Strange matter equation of state and the combustion of nuclear matter into strange matter in the quark mass-density-dependent model at $T > 0$

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We study the properties and stability of strange matter in the mass-density-dependent model at $T > 0$. We find the temperature dependence of the strange matter stability window and the critical temperature above which there is no stable strange matter. It occurs at $T_c = 34$ MeV. Also, the resulting equation of state in this approach is presented. We also study the combustion of nuclear matter into strange matter. We employ for strange matter the equation of state of this paper and for nuclear matter a set of equations of state (free neutrons, Bethe-Johnson, Lattimer-Ravenhall, and Walecka). It is shown that the results are very similar to the ones found employing the MIT bag model. However, the only equation of state which shows a noticeable dependence with temperature is the Walecka one. Moreover, contrary to the former case, the Walecka equation of state is flammable in the present model. It is found that, also at finite temperatures, the properties of strange quark matter in the quark mass-density-dependent model and in the MIT bag model are very similar.

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I. INTRODUCTION

It was first pointed out by Witten [1] that strange matter (SM) might be the ground state of QCD instead of nuclear matter. This hypothesis has many astrophysical and cosmological implications, but it is indeed interesting by itself (for a general reference see [2]). Up to date, the thermodynamical treatment of SM has been mostly carried out in the context of the MIT bag model. The stability of SM in this model at zero temperature was studied by Farhi and Jaffe [3], who found a wide stability window for the parameters of the equation of state (EOS) inside which SM is stable against decaying to ^{56}Fe. Later, the effect of temperature on bulk properties of SM was investigated in [4–6], finding that for typical values of the parameters SM remains stable even when T rises up to ≈ 30 MeV.

An alternative description of quark matter in which the confinement is treated assuming a baryon density dependence of the quark masses was introduced by Fowler, Raha, and Weiner [7] and applied by Chakrabarty [8] to study the properties of SM. Later on, the problem was reformulated (at $T = 0$) [9] showing that the properties of SM in this model are quite similar (and not very different, as stated before) to those predicted by the MIT bag one. It is the aim of this work to generalize the results of [9] to the case of $T > 0$.

In Sec. II we present the equation of state of SM at $T > 0$. In Sec. III we discuss the stability of SM in the present model showing how the stability window moves with temperature. We deal in Sec. IV with the problem of combustion of nuclear matter into SM assuming, for the latter, the EOS of this paper. Finally, we give our main conclusions in Sec. V.

II. THE EQUATION OF STATE

As usual, we assume SM to be a free Fermi gas composed of quarks u, d, s, antiquarks $\bar{u}$, $\bar{d}$, $\bar{s}$, and a fraction of electrons and positrons all in thermal equilibrium with an eighth colored gluon gas. The mass of each quark and antiquark is parametrized with the baryon number density n_B as follows:

$$m_u = m_d = m_{\bar{u}} = m_{\bar{d}} = \frac{C}{3n_B} \quad m_s = m_{\bar{s}} = m_{s0} + \frac{C}{3n_B}, \tag{1}$$

where m_{s0} is the strange quark current mass and C is a constant, both to be constrained by stability arguments (see Sec. III). This model of confinement makes the quarks very heavy (light) at low (high) baryon densities, preserving so the correct asymptotic (free particle) behavior. At $T = 0$ it was shown [9] that SM is stable inside a trianglelike region in the (C, m_{s0}) plane, where $0 < m_{s0}(\mathrm{MeV}) < 180$ and $69.05 < C(\mathrm{MeV\,fm^{-3}}) < 111.6$ (see Fig. 1 of that work).

