$O(\alpha_s)$ longitudinal spin polarization in heavy-quark production

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We present the massive one-loop QCD corrections to the production cross sections, of polarized quarks in the annihilation process $e^+e^- \rightarrow q\bar{q}(g)$ for bottom, top, and charm quarks. From the full analytical expressions for the production cross sections, Schwinger-type interpolation formulas for all parity-parity combinations (VV, VA, AA) are derived. The parity-odd interpolation formula contains the correct limit for vanishing quark masses taking into account a residual coupling of left- and right-chiral states in the massless theory. Numerical results for the total cross section and the longitudinal spin polarization demonstrate the accuracy of the interpolation formulas.

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Recent precision measurements on Z decays by the CERN e^+e^- collider LEP [1] and the SLAC Collaborations [2] permit a thorough investigation of the properties of electroweak neutral currents and thus provide a stringent test of the standard model. Moreover, the study of polarized and unpolarized observables of the annihilation process $e^+e^- \rightarrow \gamma, Z \rightarrow q\bar{q}$ to the level of strong radiative corrections offers an ideal experimental approach to confirm the validity of quantum chromodynamics (QCD).

In the present paper, we concentrate on the effects that finite quark masses have on the longitudinal polarization asymmetry P_L of heavy quarks produced through e^+e^- collisions [3]. We shall present a detailed numerical analysis of the $O(\alpha_s)$ corrections to the Born result for bottom, top, and charm quarks and then compare these results with approximations stemming from Schwinger-type interpolation formulas for the vector (V) and axial-vector (A) combinations of the production cross sections. In particular, the interpolation formulas for the parity-odd contribution σ_S^{VA} have not been given in the literature before. They provide a valuable tool for expressing the $O(\alpha_s)$ longitudinal polarization at the one-loop order of strong interactions in a condensed form.

For heavy-quark production the lowest-order γ-Z exchange cross-section formulas can be written in terms of the different parity-parity contributions. The parity-even property of the total unpolarized rate can directly be seen from the decomposition [4]

$$\sigma(e^+e^- \rightarrow \gamma, Z \rightarrow q\bar{q}) = \tfrac{1}{2}v(3-v^2)\sigma^{VV} + v^3\sigma^{AA} \ , \quad (1)$$

whereas the total parity-odd contribution relevant to the longitudinal spin polarization of the final quark q reads

$$\sigma(e^+e^- \rightarrow \gamma, Z \rightarrow q(\lambda_\pm)\bar{q}) = \pm v^2\sigma_S^{VA} \ . \quad (2)$$

Here, the two distinct helicity states of the quark are given by $\lambda_\pm = \pm\frac{1}{2}$. The velocity of the quark in the two-particle final state is $v = \sqrt{1-\xi}$ with $\xi = 4m_q^2/s$, where as usual $\sqrt{s}$ is the center-of-momentum energy in the e^+e^- system and m_q the mass of the quark q. In Eqs. (1) and (2) the vector, axial, and interference contributions are explicitly given by

$$\sigma^{VV} = \frac{4\pi\alpha^2}{s}[Q_q^2 - 2Q_q v_e v_q \mathrm{Re}\chi_Z + (v_e^2 + a_e^2)v_q^2|\chi_Z|^2] \ , \quad (3)$$

$$\sigma^{AA} = \frac{4\pi\alpha^2}{s}(v_e^2 + a_e^2)a_q^2|\chi_Z|^2 \ , \quad (4)$$

and

$$\sigma_S^{VA} = \frac{4\pi\alpha^2}{s}[Q_q v_e a_q \mathrm{Re}\chi_Z - (v_e^2 + a_e^2)v_q a_q|\chi_Z|^2] \ , \quad (5)$$

where $v_f = 2T_z^f - 4Q_f s_W^2$ and $a_f = 2T_z^f$ are the electroweak vector- and axial-vector-coupling constants for fermions (f), respectively, and Q_q denotes the fractional charge of the quark q. The quantity

$$\chi_Z(s) = g_W M_Z^2 s(s - M_Z^2 + iM_Z\Gamma_Z)^{-1}$$

characterizes the Breit-Wigner form of the Z propagator, where M_Z and Γ_Z are the mass and the total decay width of the Z boson, and $g_W \approx 4.49 \times 10^{-5}$ GeV^{-2}. Note that we do not consider polarization of the initial beam, so that the axial-vector part in σ_S^{VA} stems from the spin-projection operator in the final state, and therefore S denotes the linear dependence on the spin of the final quark.

