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Higgs-sector solitons

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We establish the existence of static, classically stable, winding solitons in a renormalizable three-dimensional gauge model, with a topologically trivial target space and vacuum manifold. They are prototypes for possible analogous particlelike excitations in an extended Higgs sector of the standard electroweak theory.

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Several authors have argued in the past that a strongly interacting Higgs sector may have solitonic excitations called "electroweak Skyrmions" [1–8]. These are characterized by the nontrivial winding of the Higgs orientation around the SU(2) manifold [9], and can be thought of as the technibaryons of an underlying technicolor model. In the minimal electroweak theory, such winding excitations are potentially unstable for at least three distinct reasons: they may lose their energy by shrinking to zero size [10]; the winding can be undone if the Higgs field passes through zero [1,8,11]; and the evolution of the gauge fields can deform the excitation to a winding-vacuum state plus radiation [2,3,6,11]. Shrinking can be avoided by adding higher-derivative (Skyrme) terms for the angular part of the Higgs doublet [9]. Stability with respect to the other two modes of decay requires on the other hand that

$$m_H\rho \gg 1 \tag{1}$$

and

$$m_W\rho \ll 1, \tag{2}$$

where m_H and m_W are the Higgs- and gauge-boson mass, and ρ the soliton size. These estimates follow if one compares the gain in gradient energy to (i) the loss in potential energy when forcing the Higgs field in the interior of the soliton to vanish, or (ii) the loss in weak magnetic energy when turning on continuously gauge fields so as to reach a winding vacuum state. They are in agreement with the more detailed numerical analysis of Refs. [3,6,8].

The presence of nonrenormalizable terms, often attributed to large quantum effects [1,4,5], renders of course any discussion of electroweak Skyrmions at best phenomenological. A consistent semiclassical expansion requires a stable solution of the classical equations derived from a renormalizable Lagrangian. One would, however, expect the scale of such a hypothetical solution to be fixed by the electroweak magnetic fields, so that $\rho \sim m_W^{-1}$ and condition (2) is not *a priori* satisfied. This heuristic argument explains why stable solutions have not been found in the minimal Weinberg-Salam model [6,12]. It also suggests one possible way out: if there is more than one Higgs doublet, the relative orientation of any two of them is gauge invariant and cannot have nontrivial winding in a vacuum state. Thus, the last instability in the above list would be absent. Such relative winding excitations have been considered before [13,7] in the context of hidden-gauge-boson models of nuclear and electroweak interactions. Though stable solitons have not thus far been found, a systematic numerical search is necessary in order to settle the issue [14].

At the same time it is, we believe, important to analyze such winding solitons in lower-dimensional models. In addition to providing a check for the above heuristic arguments and guiding the numerical search in the electroweak theory, these solitons also correspond to new types of string and wall defects in four space-time dimensions. The simplest context in which they arise is the two-dimensional (2D) Abelian-Higgs model, where their existence and properties can be established analytically [11]. The results confirm the above naive discussion, but shed no light on the shrinking instability, since the size of the soliton is fixed by an infrared cutoff or by a mass term in the relative angular direction [11]. In this Rapid Communication we will therefore go one dimension up, and show how the scale of winding solitons can be stabilized by a (massive) gauge field in a renormalizable 3D gauge model. Let us note in passing that these winding solitons differ both from Q balls and from flux vortices [15]: they are static and uncharged, and have trivial topology at large distance.

Our starting point is the three-dimensional O(3) nonlinear σ model:

$$S_0 = \frac{v^2}{2}\int d^3x\, \partial_\mu \mathbf{n}\cdot \partial^\mu \mathbf{n}, \tag{3}$$

where $\mathbf{n}$ is a three-component scalar field subject to the constraint

$$\mathbf{n}\cdot\mathbf{n} = 1. \tag{4}$$

We can solve the constraint by a stereographic projection of the three-sphere onto the complex plane:

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$$n_1+in_2=\frac{2\Omega}{1+|\Omega|^2},\quad n_3=\frac{1-|\Omega|^2}{1+|\Omega|^2}. \tag{5}$$

It is well known that the above model has static winding soliton solutions [16] given by holomorphic functions $\Omega(z)$ where $z=x_1+ix_2$. The solitons are classified by the number of times two-space wraps around the target sphere:

$$N=\frac{1}{\pi}\int d^2x\,\frac{\bar\partial\bar\Omega\partial\Omega-\bar\partial\Omega\partial\bar\Omega}{(1+|\Omega|^2)^2}, \tag{6}$$

where ∂ here stands for $\partial/\partial z$. The simplest solution,

$$\Omega^{\rm sol}=\frac{\rho e^{i\theta}}{z-z_0}+w_0, \tag{7}$$

describes the soliton with unit topological charge and energy $E^{\rm sol}=4\pi v^2$. It is characterized by six real parameters reflecting the invariance of the underlying equations under the two-dimensional conformal group SL(2,C). The complex parameters w_0 is in fact fixed by the choice of boundary conditions at infinity: $w_0=0$ if $\mathbf{n}\rightarrow(0,0,1)$. The remaining four collective coordinates correspond to translations, U(1) rotations, and scale transformations of the soliton.

