

String-motivated model

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The two-dimensional model which emerges from low-energy considerations of string theory is written down. Solutions of this classical model are noted, including some examples which have a nontrivial tachyon field. One such example represents the classical back reaction of the tachyon field on the black hole for a two parameter set of tachyon potentials. Assuming the classical black hole background in the “Eddington-Finkelstein” gauge, the tachyon equation is separable and the radial part is solved by a hypergeometric function, which is in general of complex argument. A semiclassical prescription for including the quantum effects of the tachyon field is described, and the resulting equations of motion are found. Special solutions of these equations are written down.

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I. INTRODUCTION

String theory is thought to be important to the construction of quantum gravity. The model that derives from string theory at the tree level in two dimensions [1] will be regarded here as a fundamental field theory of gravity in its own right, and methods of quantum field theory will be applied to it. This is in contrast with taking the fundamental theory to be a generally reparametrization-invariant σ model on the two-dimensional world-sheet manifold of the string and then demanding that fields configure in such a way that is consistent with conformal invariance, i.e., so that the β functions vanish. To build up this theory one would have to expand in world-sheet perturbation theory, considering also topologies, whereas we will work with the spacetime manifold. With this distinction in procedures in mind, this model will be referred to as the string-motivated model.

In semiclassical gravity the expectation value of the matter energy-momentum tensor is coupled to the gravitational field. If this coupling is to the Einstein tensor, then the Bianchi identities and energy conservation ensure mathematical consistency.¹ Physical consistency of this procedure has been amusingly questioned in [3]. Using this quantum principle of equivalence, one has approximately included the effect of the matter upon the geometry of the spacetime. The aim is to see how a black hole would develop when such back reaction is considered. This would naturally extend the original calculations [4] in which the geometry of the spacetime is treated as a fixed background. This has been done with some success both generally [5], and in the context of several

other dilaton gravity models in two dimensions [6].

We begin the second section by introducing the string-motivated model, and note classical solutions for which the tachyon field is set to zero. Two examples of solutions which have a nontrivial tachyon field are then found and written down. The first is flat space, the second represents a naked singularity. Further examples of black holes which have undergone back reaction by the tachyon field are given. An ansatz for these black holes is applied, analogous to the metric outside an evaporating star in general relativity. The general solution is found in this form. The ansatz shows the position of the apparent horizon: its relationship to the singularity and event horizon are calculable via a certain integral.

By ignoring back reaction one can solve the field equation for the tachyon in the fixed static black hole geometry. This has been done, for example in [7,8], in the Schwarzschild gauge assuming staticity. In the third section, it is found that in the ingoing null coordinates, for a particular tachyon potential, one obtains the same hypergeometric equation for the radial part as in [8], but there is also a u -dependent piece.

In the fourth section, the procedure for coupling the energy momentum of the tachyon field to the field equations is described. It is noted that this gives back the Callan-Giddings-Harvey-Strominger (CGHS) [9] equations if one works in the double-null coordinates, and drops tachyon terms. Thus the procedure used here is equivalent to adding Polyakov term for the tachyon field to the action itself. A set of semiclassical equations are found in the ingoing “Eddington-Finkelstein” gauge. Unfortunately, these equations are at least as complicated as those of [9], where numerical methods were resorted to (see, for example, [10,11]), though the model can be adjusted so as to be exactly soluble [12].

II. THE STRING MOTIVATED MODEL

The following is the action for the classical part of the string model

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¹In two dimensions, the result of applying this procedure to the Einstein equation is either de Sitter or anti-de Sitter space. The back reaction problem is completely soluble [2].

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} [R + 4\nabla\phi^2 + 4\lambda^2 - (\nabla T)^2 - V(T)] - \frac{1}{\pi} \int d\Sigma \sqrt{-h} e^{-2\phi} (K - 2\nabla_n \phi). \quad (2.1)$$

The fields present are the metric $g_{\mu\nu}$, the dilaton ϕ , and the tachyon T . There is a boundary term which makes the variational problem well defined and enables the thermodynamics of the theory to be derived. K is the trace of the second fundamental form of the metric, and \mathbf{n} is the normal vector to the boundary. The equations of motion derived from (2.1) are

$$R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi - \nabla_\mu T \nabla_\nu T = 0, \quad (2.2)$$

$$-R + 4(\nabla\phi)^2 - 4\nabla^2\phi + \nabla T^2 + V(T) - 4\lambda^2 = 0, \quad (2.3)$$

$$\nabla^2 T - 2\nabla\phi \nabla T - \frac{1}{2} \frac{dV}{dT} = 0, \quad (2.4)$$

where λ^2 defines the mass scale here but is related to the central charge in string theory, $V(T)$ is the tachyon potential.