For extracting the physics of this process at the Z, it is necessary to include radiative corrections. Obviously, QCD corrections occur only in the final state and will be proportional to α_s/π, i.e., approximately 4%, whereas the electromagnetic final-state corrections are only of the order of $3\alpha Q_q^2/4\pi < 0.1\%$ and can thus safely be neglected. Note that initial-state bremsstrahlung is significant and should be taken into account [5].

To obtain the first-order QCD corrections for the individual parity-parity contributions to the total annihilation rate, Eqs. (3)–(5), one has to include virtual-gluon

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exchange and real-gluon emission. The virtual processes have the same final state as the Born term and thus simply amount to a replacement of the $O(\alpha_s)$ massive vertex functions according to the substitutions

$$\Gamma_\mu^V = (1+A)\gamma_\mu - B\frac{(p_1-p_2)_\mu}{2m_q} ,$$

$$\Gamma_\mu^A = (1+C)\gamma_\mu\gamma_5 + D\frac{(p_1+p_2)_\mu}{2m_q}\gamma_5 , \qquad (6)$$

where A, B, C, and D are the so-called *chromomagnetic* form factors [3], and p_1 and p_2 are the momenta of quark and antiquark, respectively (the superscripts V and A refer to the parity property of the vertex function). However, in the one-loop corrections to σ_S^{VA} all terms proportional to D will eventually vanish after contracting the corresponding parity-odd hadron tensor with the usual lepton tensor.

The kinematics of a three-body final state with a polarized quark is considerably more complicated than the simple quark pair final state of the Born and virtual processes. The spin-independent phase-space integrals were first computated by Grunberg, Ng, and Tye for their study of angular distributions of heavy-quark jets [6]. In the spin-dependent case, the obstacles to overcome are twofold: not only is the integral more difficult to tackle, but also all symmetry properties of the spin-independent integrals are lost. With a suitable choice of phase-space parametrization in combination with sophisticated integration techniques, we may derive the spin-dependent integrals in a closed analytical form [3]. However, it is remarkable that all three-particle final-state phase-space integrals relevant to the computation of the production cross sections take a particularly simple form for certain limiting cases. In Table I, we display their limiting behavior for vanishing quark masses and as the center-of-momentum energy approaches the threshold value of $2m_q$.

A straightforward summation of the corresponding virtual and real contributions yields the full analytical expressions for the factors which the right-hand sides of Eqs. (3)–(5) must be multiplied by to correct those cross sections to $O(\alpha_s)$:

$$c^{VV} = 1 + \frac{\alpha_s}{2\pi}C_F\left[\left(\frac{1+v^2}{v}\ln\frac{1-v}{1+x}+2\right)\ln\left(\tfrac{1}{4}\xi\right) + F(v) + v\frac{1-v^2}{3-v^2}\ln\frac{1-v}{1+v} - \frac{4}{v}I_2 - \frac{\xi}{v}\tilde{I}_3 + \frac{4}{v(3-v^2)}I_4 + \frac{1+v^2}{v}\tilde{I}_5\right] , \qquad (7)$$

$$c^{AA} = 1 + \frac{\alpha_s}{2\pi}C_F\left[\left(\frac{1+v^2}{v}\ln\frac{1-v}{1+v}+2\right)\ln\left(\tfrac{1}{4}\xi\right) + F(v) - 2\frac{1-v^2}{v}\ln\frac{1-v}{1+v} + \frac{\xi}{v^3}I_1 - \frac{4}{v}I_2 - \frac{\xi}{v}\tilde{I}_3 + \frac{2+\xi}{v^3}I_4 - \frac{1+v^2}{v}\tilde{I}_5\right], \qquad (8)$$

