Heavy τ neutrino as the late decaying particle in the cold dark matter scenario

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The τ neutrino with a mass of about 10 MeV can be the "late decaying particle" in the cold dark matter scenario for the formation of structure in the Universe. We show how this may be realized specifically in the recently proposed doublet Majoron model.

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Study of the mass and interaction of neutrinos is a longstanding subject in particle physics and offers one of the key clues to possible new phenomena beyond the standard model. Majoron models have been attracting a lot of interest in this respect. They provide the neutrinos with Majorana masses and a new type of interaction not present in the standard model. The interaction is due to the coupling of the neutrinos and other matter to the Majoron, a Nambu-Goldstone boson associated with the violation of the lepton number symmetry. This extra interaction was originally used to facilitate the decay of a massive neutrino which would otherwise be ruled out due to the cosmological constraint on the neutrino mass [1]. In a different application, it allows a stable massive neutrino to be suitable as a dark matter candidate $[2]$. In this paper, we examine another possibility that this interaction may bring about: the τ neutrino (v_{τ}) as a candidate for the late decaying particle in the cold dark matter (CDM) scenario for the formation of structure in the Universe [3—5].

The idea of the late decaying particle was proposed to reconcile a setback of the CDM model in explaining the formation of large-scale structure in the Universe [3,4]. This setback became more evident by the recent Cosmic Background Explorer (COBE) detection of anisotropy in the temperature of the cosmic background radiation: The theoretical prediction for an $\Omega = 1$ inflationary universe on the power spectrum of the density fluctuation gives a larger power than the observation at small scales $\lambda \le 10h^{-1}$ Mpc (the Hubble constant $H_0 = 100h$ km/s Mpc) once it is normalized at large scales $\lambda \sim 10^{3} h^{-1}$ Mpc using the COBE detection [6]. A remedy is to delay the time of matter-radiation equality, at which subhorizon-sized fluctuations begin to grow. The delay reduces the power on the small scales.

A massive particle species that decays into relativistic particles can do this trick. Since the decreasing rate of the relic energy density of massive matter is smaller than that of radiation, its energy necessarily dominates the Universe if it is sufficiently long lived. Its subsequent decay into relativistic particles gets the Universe into the radiation-dominant era again with more energy, and delays the time of forthcoming matter-radiation equality.

In the late decaying particle scenario, we have two matterradiation equality epochs: first the relic v_{τ} dominates the Universe and then the cold dark matter does. They are separated by a radiation-dominated era after ν_{τ} decay. We distinguish values of cosmological variables at these epochs with subscripts EQ1 for the former and EQ2 for the latter, e.g., the age of the Universe t_{EQ1} , the temperature T_{EQ1} , the horizon size λ_{EO1} , etc.

Recently, Dodelson et al. examined a scenario in which a massive τ neutrino, with its mass in a range $m_{\nu} \sim 1-10$ MeV, may be the late decaying particle [5].(See also Ref. [7] for other candidates in particle physics models.) An important constraint for this scenario is the one from primordial nucleosynthesis: the equivalent number of massless neutrino species N_v should be less than 3.3 [8] or 3.04 [9] but a τ neutrino of this mass range may possibly contribute more than this bound [10].

This difficulty is rather easily evaded in a Majoron model, thanks to the new interaction between ν_{τ} and the Majoron φ_L . In the standard model, the relic abundance Y_∞ of ν_τ , the ratio of its number density to the entropy density (see [11] for the definition), is determined by the speed of the annihilation process via Z^0 exchange compared to the Universe expansion [11,12]. In Majoron models the process

$$
\nu_{\tau}\nu_{\tau}\to\varphi_L\varphi_L\tag{1}
$$

also works to decrease the relic abundance. The v_{τ} - φ_L coubling is proportional to $m_{\nu_{\tau}}/v_L$, where v_L is a scale for the lepton number violation. Thus the process (1) can be still active even after the Z^0 exchange process shuts off if this $m_{\nu_{\tau}}$ to v_L ratio is sufficiently large. Then the relic density can be much smaller than the one in the standard model. This can make ν_{τ} invisible with respect to the dynamical evolution of the Universe at the time of primordial nucleosynthesis and avoid the resulting constraint. In other words, the lifetime upper bound of 100 s estimated for a heavy v_{τ} to be the late decaying particle in Ref. [5] is no longer a constraint in this case.

