Dyonic black holes in effective string theory

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The spherical symmetric dyonic black hole solutions of the effective action of a heterotic string are studied perturbatively up to second order in the inverse string tension. An expression for the temperature in terms of the mass and the electric and magnetic charge of the black hole is derived and it is shown that its behavior is qualitatively different in the two special cases where the electric or the magnetic charge vanishes.

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At present, string theories are the most promising attempt to unify gravitation with the other fundamental interactions. It is therefore of great interest to study their phenomenological consequences, in particular in the context of gravitational physics.

For this reason, the modifications to black hole physics induced by string theory have been extensively investigated. Because of the difficulty of treating the full theory, a fruitful approach has been the study of its low energy limit by means of effective field theoretic actions, obtained as an expansion in the inverse string tension α'. These actions describe the dynamics of the low energy excitations of the string spectrum, which comprise in the simplest case the graviton, a dilaton, an axion, and gauge fields. In particular, some exact solutions have been obtained for the lowest order approximation to the action [1–3]. The main difference for the Einstein theory is the presence of a nonminimal coupling of the dilaton with the other fields, which changes drastically the physical properties of the black hole, and in particular the thermodynamics. For charged black holes, for example, the temperature becomes independent of the charge, at variance with the Reissner-Nordström solution of the Einstein-Maxwell theory.

The results obtained in [1–3] are, however, valid only if the order α' corrections in the gravitational sector of the effective action are neglected. These corrections consist essentially in the presence of a higher derivative Gauss-Bonnet term coupled to the dilaton. In a recent paper [4], we examined how the magnetic charged dilatonic black hole solutions of effective string theories obtained in [1] are modified when the order α' terms in the effective action are taken into account. Of particular interest were the thermodynamical properties of the solutions, which, contrary to the solutions of the leading order action, exhibit a dependence of the temperature on the magnetic charge of the black hole. In particular, these solutions can reach a vanishing temperature for a finite value of the mass, and therefore admit a stable remnant as a final state of the Hawking evaporation.

In this paper we extend our investigation to the case of dyonic black holes. When higher order corrections to the action are neglected, the dyonic solution has been obtained from the magnetic one [1] by exploiting the SL(2,R) dilaton-axion symmetry of the leading order terms in the perturbative expansion of the action [2]. Even if it has been conjectured that this symmetry might be an exact symmetry of the full theory [3], it is not, however, a symmetry of the order α' action, since, for example, the Gauss-Bonnet term in that action is not invariant under the SL(2,R) symmetry. It has been argued, however, that the symmetry could still be present in a highly nontrivial and nonlinear way and therefore not order by order in perturbation theory [5].

It is therefore interesting to obtain some explicit results also for the electric charged case. Adopting the techniques of Ref. [4], we then perform a perturbative calculation of the spherical symmetric solutions of the order α' action around the background constituted by the Schwarzschild solution with the other fields set to zero. For a detailed discussion of the choice of this particular background and other technicalities we refer to [4].

The most interesting result of our analysis is the difference in the behavior of the temperature for an electrically charged black hole with respect to the magnetic one when higher order terms are taken into account. In fact, it appears that the lowest order corrections to the temperature, which arise at order ${\alpha'}^2$, have opposite sign in the two cases, so that for a purely electric black hole at this order of approximation the temperature is a monotonic decreasing positive definite function of the mass for any value of the charge and therefore cannot give rise to stable remnants.

A similar approach was adopted in Ref. [6] for the axially symmetric charged black hole, but the calculations were performed only up to order α', so that the corrections to the temperature could not be observed.

The bosonic sector of the dimensionally reduced effective action for the heterotic string is given up to order α' by [7]

$$S_{\rm eff} = \int d^4x\sqrt{-g}[\mathcal{R} - \tfrac{1}{3}e^{-4\Phi}H_{abc}H_{abc} -2(\nabla\Phi)^2 + \alpha e^{-2\Phi}(\mathcal{S} - F^2)], \qquad (1)$$

where $\alpha \equiv \alpha'/8, F$ is the Maxwell field strength, and $\mathcal{S} \equiv \mathcal{R}^2_{abcd} - 4\mathcal{R}^2_{ab} + \mathcal{R}^2$ is the Gauss-Bonnet term. Actually, at order α, terms of the kind $H^2(\nabla\Phi)^2$, $\mathcal{R}H^2$, $\mathcal{R}(\nabla\Phi)^2$, etc., are also present in the action [8], but we shall not consider them, since they contribute to the field equations only to order α^3 or higher in the background we are considering and therefore are not relevant for our calculations. For the same reason we do not consider order α^2 corrections to the action.

