RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 51, NUMBER 3 1 FEBRUARY 1995

Pion-pair production by two photons

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(Received 26 July 1994)

The cross section for pion-pair production by two photons is calculated approximately by using the low energy theorem previously derived from the partially-conserved-axial-vector-current hypothesis and current algebra and found to agree very well with the experimental data recently obtained by the Mark II, TPC/Two-Gamma, and CLEO Collaborations.

PACS number(s): 13.60.Le, 11.40.Ha, 13.65.+i, 14.40.Aq

Recently, the two-photon processes for particle production have become a subject for extensive experimental investigations by e^+e^- colliding beams over two decades after their detailed theoretical studies [1]. In 1986, measurements of the cross section for the two-photon production of charged pion and kaon pairs had been performed by the Mark II [2] and TPC/Two-Gamma [3] Collaborations at the SLAC e^+e^- storage ring PEP and, very lately, it has been done by the CLEO Collaboration at Cornell Electron Storage Ring [4]. Especially, the last collaboration has found that the functional dependence of the measured cross section disagrees with the leading order QCD prediction by Brodsky and Lepage [5] at small values of the two-photon invariant mass but that the data show qualitatively a transition to perturbative behavior at an invariant mass of approximately 2.5 GeV. The purpose of this Rapid Communication is to calculate the cross section approximately by using the low energy theorem [6] previously derived from the partially-conserved-axial-vector-current (PCAC) hypothesis [7] and current algebra [8] and to compare it with these experimental data. The calculated cross section will be found to agree quantitatively very well with the data at a wide range of invariant masses between 1.3 and 4.5 GeV.

Let us first briefly review how to derive the low energy theorem on pion-pair production by two photons [6]. The successive application of the PCAC hypothesis [7], the soft-pion technique, and the algebra of currents [8] makes it possible to reduce a pion pair (of the momenta p and q) in the final state of the amplitude for $\gamma\gamma\to\pi^+\pi^-$:

$$\langle\pi^+(p),\pi^-(q)|T[J^\mu(x)J^\nu(0)]|0\rangle \underset{\text{soft pion}}{\cong} -2F_\pi^{-2}[2\omega_p(2\pi)^3 2\omega_q(2\pi)^3]^{-1/2}[\langle 0|T(V_3^\mu(x)V_3^\nu(0))|0\rangle - \langle 0|T(A_3^\mu(x)A_3^\nu(0))|0\rangle], \tag{1}$$

where F_π is the pion-decay constant ($F_\pi\cong 93$ MeV) and J^μ, V_3^μ, and A_3^ν are the hadronic electromagnetic current, the third component of the isovector vector current and that of the isovector axial-vector current, respectively. By using the spectral representations of the propagators, we obtain the following expression for the matrix element $M^{\mu\nu}$ for $\gamma\gamma\to\pi^+\pi^-$:

$$\begin{aligned} M^{\mu\nu} &\equiv i\int d^4x\, e^{ik_1\cdot x}\langle\pi^+(p)\pi^-(q);p+q=k_1+k_2|T(J^\mu(x)J^\nu(0))|0\rangle \\ &\underset{\text{soft pion}}{\cong} -2F_\pi^{-2}[2\omega_p(2\pi)^3 2\omega_q(2\pi)^3]^{-1/2}\left[\int dm^2\frac{\rho_V(m^2)-\rho_A(m^2)}{k_1\cdot k_2+m^2-i\varepsilon}\left(g^{\mu\nu}+\frac{k_2^\mu k_1^\nu}{m^2}\right)\right. \\ &\qquad \left. -g^{\mu 0}g^{\nu 0}\int dm^2\frac{\rho_V(m^2)-\rho_A(m^2)}{m^2} - F_\pi^2\frac{k_2^\mu k_1^\nu}{k_1\cdot k_2-i\varepsilon}+g^{\mu 0}g^{\nu 0}F_\pi^2\right], \end{aligned} \tag{2}$$

where k_1 and k_2 are the momenta of the photons, and ρ_V and ρ_A are the spectral functions of the vector and axial-vector currents, respectively. It is clearly seen in Eq. (2) that not only Lorentz covariance but also gauge invariance is maintained in the soft-pion limit if, and only if, the first sum rule of Weinberg [9],

$$\int dm^2[\rho_V(m^2)-\rho_A(m^2)]/m^2=F_\pi^2, \tag{3}$$

is valid. Therefore, we will assume the validity of the first sum rule hereafter. Then, we have the low energy theorem of

$$M^{\mu\nu} \underset{\text{soft pion}}{\cong} -2[2\omega_p(2\pi)^3 2\omega_q(2\pi)^3]^{-1/2} \times\left(g^{\mu\nu}-\frac{k_2^\mu k_1^\nu}{k_1\cdot k_2}\right)f_\pi(k_1k_2), \tag{4}$$

where $f_\pi(x)$ is the pion structure function defined by

$$f_\pi(x)\equiv\frac{1}{F_\pi^2}\int dm^2\frac{\rho_V(m^2)-\rho_A(m^2)}{x+m^2} \quad \text{with } f_\pi(0)=1. \tag{5}$$

It is clearly seen in Eq. (4) that the scattering amplitude $M^{\mu\nu}$ gives a correct Thomson limit when k_1 and k_2 vanish.