Let us work in single null coordinates,

$$ds^2 = -h(u, r) du^2 - 2du dr, \quad (2.5)$$

where h is a function on the spacetime to be determined. In these coordinates, the field equations are

$$h(h_{,rr} - 2h_{,r}\phi_{,r}) + 2h_{,r}\phi_{,u} - 2h_{,u}\phi_{,r} + 4\phi_{,uu} - 2T_{,u}^2 = 0, \quad (2.6)$$

$$h_{,rr} - 2h_{,r}\phi_{,r} + 4\phi_{,ur} - 2T_{,u}T_{,r} = 0, \quad (2.7)$$

$$2\phi_{,rr} - T_{,r}^2 = 0, \quad (2.8)$$

$$h_{,rr} - 4h_{,r}\phi_{,r} - 8\phi_{,r}\phi_{,u} + 8\phi_{,ur} + 4h\phi_{,r}^2 - 4h\phi_{,rr} + hT_{,r}^2 - 2T_{,u}T_{,r} = 4\lambda^2 - V, \quad (2.9)$$

$$hT_{,rr} - 2T_{,ur} + h_{,r}T_{,r} - 2h\phi_{,r}T_{,r} + 2\phi_{,u}T_{,r} + 2\phi_{,r}T_{,u} - \frac{1}{2} \frac{dV}{dT} = 0. \quad (2.10)$$

III. CLASSICAL SOLUTIONS

A. $T = 0$

These equations simplify if one looks for solutions with a zero tachyon field. There exists a timelike Killing vector in this case [13], and so it is no restriction to drop terms which contain derivatives of u . One then has

$$h_{,rr} - 2h_{,r}\phi_{,r} = 0, \quad (3.1)$$

$$\phi_{,rr} = 0, \quad (3.2)$$

$$h_{,rr} - 4h_{,r}\phi_{,r} + 4h\phi_{,r}^2 - 4h\phi_{,rr} - 4\lambda^2 = 0. \quad (3.3)$$

Thus there is a “linear dilaton” $\phi(r) = -\lambda r + \phi_0$ and a metric given by

$$h(r) = 1 - ae^{-2\lambda r}, \quad (3.4)$$

where a is a constant. The curvature information is in $R = -h_{,rr} = 4a\lambda^2 e^{-2\lambda r}$. There is a curvature singularity at $r \rightarrow -\infty$. It will be useful to transform solutions of the form (2.5) to null coordinates. One transforms to conformally flat null coordinates via

$$ds^2 = -\Omega^2(u, r) du dv = -h du^2 - 2du dr. \quad (3.5)$$

If h is a function of r only, then the solution is $\Omega^2 = h$. A more general case is considered later. One then finds that $h = (1 \pm e^{-\lambda(v-u)})^{-1}$, where the positive sign corresponds to $a > 0$. A further transform to “Kruskal” coordinates

$$\beta U = e^{-\lambda u}, \quad (3.6)$$

$$\alpha V = e^{\lambda v}, \quad (3.7)$$

yields, in the case of $a > 0$, the familiar metric form for the maximally extended static black hole [14],

$$ds^2 = -\frac{dU dV}{-\lambda^2/\alpha\beta - \lambda^2 UV}, \quad (3.8)$$

where $\alpha\beta < 0$. If $\alpha\beta = -\lambda^3/M$ then M is the Arnowitt-Deser-Misner (ADM) mass [9]. If $a < 0$ one finds

$$ds^2 = -\frac{dU dV}{\lambda^2/\alpha\beta - \lambda^2 UV}. \quad (3.9)$$

This latter solution represents a naked singularity, whereas the former is the black hole described in the first half of [9]. The ground state solution, which has $M = 0$, is the “linear dilaton vacuum.” This corresponds to $a = 0$ in the ingoing coordinate solution for h .