The explicit form of the axion field H_{abc} in terms of the potential B_{ab} is

$$H_{abc} = \partial_{[a}B_{bc]} + \alpha(A_{[a}\partial_b A_{c]} + \omega^{lm}{}_{[a}\partial_b\omega^{lm}{}_{c]} + \omega^{lm}{}_{[a}\omega^{mn}{}_b\omega^{nl}{}_{c]}) ,$$

where the Chern-Simons terms have been taken into account. Because of the presence of these terms a solution presenting both electric and magnetic charge will also have a nontrivial axion field [7].

It is well known that if one defines a scalar field a, dual to H, such that

$$H_{abc} = -\tfrac{1}{2}\epsilon_{abcd}e^{4\Phi}\partial_d a , \tag{2}$$

the field equations stemming from (1) can be equivalently derived from the action [2]

$$S_{\text{eff}} = \int d^4x\sqrt{-g}[\mathcal{R} - 2(\nabla\Phi)^2 - \tfrac{1}{2}e^{4\Phi}(\nabla a)^2 - \alpha e^{-2\Phi}(F^2 - \mathcal{S})] - \alpha a(F\tilde{F} - \mathcal{R}\tilde{\mathcal{R}}) , \tag{3}$$

where

$$F\tilde{F} = \tfrac{1}{2}\epsilon_{abcd}F_{ab}F_{cd}, \quad \mathcal{R}\tilde{\mathcal{R}} = \tfrac{1}{2}\epsilon_{abcd}\mathcal{R}_{abef}\mathcal{R}_{cdef} . \tag{4}$$

The field equations can then be written as

$$\begin{aligned}\mathcal{R}_{mn} = {}& 2\nabla_m\Phi\nabla_n\Phi + \tfrac{1}{2}e^{4\Phi}\nabla_m a\nabla_n a + 2\alpha e^{-2\Phi}(F_{mp}F_{np} - \tfrac{1}{4}g_{mn}F^2) \\ & + 4\alpha e^{-2\Phi}\Big[4\mathcal{R}_{p(m}\nabla_{n)}\nabla_p\Phi - 2\mathcal{R}_{mn}\nabla_p\nabla_p\Phi - g_{mn}\mathcal{R}_{pq}\nabla_p\nabla_q\Phi - \mathcal{R}\nabla_m\nabla_n\Phi \\ & + \tfrac{1}{2}g_{mn}\mathcal{R}\nabla_p\nabla_p\Phi - 2\mathcal{R}_{qmnp}\nabla_p\nabla_q\Phi\Big] \\ & - 8\alpha e^{-2\Phi}\Big[4\mathcal{R}_{p(m}\nabla_{n)}\Phi\nabla_p\Phi - 2\mathcal{R}_{mn}\nabla_p\Phi\nabla_p\Phi - g_{mn}\mathcal{R}_{pq}\nabla_p\Phi\nabla_q\Phi - \mathcal{R}\nabla_m\Phi\nabla_n\Phi \\ & + \tfrac{1}{2}g_{mn}\mathcal{R}\nabla_p\Phi\nabla_p\Phi - 2\mathcal{R}_{qmnp}\nabla_p\Phi\nabla_q\Phi\Big] + 4\alpha\Big[\epsilon_{blmn}\mathcal{R}_{ajmn}\nabla_l\nabla_j a + \epsilon_{almn}\mathcal{R}_{bjmn}\nabla_l\nabla_j a\Big] ,\end{aligned} \tag{5a}$$

$$\nabla^2\Phi = \frac{\alpha}{2}e^{-2\Phi}(\mathcal{S} - F^2) + \tfrac{1}{2}e^{4\Phi}(\nabla_p a)^2 , \tag{5b}$$

$$\nabla_p(e^{4\Phi}\nabla_p a) = \alpha(F\tilde{F} - \mathcal{R}\tilde{\mathcal{R}}) , \tag{5c}$$

$$\nabla_p(e^{-2\Phi}F_{pm}) = \tfrac{1}{2}\epsilon_{pmqr}F_{qr}\nabla_r a . \tag{5d}$$