Now, let us find the equation of state. The thermodynamical potential density is

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$$\Omega = \sum_i \Omega_i = -\sum_i \frac{g_i T}{(2\pi)^3} \int d^3p \, \ln(1 + e^{-\beta(\epsilon_i - \mu_i)}) - \frac{8}{45}\pi^2 T^4, \quad (2)$$

where the sum extends to all particles and antiparticles, $g_i = 6$ for quarks and antiquarks, $g_i = 2$ for electrons and positrons, $\epsilon_i = (p^2 + m_i^2)^{1/2}$ is the single particle energy, and μ_i the chemical potential. For antiparticles $\bar{\mu}_i = -\mu_i$. The term proportional to T^4 is the gluon contribution.

From Eq. (2) it is easy to obtain the number density of each particle, the total pressure, and energy density:

$$\Delta n_i = n_i - \bar{n}_i = \frac{g_i}{(2\pi)^3} \int d^3p \, [\eta_i(T) + \bar{\eta}_i(T)], \quad (3)$$

$$P = \sum_i \frac{g_i}{(2\pi)^3} \int \frac{d^3p}{(p^2 + m_i^2)^{1/2}} \left[\frac{p^2}{3} - \frac{Cm_i}{n_B}\right] \times [\eta_i(T) + \bar{\eta}_i(T)] + \frac{8}{45}\pi^2 T^4, \quad (4)$$

$$E_s = \sum_i \frac{g_i}{(2\pi)^3} \int \frac{d^3p}{(p^2 + m_i^2)^{1/2}} \left[(p^2 + m_i^2) + \frac{Cm_i}{n_B}\right] \times [\eta_i(T) + \bar{\eta}_i(T)] + \frac{8}{15}\pi^2 T^4. \quad (5)$$

Here $\eta_i(T)$ and $\bar{\eta}_i(T)$ are the thermal distribution functions

$$\eta_i(T) = (\exp\{[(p^2 + m_i^2)^{1/2} - \mu_i]/T\} + 1)^{-1}, \quad (6)$$

$$\bar{\eta}_i(T) = (\exp\{[(p^2 + m_i^2)^{1/2} + \mu_i]/T\} + 1)^{-1}. \quad (7)$$

We shall assume that SM is in equilibrium with respect to weak interactions:

$$d \rightleftharpoons u + e + \bar{\nu}_e, \quad s \rightleftharpoons u + e + \bar{\nu}_e, \quad u + d \rightleftharpoons u + s. \quad (8)$$

So, assuming that neutrinos freely leave the system, chemical equilibrium between the different species implies that there are only two independent chemical potentials (for example, μ and μ_e):

$$\mu_s = \mu_d = \mu; \quad \mu_s = \mu_u + \mu_e. \quad (9)$$

Furthermore, bulk SM must be electrically neutral, which gives a constraint for the particle densities:

$$2\Delta n_u - \Delta n_d - \Delta n_s - 3\Delta n_e = 0. \quad (10)$$

Finally, the baryon number density n_B is given by

$$n_B = \frac{1}{3}(\Delta n_u + \Delta n_d + \Delta n_s). \quad (11)$$

Given the values of n_B and T and assuming values for the parameters (C, m_{s0}) the equations can be solved numerically.

For studying this EOS we assume two parameter sets (see Sec. III for the problem of stability) given in Table I. The main results are summarized in Figs. 1 and 2.

TABLE I. Selected parameter sets.