B. $T \neq 0$

1. Example 1

A first example is a flat geometry bathed in a u -dependent tachyon. One starts with a linear dilaton $\phi = -\lambda r$, and it is assumed that $V(T) = aT^2$.

$$T = Ae^{-au/2\lambda}, \quad (3.10)$$

$$h = 1 - \frac{V}{4\lambda^2}. \quad (3.11)$$

Since h is a function of u only, this space is flat.

2. Example 2

If $V = \lambda = 0$, another set of solutions is of the form

$$h = \alpha r^n \quad (3.12)$$

and the dilaton and tachyon are

$$\phi = -\frac{1}{2}(1-n)\ln r = \frac{1}{2}T\sqrt{1-n}. \quad (3.13)$$

The curvature is $R = (n-1)r^{(n-2)}$. In null coordinates,

$$ds^2 = -\alpha \left(\frac{\alpha(1-n)}{2} \right)^{n/(1-n)} \frac{du dv}{(v-u)^{n/(n-1)}}. \quad (3.14)$$

In the case $n = 2$ there is a singularity free space of constant curvature. For $n = 0$ there is flat space and logarithmic $\phi = \frac{1}{2}T$. Otherwise, there is a timelike (naked) singularity on the line $v = u, r = 0$.

3. Example 3

In order to find solutions which represent the black hole perturbed classically by the tachyon field, one can assume that the metric takes the form of a black hole with a dynamical horizon. That is, solutions are of the form

$$ds^2 = -h(u, r)du^2 - 2du dr, \quad (3.15)$$

where

$$h = 1 - e^{-2\lambda[r-f(u)]}. \quad (3.16)$$

One then solves for the function $f(u)$ which gives the position of the horizon² $r_h = f(u)$. The implicit assumption

²The motivation for this ansatz comes from four-dimensional theory. The metric outside a radiating star is related to (3.16). This is called the Vaidya metric, and can be written

$$ds^2 = -\left(1 - \frac{2M(u)}{r}\right) du^2 - 2du dr + r^2 d\Omega^2 \quad (3.17)$$

The mass in the Schwarzschild metric has been upgraded from a constant to a function of the retarded time, which is reasonable if the radiation is made up of massless particles. The metric is a solution of the Einstein equations in a source field of pure radiation:

$$G_{uu} = R_{uu} = 8\pi T_{uu} = -\frac{2}{r^2} \frac{dM}{du}. \quad (3.18)$$

One might ask what is the future development of this system. To answer this, consider Stefans' law

$$\frac{dM}{du} = aAT^4 \propto M^{-2}, \quad (3.19)$$

where A is the area of the star and a is Stefans' constant. This implies that

$$\frac{dM}{du} \propto u^{-2/3}. \quad (3.20)$$

The rate of mass decrease therefore diverges at finite retarded time. This footnote will be expanded upon elsewhere.

tion that the horizon motion is u dependent corresponds to the masslessness of the tachyon field. One might try to find solutions for which the dilaton background is linear, $\phi = -\lambda r$, but the field equations then become

$$-2a\lambda^2 e^{2\lambda f} f_{,uu} - e^{2\lambda r} T_{,u}^2 = 0, \quad (3.21)$$

$$T_{,u} T_{,r} = 0, \quad (3.22)$$

$$T_{,r}^2 = 0, \quad (3.23)$$

which means $T = T(u)$. Inspection of (3.21) shows that $a = 0$ so that $T = 0$, and no progress is made.

If, by contrast, one tries the dilaton field

$$\phi = -\lambda[r - f(u)], \quad (3.24)$$

then the value of the dilaton is thus fixed on the horizon. The field equations become

$$2\lambda f_{,uu} - T_{,u}^2 = 0, \quad (3.25)$$

$$T_{,u} T_{,r} = 0, \quad (3.26)$$

$$T_{,r}^2 = 0, \quad (3.27)$$

$$8\lambda^2 f_{,u} + V(T) = 0, \quad (3.28)$$

$$4\lambda T_{,u} = -\frac{dV}{dT}. \quad (3.29)$$

Equation (3.26) implies that T is a function of u only, and given the potential $V(T)$, one can solve for T in (3.29). One can substitute into Eq. (3.25) and integrate twice to obtain the function f and hence the back-reacted metric.³

The tachyon potential is given by $V(T) = aT^2 + bT^3 + \dots$, where a and b are taken from string theory calculations, and will not be specified here.

