

Decays $H^0 \rightarrow \gamma\gamma$, γZ , and $Z \rightarrow \gamma H^0$ in the effective Lagrangian approach

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We study the decays $H^0 \rightarrow \gamma\gamma$, γZ , and $Z \rightarrow \gamma H^0$ in the framework of effective Lagrangians. We consider all dimension-six and dimension-eight fermionic and bosonic operators which are $SU_L(2) \times U_Y(1)$ gauge invariant and contribute to these decays at tree and one-loop level. Our calculations include, besides tree-level contributions, the one-loop contributions in the full theory: standard model and the underlying physics. We find that the effects due to new physics may enhance the standard model widths of these rare decays up to one order of magnitude.

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The understanding of the electroweak symmetry-breaking mechanism is a major goal for the next generation of colliders. The CERN Large Hadron Collider (LHC) offers an extraordinary opportunity to explore a completely new energy domain in which the standard model (SM) scalar sector can be tested. Nowadays there is general agreement among theorists that simple perturbation theory requires a Higgs boson mass of the order of the spontaneous symmetry breaking (SSB) scale. Also, according to several self-consistency arguments, it is generally agreed that there should be new physics beyond the SM. The one-loop $H^0 \gamma\gamma$ and $H^0 \gamma Z$ interactions predicted by the SM deserve attention as an excellent mechanism to detect possible effects of new physics. In fact, these interactions occur through the one-loop contributions induced by all the charged particles which couple to the Higgs boson [1]. It has been pointed out [2] that an intermediate-mass Higgs boson ($m_Z < m_H < 2m_Z$) could be detected through the rare decay mode $H^0 \rightarrow \gamma\gamma$ at the LHC. On the other hand, the decay $Z \rightarrow H^0 \gamma$ becomes important in the SM for a relatively light Higgs boson ($\frac{2}{3}m_Z < m_H < m_Z$). Since its branching ratio is small [3], the importance of this decay mode lies, as in the $H^0 \rightarrow \gamma Z$ case, in its sensitivity to gauge couplings and its clean signature.

The purpose of the present Rapid Communication is to report a calculation of the decay widths for $H^0 \rightarrow \gamma\gamma$, γZ , and $Z \rightarrow H^0 \gamma$ within the framework of effective Lagrangians [4–6]. In this approach the effects of new physics on the low-energy processes are parametrized with high-dimensional effective interactions constructed out of the SM fields. In the scenario that we have considered, the SM is a renormalizable theory where the low-energy processes cannot dramatically depend on the high-energy new physics [7]. Accordingly, all corrections to observables can be expressed as a power series in the inverse high-energy scale Λ , in which the underlying physics becomes apparent. Thus, we can parametrize physics beyond the SM in a model-independent manner by means of the following effective Lagrangian:

$$L_{\text{eff}} = L_0 + \sum_{n=5} \left[\sum_{i=1} \frac{\alpha_i}{\Lambda^{n-4}} (O_i^{(n)} + \text{H.c.}) \right], \quad (1)$$

where L_0 is the renormalizable SM Lagrangian and $O_i^{(n)}$ are nonrenormalizable operators of dimension $n \geq 5$ which respect the SM symmetries. There are a finite number of unknown α_i parameters determined by the required degree of accuracy. The dominant nonrenormalizable contributions at low-energy processes are induced by dimension-six operators and have been listed by Buchmüller and Wyler [4]. However, it has been pointed out recently [8] that, in the framework of a gauge and weakly coupled full theory, some of the dimension-eight operators may be competitive with the dimension-six ones. In fact, if any of the dimension-six operators are generated at one-loop level by the underlying physics, their effects on the low-energy processes are of order $(\alpha_i/16\pi^2)(v/\Lambda)^2$, where $v = \sqrt{2}\langle\phi\rangle_0$ is the SSB scale. In contrast, if some of the dimension-eight operators are generated at the tree level by the underlying physics, their effects on the low-energy processes are of order $\alpha_i(v/\Lambda)^4$. In case the underlying physics becomes apparent at a scale of order a few TeV, then the factors $(v/\Lambda)^2$ and $1/16\pi^2$ are expected to be of the same order of magnitude. Consequently, a complete study of a particular process requires to take into account the dimension-eight operators generated at the tree level.