$$\begin{aligned} c^{VA} = 1 + \frac{\alpha_s}{2\pi}C_F\Bigg[&\left(\frac{1+v^2}{v}\ln\frac{1-v}{1+v}+2\right)\ln\left(\tfrac{1}{4}\xi\right) + F(v) - \frac{1-v^2}{v}\ln\frac{1-v}{1+v} \\ &+\frac{1}{2v^2}\{(4-\xi)S_1 - (4-5\xi)S_2 - 2(4-3\xi)S_4 - \xi(1-\xi)(\tilde{S}_3-\tilde{S}_5) \\ &+\xi(S_6-S_7) - 2S_8 + (2-\xi)S_9 + (6-\xi)S_{10} - 2S_{11} + 2(1+v^2)v^2\tilde{S}_{12}\}\Bigg] , \end{aligned} \qquad (9)$$

TABLE I. The behavior of the spin-independent (I_i) and spin-dependent (S_i) phase-space integrals for the decay $Z,\gamma \to q\bar{q}g$ in the limit of vanishing quark mass ($v \to 1$) and in the asymptotic energy range near threshold ($v \to 0$). The ζ function of order 2 is $\zeta(2) = \pi^2/6$.

	I_1	I_2	$\tilde{I}_3$	I_4	$\tilde{I}_5$	
$v \to 1$	$\frac{1}{2}$	$-\ln\xi + 2\ln 2 - 1$	$\frac{2}{\xi}(\ln\xi - 2\ln 2 - 2) + 1$	$-\frac{1}{2}\ln\xi + \ln 2 - \frac{5}{4}$	$-\frac{1}{2}(\ln\xi - 2\ln 2)^2 - 4\zeta(2)$	
$v \to 0$	$O(v^5)$	$O(v^3)$	$4(-3 + 2\ln 2 + 2\ln v)v$	$O(v^5)$	$4(-3 + 2\ln 2 + 2\ln v)v$	
	S_1	S_2	$\tilde{S}_3$	S_4	$\tilde{S}_5$	$\tilde{S}_6$
$v \to 1$	1	$2\ln 2 - \ln\xi$	$\frac{4}{\xi}(\frac{1}{2}\ln\xi - \ln 2 - 1) - 1$	$\frac{1}{4}(\ln\xi - 2\ln 2)^2 + \zeta(2)$	$\frac{4}{\xi}(\frac{1}{2}\ln\xi - \ln 2) - \frac{4}{\xi^{1/2}}$	$\frac{4}{\xi} - \frac{4}{\xi^{1/2}}$
$v \to 0$	$O(v^4)$	$2v^2$	$4(2\ln v - 1 + \frac{1}{4}v^2)$	$2v^2$	$4(2\ln v + \frac{1}{4}v^2)$	$2v^2$
	S_7	S_8	$\tilde{S}_9$	S_{10}	$\tilde{S}_{11}$	$\tilde{S}_{12}$
$v \to 1$	$\frac{2}{\xi} - \frac{4}{\xi^{1/2}}$	$\frac{1}{4}$	$-\frac{1}{2}\ln\xi + \ln 2 - 1$	$\frac{1}{4}(\ln\xi - 2\ln 2 + 2)^2 + \zeta(2)$	$\frac{1}{4}(\ln\xi - 2\ln 2 + 3)^2 + \zeta(2)$	$-\frac{1}{4}(\ln\xi - 2\ln 2)^2 - 3\zeta(2)$
$v \to 0$	$O(v^4)$	$O(v^6)$	$O(v^4)$	$O(v^4)$	$O(v^6)$	$8\ln v - 4 + O(v^2)$

where I_i and S_i are, respectively, the spin-independent and spin-dependent integrals defined in Ref. [3]. As these measures are infrared safe, we have employed the notation $\tilde{I}_i$ and $\tilde{S}_i$ for the finite part of the integrals I_i and S_i, i.e., in Eqs. (7)–(9) the soft divergences of the virtual and real contributions have exactly canceled. The term $F(v)$ stems from the virtual process and is given by

$$F(v) = \left(3v - \frac{1+v^2}{2v}\ln\frac{4v^2}{1-v^2}\right)\ln\frac{1+v}{1-v} + \frac{1+v^2}{v}\mathrm{Re}\left[\mathrm{Li}_2\left(\frac{v+1}{2v}\right) - \mathrm{Li}_2\left(\frac{v-1}{2v}\right)\right] + \frac{\pi^2}{2}\frac{1+v^2}{v} - 4 \quad (10)$$

and as usual $C_F = \frac{4}{3}$ is the Casimir operator of the color group SU(3). It is easy to recognize that $c^{AV} = c^{VA}$ follows from the symmetry properties of the fermionic trace structure.