Case	C (MeV fm^{-3})	m_{s0} (MeV)
A	80	100
B	70	0

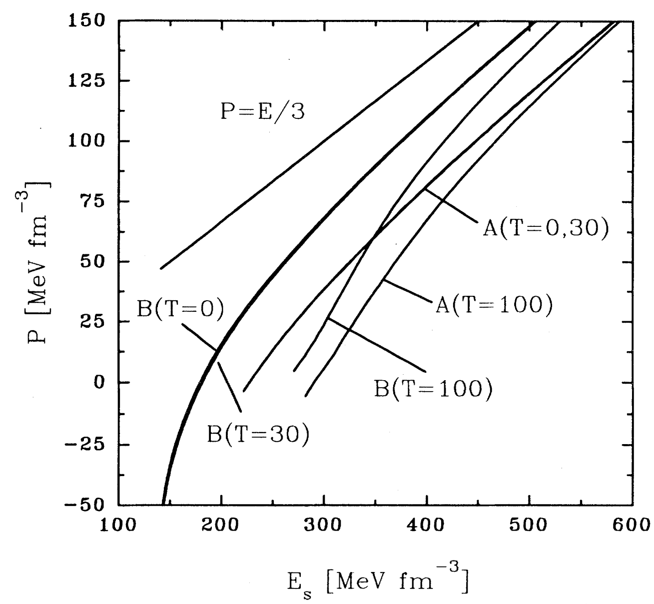

FIG. 1. The SM EOS in the quark mass-density-dependent model at $T > 0$. We show the dependence of P vs E_s for cases A and B (see Table I) for different temperatures (given in MeV). Note that for low temperatures (e.g., 30 MeV) the curves are almost indistinguishable from the one corresponding to zero temperature. This EOS asymptotically tends to the ultrarelativistic $P = E/3$, as it must.

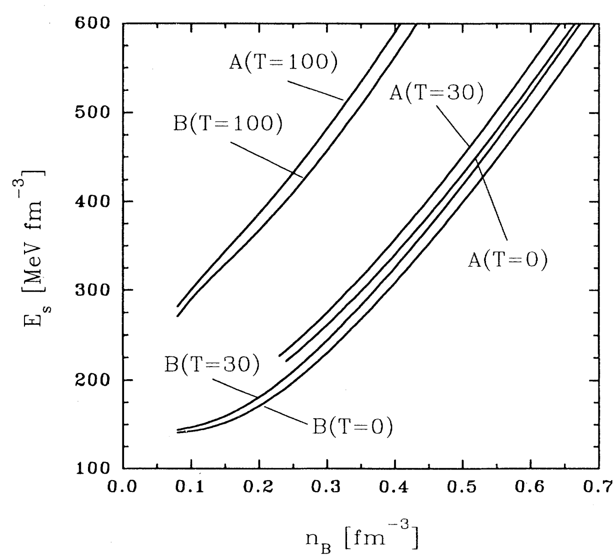

FIG. 2. The same as in Fig. 1 but for the dependence of E_s vs n_B.

In Fig. 1 we show the E_s vs P relationship for different temperatures. It can be noticed that the temperature dependence of this EOS is not very strong at least for $T \leq 30$ MeV. This is especially true for the parameter set A for which the $T = 0$ and $T = 30$ MeV cases are indistinguishable. For very high values of E_s, the EOS tends asymptotically to the ultrarelativistic limit $P = E/3$. For a given value of E_s, the higher the temperature, the lower the pressure. This behavior is similar to that found by Walecka [10] for neutron matter (note that, however, in the work of Walecka, the asymptotic behavior tends to the causal EOS $P = E$ due to nuclear interactions). In Fig. 2 we show the curves of the more temperature-dependent relation E_s vs n_B.

As is well known, in conditions of low temperature, the above integrals can be expanded in series. For the sake of completeness, we present these formulas in the Appendix.

III. STABILITY OF STRANGE MATTER AT $T > 0$

In order to investigate the stability of SM at finite temperature we shall apply the same criteria employed in the case of zero temperature [9]. The acceptable values of (C, m_{s0}) are obtained by imposing that for a given T and at $P = 0$ SM is stable ($E/n_B \leq M_{^{56}\mathrm{Fe}}/56$) and two flavor quark matter (made up of u and d quarks) is not ($E/n_B > M_{^{56}\mathrm{Fe}}/56$), $M_{^{56}\mathrm{Fe}}$ being the mass of the ^{56}Fe nucleus. We present in Fig. 3 the region in the (C, m_{s0}) plane where both conditions are satisfied at $T = 0, 30,$ and 40 MeV. It is noticeable that increasing temperature shifts the stability window towards lower values of C. If we assume that there is some correspondence between our C and the bag constant B, then we can note that this result is very similar to that found in the case of the MIT bag model (see Fig. 1 of Ref. [6]).