Our general aim in the present work is to consider all the effects of new physics on the $H^0 \gamma\gamma$ and $H^0 \gamma Z$ interactions which may be competitive with the dimension-four theory predictions. Since the $H^0 \gamma\gamma$ and $H^0 \gamma Z$ interactions occur at the one-loop level in the dimension-four theory, we consider only new effects up to the one-loop level in the full theory. Accordingly we have included only tree-level-generated dimension-six fermionic operators which induce effective $H^0 \bar{f}f$ and $\bar{f}fZ$ interactions which contribute to $H^0 \gamma\gamma$ and $H^0 \gamma Z$ interactions through fermion loops. Since there are no tree-level-generated (in the complete theory) dimension-six bosonic operators, which could generate the above interactions, we considered also only one-loop-generated

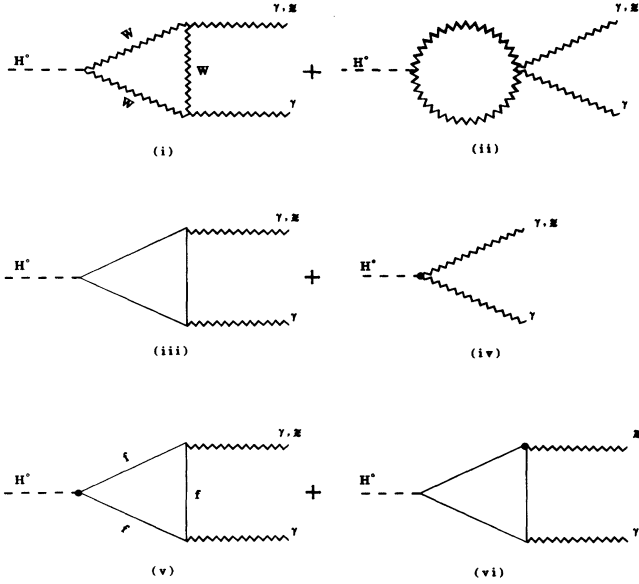


FIG. 1. Feynman diagrams contributing to the $H^0 \rightarrow \gamma\gamma$, γZ , and $Z \rightarrow \gamma H^0$ decays. The heavy dot denotes an effective vertex. There are not tree-level-generated $\bar{f}f\gamma$ effective vertices. The anomalous one-loop fermionic contributions in (v) and (vi) are finite.

dimension-six bosonic operators. Finally, we consider all the tree-level-generated dimension-eight bosonic operators which induce the $H^0 \gamma\gamma$ and $H^0 \gamma Z$ interactions.

In Fig. 1 we display the Feynman diagrams which contribute to $H^0 \rightarrow \gamma\gamma$, γZ , and $Z \rightarrow \gamma H^0$ decays up to one-loop level in the full theory. There are twelve tree-level-generated [8] dimension-six fermionic operators which contribute to these decays through the one-loop graphs shown in Figs. 1(v) and 1(vi):

$$\begin{aligned} O_{\phi U} &= i(\phi^\dagger D_\mu \phi)(\bar{U}_R \gamma^\mu U_R), \\ O_{\phi D} &= i(\phi^\dagger D_\mu \phi)(\bar{D}_R \gamma^\mu D_R), \\ O_{\phi F}^{(1)} &= i(\phi^\dagger D_\mu \phi)(\bar{F} \gamma^\mu F), \end{aligned} \quad (2)$$

$$\begin{aligned} O_{\phi F}^{(3)} &= i(\phi^\dagger D_\mu \tau^i \phi)(\bar{F} \gamma^\mu \tau^i F), \\ \hat{O}_{\phi U} &= (\phi^\dagger D_\mu \phi)(\hat{U}_R \gamma_5 \gamma^\mu U_R), \\ \hat{O}_{\phi D} &= (\phi^\dagger D_\mu \phi)(\hat{D}_R \gamma_5 \gamma^\mu D_R), \\ \hat{O}_{\phi F}^{(1)} &= (\phi^\dagger D_\mu \phi)(\hat{F} \gamma_5 \gamma^\mu F), \\ \hat{O}_{\phi F}^{(3)} &= (\phi^\dagger D_\mu \tau^i \phi)(\hat{F} \gamma_5 \gamma^\mu \tau^i F), \end{aligned} \quad (3)$$