In the high- ($v \to 1$) and low- ($v \to 0$) energy limits, the $O(\alpha_s)$ factors of Eqs. (7)–(9) greatly simplify. With the explicit results provided in Table I, we can straightforwardly find the generic form

$$c^{ij} = 1 + C_F\alpha_s\left[\frac{\pi}{2v} - f^{ij}(v)\left(\frac{\pi}{2} - \frac{\rho^{ij}}{4\pi}\right)\right] \quad \text{with } f^{ij}(1) \equiv 1 \ , \quad (11)$$

where the functions $f^{ij}(v)$ and the constants ρ^{ij} depend on the specific parity-parity combination $i, j = V, A$. The constants ρ^{ij} are derived as

$$\rho^{VV} = \rho^{AA} = 3 \ \text{ and } \rho^{VA} = 1 \ , \quad (12)$$

which directly implies that the parity-even and parity-odd one-loop corrections to the total correlation cross sections do not equal in the fermionic zero-mass limit. In fact, the one-loop factor c^{VA} receives a finite contribution from a residual coupling of left- and right-handed helicity states of the polarized quark even in the limit $m_q \to 0$. The physical origin of this $O(\alpha_s)$ effect is that transitions between both helicity states are still allowed in the massless limit through the emission of a real gluon.

The finite difference $\rho^{VV} - \rho^{VA}$ between the results of the limiting case $m_q \to 0$ and the results where the fermion mass is at the outset zero, is nothing more than a manifestation of the distinct nature of the underlying chiral symmetries in strictly massless and massive theories. Explicitly, we have

$$\rho^{VV} - \rho^{VA} = \lim_{\xi \to 0} \xi S_7 = 4\pi^2 R \ , \quad (13)$$

where $R = 1/2\pi^2$ is the absorptive part of the axial anomaly in the limit $m_q \to 0$ (the "anomaly pole" [7,8]). In the fermionic zero-mass limit, the violation of chiral invariance in triangle graphs with one axial-vector source is directly related to the breaking of chiral invariance for transitions between fermions of different helicity states [9–11].

We can connect the two boundary-value functions for vanishing quark mass and for the center-of-momentum energy squared approaching $s = 4m_q^2$ by simple interpolation formulas which are polynomials of degree n:

$$f(v) = \sum_{k=0}^{n} a_k v^k \Big/ \sum_{k=0}^{n} a_k \ , \quad (14)$$

where $a_0, \ldots, a_n$ are suitable fitting constants. Our low-order approximations for the VV and AA one-loop QCD corrections are in agreement with the results which have been presented in the literature before [12,13]:

$$c^{VV} = 1 + C_F\alpha_s\left[\frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right] \ , \quad (15)$$

$$c^{AA} = 1 + C_F\alpha_s\left[\frac{\pi}{2v} - \frac{19 - 44v + 35v^2}{10}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right] \ . \quad (16)$$

The corresponding new formula that interpolates the parity-odd contribution Eq. (9) between the exact solution for $v = 1$ and in the asymptotic energy range near threshold is

$$c^{VA} = 1 + C_F\alpha_s\left[\frac{\pi}{2v} - \frac{64 - 70v + 103v^2}{97}\left(\frac{\pi}{2} - \frac{1}{4\pi}\right)\right] \ . \quad (17)$$

In Fig. 1 we have plotted $c^{VA}(v)$ for a constant coupling $\alpha_s = 0.1$ with the correct limit $c^{VA}(1) = 1 + C_F\alpha_s/4\pi$. The solid line gives the analytic result of Eq. (9), whereas the dashed line refers to the interpolation formula Eq. (17). Up to $v = 0.7$, Eq. (17) provides an excellent approximation for the $O(\alpha_s)$ correction that multiplies with σ_S^{VA}.