For $V(T) = 0$, Eq. (3.29) implies that T is a constant, and integrating up (3.25) shows that $f(u)$ is then linear which is a static solution equivalent to (3.8), the usual static black hole.

For the quadratic $V(T)$,

$$T = \exp\left(\frac{-a}{2\lambda}(u + u_0)\right). \quad (3.30)$$

The solution for $f(u)$ is then

³It should be noted that in this form, the dilaton field is constant on the horizon. It can be shown however that the ADM mass of the solution is related to the dilaton there. The dilaton gives a coordinate independent measure of position, so if it is constant on the horizon, then the horizon is not moving. This suggests a static black hole, perturbed by the classical tachyonic back reaction.

$$\lambda f(u) = \frac{1}{8\lambda} \exp\left(\frac{-a}{\lambda}(u + u_0)\right). \quad (3.31)$$

If the $O(T^3)$ term is included, one obtains

$$T = \frac{2a}{3b} \frac{1}{\exp\left(\frac{a}{2\lambda}(u + u_0)\right) - 1}. \quad (3.32)$$

Then

$$\lambda f(u) = A \left[\ln \left| \exp\left(\frac{a}{2\lambda}(u + u_0)\right) - 1 \right| + \frac{\exp\left(\frac{a}{2\lambda}(u + u_0)\right)}{\left[\exp\left(\frac{a}{2\lambda}(u + u_0)\right) - 1 \right]^2} \right], \quad (3.33)$$

where A is a constant depending on a and b .

Note that these solutions (3.30) and (3.33) solve all the field equations at once.

The geometry has not been fixed using the metric and dilaton equations in isolation and setting $T = 0$, as is done in the following section.

To see what these geometries look like globally, one can transform to conformal null coordinates. Then the position of the horizon and singularity are easily calculable. One must find Ω in

$$ds^2 = -\Omega^2(u, r) dudv = -hdu^2 - 2dudr. \quad (3.34)$$

The following expression for Ω then obtains

$$4\Omega_{,u} = 2h\Omega_{,r} - \Omega h_{,r}. \quad (3.35)$$

For solutions of the form (3.16) one finds that $\Omega^2 = e^{-2\lambda(r+u/2)}$ and

$$\lambda v = 2e^{\lambda(r+u/2)} - \lambda c(u), \quad (3.36)$$

where $c(u) = \int du e^{2\lambda f} e^{\lambda u}$.

Thus

$$ds^2 = -\frac{du dv}{\frac{\lambda}{2}[c(u) + v]}. \quad (3.37)$$

By rescaling $V = \lambda v$, and transforming to $U = -e^{-\lambda u}$, one obtains the form

$$ds^2 = \frac{dU dV}{\frac{1}{2}U\lambda^2[V + \lambda c(U)]}. \quad (3.38)$$

For $f = 0$, the static Witten black hole given in Eq. (3.8) is recovered. If f is linear in u , this static metric still

results. Unfortunately, this transformation is difficult to perform for the solutions (3.33) and (3.31), except for the case $a = -\lambda^2$. Since the form of the geometry is that of a black hole by ansatz, and the tachyon is a scalar field so that it will remain nontrivial in any coordinate system: these are black hole solutions with classical tachyon hair. The fact that the dilaton is constant at zero on the horizon suggests that the solution is in fact static. The tachyon field however is not constant on the horizon.

IV. THE TACHYON FIELD IN FIXED GEOMETRY

Now the static solution (3.8) found by setting $T = 0$ is fed into the dilaton and gravitational field equations, and determine the tachyon equation. If one assumes that the solution is a separable function, the radial part is found to be a hypergeometric function in r , as was seen in [8], which reduces to an exponential function in flat space, while the u -dependent piece is exponential. If the constant of integration is taken to be imaginary, this becomes plane wave. One can substitute the real solution back into the field equations, expanding around the origin in r to try to find out how the tachyon back reacts upon the geometry perturbatively.