$$O_{U\phi} = (\phi^\dagger \phi)(\bar{F} U_R \bar{\phi}), \quad O_{D\phi} = (\phi^\dagger \phi)(\bar{F} D_R \bar{\phi}), \quad (4)$$

$$\hat{O}_{U\phi} = i(\phi^\dagger \phi)(\bar{F} \gamma_5 U_R \bar{\phi}), \quad \hat{O}_{D\phi} = i(\phi^\dagger \phi)(\bar{F} \gamma_5 D_R \bar{\phi}), \quad (5)$$

where U_R and D_R denote right-handed fermions of type up and down, respectively, which are SU(2) singlets, and

$\bar{F} = (\bar{U}, \bar{D})_L$ denotes the corresponding left-handed SU(2) doublet; ϕ denote the scalar SU(2) doublet, τ^i are the Pauli matrices, $\hat{\phi} = i\tau^2 \phi^*$, and D_μ is the respective covariant derivative. There are six one-loop-generated dimension-six bosonic operators [8] which induce tree-level effective $H^0 \gamma\gamma$ and $H^0 \gamma Z$ interactions:

$$O_{\phi W} = (\phi^\dagger \phi) W_{\mu\nu}^i W^{i\mu\nu}, \quad O_{\phi B} = (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu}, \quad (6)$$

$$O_{WB} = (\phi^\dagger \tau^i \phi) W_{\mu\nu}^i B^{\mu\nu},$$

$$\hat{O}_{\phi W} = (\phi^\dagger \phi) W_{\mu\nu}^i \hat{W}^{i\mu\nu}, \quad \hat{O}_{\phi B} = (\phi^\dagger \phi) B_{\mu\nu} \hat{B}^{\mu\nu}, \quad (7)$$

$$\hat{O}_{WB} = (\phi^\dagger \tau^i \phi) W_{\mu\nu}^i \hat{B}^{\mu\nu},$$

where $W_{\mu\nu}$ and $B_{\mu\nu}$ are the SU(2) and U(1) field strength tensors, respectively. We denote by $\hat{V}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} V^{\alpha\beta}$ the corresponding dual tensor. The Higgs decay channels into $\gamma\gamma$, and γZ have been previously studied using the O_{WB} operator in Refs. [5,9]. Also the above Higgs decay channels have been studied using all the CP-even operators (6) [10]. In order to get a prediction on the respective decay widths, they used the bounds on the coefficients $\epsilon_i = \alpha_i (v/\Lambda)^2$ obtained from low-energy processes, which in general are of the order 10^{-2} . However, we consider that such bounds are still rather weak since in our scenario their contributions are expected to be suppressed because the respective coefficients α_i arise from dimension-six operators induced at the one-loop level of the complete theory [8]. These studies did not take into account the fact that effective interactions can be induced at different levels in the context of a simple perturbative scheme of the full theory. Finally, there are eight tree-level-generated dimension-eight bosonic operators [8] which induce the $H^0 \gamma\gamma$ and $H^0 \gamma Z$ interactions:

$$O_{8,1} = (\phi^\dagger \tau^i \phi)(\phi^\dagger \tau^j \phi) W_{\mu\nu}^i W^{j\mu\nu},$$

$$O_{8,2} = i(D^\mu \phi^\dagger D^\nu \phi)(\phi^\dagger \tau^i \phi) W_{\mu\nu}^i, \quad (8)$$

$$O_{8,3} = (\phi^\dagger \phi)^2 B_{\mu\nu} B^{\mu\nu},$$

$$O_{8,4} = i(\phi^\dagger \phi)(D^\mu \phi^\dagger D^\nu \phi) B_{\mu\nu},$$

$$\hat{O}_{8,1} = (\phi^\dagger \tau^i \phi)(\phi^\dagger \tau^j \phi) W_{\mu\nu}^i \hat{W}^{j\mu\nu},$$

$$\hat{O}_{8,2} = i(D^\mu \phi^\dagger D^\nu \phi)(\phi^\dagger \tau^i \phi) \hat{W}_{\mu\nu}^i, \quad (9)$$

$$\hat{O}_{8,3} = (\phi^\dagger \phi)^2 B_{\mu\nu} \hat{B}^{\mu\nu},$$

$$\hat{O}_{8,4} = i(\phi^\dagger \phi)(D^\mu \phi^\dagger D^\nu \phi) \hat{B}_{\mu\nu}.$$