The doublet Majoron (DM) model which we have proposed recently [13] is very suitable for the late decaying particle scenario. An advantage of this model, compared with the singlet Majoron model, is that it allows us to use a smaller scale v_L for the lepton number violation and thus a large v_r - φ _L coupling without conflicting with the basic concept of the seesaw mechanism [14]. We estimate T_{EQ1} for various values of v_L and m_{ν} , and show that it indeed provides a possibility that v_{τ} can be the late decaying particle.

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Let us first estimate how long the temperature T_{E01} should be in order to satisfy the constraint from primordial nucleosynthesis. The precise definition for T_{E01} is the temperature at which the τ neutrino energy $\rho_{\nu} = (2\pi^2/45) m_{\nu} Y_{\infty} g_{*S} T^3$ becomes the same as the radiation energy $\rho_R = (\pi^2/30)g_* T^4$:

$$
T_{\text{EQ1}} = \frac{60}{45} \frac{g_{*s}}{g_{*}} m_{\nu_{\tau}} Y_{\infty}, \tag{2}
$$

where g_{*} and g_{*} are the statistical weights of the light degrees of freedom for the energy and entropy density, respectively. They are functions of temperature; specific values of g_* and g_{*s} at a given temperature depend on the details of the thermal history of the Universe. In the standard model with three massless neutrino species, all the light degrees of freedom, photon (γ) , electron (e) , positron (e^+) , and three neutrinos v_e , v_μ , and v_τ , keep thermal contact before the freezing out of the Z^0 exchange process, which takes place when the Universe cools down to $T \sim 1$ MeV. The statistical weights are then $g_* = g_{*s} = 10.75$. These values stay the same until e and e^+ become nonrelativisitc and annihilate in pairs at the temperature of a few tenths of MeV. After this annihilation, they decrease to $g_* = 3.36$ and $g_{*s} = 3.91$. In the late decaying particle scenario of v_{τ} based on the DM model, these values change because of the existence of the Majoron and the assumption that v_{τ} is heavy. The values we specifically use correspond to the following situation: v_{τ} was so heavy, $m_{\nu_{\tau}}$ is greater than a few MeV, that it was nonrelativistic when the Z^0 exchange process for its annihilation shuts off; its abundance is frozen before e and e^+ annihilate in pairs; i.e., let T_f denote the freezing temperature then T_f is greater than a few tenths of MeV. Thus we use $g_* = g_{*s} = 10$ (a sum of contributions from γ , e, e⁺, v_e , v_{μ} , and φ_L) for evaluating T_f and Y_{∞} . After the pair annihilation of e and e^+ , they reduce to $g_* = 3.17$ and g_{*s} =3.64. If v_{τ} is relativistic at the termination of the Z^0 process, these values change. Also there may occur a deviation in the temperatures of γ and φ_L . We, however, neglect these subtleties in this paper. For a lighter neutrino, $m_{\nu_{\tau}} \sim 1$ MeV, this may cause an error; but the numerical error in the final results of T_{EQ1} expected by this neglect is at most 30– 40% and does not affect the discussions we make in this paper.