Let us next relax the nonlinear constraint (4) by introducing a Mexican-hat potential. By Derrick's scaling argument [10] winding configurations are now unstable against shrinking to zero size. Since we are interested in *renormalizable* models, we are not allowed to stabilize the size of the soliton with explicit higher-derivative terms in the action [17]. We must thus try to evade Derrick's argument by introducing gauge interactions. The simplest possibility is to gauge a U(1) subgroup of the global O(3) symmetry of the model. The corresponding gauge field can furthermore be massive without violating renormalizability, provided it couples to a conserved current. We are thus led to consider the action

$$S=\int d^3x\left[\frac{1}{2}|(\partial_\mu+ieA_\mu)(\Phi_1+i\Phi_2)|^2+\frac{1}{2}\,\partial^\mu\Phi_3\partial_\mu\Phi_3 -V(\Phi)-\frac{1}{4}\mathcal{F}^2_{\mu\nu}+\frac{m^2}{2}A^\mu A_\mu\right], \tag{8}$$

with

$$V(\Phi)=\frac{\lambda}{4}\left(\sum_a \Phi_a\Phi_a-v^2\right)^2+\frac{\kappa^2}{8}(\Phi_3-v)^4. \tag{9}$$

This model may look somewhat contrived. Indeed (i) the scalar potential is not the most general one consistent with the residual O(2) invariance, and (ii) the mass of the gauge field would arise more naturally by coupling it to an extra complex scalar. Our choice here is simply one of convenience: we try to use the minimum number of fields and coupling constants. The solitons we will describe should, however, continue to exist in a much larger class of 3D models.

The model defined by Eqs. (8) and (9) has trivial topology, both in its target space and in its vacuum manifold. It reduces, however, to the ungauged O(3) nonlinear σ model in the naive

$$\lambda\rightarrow\infty \quad\text{and}\quad e,\kappa\rightarrow 0 \tag{10}$$

limit. Our strategy will therefore be to show that for some range of parameters it has classically stable solitons, which are small deformations of the configuration (7) with $w_0=0$ and fixed size. To this end, let us decompose the scalar triplet into a radial and an angular part: $\Phi_a=Fn_a$, with $\mathbf{n}$ a vector of unit length which can be expressed through Ω as in Eq. (5). Working in units of the gauge-boson mass, $m=1$, and rescaling $F\rightarrow F/\sqrt{2\lambda}$ and $A_\mu\rightarrow A_\mu/\sqrt{2\lambda}$, brings the action to the form

$$S=\frac{1}{2\lambda}\int d^3x\left[\frac{1}{2}(\partial_\mu F)^2+\frac{1}{2}\,F^2|(\partial_\mu+i\tilde{e}A_\mu)(n_1+in_2)|^2+\frac{1}{2}\,F^2(\partial_\mu n_3)^2 -\frac{1}{8}(F^2-m_H^2)^2-\frac{\tilde\kappa^2}{8}(Fn_3-m_H)^4-\frac{1}{4}\mathcal{F}^2_{\mu\nu}+\frac{1}{2}\,A^\mu A_\mu\right], \tag{11}$$

with

$$m_H\equiv\sqrt{2\lambda}v,\quad \tilde{e}\equiv e/\sqrt{2\lambda},\quad\text{and}\quad \tilde\kappa\equiv\kappa/\sqrt{2\lambda}. \tag{12}$$

The above rewriting demonstrates that $\tilde\kappa$, $\tilde e$, and m_H are the only classically relevant parameters of the model. The quartic scalar coupling λ on the other hand plays the role of Planck's constant $\hbar$, and can be taken to zero independently in order to approach a semiclassical limit. The existence of classically stable winding solitons will not therefore be tied to the presence of a strongly interacting scalar sector.

