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Novel "no-scalar-hair" theorem for black holes

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We formulate a new "no-hair" theorem for black holes in general relativity which rules out a multicomponent scalar field dressing of any asymptotically flat, static, spherically symmetric black hole. The field is assumed to be minimally coupled to gravity, and to bear a non-negative energy density as seen by any observer, but its field Lagrangian need not be quadratic in the field derivatives. The proof centers on energy-momentum conservation and the Einstein equations. One kind of field ruled out is the Higgs field with a double (or multiple) well potential. The theorem is also proved for scalar-tensor gravity.

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Wheeler's dictum [1] "a black hole has no hair" has had great influence in the development of black hole physics. One aspect of it, that a stationary black hole should be described by only a few parameters, proved crucial in the formulation of black hole thermodynamics [2]. In its original form, "black holes have no hair" held that a black hole can be "dressed" only by fields, like the electromagnetic one, which are associated with a Gauss-like law. Early "no-hair" theorems, which excluded scalar [3], massive vector [4], and spinor [5] fields from a stationary black hole's exterior, buttressed the dictum. With later day developments in particle physics, the settings assumed by the original theorems have become outdated, and as a consequence solutions for black holes with various "hairs" have been found. Among them are black holes dressed with Yang-Mills, Proca-type Yang-Mills, and Skyrme fields in various combinations with Higgs fields (for reviews see [6–8]). Some, but not all, of these black holes are unstable.

Scalar fields were also covered by the original no-hair theorems, which, however, limited themselves to the Klein-Gordon minimally coupled field. Conformal coupling to gravity permitted the early discovery [9,10] of a black hole solution with extremal Reissner-Nordström geometry, a conformal scalar field and the corresponding charge, as well as electric charge. The solution admits a generalization possessing a magnetic monopole [11]. These black holes sidestep the no-hair theorems because the scalar field diverges at the horizon [9], as certified by a recent no-hair theorem of Zannias [12]. On the other hand, it has long been recognized [10] that the divergence of the scalar field is physically innocuous. There remains the possibility that this solution is unstable [13], and the suspicion that it is isolated in the sense of having exactly the same number of free parameters as the traditional Reissner-Nordström black hole family and not one more [14]. All these points suggest that the conformal scalar hair black hole does not compromise the spirit of "black holes have no hair."

Another shortcoming of the original "no-scalar hair" theorems is that they do not take cognizance of the possibility that the scalar field may have a potential with some complicated shape. Developments in particle physics have shown this to be a likely circumstance. In simplest form, the idea of the original theorems was to start from the action

$$S_\psi=-\frac{1}{2}\int [\psi_{,\alpha}\psi^{,\alpha}+V(\psi^2)](-g)^{1/2}\, d^4x \tag{1}$$

for a static scalar field in a static black hole background, derive the field equation, multiply it by ψ and integrate over the black hole exterior at a given time. A physical argument allows one to show that the boundary terms at infinity and horizon must vanish. The result is

$$\int [g^{ab}\psi_{,a}\psi_{,b}+\psi^2 V'(\psi^2)](-g)^{1/2}\, d^3x=0, \tag{2}$$

with the integration extending over the black hole exterior, and the indices a and b running over the space coordinates only. The restricted metric g^{ab} is positive definite. If $V'(\psi^2)$ is everywhere non-negative and can vanish only at some discrete values ψ_j, then it is clear that the field ψ must be constant everywhere outside the black hole, taking on one of the values $\{0,\psi_j\}$. The theorem works for the Klein-Gordon field for which $V'(\psi^2)=\mu^2$ where μ is the field's mass. However, there are examples in particle physics, i.e., Higgs field with a double well potential, for which $V'(\psi^2)$ is negative in some region. The theorem then fails. Modern "no scalar-hair" theorems (see Ref. [8] for review) have left the question of this type of Higgs hair open.

In this connection Bechmann and Lechtenfeld (BL) [15] have claimed that exponentially decaying scalar hair can be attached to a static spherical black hole. Their field's potential, which is not known as a closed expression, does indeed violate the condition $V'(\psi^2)\geqslant 0$. However, the BL solution has regions with negative scalar's energy density; one can argue that this makes the BL black hole solution unphysical. BL contend that their solution evades the no-hair theorems only by virtue of having $V'(\psi^2)\leqslant 0$ in some region. Were this view correct, it might be possible to deform the BL potential so as to produce a physical black hole solution with scalar hair and with positive energy density everywhere. With the demonstration in the present paper that the positiveness of the field's energy density is sufficient to rule out a black hole with scalar hair, the hope engendered by the BL solution is dashed. One more application of our results is to

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rule out as black hole hair any Higgs field whose potential has two or more wells of finite depth.