Calculation of the Feynman diagrams shown in Fig. 1 leads to the following effective widths for $H^0 \rightarrow \gamma\gamma$, γZ , and $Z \rightarrow \gamma H^0$ decays:

$$\Gamma_{\text{eff}}(H^0 \rightarrow \gamma\gamma) = a \left\{ \left| \sum_f Q_f^2 N_C \left[F_0^f + \left(\frac{v}{\Lambda} \right)^2 F_a^f \right] + F_0^b + \left(\frac{v}{\Lambda} \right)^2 \left[F_{a6}^b + \left(\frac{v}{\Lambda} \right)^2 F_{a8}^b \right] \right|^2 + \left| \left(\frac{v}{\Lambda} \right)^2 \left[\sum_f Q_f^2 N_C \hat{F}_a^f + \hat{F}_{a6}^b + \left(\frac{v}{\Lambda} \right)^2 \hat{F}_{a8}^b \right] \right|^2 \right\}, \quad (10)$$

$$\Gamma_{\text{eff}}(H^0 \rightarrow \gamma Z) = b \left\{ \left| \sum_f Q_f N_C \left[A_0^f + \left(\frac{v}{\Lambda} \right)^2 A_a^f \right] + A_0^b + \left(\frac{v}{\Lambda} \right)^2 \left[A_{a6}^b + \left(\frac{v}{\Lambda} \right)^2 A_{a8}^b \right] \right|^2 + \left| \left(\frac{v}{\Lambda} \right)^2 \left[\sum_f Q_f N_C \hat{A}_a^f + \hat{A}_{a6}^b + \left(\frac{v}{\Lambda} \right)^2 \hat{A}_{a8}^b \right] \right|^2 \right\}, \quad (11)$$

$$\Gamma_{\text{eff}}(Z \rightarrow \gamma H^0) = -\frac{1}{3} \frac{m_H^3}{m_Z^3} \Gamma_{\text{eff}}(H^0 \rightarrow \gamma Z), \quad (12)$$

$$F_{a8}^b = -\frac{4\pi}{\alpha} (2s_w^2 \alpha_{8,1} + 2c_w^2 \alpha_{8,3}),$$

where

$$a = \frac{\alpha^2 G_F m_H^3}{64\sqrt{2}\pi^3}, \quad b = \frac{\alpha G_F^2 m_Z^2 m_H^3}{128\pi^4} \left(1 - \frac{m_Z^2}{m_H^2} \right)^3, \quad (13)$$

$$\hat{F}_a^f = -2\tau_f I^2(\tau_f) (\hat{\alpha}_{U\phi} + \hat{\alpha}_{D\phi}) \mp [1 - \tau_f I^2(\tau_f)] (\hat{\alpha}_{\phi F}^{(1)} + \hat{\alpha}_{\phi F}^{(3)}) + [1 - \tau_f I^2(\tau_f)] (\hat{\alpha}_{\phi U} + \hat{\alpha}_{\phi D}), \quad (16)$$

and the parametric functions are given by

$$F_0^f = -2\tau_f [1 + (1 - \tau_f) I^2], \quad (14)$$

$$\hat{F}_{a6}^b = \frac{1}{4\pi\alpha} (2s_w^2 \hat{\alpha}_{\phi W} + 2c_w^2 \hat{\alpha}_{\phi B} - s_{2w} \hat{\alpha}_{WB}), \quad (17)$$

$$F_a^f = -\frac{3s_w}{2\sqrt{2}\pi\alpha} \sqrt{\frac{\tau_w}{\tau_f}} (\alpha_{U\phi} + \alpha_{D\phi}) F_0^f,$$

$$\hat{F}_{a8}^b = \frac{4\pi}{\alpha} (2s_w^2 \hat{\alpha}_{8,1} + 2c_w^2 \hat{\alpha}_{8,3}),$$

$$F_0^b = 2 + 3\tau_w + 3\tau_w(2 - \tau_w) I^2(\tau_w),$$

where in our notation subindices and superindices denote the dimension and the type of operator, respectively—the dimension-four theory contributions are denoted by the subindex zero. The parametric functions for the $H^0 \rightarrow \gamma Z$ mode are given by

