f)$ disagrees with [11] but agrees with [12], while $F_3(\bar{m}_f)$ Eq. (6.3) disagrees with both. The very close agreement for $F_3(\bar{m}_f)$ with the numerical results in Table I, shows that the formula is accurate up to $\bar{m}_f$ of the order 0.2.

VII. CROSS SECTIONS AND ASYMMETRIES

The cross section for polarized electrons is obtained as in Eq. (3.7), now from Eq. (2.5) and Eq. (4.7):

TABLE I. Exact values of the form factor $(4\beta^2/3)F_3(\bar{m}_f)$, compared to the $O(\bar{m}_f^2)$ expansion, Eq. (A3), for small $\bar{m}_f$.

	$\frac{4\beta^2}{3}F_3(\bar{m}_f)$			$\frac{4\beta^2}{3}F_3(\bar{m}_f)$			$\frac{4\beta^2}{3}F_3(\bar{m}_f)$		$\frac{4\beta^2}{3}F_3(\bar{m}_f)$
$\bar{m}_f$	Exact	Expansion	$\bar{m}_f$	Exact	Expansion	$\bar{m}_f$	Exact	$\bar{m}_f$	Exact
0.01	0.0290	0.0283	0.175	0.6464	0.6449	0.365	1.4714	0.705	2.5633
0.02	0.0589	0.0585	0.185	0.6887	0.6866	0.385	1.5568	0.725	2.5830
0.03	0.0905	0.0903	0.195	0.7313	0.7289	0.405	1.6411	0.745	2.5937
0.04	0.1235	0.1234	0.205	0.7741	0.7705	0.425	1.7241	0.765	2.5946
0.05	0.1592	0.1576	0.215	0.8172	0.8125	0.445	1.8053	0.785	2.5844
0.06	0.1941	0.1929	0.225	0.8605	0.8546	0.465	1.8847	0.805	2.5615
0.07	0.2299	0.2290	0.235	0.9040	0.8966	0.485	1.9618	0.825	2.5242
0.08	0.2666	0.2659	0.245	0.9476	0.9386	0.505	2.0365	0.845	2.4702
0.09	0.3041	0.3036	0.255	0.9913	0.9804	0.525	2.1083	0.865	2.3965
0.10	0.3423	0.3419	0.265	1.0351	1.0221	0.545	2.1770	0.885	2.2993
0.105	0.3617	0.3613	0.275	1.0789	1.0635	0.565	2.2423	0.905	2.1729
0.115	0.4009	0.4006	0.285	1.1228	1.1047	0.585	2.3036	0.925	2.0089
0.125	0.4406	0.4403	0.295	1.1667	1.1456	0.605	2.3606	0.945	1.7927
0.135	0.4809	0.4805	0.305	1.2105	1.1860	0.625	2.4129	0.965	1.4955
0.145	0.5217	0.5211	0.325	1.2980	1.2658	0.645	2.4599	0.985	1.0338
0.155	0.5629	0.5621	0.345	1.3850	1.3436	0.665	2.5010	1.000	0.0000
0.165	0.6045	0.6033				0.685	2.5358		

$$\frac{d^2\sigma_f}{d\Omega} = \frac{3}{4}\left(\frac{\alpha}{4\sin^2 2\theta_W}\right)^2 \frac{\beta}{\Gamma^2}\Bigg\{(v^2+a^2-2avP_-) \times \left[(v_f^2+a_f^2)\left(1+\frac{\bar{m}_f^2}{2}\right)\left(1+\frac{4\alpha_s}{3\pi}F_1(\bar{m}_f)\right) - \frac{3}{2}a_f^2\bar{m}_f^2\left(1+\frac{4\alpha_s}{3\pi}F_4(\bar{m}_f)\right)\right](1+\cos^2\theta)$$
$$+(v^2+a^2-2avP_-)\left[(v_f^2+a_f^2)\left(\frac{\bar{m}_f^2}{2}+\frac{4\alpha_s}{3\pi}F_2(\bar{m}_f)\right) - a_f^2\frac{\bar{m}_f^2}{2}\left(1+\frac{4\alpha_s}{3\pi}F_5(\bar{m}_f)\right)\right](1-3\cos^2\theta)$$
$$+[2av-(v^2+a^2)P_-]4a_fv_f\beta\left(1+\frac{4\alpha_s}{3\pi}F_3(\bar{m}_f)\right)\cos\theta\Bigg\}\ . \tag{7.1}$$

This equation gives all information concerning cross sections and asymmetries. Together with the expressions for the form factors Eqs. (6.1)–(6.5), we can obtain sufficient accuracy for cross sections and asymmetries, with b-quark mass $\bar{m}_b = 0.140 \pm 0.005$.