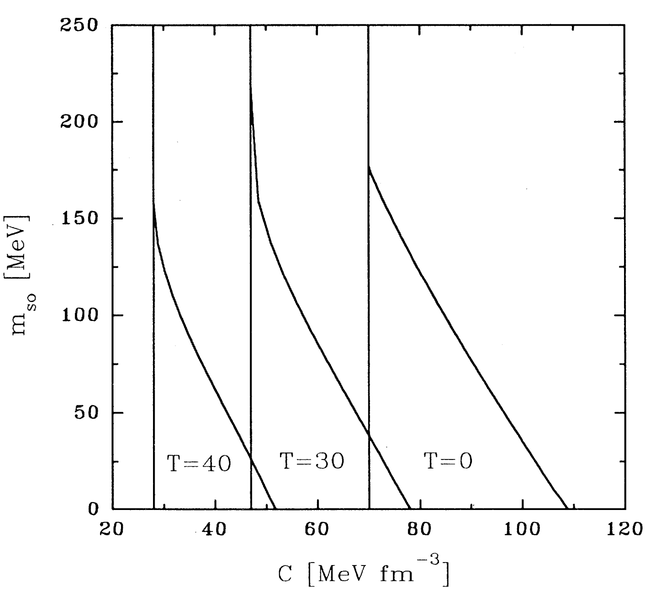

FIG. 3. The stability window of SM in the quark mass-density-dependent model at zero pressure and different temperatures (given in MeV). The stability region is where the energy per particle is lower than 930 MeV and two flavor quark matter is unstable. The behavior of this window is very similar to that of the MIT bag model window found in [6].

If we want SM to be stable at zero temperature we must choose a pair (C, m_{s0}) inside the "$T = 0$ triangle" in Fig. 3. As the window moves to the left with increasing temperature, there is a critical temperature T_c for which the selected point (C, m_{s0}) lies just at the edge of the window corresponding to T_c. Above this T_c there is no stable SM for that choice of parameters (note that this condition holds at $P = 0$; under external pressure the decay to ^{56}Fe would begin at higher temperatures). The maximum temperature for which SM (stable at $T = 0$) remains stable corresponds to the choice ($C = 70$ MeV fm^{-3}, $m_{s0} = 0$ fm^{-3}) and $T_c = 34$ MeV. At higher temperatures we find a shifted window, but it does not overlap the one corresponding to $T = 0$ (see Fig. 3).

IV. COMBUSTION OF NUCLEAR MATTER INTO STRANGE MATTER

The problem of SM formation has already been studied [11] in the framework of relativistic hydrodynamic combustion theory. This problem is very important in predicting signals of SM that could be experimentally detected. The conversion process must be exothermic if we want it to propagate spontaneously to other regions of the fluid, so we have a necessary condition to be satisfied [12]:

$$E_{\mathrm{burnt}}(P, X) < E_{\mathrm{unburnt}}(P, X), \qquad (12)$$

where $E_{\mathrm{burnt}}(P, X)$ and $E_{\mathrm{unburnt}}(P, X)$ are the energy densities of the respective fluids, both evaluated at the same thermodynamic state.

We should remark that this kind of analysis only allows us to assert at which conditions this process *cannot* take place and at which ones it *may* occur. In other words, condition (12) is necessary but not sufficient.

Condition (12) was explored in [11] adopting the MIT bag model EOS for SM and a set of nuclear matter EOS's in order to cover all the range of stiffness up to now published. It is well known that there is a large uncertainty about nuclear matter properties specially above saturation density. It was found on very general grounds that there is an absolute lower density limit for the combustion to be possible. Assuming free neutrons, Bethe-Johnson [13], Lattimer-Ravenhall [14], and Walecka [10] EOS's for nuclear matter, it was also shown that combustion is possible only in a finite range of nuclear matter baryon densities except for the case of the Walecka EOS for which the combustion was found to be endothermic at any n_B due to its extreme stiffness. In this section we generalize these results employing the same set of nuclear matter EOS's for the nuclear phase and the EOS of the present paper for the SM one.