$$F_{a6}^b = -\frac{1}{4\pi\alpha} (2s_w^2 \alpha_{\phi W} + 2c_w^2 \alpha_{\phi B} - s_{2w} \alpha_{WB}), \quad (15)$$

$$A_0^f = -2g_V^f \frac{\sigma_f \tau_f}{\sigma_f - \tau_f} \left\{ 1 + \left(1 - \frac{\sigma_f \tau_f}{\sigma_f - \tau_f} \right) [I^2(\tau_f) - I^2(\sigma_f)] - \frac{2\tau_f}{\sigma_f - \tau_f} [J(\tau_f)I(\tau_f) - J(\sigma_f)I(\sigma_f)] \right\}, \quad (18)$$

$$A_a^f = -\frac{1}{2} \left[\alpha_{\phi U} + \alpha_{\phi D} + \alpha_{\phi F}^{(1)} + \alpha_{\phi F}^{(3)} + \frac{3s_w}{\sqrt{8}\pi\alpha} \sqrt{\frac{\tau_w}{\tau_f}} (\alpha_{U\phi} + \alpha_{D\phi}) \right] A_0^f,$$

$$A_0^b = \frac{2\sigma_w \tau_w}{\sigma_w - \tau_w} \left\{ x - \left(y + x \frac{\sigma_w \tau_w}{\sigma_w - \tau_w} \right) [I^2(\tau_w) - I^2(\sigma_w)] - 2x \frac{\tau_w}{\sigma_w - \tau_w} [J(\tau_w)I(\tau_w) - J(\sigma_w)I(\sigma_w)] \right\}, \quad (19)$$

$$A_{a6}^b = -\frac{s_{2w}}{4\pi\alpha} (s_{2w} \alpha_{\phi W} - s_{2w} \alpha_{\phi B} - 2c_{2w} \alpha_{WB}), \quad A_{a8}^b = -\frac{4\pi s_{2w}}{\alpha} \left(s_{2w} \alpha_{8,1} - s_{2w} \alpha_{8,3} - \frac{\sqrt{\pi\alpha}}{4c_w} \alpha_{8,2} - \frac{\sqrt{\pi\alpha}}{4s_w} \alpha_{8,4} \right),$$

$$\hat{A}_a^f = -2 \left\{ \left[g_A^f \pm g_V^f \left(1 - \frac{\sigma_f \tau_f}{\sigma_f - \tau_f} [I^2(\tau_f) - I^2(\sigma_f)] \right) \right] (\hat{\alpha}_{\phi F}^{(1)} + \hat{\alpha}_{\phi F}^{(3)}) - \frac{3s_w g_V^f}{\sqrt{2}\pi\alpha} \sqrt{\frac{\tau_w}{\tau_f}} \frac{\sigma_f \tau_f}{\sigma_f - \tau_f} [I^2(\tau_f) - I^2(\sigma_f)] (\hat{\alpha}_{U\phi} + \hat{\alpha}_{D\phi}) + \left[g_A^f - g_V^f \left(1 - \frac{\sigma_f \tau_f}{\sigma_f - \tau_f} [I^2(\tau_f) - I^2(\sigma_f)] \right) \right] (\hat{\alpha}_{\phi U} + \hat{\alpha}_{\phi D}) \right\}, \quad (20)$$