Obviously left-handed electrons, P_- negative, coupling directly to the left-handed natural polarization $-2av/(v^2+a^2)$, gives higher Z^0 polarization, Eq. (2.6), than right-handed electrons. The effect of a left-handed electron polarization $-|P_-|$ is then to increase the total cross section by the factor

$$1+\frac{2av}{v^2+a^2}|P_-|\ ,$$

which gives a small increase.

The total cross section including radiative corrections has been obtained to a high degree or precision in [19].

The *left-right asymmetry* defined by

$$A_{LR} = \frac{\sigma(-P_-)-\sigma(P_-)}{\sigma(-P_-)+\sigma(P_-)} = \frac{2av}{v^2+a^2}\ , \tag{7.2}$$

has no QCD radiative or quark mass corrections, which is easily understood, since integration over the angle θ removes the Z^0 polarization dependence from the decay probability Eq. (4.7). The left-right asymmetry depends only on the initial production of Z^0.

The *quark forward-backward asymmetry* is initiated by a polarized electron

$$A_{\rm FB}(P_-) = \frac{\sigma_F(P_-)-\sigma_B(P_-)}{\sigma_F(P_-)+\sigma_B(P_-)} = \frac{P_{Z^0}(P_-)}{P_{Z^0}(0)}A_{\rm FB}(0)\ , \tag{7.3}$$

where $P_{Z^0}(P_-)$ and $P_{Z^0}(0)$ are the Z^0 polarizations initiated by polarized electrons and unpolarized electrons, respectively, Eqs. (2.6) and (2.7), and where $A_{\rm FB}(0)$ is the forward-backward asymmetry initiated by unpolarized electrons. $A_{\rm FB}(0)$ was first derived by Djouadi *et al.* [11], however, with a result which differs somewhat from ours. We are indebted to Dr. Zerwas for discussions on this point.

From Eq. (7.1) we find the asymmetry for unpolarized electrons:

$$A_{\rm FB}(0) = \frac{3}{4}\beta\frac{2av}{v^2+a^2}2v_fa_f\left(1+\frac{4\alpha_s}{3\pi}F_3(\bar{m}_f)\right) \times\left\{(v_f^2+a_f^2)\left(1+\frac{\bar{m}_f}{2}\right)\left(1+\frac{4\alpha_s}{3\pi}F_1(\bar{m}_f)\right) -\frac{3}{2}a_f^2\bar{m}_f^2\left(1+\frac{4\alpha_s}{3\pi}F_4(\bar{m}_f)\right)\right\}^{-1}\ . \tag{7.4}$$

We write it in the from

$$A_{\rm FB}(0) = A^0_{\rm FB}(0)(1-\Delta^f_{\rm FB})\ , \tag{7.5}$$

with the exact lowest-order asymmetry

$$A^0_{\rm FB}(0) = \frac{3}{4}\beta\frac{2av}{v^2+a^2}\frac{2a_fv_f}{v_f^2\frac{3-\beta^2}{2}+\beta^2a_f^2}\ , \tag{7.6}$$

and the radiative correction, to first order in α_s and second order in $\bar{m}_f$,

$$\Delta^f_{\rm FB} = \frac{\alpha_s}{\pi}\left[1-\frac{8}{3}\bar{m}_f-\frac{\bar{m}_f^2}{3} \times\left(7+\frac{\pi^2}{6}-2\ln\frac{\bar{m}_f}{2}+\ln^2\frac{\bar{m}_f}{2}\right) +3\bar{m}_f^2\frac{v_f^2-2a_f^2\ln\frac{\bar{m}_f}{2}}{v_f^2+a_f^2}\right]\ . \tag{7.7}$$

For $\alpha_s(M_Z)=0.118$, $\sin^2 2\theta_W = 0.23$, and $\bar{m}_b = 0.104 \pm 0.005$ we find $\Delta^b_{\rm FB} = 0.029\,25 \mp 0.000\,67$. The radiative correction differs from the result of Djouadi *et al.* [11] because of the disagreement concerning $F_3(\bar{m}_f)$ mentioned in Sec. VI.