To look for static minima of the energy we will proceed in two steps: we first keep the angular degree of freedom $\mathbf{n}$ fixed and time independent, and minimize the energy with respect to the radial and gauge fields F and A_μ. Assuming these stay close to their vacuum values one finds

$$F\simeq m_H\left[1-\frac{1}{m_H^2}\,\partial_i\mathbf{n}\cdot\partial_i\mathbf{n}\right], \tag{13}$$

$$A_0=0,\quad\text{and}\quad A_k(x)\simeq 2\tilde{e}m_H^2\int d^2y\; G_{kl}(x-y)J_l(y), \tag{14}$$

where

$$J_l=\frac{1}{2}(n_2\partial_l n_1-n_1\partial_l n_2) \tag{15}$$

is the U(1) current of the scalars, and

$$G_{kl}(x)=\int \frac{d^2p}{(2\pi)^2}\, e^{-i\vec{p}\cdot\vec{x}}\frac{\delta_{kl}+p_kp_l}{\vec{p}^2+1} \tag{16}$$

is the two-dimensional massive Green function. Consistency of our approximation requires that

$$\frac{1}{m_H\rho}\ll 1, \quad \tilde{\kappa} m_H\rho \ll 1, \quad \text{and} \quad \tilde{e} m_H \min(\rho,1)\ll 1, \tag{17}$$

with ρ the typical scale over which $\mathbf{n}$ varies. These conditions ensure in particular that $F-m_H\ll m_H$, and that $\tilde{e}A_i\mathbf{n}\ll\partial_i\mathbf{n}$. They give a precise meaning to the naive limit, Eq. (10). Since ρ will be determined dynamically, we must *a posteriori* check that these constraints can indeed be satisfied.

Eliminating F and A_μ with the help of Eqs. (13) and (14) we arrive at an energy functional that depends only on the angular degrees of freedom. It is of the form

$$E=E_0-\mathscr{E}, \tag{18}$$

where

$$E_0=\frac{m_H^2}{2\lambda}\int d^2x\, \frac{1}{2}\,\partial_i\mathbf{n}\cdot\partial_i\mathbf{n} \tag{19}$$

is the energy in the nonlinear σ-model limit, while

$$\mathscr{E}=\frac{m_H^2}{2\lambda}\left[\frac{1}{2m_H^2}\int d^2x(\partial_i\mathbf{n}\cdot\partial_i\mathbf{n})^2-\frac{1}{8}\,\tilde{\kappa}^2m_H^2\int d^2x(n_3-1)^4 + \tilde{e}^2m_H^2\int d^2x\int d^2yJ_i(x)G_{ik}(x-y)J_k(y)\right] \tag{20}$$

is a small perturbation under the above assumptions.

Let us here pause for a minute and consider a simple calculus problem: we are asked to minimize a function of two variables $G(u,v)=G_0(u,v)-\mathscr{G}(u,v)$, where G_0 has a line of degenerate minima along the u axis, while $\mathscr{G}$ is a small perturbation. Minimizing first with respect to v yields a line $\bar{v}(u)$ which lies *a priori* close to the u axis. Along this line one finds easily

$$G(u,\bar{v}(u))\simeq\mathscr{G}-\frac{1}{2}\mathscr{G}'(G_0'')^{-1}\mathscr{G}'+O(\mathscr{G}^3), \tag{21}$$

where the primes stand for derivatives with respect to v and all the functions on the right-hand side are evaluated at $v=0$. As shown by this formula, for the expansion in powers of $\mathscr{G}$ to be valid G_0'' must stay bounded away from zero, meaning that the valley must not become too shallow in the transverse direction. In this case the first term of the series dominates, and the minima of the function G are given by the minima of the perturbation $\mathscr{G}$ along the u axis.

Going back to the energy functional, Eq. (18), one notes that the role of u is played by the zero modes of $\Omega^{\rm cl}$, which is a local minimum of E_0, while the role of v is played by the infinite number of transverse fluctuations. Let us write $\mathbf{n}=\mathbf{n}^{\rm cl}\sqrt{1-(\delta\mathbf{n})^2}+\delta\mathbf{n}$ with $\mathbf{n}^{\rm cl}\cdot\delta\mathbf{n}=0$, and consider fluctuations which can be normalized on a sphere of radius ρ, i.e., with respect to the inner product

$$\langle\delta\mathbf{n},\delta\mathbf{n}'\rangle\equiv\int d\mu(x)\,\delta\mathbf{n}\cdot\delta\mathbf{n}',$$

$$\text{with } d\mu(x)\equiv\frac{1}{\pi\rho^2}\frac{d^2x}{(1+|x|^2/\rho^2)^2}\,. \tag{22}$$