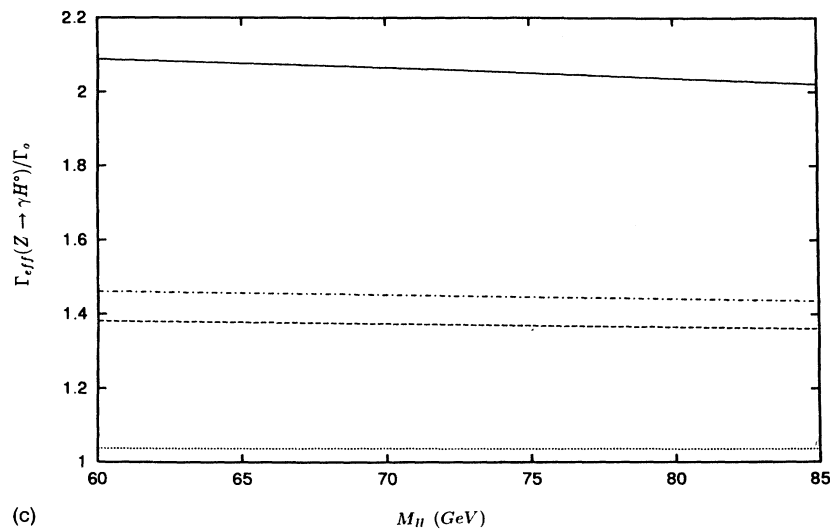
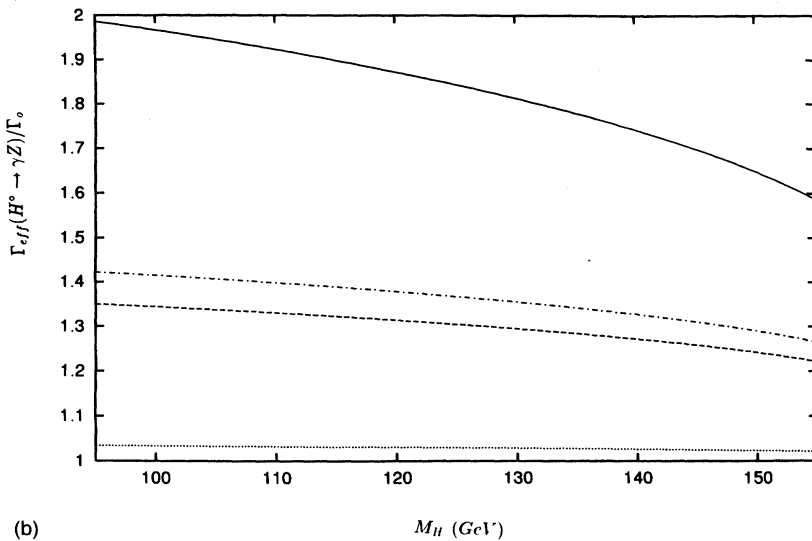
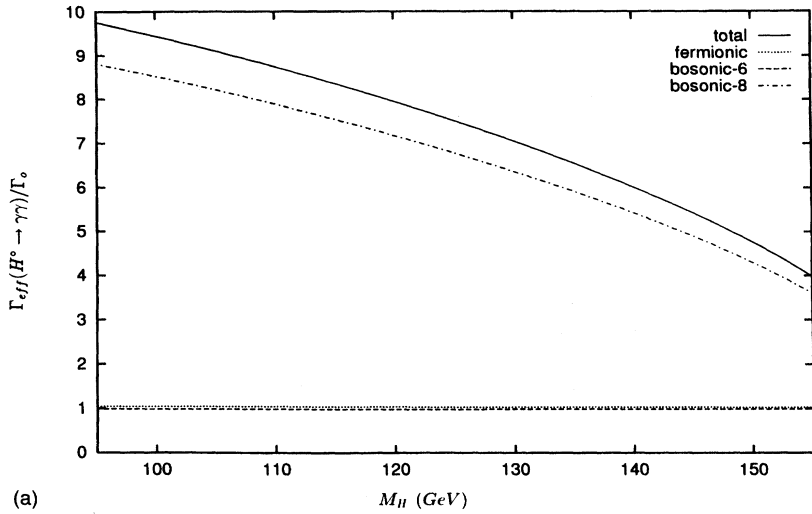


FIG. 2. The fractions $\Gamma_{\text{eff}}(H^0 \rightarrow \gamma\gamma)/\Gamma_0(H^0 \rightarrow \gamma\gamma)$, $\Gamma_{\text{eff}}(H^0 \rightarrow \gamma Z)/\Gamma_0(H^0 \rightarrow \gamma Z)$, and $\Gamma_{\text{eff}}(Z \rightarrow \gamma H^0)/\Gamma_0(Z \rightarrow \gamma H^0)$ as a function of m_H for $m_t = 174$ GeV, $\Lambda = 1$ TeV, and for all the coefficients taken as unity. In each case, fermionic (dotted), dimension-six bosonic (dashed), dimension-eight bosonic (dot-dashed), and total (solid line) contributions are displayed.