As seen from Eqs. (7.6) and (7.8) the radiative correction does not depend on the electron polarization. In fact electron polarization changes the asymmetry Eq. (7.8) by replacing the Z^0 "natural" polarization $P_{Z^0}(0)$ by the P_- dependent polarization $P_{Z^0}(P_-)$, factorizing the electron polarization-dependent asymmetry in a lepton-dependent part and a quark-dependent part:

$$A_{\rm FB}(P_-) = \frac{3}{4}\beta\frac{2av-(v^2+a^2)P_-}{v^2+a^2-2avP_-} \times\frac{2a_fv_f}{v_f^2\frac{3-\beta^2}{2}+\beta^2a_f^2}(1-\Delta^f_{\rm FB})\ . \tag{7.8}$$

The *forward-backward, left-right asymmetry* (FB,LR) defined by [20]

$$A_{\mathrm{FB},LR}(P_-)$$

$$= \frac{\sigma_F(-P_-) - \sigma_B(-P_-) - \sigma_F(P_-) + \sigma_B(P_-)}{\sigma_F(-P_-) + \sigma_B(-P_-) + \sigma_F(P_-) + \sigma_B(P_-)} , \quad (7.9)$$

is found to be given by

$$A_{\mathrm{FB},LR}(P_-) = \frac{P_-}{P_{Z^0}(0)} A_{\mathrm{FB}}(0) , \quad (7.10)$$

with $A_{\mathrm{FB}}(0)$ given by Eq. (7.8). The FB,LR asymmetry is therefore obtained from Eq. (7.9) by replacing $P_{Z^0}(0) = -2av/(v^2 + a^2)$ by $-P_-$:

$$A_{\mathrm{FB},LR}(P_-) = -\frac{3}{4}\beta P_- \frac{2a_f v_f}{v_f^2 \frac{3-\beta^2}{2} + \beta^2 a_f^2}(1 - \Delta^f_{\mathrm{FB}}) . \quad (7.11)$$

Compared to $A_{\mathrm{FB}}(0)$ and $A_{\mathrm{FB}}(P_-)$ then, the lepton coupling coefficients v and a are removed.

It is interesting to note that the forward-backward asymmetries are related by

$$A_{\mathrm{FB}}(P_-) : A_{\mathrm{FB}}(0) : A_{\mathrm{FB},LR}(P_-)$$

$$= P_{Z^0}(P_-) : P_{Z^0}(0) : P_- , \quad (7.12)$$

with the same quark dependent factor of proportionality:

$$\frac{A_{\mathrm{FB}}(P_-)}{P_{Z^0}(P_-)} = -\frac{3}{4}\beta \frac{2a_f v_f}{v_f^2 \frac{3-\beta^2}{2} + \beta^2 a_f^2}(1 - \Delta^f_{\mathrm{FB}}) . \quad (7.13)$$

Note added. After submission of the paper an erratum to [12] came to our attention: A. B. Arbuzov, D. Yu Bardin, and A. Leike, Mod. Phys. Lett. A **9**, 1515(E) (1994). We are happy to note that the corrected radiative correction in the limit of $\bar{m}_f$ small agrees with our result above Eq. (6.3).

