

Derivative expansion of the effective action and vacuum instability for QED in 2+1 dimensions

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We investigate the effective action of (2+1)-dimensional charged spin-1/2 fermions and spin-0 bosons in the presence of a U(1) gauge field. We evaluate terms in an expansion up to second order in derivatives of the field strength, but exactly in the mass parameter and in the magnitude of the nonvanishing constant field strength. We find that in a strong uniform magnetic field background, space-derivative terms lower the energy, and there arises an instability toward inhomogeneous magnetic fields.

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The effective action provides an extremely useful tool for the investigation of instabilities such as spontaneous symmetry breakdown in quantum field theory. These phenomena are frequently driven by low momentum dynamics, so that a small momentum approximation to the effective action may suffice. Electrodynamics in (2+1)-dimensional space-time (QED₃) with massless or massive charged spin-1/2 fermions or spin-0 bosons, especially in the presence of a strong uniform magnetic field, is a field theory model with potentially many applications in condensed matter and particle physics [1]. Its dynamics appears intricate and incompletely understood, and may well reveal exciting new physical phenomena.

In the present Rapid Communication, we evaluate the effective action for spin-1/2 fermions or spin-0 bosons of charge e in the presence of a U(1) gauge field in a derivative expansion [2]. We obtain contributions with no derivatives and two derivatives (total) on the field strength, but our result is exact in the mass of the charged particle, and is also exact in the magnitude of the field strength. This calculation—in fact to all orders in derivatives—results entirely from one-loop effects, and reduces to the evaluation of functional determinants of spin-1/2 and spin-0 gauge covariant derivatives [3]

$$i \int dx \mathcal{L}_{\mp} = \pm \ln \text{Det} \{ D_{\mu} D^{\mu} + m^2 + e \Sigma_{\mp}^{\mu\nu} F_{\mu\nu} \}. \quad (1)$$

Here, \mathcal{L}_{\mp} are the effective Lagrangians for fermions (−) and bosons (+), and $D_{\mu} = \partial_{\mu} + ieA_{\mu}$. For \mathcal{L}_{-} , $\Sigma^{\mu\nu} = (i/4)[\gamma^{\mu}, \gamma^{\nu}]$ produces the effective action for a 4-component spinor consisting of 2-component spinors of masses m and $-m$, respectively, whereas for \mathcal{L}_{+} , $\Sigma^{\mu\nu} = 0$ produces the effective action for spin-0 complex scalars, for which the spin coupling term is, of course, absent. The ef-

fective action for a single massive 2-component spinor fermion differs from \mathcal{L}_{-} through the inclusion of the parity-violating Chern-Simons term [4]. To obtain a sensible derivative expansion, we assume either that the derivatives (i.e., the momenta) are small compared to the mass m , or that they are small compared to the background field strength magnitude, such as the magnitude of a constant magnetic field, or both. Under these circumstances, we have the expansion (valid for both actions in (2+1)-dimensional space-time)

$$\begin{aligned} \mathcal{L}^{(2)} = & p_0 + e^4 [\partial_{\mu} F^2 \partial^{\mu} F^2] p_1 + e^2 [\partial_{\mu} F_{\nu} \partial^{\mu} F^{\nu}] p_2 \\ & + e^2 [\partial_{\mu} F_{\nu} \partial^{\nu} F^{\mu}] p_3 + e^6 [F^{\mu} \partial_{\mu} F^2 F^{\nu} \partial_{\nu} F^2] p_4 \\ & + e^4 [F^{\mu} \partial_{\mu} F_{\alpha} F^{\nu} \partial_{\nu} F^{\alpha}] p_5 + e^4 [\partial_{\mu} F^2 F^{\nu} \partial_{\nu} F^{\mu}] p_6 \end{aligned} \quad (2)$$