$$\begin{aligned}\hat{A}_{a6}^b &= \frac{s_{2w}}{4\pi\alpha} (s_{2w}\hat{\alpha}_{\phi W} - s_{2w}\hat{\alpha}_{\phi B} - 2c_{2w}\hat{\alpha}_{WB}), \\ \hat{A}_{a8}^b &= \frac{4\pi s_{2w}}{\alpha} \left(s_{2w}\hat{\alpha}_{8,1} - s_{2w}\hat{\alpha}_{8,3} - \frac{\sqrt{\pi\alpha}}{4c_w} \hat{\alpha}_{8,2} - \frac{\sqrt{\pi\alpha}}{4s_w} \hat{\alpha}_{8,4} \right),\end{aligned}\quad (21)$$

where $\tau_i = 4m_i^2/m_H^2$, $\sigma_i = 4m_i^2/m_Z^2$, g_V^f and g_A^f are the vector and axial-vector coupling coefficients for fermions to the Z , respectively, $s_w = \sin\theta_w$, $c_w = \cos\theta_w$, $s_{2w} = \sin 2\theta_w$, $c_{2w} = \cos 2\theta_w$, and we have taken the following replacement: $\alpha_i \rightarrow \alpha_i/16\pi^2$ in the one-loop generated operators (6), (7). In our notation the signs $+$ ($-$) correspond to down (up) type fermions. It is important to notice that the one-loop effective contributions of the fermionic operators, which are generated at the tree level, give a finite result. On the contrary, in a previous calculation the W -boson loop diagram, involving one-loop generated $H^0 WW$ and $WW\gamma$ vertices, lead to a logarithmically divergent result [9]. The functions I , J , x , and y [1,2] are given by

$$I = \begin{cases} \arctan\left(\frac{1}{J}\right), & \lambda_i > 1, \\ \frac{1}{2} \left[\ln \frac{1-iJ}{1+iJ} + i\pi \right], & \lambda_i < 1, \end{cases} \quad (22)$$

where

$$J(\lambda_i) = \begin{cases} \sqrt{\lambda_i - 1}, & \lambda_i > 1, \\ i\sqrt{1 - \lambda_i}, & \lambda_i < 1 \end{cases} \quad (23)$$

and

$$\begin{aligned}x &= -\frac{c_w^2}{2} \left[\left(1 + \frac{2}{\tau_w} \right) \tan^2 \theta_w - \left(5 + \frac{2}{\tau_w} \right) \right], \\ y &= -2c_w^2 (3 - \tan^2 \theta_w),\end{aligned}\quad (24)$$

with $\lambda_i = \tau_i, \sigma_i$.

In Fig. 2 we have displayed the fractions $\Gamma_{\text{eff}}/\Gamma_0$ as a function of m_H for $m_t = 174$ GeV, $\Lambda = 1$ TeV, and we took all the coefficients equal to 1. In the fermionic part, only the quark top contributions have been taken into account. In each case we present the contributions coming from dimension-six fermionic operators, dimension-six bosonic operators, dimension-eight bosonic operators, and the overall contribution. We can appreciate that the rare decays are insensitive to both fermionic and bosonic dimension-six contributions. In contrast, the dimension-eight tree-level-generated contributions enhance up to one order of magnitude the SM width $H^0 \rightarrow \gamma\gamma$, while the corresponding widths for $H^0 \rightarrow \gamma Z$ and $Z \rightarrow \gamma H^0$ may be enhanced up to a factor of 2. We have found that different scenarios for the values of the α_i coefficients do not modify appreciably this result.

In conclusion, our analysis for the rare decays in the framework of effective Lagrangians confirms the result obtained in previous studies [5,9,10] that those decay modes are rather sensitive to the structure of the couplings induced by physics beyond the SM. However, we have found that a systematic bookkeeping of the α_i anomalous coefficients leads us to expect that an enhancement of these radiative decays up to one order of magnitude can possibly arise from the contribution of tree-level-generated dimension-eight bosonic operators. On the other hand, as in the SM case, the anomalous contributions to the decay widths $H^0 \rightarrow \gamma Z$ and $Z \rightarrow \gamma H^0$ are marginally important with respect to the $H^0 \rightarrow \gamma\gamma$ mode. Our results are consistent within the perturbative framework of the full theory, where the tree-level-generated dimension-eight operators induce the most important contribution.

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