In the vicinity of $\mathbf{n}^{\rm cl}$ the energy reads, in obvious notation,

$$E-E^{\rm cl}\simeq-\mathscr{E}(\mathbf{n}^{\rm cl})-\int d\mu\,\frac{1}{2}\,\delta\mathbf{n}^T\cdot E_0''\cdot\delta\mathbf{n} -\int d\mu\,\mathscr{E}'\cdot\delta\mathbf{n}+O(\delta\mathbf{n}^3,\mathscr{E}\delta\mathbf{n}^2). \tag{23}$$

The matrix of quadratic fluctuations E_0'' has been shown in Ref. [18] to have a discrete spectrum: $\lambda^{(j,\alpha)}=j(j+1)-2$, where $j=1,2,\ldots$ and α labels some finite degeneracy. It is furthermore straightforward to check that with the inner product (22) the first variation of the perturbation $\mathscr{E}'$ can be normalized. The analysis of the calculus problem is under these conditions easily extended to show that we need only minimize the energy in the space of zero modes of the unperturbed soliton, since transverse fluctuations affect the equations at higher orders.

Translation and rotation invariance ensures in fact that the energy does not depend on the U(1) orientation and position. For any nonzero value of $\tilde{\kappa}$ on the other hand, the energy is infinite unless $w_0=0$. The only relevant collective coordinate is thus the scale, and after a straightforward calculation we find

$$E(\rho)=\frac{2\pi m_H^2}{\lambda}\left[1+\frac{1}{6}\,\tilde{\kappa}^2m_H^2\rho^2-\frac{8}{3m_H^2\rho^2} -\tilde{e}^2m_H^2\rho^2\int_0^\infty dx\,\frac{x^3K_0^2(x)}{x^2+\rho^2}\right], \tag{24}$$

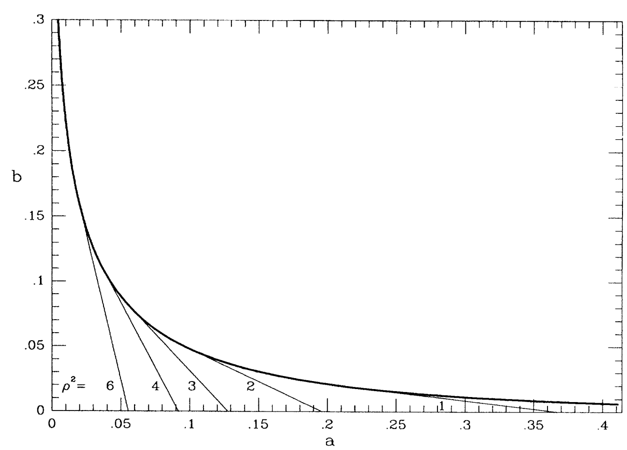

FIG. 1. For values of the parameters a and b below the thick line there exist classically stable solitons. Their size ρ is determined in our approximation by the value labeling the corresponding tangent to the thick line, as shown.

with K_0 the modified Bessel function. The shape of the function $E(\rho)$, up to overall multiplicative and additive factors, depends only on the two parameters

$$a \equiv \frac{\tilde{\kappa}^2}{\tilde{e}^2} \text{ and } b \equiv \frac{2}{\tilde{e}^2 m_H^4} . \qquad (25)$$

In the region above the thick line of Fig. 1, E grows monotonically with ρ so that, to the extent that our approximations are valid, we conclude that the would-be soliton is unstable against shrinking. In the region below this thick line, on the other hand, the function develops a local minimum at some size $\bar{\rho}(a,b)$ at which the soliton is stabilized. The tangents to the boundary of stability are lines of constant $\bar{\rho}$ as shown in the figure. To complete our proof of the existence of stable solitons, we must still make sure that conditions (17) can be satisfied. This can, however, always be arranged by taking m_H sufficiently large, while keeping a and b fixed anywhere below the thick line.

We have verified independently the existence of these winding solitons, by solving numerically the equations of motion. Determining the complete region of stability in the $(m_H, \tilde{\kappa}, \tilde{e})$ space is straightforward but beyond the scope of the present work. Let us conclude instead with a comment on the potential importance of such nontopological solitons, should they turn out to exist in the electroweak model. Since they decay by quantum tunneling, they could be stable on cosmological time scales. Assuming they are quantized as bosons, they should have zero charge and higher moments in their ground state. Furthermore their expected size is $\sim 1/m_W$, their expected mass in the TeV range, while their annihilation cross section, being essentially geometrical, should be somewhat larger than weak cross sections. These properties would make them serious candidates for cold dark matter in the Universe.

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