We shall consider a multiplet of scalar fields, $\psi,\chi,\ldots,$ subject to the action

$$S_{\psi,\chi,\ldots}=-\int \mathcal{E}(\mathcal{I},\mathcal{J},\mathcal{K},\ldots,\psi,\chi,\ldots)(-g)^{1/2}\,d^4x. \tag{3}$$

Here $\mathcal{E}$ is a function, and $\mathcal{I}\equiv g^{\alpha\beta}\psi_{,\alpha}\psi_{,\beta}$, $\mathcal{J}\equiv g^{\alpha\beta}\chi_{,\alpha}\chi_{,\beta}$, and $\mathcal{K}\equiv g^{\alpha\beta}\chi_{,\alpha}\psi_{,\beta}$ are examples of the invariants that can be formed from first derivatives of $\psi,\chi,\ldots$. We do not assume that the kinetic part of the scalar's Lagrangian density can be separated out, nor that it is a quadratic form in first derivatives. One reason for allowing this more general form is that the known scalar fields in nature are not elementary fields. Their actions must thus be effective actions obtained by integrating the functional integral of the elementary fields in nature over some of the fields. Thus under some circumstances the more general form of the action, Eq. (3), may arise. Actions such as Eq. (3) have also been considered in astrophysical contexts [16]; many manipulations here are inspired by this last reference. In spite of the generality we do assume minimal coupling to gravity: the scalar's action does not include the curvature in any form.

We shall assume that the energy density carried by the scalar field multiplet is non-negative. When there are only two field components, as we shall assume from now on (it is straightforward to generalize to any number), the energy-momentum tensor corresponding to $S_{\psi,\chi}$ is

$$T_\alpha{}^\beta=-\mathcal{E}\delta_\alpha{}^\beta+2(\partial\mathcal{E}/\partial\mathcal{I})\psi_{,\alpha}\psi^{,\beta}+2(\partial\mathcal{E}/\partial\mathcal{J})\chi_{,\alpha}\chi^{,\beta}+(\partial\mathcal{E}/\partial\mathcal{K})(\chi_{,\alpha}\psi^{,\beta}+\psi_{,\alpha}\chi^{,\beta}). \tag{4}$$

An observer with a four-velocity U^α ($U^\alpha U_\alpha=-1$) observes the local energy density

$$\rho=\mathcal{E}+2[(\partial\mathcal{E}/\partial\mathcal{I})(\psi_{,\alpha}U^\alpha)^2+(\partial\mathcal{E}/\partial\mathcal{J})(\chi_{,\alpha}U^\alpha)^2+(\partial\mathcal{E}/\partial\mathcal{K})\chi_{,\alpha}U^\alpha\psi_{,\beta}U^\beta]. \tag{5}$$

Suppose that, as in our case, the field has a timelike Killing vector, as would be the case for a static black hole with scalar hair. If the observer moves along this Killing vector, $\psi_{,\alpha}U^\alpha=0$, etc., so that $\rho=\mathcal{E}$. Thus

$$\mathcal{E}\geqslant 0, \tag{6}$$

at least for a field with the said symmetry.

Consider now a second observer moving relative to the Killing-vector observer with a three-velocity $\mathbf{v}$. In a freely falling frame of reference co-moving momentarily with the first observer $U^0=1/(1-\mathbf{v}^2)^{1/2}$, while $\mathbf{U}=\mathbf{v}/(1-\mathbf{v}^2)^{1/2}$. When $|\mathbf{v}|\rightarrow 1$, all the terms involving derivatives in Eq. (5) obviously dominate $\mathcal{E}$; thus together they must be non-negative. The conditions for positivity of the resulting quadratic form in $\psi_{,\alpha}U^\alpha$ and $\chi_{,\alpha}U^\alpha$ are

$$\partial\mathcal{E}/\partial\mathcal{I}>0, \quad \partial\mathcal{E}/\partial\mathcal{J}>0, \tag{7}$$

and

$$(\partial\mathcal{E}/\partial\mathcal{K})^2\leqslant 4(\partial\mathcal{E}/\partial\mathcal{I})(\partial\mathcal{E}/\partial\mathcal{J}). \tag{8}$$