The tachyon equation of the string model was

$$\nabla^2 T - 2\nabla\phi\nabla T = \frac{1}{2} \frac{dV}{dT}, \quad (4.1)$$

which in the coordinates (3.5) with $f = 0$ becomes

$$hT_{,rr} + 2\lambda T_{,r} - \frac{1}{2} \frac{dV}{dT} - 2T_{,ru} - 2\lambda T_{,u} = 0. \quad (4.2)$$

Let $U = e^{\lambda r} T$, and look for solutions $U = \rho(r)\xi(u)$, assuming quadratic tachyon potential with coefficient $a = -\lambda^2$. One then finds that the function $\xi = e^{cu/2}$, where c is a constant. The equation for $\rho(r)$ is

$$x(1-x)\rho'' + \left(1 + \frac{c}{2\lambda} - 2x\right)\rho' - \frac{1}{4}\rho = 0. \quad (4.3)$$

This is a hypergeometric equation. The solutions are

$$\rho = AF\left(\frac{1}{2}; \frac{1}{2}; 1 + \frac{c}{2\lambda}; e^{-2\lambda r}\right) + BF\left(\frac{1}{2}; \frac{1}{2}; 1 + \frac{c}{2\lambda}; 1 - e^{-2\lambda r}\right). \quad (4.4)$$

This gives T immediately. We now return to the gravitational and dilaton equations. It is expected that the dilaton and metric to be perturbed near the origin by this T field which is fed into the field equations. One finds that the new dilaton and metric must be static. The static field equations with $V(T) = aT^2$ are

$$h_{,rr} - 2h_{,r}\phi_{,r} = 0, \quad (4.5)$$

$$2\phi_{,rr} - T_{,r}^2 = 0, \quad (4.6)$$

$$h_{,rr} - 4h_{,r}\phi_{,r} + 4h\phi_{,r}^2 - 4h\phi_{,rr} + hT_{,r}^2 - 4\lambda^2 + aT^2 = 0, \quad (4.7)$$

$$hT_{,rr} + h_{,r}T_{,r} - 2h\phi_{,r}T_{,r} - aT = 0. \quad (4.8)$$

T is known, hence (4.6) implies $\phi(r)$ and (4.7) and (4.8) act as a check for this solution. One can calculate the power series solution for metric and dilaton around the origin of r . This naturally depends on the expansion coefficients of the hypergeometrical tachyon, and is not very instructive.

In summary, the black hole solution has been taken as a fixed background in which the tachyon moves. This gives a hypergeometric function. When one iterates this solution, one can find an expansion for the perturbed dilaton and metric near the origin. The metric and dilaton must remain static, though in which global configuration we do not know. The initial assumption of $f(u) = 0$ as a fixed background followed by many iterations does not necessarily lead to the result which one would obtain if one were to solve the equations of motion at once, and is thus of limited value.

V. SEMICLASSICAL TREATMENT OF THE STRING MOTIVATED MODEL

In this section the tachyon field is treated as a quantum field. One simply adds to the expression for its classical stress tensor the quantum stress tensor, which is derivable in two dimensions using the trace anomaly and the conservation equations. The additional term might be produced by including a term in the action. This term is nonlocal, and it need not be specified here. The other fields are still treated classically, but one would need later to include dilaton and graviton loops. This question was addressed in the CGHS model by proliferating the number of scalar fields, which rendered other terms small and the semiclassical approximation exact.

The tachyon in (2.1) is not coupled as a standard scalar field, i.e.,

$$\mathcal{L} = \sqrt{-g}[(\nabla T)^2 - (m^2 + \xi R)T^2], \quad (5.1)$$

where ξ is a numerical factor, which is zero for both minimal and conformal couplings in two dimensions, but rather,

$$\mathcal{L}_T = \sqrt{-g}e^{-2\phi}[(\nabla T)^2 + V(T)]. \quad (5.2)$$

The Lagrangian (5.2) for the tachyon field is clearly conformally coupled. The factor $e^{-2\phi}$ cannot be removed by a conformal transformation in two dimensions.

Using dimensional analysis the trace anomaly for this form of field must be

$$\alpha R + \beta \quad (5.3)$$

where R is the Ricci scalar; α and β are constants found in the explicit calculation through the heat equation.