For higher precision, we give in Table II polynomial approximations of all possible parity-parity combinations (ij) up to fifth order. The coefficients a_k^{ij} and ρ^{ij} com-

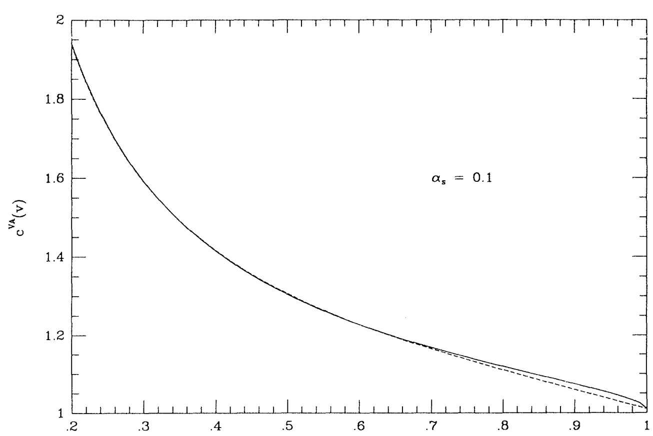

FIG. 1. The $O(\alpha_s)$ correction factor $c^{VA} = \sigma_S^{VA}/\sigma_S^{VA}$ (Born) given as a function of $v = \sqrt{1 - 4m_q^2/s}$. The solid line represents the correct numerical values whereas the dashed line corresponds to the low-order Schwinger-type interpolation formula Eq. (17).

TABLE II. One-loop QCD corrections for the individual parity-parity combinations in the process $e^+e^- \to \gamma, Z \to q\bar{q}$. The functions $f^{ij}(v)$ give the nontrivial ingredient in the interpolation formulas for the total correlation cross sections.

$f^{ij}(v)$	a_0	a_1	a_2	a_3	a_4	a_5
	95	−82	173	−85		
VV	95	−85	185	−104	9	
	95	−104	313	−430	363	−137
	60	−17	−14	71		
VA	67	−148	538	−735	379	
	6	−1	−35	150	−201	91
	43	−30	15	71		
AA	49	−127	426	−536	288	
	5	−7	3	49	−82	43

pletely determine the structure of c^{ij} in Eq. (11).

To demonstrate the accuracy of the entirely new Schwinger-type interpolation formulas for the one-loop correction c^{VA}, we have plotted only the corresponding nontrivial terms $f^{VA}(v)$ as a function of v. Figure 2 displays the numerical results for the exact analytical form Eq. (9) (solid line), for the low-order approximation Eq. (17) (long-dashed line), and for the various polynomial interpolations presented in Table II. We have drawn the polynomials of third order (short-dashed line), fourth order (dot-dashed line), and fifth order (dotted line).

In Table III, we present the $O(\alpha_s)$ numerical estimates for the total unpolarized rate $\sigma_{\text{tot}} = \sigma(e^+e^- \to \gamma, Z \to c\bar{c}(g))$ and for the longitudinal spin polarization $\langle P_L \rangle$ defined by

$$\langle P_L \rangle = \frac{\sigma(\lambda_+) - \sigma(\lambda_-)}{\sigma_{\text{tot}}} \,, \tag{18}$$

where the following shorthand notation has been used:

$$\begin{aligned} \sigma(\lambda_\pm) &= \sigma(e^+e^- \to \gamma, Z \to q(\lambda_\pm)\bar{q}(g)) \\ &= \pm v^2 c^{VA} \sigma_S^{VA} \,. \end{aligned} \tag{19}$$