up to terms involving at least three derivatives. We find it more convenient to use the “dual” field strength $F^{\mu} = \frac{1}{2} \epsilon^{\mu\nu\kappa} F_{\nu\kappa}$, and $F^2 = F_{\mu} F^{\mu} = \frac{1}{2} F_{\mu\nu} F^{\mu\nu}$, with $B = -F_0$, $E^i = -\epsilon^{0ij} F_j$ the usual magnetic and electric fields. We have made use of the fact that F^{μ} satisfies the Bianchi identities $\partial_{\mu} F^{\mu} = i(\partial E - \vec{\partial} \vec{E}) - \vec{B} = 0$ to restrict possible contributions in (2). The coefficients p_i are functions of $e^2 F^2 = e^2 (B^2 - \vec{E}^2)$, and of the mass parameter m . They are Lorentz and gauge invariant and even under $F^{\mu} \rightarrow -F^{\mu}$. Notice that in (2) we have not retained terms that are odd under $F^{\mu} \rightarrow -F^{\mu}$, even though they would be allowed by Lorentz and gauge invariance (for example $\epsilon^{\mu\nu\kappa} \partial_{\mu} F^{\alpha} \partial_{\alpha} F_{\nu} F_{\kappa}$ and $\epsilon^{\mu\nu\kappa} \partial_{\mu} F^2 F^{\alpha} \partial_{\alpha} F_{\nu} F_{\kappa}$). Such terms have vanishing contribution in view of charge conjugation symmetry of (1), a property usually referred to as Furry’s theorem.

The above expansion may easily be rewritten in terms of \vec{E} and B , which may be particularly useful when investigating dynamics around large constant magnetic fields. We can, in fact, exploit the global Lorentz invariance of the effective action to express the effective Lagrangian (2) in a Lorentz frame in which the constant part of \vec{E} vanishes [5]

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$$\begin{aligned} \mathcal{L}^{(2)} = & q_0 + e^2[\bar{\partial}B\partial B]q_1 + e^2[\bar{E}\dot{E}]q_2 + e^2[\bar{\partial}\dot{E}\partial E]q_3 \\ & + e^2[\bar{\partial}E\partial\dot{E}]q_4 + \frac{1}{2}e^2[\partial E\partial E + \bar{\partial}\dot{E}\bar{\partial}\dot{E}]q_5 \\ & - \frac{i}{2}e^2[\partial B\dot{E} - \bar{\partial}B\dot{E}]q_6, \end{aligned} \quad (3)$$

where the q_i are functions of eB with the renormalization condition $q_0(0)=0$. The functions p_i and q_i are algebraically related as follows:

$$\begin{aligned} p_0 &= q_0, \\ p_1 &= -(q_1 + q_3 + q_4 + q_5)/(16e^2F^2), \\ p_2 &= (q_3 + q_4 + q_5)/4, \\ p_3 &= -(q_3 - q_4 + q_5)/4, \\ p_4 &= (q_1 + 4q_2 + q_3 + q_4 - q_5 - 2q_6)/(4e^2F^2)^2, \\ p_5 &= -(4q_2 + q_3 + q_4 + q_5)/(4e^2F^2), \\ p_6 &= (q_3 - q_4 + q_5 + q_6)/(4e^2F^2). \end{aligned} \quad (4)$$

In the remainder of the Rapid Communication, we shall determine the functions p_i and q_i explicitly, using the Schwinger proper time method [6], reformulated in terms of quantum mechanical path integrals over closed loops $y^\mu(\tau+T)=y^\mu(\tau)$ and for fermions also with additional one-dimensional Grassmann variables [7] $\psi^\mu(\tau+T)=-\psi^\mu(\tau)$. This method is particularly convenient to use in the background of constant \vec{E} and B fields, where ordinary Feynman diagram techniques are cumbersome [4,8]. The effective Lagrangian is given in dimensional regularization around $d=3$ by the following expectation value [9]

$$\mathcal{L}_\mp = C_\mp \int_0^\infty \frac{dT}{T} (2\pi\mathcal{E}T)^{-d/2} e^{-m^2\mathcal{E}T/2} \langle e^{\int_0^T d\tau L_\mp^I} \rangle_{L_\mp^{\text{free}}} \quad (5)$$

with $C_- = -1$ for a 2-component spinor, $C_- = -2$ for a 4-component spinor and $C_+ = 1$ for a complex scalar. Here the free and interacting Lagrangians are given by

$$\begin{aligned} L_+^{\text{free}} &= \frac{1}{2\mathcal{E}} \dot{y}_\mu \dot{y}^\mu & L_+^I &= -ieA_\mu(x_0+y)\dot{y}^\mu \\ L_-^{\text{free}} &= L_+^{\text{free}} + \frac{1}{2}\psi_\mu \dot{\psi}^\mu & L_-^I &= L_+^I + \frac{ie}{2}\mathcal{E}\psi^\mu F_{\mu\nu}(x_0+y)\psi^\nu. \end{aligned} \quad (6)$$