We now assume the existence of a self-consistent, asymptotically flat solution of the Einstein and scalar field equations with the character of static, spherically symmetric black hole. Using the symmetries, the metric outside the horizon may be taken as

$$ds^2=g_{\alpha\beta}dx^\alpha dx^\beta=-e^\nu dt^2+e^\lambda dr^2+r^2(d\theta^2+\sin\theta^2 d\varphi^2), \tag{9}$$

with ν and λ depending on r only and obeying $\nu(r),\lambda(r)=O(r^{-1})$ as $r\rightarrow\infty$ because of asymptotic flatness. Assuming the scalar field is nontrivial, we must also have $\psi=\psi(r)$ and $\chi=\chi(r)$. As usual, the event horizon radius is at $r=r_h$ where $\exp[\nu(r_h)]=0$ (in case there are several such zeros, the horizon corresponds to the outer one).

The energy-momentum tensor, Eq. (4), must obey the conservation law

$$T_\mu{}^\nu{}_{;\nu}=0, \tag{10}$$

which follows from the coordinate invariance of the scalar's action. The r component of this law takes the form [17]

$$[(-g)^{1/2}\,T_r{}^r]'-(1/2)(-g)^{1/2}\,(\partial g_{\alpha\beta}/\partial r)T^{\alpha\beta}=0, \tag{11}$$

where the prime denotes $\partial/\partial r$. Because of the static and spherical symmetry of the solution, $T_\mu{}^\nu$ must be diagonal and $T_\theta{}^\theta=T_\varphi{}^\varphi$. These conditions allow us to rewrite Eq. (11) in the form

$$(e^{\frac{\lambda+\nu}{2}}r^2T_r{}^r)'-(1/2)e^{\frac{\lambda+\nu}{2}}r^2[\nu' T_t{}^t+\lambda' T_r{}^r+4T_\theta{}^\theta/r]=0. \tag{12}$$

The terms containing λ' cancel out so that

$$(e^{\nu/2}r^2T_r{}^r)'=(1/2)e^{\nu/2}r^2[\nu' T_t{}^t+4T_\theta{}^\theta/r], \tag{13}$$

but by Eq. (4) and the symmetries, $T_t{}^t=T_\theta{}^\theta=-\mathcal{E}$. Substituting this in the right-hand side (RHS) of Eq. (13) and rearranging the derivatives we get our key expression:

$$(e^{\nu/2}r^2T_r{}^r)'=-(e^{\nu/2}r^2)'\mathcal{E}. \tag{14}$$

Let us now integrate Eq. (14) over r from $r=r_h$ to a generic r. The boundary term at the horizon vanishes because $e^\nu=0$ and $T_r{}^r$ is finite there. The reason for the last is that in order for the surface $e^\nu=0$ to be a regular horizon, physical invariants, such as $T_{\alpha\beta}\,T^{\alpha\beta}$, must be finite there. In the coordinates of the metric (9) $T_{\alpha\beta}\,T^{\alpha\beta}=(T_t{}^t)^2+(T_r{}^r)^2+(T_\theta{}^\theta)^2+(T_\varphi{}^\varphi)^2$ so that $T_r{}^r$ (and also $T_t{}^t=-\mathcal{E}$) must be finite at $r=r_h$. The result of the integration is

$$T_r{}^r(r)=-\frac{e^{-\nu/2}}{r^2}\int_{r_h}^{r}(r^2e^{\nu/2})'\,\mathcal{E}dr. \tag{15}$$

Now, since e^ν vanishes at $r=r_h$ and must be positive outside it, $r^2e^{\nu/2}$ must grow with r sufficiently near the horizon. It is then immediately obvious from Eq. (15) and the positivity of $\mathcal{E}$ that, sufficiently near the horizon, $T_r{}^r<0$.