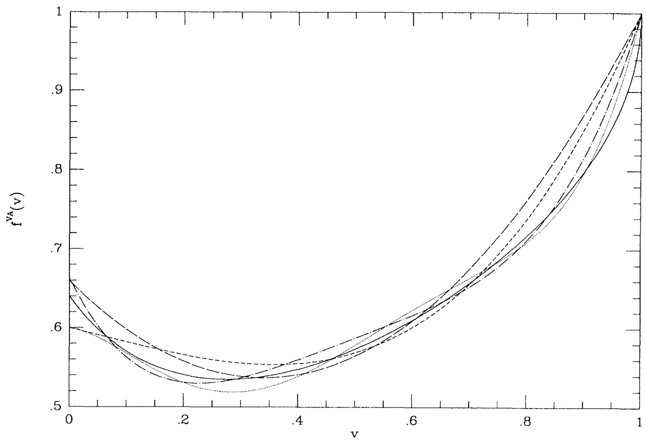

FIG. 2. The v dependence of the nontrivial term $f^{VA}(v)$ in the one-loop QCD vector axial-vector correction factor c^{VA} as defined in Eq. (11). Shown are the exact result Eq. (9) (solid line), the low-order interpolation Eq. (17) (long-dashed line), and the polynomial approximations of third (short-dashed line), fourth (dot-dashed line), and fifth (dotted line) order given in Table II.

In the calculation we have incorporated the running of the quark mass and the q^2 evolution of the strong coupling according to Ref. [14] with $\alpha_s(M_Z^2) = 0.12$ and $\Lambda_{\overline{\text{MS}}} = 0.238$ GeV, where $\overline{\text{MS}}$ denotes the modified minimal subtraction scheme. Column I corresponds to the exact analytical expressions using Eqs. (7)–(9), whereas columns II and III employ the low-order approximation Eqs. (15)–(17) and the third-order interpolation formulas c^{ij} with the coefficients a_k^{ij} of Table II. We find that already the low-order approximation provides a very accurate interpolation to the exact analytical results and even for $\sqrt{s} < 100$ GeV the higher-order approximations give only little improvement. In Tables IV and V, the exact results (I) are compared with the low-order (II)

TABLE III. $O(\alpha_s)$ production cross section and longitudinal polarization asymmetry for the process $e^+e^- \to \gamma, Z \to c\bar{c}$ as a function of the c.m.s. energy. Column I gives the exact numerical results, whereas columns II and III use the low-order approximation, Eqs. (15)–(17), and the third-order interpolation formulas of Table II, respectively.

$\sqrt{s}$ (GeV)	σ_{tot} (pb)			$\langle P_L \rangle = \frac{\sigma(\lambda_+)-\sigma(\lambda_-)}{\sigma_{\text{tot}}}$ (%)		
	I	II	III	I	II	III
85	284.184	284.145	284.142	−59.584	−59.536	−59.538
90	3 873.822	3 873.387	3 873.342	−66.355	−66.308	−66.310
91	7 302.698	7 301.787	7 301.705	−66.989	−66.943	−66.944
91.173	7 475.076	7 474.148	7 474.064	−67.077	−67.031	−67.032
92	5 278.417	5 277.777	5 277.719	−67.410	−67.365	−67.366
95	767.018	766.934	766.926	−67.578	−67.536	−67.537
100	176.078	176.062	176.060	−65.341	−65.304	−65.305
200	5.429	5.429	5.429	−30.795	−30.791	−30.791
300	2.206	2.206	2.206	−26.631	−26.630	−26.630
500	0.769 7	0.769 7	0.769 7	−24.712	−24.712	−24.712
800	0.296 81	0.296 81	0.296 81	−24.153	−24.153	−24.153
1000	1.189 34	0.189 34	0.189 34	−24.032	−24.032	−24.032

TABLE IV. $O(\alpha_s)$ production cross section and longitudinal polarization for the process $e^+e^- \to \gamma, Z \to b\bar{b}$; column I, exact results; column II, low-order approximation; column III, fifth-order interpolation.