The vacuum expectation value in (5) is taken with respect to the free Lagrangian, x_0 is the average position of the closed loop at which \mathcal{L}_\mp is evaluated and $\int_0^T d\tau y(\tau)=0$. To evaluate \mathcal{L}_\mp in a derivative expansion, we expand L_\mp^I in derivatives of A and F . In the Fock-Schwinger gauge, we have

$$\begin{aligned} A_\mu(x_0+y) &= \frac{1}{2}y^\rho F_{\rho\mu}(x_0) + \frac{1}{3}y^\sigma y^\rho \partial_\sigma F_{\rho\mu}(x_0) \\ &+ \frac{1}{8}y^\omega y^\sigma y^\rho \partial_\omega \partial_\sigma F_{\rho\mu}(x_0) + \dots \end{aligned} \quad (7)$$

We need to retain second order derivatives in this expansion to linear order, since by integration by part in x_0 , they yield terms bilinear in single derivatives of F .

As is well known, the constant $F_{\mu\nu}$ problem is quadratic and may be solved completely [6,10]. The derivative expansion we are interested in is thus a perturbation around constant $F_{\mu\nu}$. We denote by L^n for $n \geq 0$, the contribution to the interaction Lagrangian L^I resulting from the n th expansion term in (7), containing n derivatives on F . It is convenient to rearrange the expectation value of (5) as

$$\begin{aligned} & \left\langle \exp \int_0^T d\tau L^I \right\rangle_{L^{\text{free}}} \\ &= \left\langle \exp \int_0^T d\tau L^0 \right\rangle_{L^{\text{free}}} \left\langle \exp \int_0^T d\tau (L^I - L^0) \right\rangle_{L^{\text{free}}+L^0}. \end{aligned} \quad (8)$$

To the order we are interested in, the exponential in the second factor may be expanded, and we get

$$\begin{aligned} & \left\langle \exp \int_0^T d\tau (L^I - L^0) \right\rangle_{L^{\text{free}}+L^0} \\ &= 1 + \int_0^T d\tau \langle L^2 \rangle_{L^{\text{free}}+L^0} \\ &+ \frac{1}{2} \int_0^T d\tau \int_0^T d\tau' \langle L^1(\tau)L^1(\tau') \rangle_{L^{\text{free}}+L^0}. \end{aligned} \quad (9)$$

The first factor in (8) is just the constant electromagnetic field problem, and is easily evaluated

$$\begin{aligned} & \left\langle \exp \int_0^T d\tau L_\mp^0 \right\rangle_{L_\mp^{\text{free}}} \\ &= \begin{cases} (bT/2)\coth(bT/2) & \text{for } (-) \text{ fermions} \\ (bT/2)/\sinh(bT/2) & \text{for } (+) \text{ bosons.} \end{cases} \end{aligned} \quad (10)$$

We shall henceforth use the abbreviation $b=e(F^2)^{1/2}$. To obtain the correction from derivative terms of F in (9), we need y^μ and ψ^μ propagators, in the presence of constant $F_{\mu\nu}$ fields. They are

$$\begin{aligned} \langle y^\mu y^\nu \rangle &= (\eta^{\mu\nu} - \hat{F}^\mu \hat{F}^\nu)G_0 + \hat{F}^\mu \hat{F}^\nu G_1 + i\epsilon^{\mu\nu\kappa} \hat{F}_\kappa G_2, \\ \langle \psi^\mu \psi^\nu \rangle &= (\eta^{\mu\nu} - \hat{F}^\mu \hat{F}^\nu)S_0 + \hat{F}^\mu \hat{F}^\nu S_1 + i\epsilon^{\mu\nu\kappa} \hat{F}_\kappa S_2, \end{aligned} \quad (11)$$

where $\hat{F}^\mu = F^\mu(F^2)^{-1/2}$. The scalar functions G and S are given as functions of $\tau = \tau_1 - \tau_2$

$$\begin{aligned} G_0(\tau) &= -\frac{1}{2b} \frac{\cosh(b|\tau| - bT/2)}{\sinh(bT/2)} + \frac{1}{b^2 T}, \\ G_1(\tau) &= -\frac{1}{2T} |\tau|^2 + \frac{1}{2} |\tau| - \frac{T}{12}, \\ S_0(\tau) &= -\frac{1}{2} \epsilon(\tau) \frac{\cosh(b|\tau| - bT/2)}{\cosh(bT/2)}, \end{aligned} \quad (12)$$

$$S_1(\tau) = -\frac{1}{2}\epsilon(\tau).$$

Here $\epsilon(\tau) = +(-)$ for $\tau > 0 (< 0)$, and the functions G_2 and S_2 will not be needed explicitly, except for the fact that $\dot{G}_2 = -bG_0, \dot{S}_2 = -bS_0$.