By using this theorem, the differential cross section for $\gamma\gamma\to\pi^+\pi^-$ at low energies is approximately given by

$$d\sigma(\gamma\gamma\to\pi^+\pi^-) \cong \frac{(4\pi\alpha)^2}{s}[f_\pi(\tfrac{1}{2}s)]^2\frac{d^3pd^3q}{2\omega_p(2\pi)^3 2\omega_q(2\pi)^3} \times(2\pi)^4\delta(k_1+k_2-p-q), \tag{6}$$

where s is the square of the total energy of two incident photons or a pair of produced pions in the center-of-mass system, i.e., $s=(k_1+k_2)^2=(p+q)^2$. After the easy integration of phase space, we obtain the differential cross section with respect to the scattering angle θ of

$$\frac{d\sigma}{d\cos\theta}(\gamma\gamma\to\pi^+\pi^-)\cong\frac{\pi\alpha^2}{s}\left(1-\frac{4m_\pi^2}{s}\right)^{1/2}[f_\pi(\tfrac{1}{2}s)]^2, \tag{7}$$

and the total cross section of

$$\sigma(\gamma\gamma\to\pi^+\pi^-)\cong\frac{2\pi\alpha^2}{s}\left(1-\frac{4m_\pi^2}{s}\right)^{1/2}[f_\pi(\tfrac{1}{2}s)]^2, \tag{8}$$

where m_π is the pion mass. We should note that the differential cross section for $\gamma\gamma\to\pi^+\pi^-$ at low energies is approximately isotropic.

In order to estimate the pion structure function $f_\pi(x)$, let us assume that the spectral functions $\rho_V(m^2)$ and $\rho_A(m^2)$ are dominated by a single pole as

$$\rho_V(m^2)\cong\frac{m_V^4}{f_V^2}\delta(m^2-m_V^2)$$

and (9)

$$\rho_A(m^2)\cong\frac{m_A^4}{f_A^2}\delta(m^2-m_A^2),$$

where $m_V(m_A)$ and $f_V(f_A)$ are the mass and coupling constant of a vector (axial-vector) meson such as $\rho(a_1)$, respectively. Then, the pion structure function can be approximated by

$$f_\pi(x)\cong\frac{1}{F_\pi^2}\left[\frac{m_V^4}{f_V^2(x+m_V^2)}-\frac{m_A^4}{f_A^2(x+m_A^2)}\right] \tag{10}$$

and satisfies the first Weinberg sum rule (3) if

$$f_\pi(0)\cong\frac{1}{F_\pi^2}\left(\frac{m_V^2}{f_V^2}-\frac{m_A^2}{f_A^2}\right)=1. \tag{11}$$

Therefore, it can be approximately parametrized as

$$f_\pi(x)\cong\frac{1}{F_\pi^2}\left[\frac{m_V^4}{f_V^2(x+m_V^2)}-\left(\frac{m_V^2}{f_V^2}-F_\pi^2\right)\frac{m_A^2}{x+m_A^2}\right]. \tag{12}$$

Let us also assume the validity of the second Weinberg sum rule

$$\int dm^2[\rho_V(m^2)-\rho_A(m^2)]=0, \tag{13}$$

which indicates

$$\frac{m_V^4}{f_V^2}-\frac{m_A^4}{f_A^2}\cong 0 \tag{14}$$

in the approximate parametrization of (9). Then we can reduce the number of independent parameters as

$$f_\pi(x)\cong m_V^4/\{(x+m_V^2)[(1-f_V^2F_\pi^2/m_V^2)x+m_V^2]\} \tag{15}$$

since, from Eqs. (11) and (14), we obtain the Weinberg relation [9]

$$m_A^2\cong m_V^2/(1-f_V^2F_\pi^2/m_V^2). \tag{16}$$

For simplicity, let us further assume the validity of the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation [10]

$$m_V^2\cong 2f_V^2F_\pi^2, \tag{17}$$

which changes the relation (16) into the Weinberg mass relation [9]

$$m_A=\sqrt{2}m_V. \tag{18}$$

Then, we can finally obtain the following simplest parametrization with the only one parameter, m_V^2:

$$f_\pi(x)\cong\frac{m_V^4}{(x+m_V^2)(\frac{1}{2}x+m_V^2)}. \tag{19}$$

Thus, the final result for the differential cross section becomes

$$\frac{d\sigma}{d\cos\theta}(\gamma\gamma\to\pi^+\pi^-)\cong\frac{\pi\alpha^2}{s}\left(1-\frac{4m_\pi^2}{s}\right)^{1/2} \times\left[\frac{m_V^4}{(\frac{1}{2}s+m_V^2)(\frac{1}{4}s+m_V^2)}\right]^2 \tag{20}$$

while that for the total cross section is

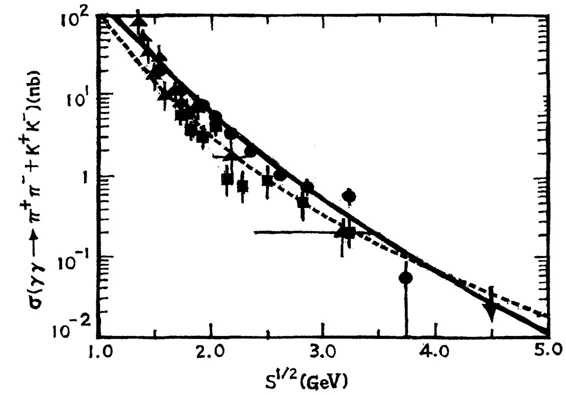

FIG. 1. The total cross section for the processes of $\gamma\gamma\rightarrow\pi^+\pi^-$ and $\gamma\gamma\rightarrow K^+K^-$, $\sigma(\gamma\gamma\rightarrow\pi^+\pi^-+K^+K^-)$, with the scattering angle θ restricted in a certain region as a function of the total energy of two photons or a pair of pions or kaons in the center-of-mass system $s^{1/2}$. The experimental data points are taken from the Mark II Collaboration [2] with $|\cos\theta|<0.5$, from the TPC/Two-Gamma Collaboration [3] with $|\cos\theta|<0.3$ for $\pi^+\pi^-$ and $|\cos\theta|<0.6$ for K^+K^- and from the CLEO Collaboration [4] with $|\cos\theta|<0.6$. They are denoted by the square, triangle, and circle dots with error bars, respectively. The solid and broken lines denote the current algebraic prediction in the present paper and the leading order QCD prediction by Brodsky and Lepage [5], respectively. Both of these theoretical curves are for $|\cos\theta|<0.6$.

$$\sigma(\gamma\gamma\rightarrow\pi^+\pi^-)\cong\frac{2\pi\alpha^2}{s}\left(1-\frac{4m_\pi^2}{s}\right)^{1/2}\times\left[\frac{m_V^4}{(\frac{1}{2}s+m_V^2)(\frac{1}{4}s+m_V^2)}\right]^2. \tag{21}$$

Now, we are ready to compare our result with the experimental data. Unfortunately, none of the Mark II, TPC/Two-Gamma, and CLEO Collaborations [2–4] have provided us with the data on the differential cross section for the whole region of the scattering angle of $0\leqslant|\cos\theta|\leqslant 1$ and/or the total cross section. Furthermore, the latter two collaborations have not been able to distinguish between pions and kaons so that they have given us the data on the differential and total cross sections for the processes of $\gamma\gamma\rightarrow\pi^+\pi^-$ and $\gamma\gamma\rightarrow K^+K^-$, $(d\sigma/d\cos\theta)(\gamma\gamma\rightarrow\pi^+\pi^-+K^+K^-)$ and $\sigma(\gamma\gamma\rightarrow\pi^+\pi^-+K^+K^-)$. More precisely, the Mark II Collaboration has presented the data on $(d\sigma/d\cos\theta)(\gamma\gamma\rightarrow\pi^+\pi^-+K^+K^-)$ and $\sigma(\gamma\gamma\rightarrow\pi^+\pi^-+K^+K^-)$ for $|\cos\theta|<0.5$ at the mass range of $s^{1/2}=1.7$–3.5 GeV, the TPC/Two-Gamma Collaboration the data on $\sigma(\gamma\gamma\rightarrow\pi^+\pi^-)$ for $|\cos\theta|<0.3$ and $\sigma(\gamma\gamma\rightarrow K^+K^-)$ for $|\cos\theta|<0.6$ at the mass range of $s^{1/2}=1.3$–3.5 GeV, and the CLEO Collaboration the data on $(d\sigma/d\cos\theta)(\gamma\gamma\rightarrow\pi^+\pi^-+K^+K^-)$ and $\sigma(\gamma\gamma\rightarrow\pi^+\pi^-+K^+K^-)$ for $|\cos\theta|<0.6$ at the mass range of $s^{1/2}=1.5$–4.5 GeV.