The most direct way to apply condition (12) now is to

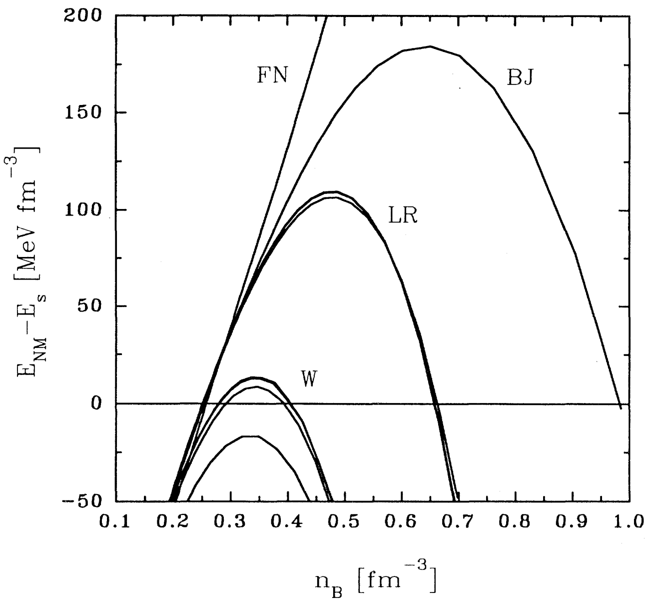

FIG. 4. The flammability condition for the combustion of nuclear matter into SM in the quark mass-density-dependent model. FN, BJ, LR, and W correspond to free neutrons, Bethe-Johnson, Lattimer-Ravenhall, and Walecka EOS's, respectively. For LR and W EOS's we included the cases of $T = 0$, 30, and 100 MeV. In LR the curves of $T = 0$ and 30 MeV are superimposed, while the 100 MeV one is below. The same thermal dependence is shown by the W EOS though it is much more steep.

calculate $E_{NM}(P,X) - E_s(P,X)$ for the range of densities and temperatures where we expect this process to take place. The results are shown in Fig. 4 assuming for SM the set of values (C, m_{s0}) labeled as A in Table I. It can be noticed that the results are very similar to those found in our previous work. The region in which combustion is possible is limited at low densities by a value of $n_B \approx 0.25$ fm^{-3} for not too stiff nuclear EOS's. This is just what we found employing the MIT bag model assuming $B = 60$ MeV fm^{-3}. Moreover, for the cases of free neutrons, Bethe-Johnson, and Lattimer-Ravenhall EOS's the density range of exothermic conversion from nuclear matter into SM is almost the same found with the MIT bag model although the upper limiting densities are slightly pushed upwards. Nevertheless, there are some differences. First, the density range for exothermic SM formation shows a weaker dependence on temperature. In Fig. 4 we have included the results for Lattimer-Ravenhall and Walecka EOS's at $T = 0$, 30, and 100 MeV. The Lattimer-Ravenhall EOS curves show almost no dependence on T, while Walecka EOS curves are slightly dependent. On the other hand, in contrast to the case of the MIT bag model, the Walecka EOS is now flammable although in a very restricted density range and at not too high temperatures (e.g., at 100 MeV the conversion is no longer exothermic).

By means of the previous analysis, we cannot calculate the velocity of propagation of the flame (and the time scales of the reaction processes inside it). For this purpose we would need to perform a kinetic study of the problem such as the ones presented in [16–18] and consider the "strangeness creation" (weak) reactions. Anyway, we note that the flame can burn nuclear matter at velocities orders of magnitude larger if there is turbulence inside the flame, because its corrugation increases the effective surface of burning. This seems to be the actual case as discussed in [11] because the hydrodynamic time scales are larger by several orders of magnitude compared to kinetic time scales.