From the above formalism, we obtain the following results for the functions q_i in (3):

$$q_i = \left(\frac{1}{4\pi eB}\right)^{3/2} \int_0^\infty ds e^{-(m^2/eB)s} s^{-3/2} f_i(s). \quad (13)$$

In the fermion case (2-component spinor), they are expressed in terms of the function $\ell_-(s) \equiv s \coth s$:

$$f_0 = -(eB)^3 \frac{1}{s} (\ell_- - 1),$$

$$f_1 = -\frac{s}{2} \ell_-''',$$

$$f_2 = -\frac{3}{4s} \ell_-'' + \frac{1}{2},$$

$$f_3 = -\frac{1}{6}(s^2 - 3)\ell_-'' - \frac{1}{3}(s^2 + 3)\frac{1}{s}\ell_-'' + \frac{1}{3}\ell_-',$$

$$f_4 = \frac{1}{8}\ell_-'' + \frac{1}{2s}\ell_-'' - \frac{1}{6}\ell_-' - \frac{1}{4},$$

$$f_5 = -\frac{1}{12}(3 - 2s^2)\ell_-'' - \frac{1}{6}(s^2 - 3)\frac{1}{s}\ell_-'' + \frac{1}{3}\ell_-' - \frac{1}{2},$$

$$f_6 = -\frac{1}{4}\ell_-'' - \frac{1}{6}(s^2 + 3)\frac{1}{s}\ell_-'' + \frac{1}{2}, \quad (14a)$$

whereas in the boson case, they are written in terms of $\ell_+ \equiv s/\sinh s$:

$$f_0 = (eB)^3 \frac{1}{s} (\ell_+ - 1),$$

$$f_1 = \frac{s}{2} \ell_+'' + \frac{s}{2} \ell_+',$$

$$f_2 = \frac{3}{4s} \ell_+'' + \frac{1}{4} \ell_+',$$

$$f_3 = \frac{1}{6}(s^2 - 3)\ell_+'' - \frac{1}{6}(s^2 - 6)\frac{1}{s}\ell_+'' + \frac{1}{6}\ell_+',$$

$$f_4 = -\frac{1}{8}\ell_+'' - \frac{1}{2s}\ell_+'' - \frac{5}{24}\ell_+',$$

$$f_5 = \frac{1}{12}(3 - 2s^2)\ell_+'' + \frac{1}{6}(s^2 - 3)\frac{1}{s}\ell_+'' - \frac{1}{12}\ell_+',$$

$$f_6 = \frac{1}{4}\ell_+'' + \frac{1}{6}(s^2 + 3)\frac{1}{s}\ell_+'' + \frac{1}{4}\ell_+'. \quad (14b)$$

The integrals in (13) can all be expressed explicitly in terms of generalized Riemann ζ functions [11].

It is instructive to consider two important physical limits. In the massless limit, $m = 0$, we have

$$[\mathcal{L}_{\mp}^{(2)}]_{m=0} = -\frac{(eB)^{3/2}}{\sqrt{2}(2\pi)^2} \left(\alpha_{\mp} + \beta_{\mp} \frac{\bar{\partial}B \partial B}{eB^3} \right) \quad (15)$$

with $\alpha_- = \zeta(3/2) \approx 2.6$, $\beta_- = -(15/16\pi) \zeta(5/2) \approx -0.4$ and $\alpha_+ = (1 - 1/\sqrt{2}) \zeta(3/2) \approx 0.8$, $\beta_+ = -(\sqrt{2} - 1) \pi/4 \zeta(1/2) - [1 - 1/(2\sqrt{2})](15/16\pi) \zeta(5/2) \approx 0.2$. Notice that $\mathcal{L}^{(2)}$ diverges in the $B \rightarrow 0$ limit for massless particles. Rather, the small B limit should be taken relative to the scale set by the fermion or boson mass. Thus, one should expand the effective Lagrangian in terms of the ratio of the cyclotron energy scale eB/m and the rest mass energy scale m :

$$\mathcal{L}_-^{(2)} = -\frac{m^3}{24\pi} \left(\frac{eB}{m^2}\right)^2 + \frac{\bar{\partial}B \partial B}{eB^3} \frac{m^3}{60\pi} \left(\frac{eB}{m^2}\right)^3 + \dots \quad (16a)$$

$$\mathcal{L}_+^{(2)} = -\frac{m^3}{48\pi} \left(\frac{eB}{m^2}\right)^2 + \frac{\bar{\partial}B \partial B}{eB^3} \frac{m^3}{240\pi} \left(\frac{eB}{m^2}\right)^3 + \dots \quad (16b)$$