Further, carry out the differentiation in Eq. (14) and rearrange terms to obtain

$$(T_r{}^r)'=-e^{-\nu/2}r^{-2}\,(r^2e^{\nu/2})'\,(\mathcal{E}+T_r{}^r). \tag{16}$$

From Eq. (4) we obtain

$$\mathscr{E}+T_r{}^r=2e^{-\lambda}[(\partial\mathscr{E}/\partial\mathscr{T})\psi,_r^2+(\partial\mathscr{E}/\partial\mathscr{Y})\chi,_r^2+(\partial\mathscr{E}/\partial\mathscr{K})\chi,_r\psi,_r]. \quad (17)$$

But conditions (7) and (8) precisely insure the positive definiteness of the quadratic form in Eq. (17), so that $\mathscr{E}+T_r{}^r\geqslant 0$ everywhere. It then follows from Eq. (16) and our previous conclusion about $r^2e^{\nu/2}$ that sufficiently near the horizon $(T_r{}^r)'<0$ as well.

Asymptotically $e^{\nu/2}\to 1$. When this is put in Eq. (16) it tells us that $(T_r{}^r)'<0$ asymptotically. Now for $r\to\infty$, $\mathscr{E}$ must decrease at least as r^{-3} to guarantee asymptotic flatness of the solution [see Eq. (20) below and the subsequent comments]. Thus the integral in Eq. (15) converges and $|T_r{}^r|$ decreases asymptotically as r^{-2}. But since $(T_r{}^r)'<0$ asymptotically, we deduce that $T_r{}^r$ must be positive and decreasing with increasing r as $r\to\infty$. Now we found that near the horizon $T_r{}^r<0$ and $(T_r{}^r)'<0$. All these facts together tell us that in some intermediate interval $[r_a,r_b]$, $(T_r{}^r)'>0$ and also that $T_r{}^r$ itself changes sign at some r_c, with $r_a<r_c<r_b$, being positive in $[r_c,r_b]$ (there may be several such intervals $[r_a,r_b]$). To show that this conclusion is gravitationally untenable, we now turn, for the first time, to the Einstein equations.

The relevant ones are

$$e^{-\lambda}(r^{-2}-r^{-1}\lambda')-r^{-2}=8\pi GT_t{}^t=-8\pi G\mathscr{E}, \quad (18)$$

$$e^{-\lambda}(r^{-1}\nu'+r^{-2})-r^{-2}=8\pi GT_r{}^r. \quad (19)$$

We solve the first Einstein equation with

$$e^{-\lambda}=1-8\pi Gr^{-1}\int_{r_h}^{r}\mathscr{E}r^2dr-2GMr^{-1}, \quad (20)$$

where M is a constant of integration. Asymptotic flatness requires that $\mathscr{E}=O(r^{-3})$ asymptotically so that $\lambda=O(r^{-1})$. We also require that $e^{\lambda}\to\infty$ for $r\to r_h$ (horizon located at $r=r_h$) so that $2GM=r_h$; evidently M can be interpreted as the bare mass of the black hole. It follows from Eq. (20) that $e^{\lambda}\geqslant 1$ throughout the black hole exterior (e^{λ} cannot switch sign through infinity outside the horizon; since $e^{\nu}>0$ this would imply a change of the metric's signature, and would be incompatible with a regular black hole solution).

We now rewrite the second Einstein equation (19) in the form

$$e^{-\nu/2}r^{-2}\,(r^2e^{\nu/2})'=[4\pi rGT_r{}^r+(1/2r)]e^{\lambda}+3/2r>4\pi rGT_r{}^re^{\lambda}+2/r, \quad (21)$$

where the inequality results because $e^{\lambda}/2+3/2>2$. We found that in $[r_c,r_b]$, $T_r{}^r>0$. Thus $e^{-\nu/2}r^{-2}\,(r^2e^{\nu/2})'>0$ there. According to Eq. (16) this means that $(T_r{}^r)'<0$ throughout $[r_c,r_b]$. However, we determined that $(T_r{}^r)'>0$ throughout the encompassing interval $[r_a,r_b]$. Thus there is a contradiction.

The only way to resolve the contradiction is to accept that the scalar field components $\psi,\chi,\ldots$ must be constant throughout the black hole exterior, taking on values such that all components of $T_\mu{}^\nu$ vanish identically, namely values such that [see Eq. (4)]

$$\mathscr{E}(0,0,0,\ldots,\psi,\chi,\ldots)=0. \quad (22)$$

Such values must exist in order that a trivial solution of the scalar equation be possible in free empty space. It is this solution which served as an asymptotic boundary condition in our argument. The black hole solution must thus be identically Schwarzschild. Were the black hole electrically and/or magnetically charged, and the scalar fields uncoupled to electromagnetism so that Eq. (10) applied, a similar argument would show that the black hole must be a Reissner-Nordström black hole. This establishes the no-hair theorem for static, spherically symmetric holes.