V. DISCUSSION AND CONCLUSIONS

In this work we have addressed the problem of the equation of state of strange quark matter in the quark mass-density-dependent model at finite temperature.

We have investigated the properties of the parameter window inside which the equation of state found here is stable against decaying to ^{56}Fe. As stated above, if we assume some correspondence between our C and the bag constant B, the behavior of our window is almost the same as that found by Chmaj and Slomiński [6].

The equation of state approaches asymptotically the ultrarelativistic $P = E/3$ but at low pressures it is significantly stiffer than the bag model equation of state as found in [9] (see Fig. 4 of that work). At very high temperatures and low pressures the equation of state gets softer.

Also we have calculated the density and temperature ranges inside which this matter can be formed from combustion of nuclear matter. For the nuclear matter equations of state employed (free neutrons, Bethe-Johnson, Lattimer-Ravenhall, and Walecka) it is found that the combustion can occur at ranges of densities (for each nuclear EOS) very similar to the ones found in the bag model (see Ref. [11]). The main difference is that here the Walecka EOS can also be burnt although in a very narrow density range, contrary to that found in the framework of the MIT bag model where the combustion is always endothermic.

It is our main conclusion that (as in our previous work on this subject [9]) the properties of strange quark matter in the quark mass-density-dependent model and in the MIT bag one are very similar. It seems that, at the level of our knowledge on this topic, the predictions of both models are hardly distinguishable. Perhaps the most significant difference appears in the velocity of sound which at low pressures is $\approx 50\%$ larger than in the MIT bag model.

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APPENDIX: SOMMERFELD APPROXIMATION TO THE INTEGRALS

In conditions of strong degeneracy and low temperature (more precisely when $\exp[(\mu-m)/T] \gg 1$), it is well known that integrals such as those of Eqs. (3), (4), and (5) can be expanded by employing Sommerfeld's lemma [15]. For brevity we give here the expressions of the particle contibution up to the second term only. The antiparticle contribution is obtained by changing the sign of the chemical potential. Then $x = [(\mu/m)^2 - 1]^{1/2}$ is unchanged and the result is exactly the same as for particles. The expressions are

$$\int_0^\infty d^3p\, \eta(T) = 4\pi m^3 \left(\frac{x^3}{3} + \pi^2 \left(\frac{T}{m}\right)^2 \frac{2x^2+1}{6x} \right), \tag{A1}$$

$$\int_0^\infty \frac{p^2 d^3p}{(p^2+m^2)^{1/2}}\, \eta(T) = \frac{\pi m^4}{2} \left[x(x^2+1)^{1/2}(2x^2-3) + 3\,\mathrm{arcsinh}(x) + 4\pi^2 \left(\frac{T}{m}\right)^2 x(x^2+1)^{1/2} \right], \tag{A2}$$

$$\int_0^\infty \frac{d^3p}{(p^2+m^2)^{1/2}}\, \eta(T) = 2\pi m^2 \left[[x(x^2+1)^{1/2} - \mathrm{arcsinh}(x)] + \frac{\pi^2}{3} \left(\frac{T}{m}\right)^2 \frac{(x^2+1)^{1/2}}{x} \right], \tag{A3}$$

$$\int_0^\infty d^3p (p^2+m^2)^{1/2}\, \eta(T) = \frac{\pi m^4}{2} \left[[x(x^2+1)^{1/2}(1+2x^2) - \mathrm{arcsinh}(x)] + \frac{4\pi^2}{3} \left(\frac{T}{m}\right)^2 (x^2+1)^{1/2} \frac{3x^2+1}{x} \right]. \tag{A4}$$

For $\exp[(\mu-m)/T] \ll 1$, $\eta(T)$ can be expanded in series in order to simplify the calculation (see [15]).

For more details on the treatment of this kind of integral the reader is referred to the work of Cloutman [19].

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