The most immediate physical consequence to be drawn from this work concerns the stability of a state in which the background electric and magnetic fields are nonzero. In four-dimensional QED, a uniform electric background field produces an instability which leads to the spontaneous creation of electron-positron pairs. In the present case of three-dimensional QED, the same instability exists for electric fields. In addition, however, there now also arises an instability related purely to magnetic fields.

For both bosons and fermions, the presence of a uniform magnetic field increases the energy of the state for all values of the mass, as can be seen directly from (13) and (14) [$f_0(s)$ is a negative function for fermions and for bosons]. On the other hand, the presence of inhomogeneities in the magnetic field may lower the energy. This is determined by the functions $f_1(s)$ appearing in (14). For large mass [$m \gg (eB)^{1/2}$], the leading derivative terms in the effective action (16) have a positive coefficient and lower the energy as soon as inhomogeneities are introduced. For bosons, this phenomenon disappears when the mass falls below a certain critical value $m \approx 0.9(eB)^{1/2}$. For fermions, however, the sign of the derivative term does not change with mass and inhomogeneities in the magnetic field always lower the energy. Our conclusions are of course limited to the approximation in which the derivatives on the magnetic field are much smaller than either $(eB)^{1/2}$ or m . Our result should be compared with the case of 3+1 dimensional QED, where no such instability has been found [2].

Physically, the magnetic field itself is dynamical and we briefly discuss how the above conclusions are modified. Dynamics in QED₃ is usually introduced through the Maxwell Lagrangian $(\vec{E}^2 - B^2)/2$ or through the Abelian Chern Simons term, or both. If only the Maxwell term is added, the conclusions of the preceding paragraph are not modified. Indeed, we may then focus on perturbations around constant magnetic field that are time-independent, so that no fluctua-

tions in the electric field arise to this order. Again, these perturbations will be inhomogeneous, they will lower the energy and create an instability. When a Chern-Simons term is present, the electric field couples directly to the magnetic field and all terms in the effective action (3) should be retained in the analysis. It is possible that under these circumstances, the constant magnetic background state is stabilized, but we have not completed the investigation of this effect.

The above conclusions may be relevant to some recent proposal concerning the stability of the $B=0$ state in QED₃ and the possible associated breaking of Lorentz invariance. In [12], a version of QED₃ is proposed with chiral fermions and a bare Chern-Simons term, arranged precisely in such a way as to cancel the induced Chern-Simons term. It is argued in [12] that dynamical fluctuations in the electromagnetic fields are responsible for an instability of the $B=0$ state and that a state with nonzero uniform magnetic field is the correct ground state.

We reconsider these assertions in light of the above results. First of all, our use of the effective action has the advantage that Lorentz invariance is preserved at all stages of the calculation. Then, if indeed a ground state were to arise with nonzero and uniform vacuum expectation value for the magnetic field, we can use the above analysis to study the dynamics of small fluctuations around the proposed state. As shown above, time-independent fluctuations produce an instability towards inhomogeneities in the magnetic field. It appears that the uniform magnetic field state is not a stable one, but restructures itself in an inhomogeneous pattern with lower energy. Thus, the conclusions of [12], based upon the

assumption that the ground state is supported by a uniform magnetic field appear to deserve further investigation. For example, it is important to understand the effect of a chemical potential [13] on our calculation of the effective potential.

The present analysis itself may, however, shed light on the nature of the true ground state of QED₃. For example, our analysis could be used to extend the results of [14], concerning dynamical flavor symmetry breaking in QED₃ by a magnetic field, to the case where inhomogeneities are present. Furthermore, from some points of view, this theory is similar to four-dimensional QCD [15]. It was shown in [16] that (compact) QED₃ confines electric charges with a linear potential, just as in QCD. This confinement comes about because instantons disorder magnetic and electric fields. From this analogy, one may reasonably conjecture that the true ground state of three-dimensional QED is more like the QCD ground state with disordered magnetic fields than like an ordered uniform magnetic field. Our calculations indeed show an instability of the uniform magnetic field state towards a more disordered state with inhomogeneous magnetic fields.

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