By functionally differentiating the tachyon part of the Lagrangian with respect to the metric one finds the classical stress tensor for the tachyon is

$$T_{\mu\nu} = e^{-2\phi}[\nabla_\mu T \nabla_\nu T - \frac{1}{2}g_{\mu\nu}(\nabla T^2 + V)]. \quad (5.4)$$

Using the field equations, this can be written

$$T_{\mu\nu} = 2e^{-2\phi}[g_{\mu\nu}(\nabla\phi^2 - \nabla^2\phi - \lambda^2) + \nabla_\mu\nabla_\nu\phi]. \quad (5.5)$$

The simple step that we propose in including quantum effects is to take the left-hand side of this equation to be the sum of the classical and quantum stress tensors for the tachyon. For completeness, the equations of motion in the gauge (2.5) are

$$\begin{aligned} \epsilon^{2\phi}T_{uu} &= hh_{,r}\phi_{,r} + h_{,r}\phi_{,u} - h_{,u}\phi_{,r} - 4h\phi_{,ru} \\ &\quad + 2h^2\phi_{,rr} - 2h^2\phi_{,r}^2 + 4h\phi_{,r}\phi_{,u} + 2\phi_{,uu} + 2h\lambda^2, \end{aligned} \quad (5.6)$$

$$\begin{aligned} e^{2\phi}T_{ur} &= h_{,r}\phi_{,r} - 2\phi_{,ru} + 2h\phi_{,rr} - 2h\phi_{,r}^2 \\ &\quad + 4\phi_{,r}\phi_{,u} + 2\lambda^2, \end{aligned} \quad (5.7)$$

$$e^{2\phi}T_{rr} = 2\phi_{,rr}, \quad (5.8)$$

where $T_{\mu\nu} = T_{\mu\nu}^{\text{cl}} + \langle T_{\mu\nu}^q \rangle$.

The classical components of $T_{\mu\nu}$ are

$$T_{rr}^{\text{cl}} = e^{-2\phi}T_{,r}^2, \quad (5.9)$$

$$T_{ur}^{\text{cl}} = \frac{1}{2}e^{-2\phi}[hT_{,r}^2 + V(T)], \quad (5.10)$$

$$T_{uu}^{\text{cl}} = e^{-2\phi}\{T_{,u}^2 + \frac{1}{2}h[hT_{,r}^2 - 2T_{,u}T_{,r} + V(T)]\}. \quad (5.11)$$

If one works in the ingoing null coordinate gauge, it may be seen that it is not possible to solve for the energy-momentum tensor components for general h [see (2.5)], but if one tries the ansatz (3.16), the quantum piece of the energy momentum tensor can be found. If the trace anomaly for the tachyon field is αR , then this is

$$\langle T_{rr}^q \rangle = 2\lambda^2\alpha + \xi, \quad (5.12)$$

$$\langle T_{ur}^q \rangle = -3\lambda^2\alpha e^{-2\lambda(r-f)} + \lambda^2\alpha + \frac{1}{2}\xi(1 - e^{-2\lambda(r-f)}), \quad (5.13)$$

$$\begin{aligned} \langle T_{uu}^q \rangle &= -e^{-2(r-f)}(2\lambda^2\alpha f + 4\lambda^2\frac{1}{2}\xi) \\ &\quad + 3\lambda^2\alpha e^{-4\lambda(r-f)} + \frac{1}{4}\xi + t(u), \end{aligned} \quad (5.14)$$

where $\xi = Be^{2\lambda(2r+u)}$, and $t(u)$ is an arbitrary function of u determined by the boundary conditions. Keeping terms involving ξ , there will be large distance divergences in the components, so one sets $B = 0$.

These terms are added to the classical stress tensor, and substituted into field equations (5.6)–(5.8). When α is set to zero, one recovers the classical field equations (2.6)–(2.9). These equations are clearly quite complicated, and one cannot find closed form solutions. Numerical solutions might be interesting, but this is not pursued here.