In Fig. 1, we illustrate (by the dots with error bars) these data [2–4] on the total cross section as a function of the total energy of two photons or a pair of pions or kaons in the center-of-mass system, and (by the broken line) the leading order QCD prediction of Brodsky and Lepage [5] in the form transformed by the CLEO Collaboration [4] for $|\cos\theta|<0.6$. In order to compare our results with these data, we must also transform our predictions of $(d\sigma/d\cos\theta)(\gamma\gamma\rightarrow\pi^+\pi^-)$ in Eq. (20) and $\sigma(\gamma\gamma\rightarrow\pi^+\pi^-)$ in Eq. (21) in the following two steps. (1) Assume that the ratio of the cross section for $\gamma\gamma\rightarrow K^+K^-$ to that for $\gamma\gamma\rightarrow\pi^+\pi^-$ be about two. This assumption can be justified either by the careful comparison of the TPC/Two-Gamma Collaboration data on $\sigma(\gamma\gamma\rightarrow\pi^+\pi^-)$ for $|\cos\theta|<0.3$ and $\sigma(\gamma\gamma\rightarrow K^+K^-)$ for $|\cos\theta|<0.6$ or by the reasonable prediction of $d\sigma(\gamma\gamma\rightarrow K^+K^-)/d\sigma(\gamma\gamma\rightarrow\pi^+\pi^-)=(F_K/F_\pi)^4$ ($\cong 2.2$ where F_K is the kaon decay constant, $F_K\cong 113$ MeV) by Brodsky and Lepage [5]. Then, the cross sections for $\gamma\gamma\rightarrow\pi^+\pi^-$ and $\gamma\gamma\rightarrow K^+K^-$ is given by about three times (or 3.2 times) that for $\gamma\gamma\rightarrow\pi^+\pi^-$. (2) Assume that the differential cross sections for $\gamma\gamma\rightarrow\pi^+\pi^-$ and $\gamma\gamma\rightarrow K^+K^-$ are approximately isotropic at the relatively low energy range of $s^{1/2}=1.3$–4.5 GeV. This assumption can be justified by our prediction of the approximate isotropy of the differential cross section for $\gamma\gamma\rightarrow\pi^+\pi^-$ at low energies which has previously been mentioned after Eq. (7) and by its easy transformation into a similar prediction for $\gamma\gamma\rightarrow K^+K^-$. Then, the total cross section for $\gamma\gamma\rightarrow\pi^+\pi^-$ and $\gamma\gamma\rightarrow K^+K^-$ with the restriction of $|\cos\theta|<0.6$ is finally given by about 3.2×0.6 ($\cong 1.9$) times the total cross section for $\gamma\gamma\rightarrow\pi^+\pi^-$ in Eq. (21). Our current algebraic prediction for $\sigma(\gamma\gamma\rightarrow\pi^+\pi^-+K^+K^-)$ for $|\cos\theta|<0.6$ thus obtained is illustrated by the solid line in Fig. 1 for the parameter of $m_V^2=2$ GeV^2 adjusted to get a best fit to the data.

Remarkably, it is found in Fig. 1 that our calculated total cross section agrees quantitatively very well with the data at a wide range of invariant masses between 1.3 and 4.5 GeV and does better than the predicted total cross section by Brodsky and Lepage. The determined value of the single parameter for the best fit, $m_V\cong 1.4$ GeV, seems to be very reasonable since it should be a kind of weighted average mass of all isovector vector mesons such as $\rho(770)$, $\rho(1450)$, $\rho(1700)$, and so on in the spirit of the generalized vector-meson-dominance model [11]. This strongly suggests that the PCAC hypothesis and current algebra, the heritage from the 1960's, are very powerful (and still more useful and convenient than QCD in the leading order) for predicting and explaining some hadronic processes involving pseudoscalar mesons and electromagnetic (or weak) currents.

In conclusion, let us emphasize that the validity of our calculation in this Rapid Communication should be limited to a relatively low energy region. In fact, it can be seen in Fig. 1 that for invariant masses larger than 4.5 GeV our prediction deviates significantly from the leading order QCD prediction by Brodsky and Lepage which indicates a slower falloff for larger invariant masses and which should be taken as a better approximation at high energies. Such possible deviation and clearer distinction between the Brodsky-Lepage predictions and ours can be found in the future e^+e^- colliding beam experiments for the two-photon processes of $\gamma\gamma\rightarrow\pi^+\pi^-$ (and $\gamma\gamma\rightarrow K^+K^-$) at higher energies at KEK TRISTAN, the CERN e^+e^- colliders LEP and LEP II, the Next Linear Collider, and the Japan Linear Collider.

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