The temperature at which primordial nucleosynthesis commences is about 1 MeV $[11]$. At this temperature, we need to satisfy the condition g_* <11.3 or 10.82, which corresponds to the bounds N_v <3.3 [8] or 3.04 [9], respectively. Since $g_* = 10$ at $T \sim 1$ MeV, the v_{τ} fraction in the energy density must satisfy

$$
\frac{\rho_{\nu_{\tau}}}{\rho_R} \le 0.13 \quad \text{or} \quad 0.08,\tag{3}
$$

respectively. On the other hand, the fraction in terms of T_{EQ1} is given by

$$
\frac{\rho_{\nu_{\tau}}}{\rho_R} \simeq \frac{g_{*S}(1 \text{ MeV})}{g_{*}(1 \text{ MeV})} \frac{g_{*}(T_{\text{EQ1}})}{g_{*S}(T_{\text{EQ1}})} \frac{T_{\text{EQ1}}}{1 \text{ MeV}} \sim \frac{T_{\text{EQ1}}}{1 \text{ MeV}} \tag{4}
$$

as long as T_f is higher than 1 MeV. Thus T_{EQ1} needs to be less than 0.08 MeV.

To evaluate Y_∞ , we use formulas given by Kolb and Turner [11]:

$$
Y_{\infty} = \frac{3.79(n+1)x_f^{n+1}}{(g_{*S}/g_*^{1/2})m_{\text{Pl}}m_{\nu_\tau}\sigma_0},\tag{5}
$$

and

$$
x_f = \ln[0.038(n+1)(g/g_*^{1/2})m_{\rm Pl}m_{\nu_\tau}\sigma_0]
$$

$$
-(n+\frac{1}{2})\ln{\{\ln[0.038(n+1)(g/g_*^{1/2})m_{\rm Pl}m_{\nu_\tau}\sigma_0]\}},
$$
 (6)

where $m_{\text{Pl}} = 1.2 \times 10^{19}$ GeV is the Planck mass, $g = 2$ for v_{τ} , *n* and σ_0 are read off from the form of the thermally averaged cross section for the process

$$
\langle \sigma v_{\rm rel} \rangle = \sigma_0 (T/m_{\nu_\tau})^n. \tag{7}
$$

The freezing temperature is given by $T_f \approx m_{\nu} x_f^{-1}$.

The evaluation of the cross section is straightforward. The appropriate interaction terms are given by

$$
\mathcal{L} = \frac{\partial^{\mu} \varphi_L}{2v_L} (\nu_{\tau}^{\dagger} \tilde{\sigma}_{\mu} \nu_{\tau}) + \frac{(\partial^{\mu} \varphi_L)(\partial_{\mu} \varphi_L)}{2v_L} (\cos \alpha \phi_{-} + \sin \alpha \phi_{+})
$$

+
$$
\frac{m_{\nu_{\tau}}}{2v_L} (\cos \alpha \phi_{-} + \sin \alpha \phi_{+}) \{ (\nu_{\tau}^T i \sigma^2 \nu_{\tau}) - (\nu_{\tau}^{\dagger} i \sigma^2 \nu_{\tau}^{*}) \},
$$
(8)

where ϕ_{\pm} are neutral scalar fields, α and β are mixing angles, which are the same as those defined in Ref. [13].The corresponding cross section in the singlet Majoron model has been evaluated in Ref. [15].We calculate the same Feynman diagrams as those in [15]: two diagrams with the v_τ - φ_L vertices and two scalar-exchange diagrams. The freezing of the abundance mostly takes place when the initial two v_r 's are nonrelativistic and their energy is much smaller than the mass of ϕ . In this energy region, the ϕ ₊-exchange diagrams are negligible and we obtain the same result as the one in Ref. [15],

$$
\sigma = \frac{1}{96\pi} \frac{m_{\nu_{\tau}}^2 |{\bf p}|}{v_L^4 E},
$$
 (9)

in terms of the energy E and the momentum **p** of one of the initial neutrinos in the center of mass frame. A characteristic behavior of σ , i.e., it vanishes at vanishing $|\mathbf{p}|$, comes from a combined effect of the statistics of identical particles and the conservation of both angular momentum and \mathcal{CP} ; the latter can be defined as a discrete symmetry of the v_{τ} - φ_L interaction and forbids the s-wave contribution. The thermally averaged cross section is given by the integration over the distribution function for the relative velocity v_{rel} = $2|{\bf p}|/E:$ ¹