One immediate application is to a Higgs field, a complex scalar field ψ with an action analogous to Eq. (1). Suppose the potential $V(|\psi|^2)$ has several wells; we assume its global minimum is $V=0$. One of the values of ψ for which $V=0$, call it ψ_0, serves as boundary condition for our asymptotically flat solution which, as mentioned, requires that the energy density vanish far away. Obviously the field's energy density is positive definite. Our theorem thus requires that $\psi=\psi_0$ thoughout the black hole exterior, and this excludes Higgs hair.

Broad as the theorem's assumptions are, there are fields that are not subject to it. An example is the conformal scalar field; its action includes the scalar curvature and is not of the form of Eq. (3). Put another way, although the Einstein equations in the presence of this field can be put in a form not involving extra curvature terms [9], the energy density is not necessarily positive definite, so the theorem does not apply. This is what permits the conformal scalar hair black hole solution [10] to exist.

The theorem does apply within the context of the scalar tensor (ST) gravitation theory [18,19]. In a ST theory there is an additional gravitational field, a scalar one ϕ. Such a theory can always be expressed either as a metric theory (Brans-Dicke conformal frame [18,19]) whose metric $g_{\alpha\beta}$ is measured by material rods and atomic clocks, or else expressed in the Dicke conformal frame [20] in which the gravitational action contains a term exactly like the Einstein-Hilbert action for a new metric $\hat{g}_{\alpha\beta}\equiv\phi g_{\alpha\beta}$. In the latter frame the total action is

$$S=\frac{1}{16\pi G}\int[R(\hat{g}_{\alpha\beta})-(\omega+3/2)\phi^{-2}\hat{g}^{\alpha\beta}\phi,_\alpha\phi,_\beta]\times(-\hat{g})^{1/2}\,d^4x+S_{\psi,\chi}, \quad (23)$$

with $\omega(\phi)$ a function and $S_{\psi,\chi}$ still expressed as in Eq. (3) in terms of $g_{\alpha\beta}$. Since ϕ plays the role of conformal factor between the two frames, we assume that $0<\phi<\infty$ everywhere outside the black hole.

We further restrict ourselves to ST theories with $2\omega+3>0$ for all ϕ. This is one way to ensure the physical requirement that the locally measured Newtonian gravitational constant [18], $G_N=G\phi^{-1}(2\omega+4)/(2\omega+3)$, be positive everywhere. The other way to ensure this, requiring

$\omega+2<0$ for all ϕ, is not very interesting since empirically we know that in the solar system, binary pulsars and cosmology, $\omega\gg 1$.

Suppose we replace $g_{\alpha\beta}\to\phi^{-1}\hat{g}_{\alpha\beta}$ in $S_{\psi,\chi}$. If we now combine the ϕ action with $S_{\psi,\chi}$ we obtain a new action for three fields, $\hat{S}_{\psi,\chi,\phi}$, of the form of Eq. (3), but with respect to the new metric $\hat{g}_{\alpha\beta}$. The problem is then reduced to that of three scalar fields, ψ, χ, and ϕ, in general relativity. The energy-momentum tensor $\hat{T}_\alpha{}^\beta$ obtained from $\hat{S}_{\psi,\chi,\phi}$ by varying the metric $\hat{g}_{\alpha\beta}$ and lowering an index with it is the sum of a part which differs from Eq. (4) only by a factor ϕ^{-1}, so that it inherits the assumed positivity of energy, and the term

$$\hat{\tau}_\alpha{}^\beta=(2\omega+3)\ \phi^{-2}[\phi,_\alpha\phi,^\beta-(1/2)\phi,_\gamma\phi,^\gamma\hat{\delta}_\alpha{}^\beta], \tag{24}$$

which is of the form of Eq. (4) with $\mathcal{E}=(\omega+3/2)\phi^{-2}\hat{g}^{\alpha\beta}\phi,_\alpha\phi,_\beta$ so that by the tests (6)–(8) it bears positive energy. We may thus repeat the procedure spanned by Eqs. (10)–(22) but with quantities replaced by their careted counterparts.