VI. SOLUTIONS IN THE CONFORMAL GAUGE

One can work in Kruskal double null coordinates, i.e., (3.5). The equations of motion are then

$$e^{2\rho}(-4\lambda^2 + V) - 8\rho_{,uv} + 16\phi_{,uv} - 16\phi_{,u}\phi_{,v} - 4T_{,u}T_{,v} = 0, \quad (6.1)$$

$$\alpha e^{2\phi}[\rho_{,uu} - \rho_{,u}^2 - t_u(u)] + 4\rho_{,u}\phi_{,u} - 2\phi_{,uu} + T_{,u}^2 = 0, \quad (6.2)$$

$$e^{2\rho}(-4\lambda^2 + V) - 4\alpha e^{2\phi}\rho_{,uv} + 8\phi_{,uv} - 16\phi_{,u}\phi_{,v} = 0. \quad (6.3)$$

These equations reduce to the CGHS equations if one removes tachyon terms.

Another approach is to define

$$\Theta_{\mu\nu}^{\text{cl}}(\tilde{T}) = \Theta_{\mu\nu}^{\text{cl}}(T) + \Theta_{\mu\nu}^q(T) \quad (6.4)$$

where the quantity \tilde{T} takes into account both the quantum and classical contributions to the energy-momentum tensor of the tachyon field. It is this field then that appears in the action (2.1). Taking $\tilde{T} = 0$, so that $V(\tilde{T}) = 0$, one has the classical CGHS equations with no matter. The solution to these is a one parameter family of static black holes, with a vacuum state, the linear dilaton, given by the zero mass case (3.8). But relations (6.4) will give equations for the potential $V(T)$ in terms of the conformal factor and the dilaton,

$$e^{-2\phi}V(T) = \alpha e^{-2\rho}\rho_{,uv} \quad (6.5)$$

which are known, and which will determine the potential $V(T)$ if one states the form of T . This will then determine the constraint functions t_u and t_v .

$$T_{,u}^2 = \alpha e^{2\phi}[\rho_{,uu} - \rho_{,u}^2 - t_u(u)], \quad (6.6)$$

and similarly for the advanced constraint equation in v .

One can choose the tachyon field to cancel out the quantum piece after combining (6.1) and (6.3): i.e.,

$$T_{,u}T_{,v} = \alpha e^{2\phi}\rho_{,uv}. \quad (6.7)$$

Then if one works in the gauge $\rho = \phi$ and chooses

$$V = 2\alpha\rho_{,uv}, \quad (6.8)$$

the remaining equation is just the dynamical equation of the Russo-Susskind-Thorlacius (RST) model:

$$-4\lambda^2 e^{2\rho} - 2\alpha e^{2\rho}\rho_{,uv} + 8\rho_{,uv} - 16\rho_{,u}\rho_{,v} = 0. \quad (6.9)$$

These are the RST black holes, but generated by the tachyon field and its potential. The relations (6.7) and (6.8) imply the form of the tachyon potential in terms of T .

To summarize, the equations for the string-motivated model have been found, which correspond to those of the

CGHS model but in ingoing coordinates which give the position of the apparent horizon. It seems that one has to resort to numerical solutions, where one could consider tachyonic ingoing matter, for various potentials. Equilibrium static solutions and contact with RST black holes are found by working in the conformal gauge and shaping the tachyonic terms.

VII. CONCLUSIONS

One can try to simulate the black hole formation and evaporation in two dimensions: the hope is that the results will have bearing upon more realistic descriptions, as other scientific work in two dimensions often has.

In this paper, a model of gravity arising from string theory is treated as a quantum field theory. First classical solutions are noted for zero and nontrivial tachyon field configurations. Then, the equation of motion for the tachyon in a fixed flat space and black hole geometry is solved, and is iterated into the dilaton and gravitational field equations. Finally, the quantum stress tensor for the tachyon field is found and coupled appropriately to the classical field equations in another gauge from that which has usually been used. The aim was to consider an analogous coordinate system to that which highlights most clearly the behavior of a radiating star in four dimensions. The solutions then immediately tell us where the apparent horizon is. Working in this gauge was useful in finding classical solutions and considering the behavior of the tachyon in a fixed geometry. However, although this is another example of a coordinate system in which one can calculate the quantum stress tensor components, and thus derive semiclassical equations of motion, it does not yield simpler equations than those found in the conformal gauge.

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