$\sqrt{s}$ (GeV)	$\sigma_{\rm tot}$ (pb)			$\langle P_L \rangle = \frac{\sigma(\lambda_+)-\sigma(\lambda_-)}{\sigma_{\rm tot}}$ (%)		
	I	II	III	I	II	III
85	348.187	347.831	347.858	−90.208	−89.740	−89.805
90	4 975.763	4 971.125	4 971.473	−91.884	−91.433	−91.492
91	9 387.988	9 379.413	9 380.050	−92.012	−91.564	−91.622
91.173	9 608.702	9 599.957	9 600.606	−92.029	−91.582	−91.638
92	6 775.832	6 769.770	6 770.217	−92.085	−91.641	−91.696
95	969.131	968.317	968.375	−92.019	−91.587	−91.638
100	211.230	211.071	211.082	−91.223	−90.813	−90.858
200	3.375	3.375	3.375	−70.855	−70.745	−70.752
300	1.239	1.239	1.239	−66.127	−66.083	−66.086
500	0.411 5	0.411 5	0.411 5	−63.794	−63.629	−63.630
800	0.156 27	0.156 26	0.156 26	−63.043	−62.897	−62.898
1000	0.099 338	0.099 337	0.099 337	−62.883	−62.744	−62.745

and fifth-order (III) approximations for bottom- and top-quark production, respectively.

In the standard model the longitudinal spin polarization on the Z peak is fairly large yielding for the strictly massless Dirac theory a Born value of $\langle P_L\rangle_{m_{c,t}=0} = -68.5\%$ (up-type quarks) and $\langle P_L\rangle_{m_b=0} = -93.9\%$ (down-type quarks). As pointed out before, at the one-loop order of quantum corrections a correct theory for vanishing quark masses should always be regarded as the limit of a massive theory, thus including the anomalous contribution of Eq. (13). With $R \neq 0$ one obtains, in this limit,

$$\langle P_L\rangle_{m_q\to 0} = \frac{1 + C_F\alpha_s(\rho^{VV} - 4\pi^2 R)/4\pi}{1 + C_F\alpha_s\rho^{VV}/4\pi}\langle P_L\rangle_{m_q=0} \ , \tag{20}$$

which gives a sizable effect of the order of 3%. We anticipate that future experimental analyses of $\langle P_L\rangle$ via the decay products of the final quark pair will be able to detect this effect. The angular distributions of the charged leptons from semileptonic decays of the charmed Λ [15] would serve as spin analyzers for the heavy quark, since in the heavy-quark limit the polarization information of the quark q is completely transferred to the corresponding lambda baryon Λ_q [16].

To summarize, we have presented the analytical expressions for the total unpolarized and polarized production cross sections for the process $e^+e^- \to \gamma, Z \to q\bar{q}$ up to one-loop order of strong interactions. The numerical results for the total rate and longitudinal spin polarization are compared with estimates Schwinger-type interpolation formulas give. The simple second-order approximation for the parity-odd correlation cross section already yields very accurate results. Use of this compact formula may be expected to provide accurate numerical estimates in an uncomplicated way.

In this context, we have also pointed out that finite contributions from the axial anomaly are relevant in the limit of vanishing quark mass. The breaking of chiral invariance for transitions between fermions of different helicity states fundamentally alters the high-energy behavior of parity-odd observables, such as the longitudinal polarization asymmetry. For heavy-quark production, these chirality-violating mass effects reduce the longitudinal polarization by approximately 3% and thus should be observable at future TeV e^+e^- colliders by analyzing

TABLE V. $O(\alpha_s)$ production cross section and longitudinal polarization for the process $e^+e^- \to \gamma, Z \to t\bar{t}$ with $m_t = 174$ GeV; column I, exact results; column II, low-order approximation; column III, fifth-order interpolation.

$\sqrt{s}$ (GeV)	$\sigma_{\rm tot}$ (pb)			$\langle P_L \rangle = \frac{\sigma(\lambda_+)-\sigma(\lambda_-)}{\sigma_{\rm tot}}$ (%)		
	I	II	III	I	II	III
350	0.929 4	0.927 7	0.929 7	−8.504	−8.523	−8.518
400	0.932 9	0.930 8	0.933 2	−14.149	−14.187	−14.114
500	0.685 7	0.686 5	0.685 8	−18.644	−18.496	−18.655
800	0.288 07	0.288 37	0.287 99	−22.341	−21.997	−21.316
1000	0.186 16	0.186 23	0.186 07	−23.037	−22.701	−22.950

the charged lepton spectrum of the semileptonic decay mode.

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