 $\frac{I_{\text{EQ1}}}{I_{\text{MeV}}}$ (4) ¹Note that the initial neutrinos in the process (1) are identical Majorana particles. Thus the event rate of the process per unit co-

FIG. 1. Contour plot of $log_{10}[T_{EQ1}/1 \text{ GeV}]$. The contours correspond to $log_{10}[T_{EQ1}/1 \text{ GeV}]=-5, -6, -7, -8, -9, \text{ and } -10$ from top to bottom.

$$
\langle \sigma v_{\text{rel}} \rangle = \int d^3 v_{\text{rel}} \left(\frac{m_{\nu_{\tau}}}{4 \pi T} \right)^{3/2} e^{-m_{\nu_{\tau}} v_{\text{rel}}^2 / 4T} \sigma v_{\text{rel}}
$$

$$
= \frac{1}{32 \pi} \frac{m_{\nu_{\tau}} T}{v_L^4} . \tag{10}
$$

This result gives $\sigma_0 = (1/32\pi)(m_{\nu_p}^2/v_L^4)$ and $n = 1$ in (7).

We plot the values of T_{EQ1} obtained by Eqs. (2), (5), and (6) for various values of v_L and m_{ν_τ} as contours in Fig. 1. The thermal history of the Universe we have assumed based on the DM model is correct for most of the range of values of $m_{\nu_{\tau}}$ and v_L shown in Fig. 1. In order to satisfy the constraint from the primordial nucleosynthesis, a parameter region with T_{EQ1} less than 0.01 MeV is safe: This stringent value, when compared with Eqs. (3) and (4), is extracted by an observation that the formulas (5) and (6) are not good for v_{τ} in a region around $m_{v} \sim 4$ MeV and $v_L \sim 30$ GeV; it is dangerous to determine if v_{τ} in this region can be a late decaying particle without an detailed numerical analysis on the abundance. For definiteness, we will use this stringent constraint hereafter. Obviously T_{EO1} needs to be higher than T_{EQ2} , which is about 1 eV, in order that ν_{τ} plays the role of the late decaying particle. (Note that v_{τ} with T_{E01} less than 1 eV can be a candidate of dark matter [2].) Thus the parameter region in Fig. 1 for accommodating the late decaying particle is²

$$
-9 < log10 \left(\frac{T_{\text{EQ1}}}{1 \text{ GeV}} \right) \le -5. \tag{11}
$$

In the DM model, v_L smaller than 10 GeV predicts a light scalar boson ϕ ₋, which can be a rare decay product in $Z^0 \rightarrow \phi_- +$ (a fermion pair), and contradicts the known lower mass bound, about 60 GeV, for the standard Higgs boson [13].³ Thus a region $v_L \ge 10$ GeV is left for the τ neutrino to be the late decaying particle. The temperature T_{EQ1} is, then, 1—10 keV.

The lifetime of the τ neutrino, τ_{ν} , should be adjusted so that it generates an appropriate amount of radiation energy in its decay. Let us estimate the required lifetime. We use the sudden-decay approximation and assume v_{τ} decays all at once at the age $t_D \approx \tau_{\nu_\tau}$ and temperate T_D . The total radiation energy density after the decay, including the decay product R' , is

$$
\rho_{R+R'} = \frac{\pi^2}{30} g_* T^4 \bigg(1 + \frac{T_{\text{EQ1}}}{T_D} \bigg). \tag{12}
$$