The metric is now (new radial coordinate)

$$d\hat{s}^2=\phi\ ds^2=-e^{\hat{\nu}}dt^2+e^{\hat{\lambda}}d\hat{r}^2+\hat{r}^2(d\theta^2+\sin\theta^2d\varphi^2). \tag{25}$$

Asymptotic flatness is stated in terms of the physical line element ds^2; however, by a suitable choice of units we can make $\phi(\infty)=1$ so that $\hat{\nu}$ and $\hat{\lambda}$ are subject to our original conditions. Two steps require care. The horizon is at $\hat{r}=r_h$ with $e^{\nu(\hat{r}_h)}=0$. But the calculation to get the analogue of Eq. (15) assumes rather that $e^{\hat{\nu}(r_h)}=0$. However, $e^{\hat{\nu}(r_h)}=\phi(r_h)e^{\nu(r_h)}=0$, the last equality because $\phi<\infty$. We also require finiteness of $\hat{T}_{\hat{r}}{}^{\hat{r}}$ at the horizon. For the contribution of ψ and χ this follows from the regularity of their physical $T_{\alpha\beta}\,T^{\alpha\beta}$ at the horizon which is identical to $\hat{T}_{\hat{\alpha}\hat{\beta}}\,\hat{T}^{\hat{\alpha}\hat{\beta}}$. The contribution $\hat{\tau}_{\hat{r}}{}^{\hat{r}}$ must likewise be finite at the horizon; otherwise, the divergence of the curvature invariants of $\hat{g}_{\alpha\beta}$ it would induce via "Einstein's equations" would inexorably afflict the physical curvature (of $g_{\alpha\beta}$), and the solution would not be a black hole. With these input the previous proof goes through: $\{\psi,\chi,\ldots\}$ must all be constant and $\phi=1$ throughout the black hole exterior, which must thus be Schwarzschild.

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[1] R. Ruffini and J. A. Wheeler, Phys. Today **24** (1), 30 (1971).
[2] J. D. Bekenstein, Phys. Today **33**, 24 (1980).
[3] J. E. Chase, Commun. Math. Phys. **19**, 276 (1970); J. D. Bekenstein, Phys. Rev. Lett. **28**, 452 (1972); C. Teitelboim, Lett. Nuovo Cimento **3**, 326 (1972).
[4] J. D. Bekenstein, Phys. Rev. D **5**, 1239 (1972); **5**, 2403 (1972).
[5] J. Hartle, Phys. Rev. D **3**, 2938 (1971); C. Teitelboim, Lett. Nuovo Cimento **3**, 397 (1972).
[6] K. Maeda, T. Tachizawa, T. Torii, and T. Maki, Phys. Rev. Lett. **72**, 450 (1994).
[7] P. Bizoń, Acta Phys. Pol. **B25**, 877 (1994).
[8] G. W. Gibbons, in *The Physical World*, Lecture Notes in Physics Vol. 383 (Springer, Berlin, 1991).
[9] J. D. Bekenstein, Ann. Phys. (N.Y.) **82**, 535 (1974).
[10] J. D. Bekenstein, Ann. Phys. (N.Y.) **91**, 72 (1975).
[11] K. S. Virbhadra and J. C. Parikh, Phys. Lett. B **331**, 302 (1994).
[12] T. Zannias, Report No. gr-qc/9409030 (unpublished).
[13] N. Bocharova, K. Bronikov, and V. Melnikov, Vestn. Mosk. Univ. Fiz. Astron. **6**, 706 (1970).
[14] B. C. Xanthopoulos and T. Zannias, J. Math. Phys. **32**, 1875 (1991).
[15] O. Bechmann and O. Lechtenfeld, Report No. gr-qc/9502011 (unpublished).
[16] J. D. Bekenstein and R. H. Sanders, Astrophys. J. **429**, 480 (1994).
[17] L. D. Landau and E. M. Lifshitz, *Classical Theory of Fields*, 4th ed. (Pergamon, London, 1975), p. 280.
[18] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).
[19] P. G. Bergmann, Int. J. Theor. Phys. **1**, 25 (1968); R. V. Wagoner, Phys. Rev. D **1**, 3209 (1970).
[20] R. H. Dicke, Phys. Rev. **125**, 2163 (1962).