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APPENDIX

In this appendix we give explicitly the expansion of $F_3(\bar{m}_f)$ in powers of $\bar{m}_f$ for reasons explained in Sec. VI. The simplest way is to note that from Eq. (4.6) it follows that, to order $\bar{m}_f^2$

$$F_3(\bar{m}_f) = F_1(\bar{m}_f) + (1 + \bar{m}_f^2)\Psi_3(\bar{m}_f, \bar{\mu}) - \Psi_1(\bar{m}_f, \bar{\mu}) - \frac{6}{2 + \bar{m}_f^2}\mathrm{Re}F_V'(\beta) ,$$

which shows that this particular Ψ_1, Ψ_3 combination is infrared finite. We write the combination in the form

$$(1 + \bar{m}_f^2)2\Psi_3(\bar{m}_f, \bar{\mu}) - 2\Psi_1(\bar{m}_f, \bar{\mu}) \simeq (1 + \bar{m}_f^2)J_0 + \sum_{i=1}^{4} J_i ,$$

and find, from Eq. (4.5),

$$\begin{aligned}
J_0 &= -2\int_{\bar{m}_f}^1 dx \int_{\bar{x}_-}^{\bar{x}_+} d\bar{x} \frac{\bar{x}}{\sqrt{x^2 - \bar{m}_f^2}} , \\
J_1 &= \int_{\bar{m}_f}^1 dx \int_{\bar{x}_-}^{\bar{x}_+} d\bar{x} \frac{\bar{x}^2 - x^2}{(1-x)(1-\bar{x})} \frac{x}{\sqrt{x^2 - \bar{m}_f^2}} , \\
J_2 &= \bar{m}_f^2 \int_{\bar{m}_f}^1 dx \int_{\bar{x}_-}^{\bar{x}_+} \times d\bar{x} \left[\frac{1}{x} + \frac{1-x}{1-\bar{x}}\left(2 + \frac{1}{\sqrt{x^2 - \bar{m}_f^2}}\right)\right] , \qquad \text{(A1)} \\
J_3 &= \frac{\bar{m}_f^2}{2}\int_{\bar{m}_f}^1 dx \int_{\bar{x}_-}^{\bar{x}_+} d\bar{x} \left[1 + \frac{\bar{m}_f^2}{2} - \frac{x - \bar{m}_f^2(1-x)}{\sqrt{x^2 - \bar{m}_f^2}}\right] \times \left(\frac{1}{(1-x)^2} + \frac{1}{(1-\bar{x})^2}\right) , \\
J_4 &= \bar{m}_f^2 \int_{\bar{m}_f}^1 dx \int_{\bar{x}_-}^{\bar{x}_+} d\bar{x} \frac{\bar{x} - x}{(1-x)(1-\bar{x})} \frac{1}{\sqrt{x^2 - \bar{m}_f^2}} ,
\end{aligned}$$

where

$$1 - \bar{x}_\pm = (1-x)\frac{2(x \mp \sqrt{x^2 - \bar{m}_f^2}) - \bar{m}_f^2}{4(1-x) + \bar{m}_f^2} .$$

The results are

$$\begin{aligned}
J_0 &= -\frac{3}{2} + 2\bar{m}_f - \frac{\bar{m}_f^2}{4} , \\
J_1 &= 2\bar{m}_f + \bar{m}_f^2\left[\frac{\pi^2}{12} - \frac{1}{4} + \left(3 + \frac{1}{2}\ln\frac{\bar{m}_f}{2}\right)\ln\frac{\bar{m}_f}{2}\right] , \\
J_2 &= \bar{m}_f^2\left[\frac{\pi^2}{6} - \frac{1}{2} + \ln^2\frac{\bar{m}_f}{2}\right] , \qquad \text{(A2)} \\
J_3 &= \bar{m}_f^2\left[\frac{3}{2} + \ln\frac{\bar{m}_f}{2}\right] , \\
J_4 &= -\bar{m}_f^2\left[\frac{\pi^2}{6} + 2\ln\frac{\bar{m}_f}{2} + \ln^2\frac{\bar{m}_f}{2}\right] ,
\end{aligned}$$

which gives

$$F_3(\bar{m}_f) = 2\bar{m}_f + \bar{m}_f^2\left(\frac{7}{4} + \frac{\pi^2}{24} - \frac{1}{2}\ln\frac{\bar{m}_f}{2} + \frac{1}{4}\ln^2\frac{\bar{m}_f}{2}\right) . \quad \text{(A3)}$$

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