To fit the predicted power spectrum of the density fluctuation to the observation, this needs to be about 3 times bigger than the radiation energy in the standard prediction [5]. Thus $T_{\text{EQ1}}/T_D \approx 2$. Taking into account the relation $T \propto t^{-2/3}$ in the v_{τ} -dominated era, we get $\tau_{v_{\tau}} \approx 3t_{\text{EQ1}}$. Until the age t_{EO1} , the Universe was radiation dominated. Thus it is estimated by $t_{\text{EQ1}} \approx \frac{1}{2} H_0^{-1} \Omega_R^{-1/2} (R_{\text{EQ1}}/R_0)^2$ in terms of the density parameter Ω_R , the ratio of the radiation energy density to the critical one, and the cosmic scale factors R_0 at the present and $R_{\text{EO}1}$ at $T_{\text{EO}1}$. Since the photon temperature is inversely proportional to the scale parameter after e and e^+ annihilate in pairs, it is written as

$$
t_{\text{EQ1}} \approx 2.4 \times 10^{19} \left(\frac{T_0}{T_{\text{EQ1}}} \right)^2
$$
 s, (13)

where $T_0 = 2.735$ K is the present (photon) temperature of the Universe. $\tau_{\nu_{\tau}}$ is then estimated as $10^4 - 10^6$ s. The lifetime in the DM model is given by [13]

$$
\tau_{\nu_{\tau}}^{-1} = \frac{1}{64\pi} |R_{\nu_{\tau}\nu_{a}}|^{2} \frac{m_{\nu_{\tau}}^{3}}{\nu_{L}^{2}},
$$
\n(14)

where $R_{\nu_{\tau}\nu_{a}}$ is the flavor-changing matrix element between v_{τ} and lighter neutrinos $(v_a = v_3, v_{\mu})$. If we take $v_L \approx 20$ GeV and $m_{\nu} \approx 10$ MeV to get an idea of the magnitude of R|, it resides in a range $|R| \sim 10^{-9} - 10^{-10}$. The smallness of these values can be regarded as a result of the seesaw mechanism: $|R|$ can be parametrized as $[(m_{\nu}/M)\sin\theta]$ with M the mass scale of the gauge singlet neutrino and θ a mixing angle; the values we used for $m_{\nu_{\tau}}$ and v_L imply $M \sim 10 \text{ TeV}$ and $|R| \sim 10^{-6} \theta$.

In a late-decaying-particle scenario for structure formation, we necessarily have an extra small scale corresponding

moving volume is $\frac{1}{2}$ of $\langle \sigma v_{rel} \rangle$ defined this way. In the Boltzman equation, which is the basis to derive the formulas (5) and (6) [11], this factor $\frac{1}{2}$ is canceled by another factor 2 that represents the fact that two neutrinos annihilate in the process.

 ${}^{2}T_{f}$ is lower than 1 MeV for ν_{τ} with some values of the parameters allowed by this inequality, and Eq. (4) does not apply for its energy fraction. But the fraction evaluated by using the equilibrium distribution satisfies the constraint (3) at $T \sim 1$ MeV.

³Since $\phi = -Z^0 - Z^0$ coupling is proportional to $(-\cos \beta \sin \alpha$ $+\sin \beta \cos \alpha$, we can fix this problem by tuning the two mixing parameters in the model, i.e., $\alpha \approx \beta$; but we will not pursue it in this paper.

to the horizon at t_{EO1} . Its size λ_{EO1} at T_{EO1} , after being scaled up to the present taking into account the expansion of the Universe, is

$$
\lambda_{\text{EQ1}} \simeq 4.8 \times 10^5 \left(\frac{T_0}{T_{\text{EQ1}}} \right) \text{ Mpc.}
$$
 (15)

Thus $\lambda_{E01} \approx 10-100$ kpc for the allowed parameters in the DM model. The consequence of the existence of this scale for structures in the Universe needs to be clarified by further investigations. It may be related to the dwarf galaxies [3].

In summary, we have shown that the DM model represents an appropriate particle physics model for realizing a possibility that the τ neutrino, with a mass of about 10 MeV and a lifetime in the range $10⁴$ to $10⁶$ s, is the late decaying particle in the CDM scenario for the formation of structure in the Universe.

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