It is convenient to define a new scalar field b, such that

$$\nabla_p b = e^{4\Phi}\nabla_p a . \tag{6}$$

This simplifies the field equation. In particular,

$$\nabla^2\Phi = \frac{\alpha}{2}e^{-2\Phi}(\mathcal{S} - F^2) + \tfrac{1}{2}e^{-4\Phi}(\nabla b)^2 , \tag{7a}$$

$$\nabla^2 b = \alpha(F\tilde{F} - \mathcal{R}\tilde{\mathcal{R}}) . \tag{7b}$$

We want to find a perturbative expansion of the solution to (5) around the background constituted by the Schwarzschild metric of mass m with vanishing dilaton and axion. Our expansion will be in the parameter α or, more correctly, in α/m^2. We adopt a spherically symmetric ansatz for the metric:

$$ds^2 = -\lambda^2 dt^2 + \lambda^{-2}dr^2 + R^2 d\Omega^2 , \tag{8}$$

where $\lambda = \lambda(r)$, $R = R(r)$. This particular form of the metric is suggested from the exact solution found in [2] in the absence of the Gauss-Bonnet term (see also [4]).

The general spherical symmetric solution of the generalized Maxwell equation (5d) containing both electric and magnetic charge, q_e and q_m, respectively, is given in orthonormal coordinates by

$$F_{ij} = \frac{q_m}{R^2}, \quad F_{01} = \frac{q_e + q_m a}{R^2}e^{2\Phi} . \tag{9}$$

We can now expand the fields in α as

$$\begin{aligned}&\lambda = \lambda_0(1 + \alpha\psi_1 + \alpha^2\psi_2 + \cdots), \\ &R = r + \alpha\rho_1 + \alpha^2\rho_2 + \cdots , \\ &\Phi = \alpha\phi_1 + \alpha^2\phi_2 + \cdots , \quad b = \alpha\chi_1 + \alpha^2\chi_2 + \cdots ,\end{aligned} \tag{10}$$

where $\lambda_0 = (1 - 2m/r)^{1/2}$ and ψ_i, ρ_i, ϕ_i, and χ_i are functions of r. Moreover, from (9),

$$\begin{aligned}&F_{ij} \sim \frac{q_m}{r^2}\left(1 - \frac{2\alpha\rho_1}{r} + \cdots\right) , \\ &F_{01} \sim \frac{q_e}{r^2} + \alpha\left[\frac{q_e}{r^2}\left(2\phi_1 - \frac{2\rho_1}{r}\right) + \frac{q_m}{r^2}\chi_1\right] + \cdots ,\end{aligned} \tag{11}$$

and hence

$$F^2 \sim 2\frac{q_m^2 - q_e^2}{r^4} + 2\alpha\left[-4q_m^2\frac{\rho_1}{r^5} + 4q_e^2\left(\frac{\rho_1}{r^5} - \frac{\phi_1}{r^4}\right) - 2q_e q_m \frac{\chi_1}{r^4}\right] + \cdots , \tag{12a}$$

$$F\tilde{F} \sim 4\frac{q_e q_m}{r^4} + 4\alpha\left[q_m^2\frac{\chi_1}{r^4} + 2q_e q_m\left(\frac{\phi_1}{r^4} - \frac{2\rho_1}{r^5}\right)\right] + \cdots . \tag{12b}$$

The field equations can now be expanded by inserting (10) and (11). At first order in α one has

$$[r(r-2m)\phi_1']' = \frac{q_e^2 - q_m^2}{r^2} + \frac{24m^2}{r^4} , \tag{13a}$$

$$[r(r-2m)\chi_1']' = 4\frac{q_e q_m}{r^2} , \tag{13b}$$

$$\rho_1'' = 0 , \tag{13c}$$

$$[(r-2m)\psi_1]' = -\frac{m}{r^2}\rho_1 - \frac{q_e^2 + q_m^2}{2r^2} . \tag{13d}$$

If one requires asymptotic flatness, a solution of (13) can be obtained such that $\psi_1 = 0$:

$$\phi_1 = -\frac{1}{m}\left(\frac{2 + q_e^2 - q_m^2}{2r} + \frac{m}{r^2} + \frac{4m^2}{3r^3}\right) , \quad \chi_1 = -\frac{2q_e q_m}{mr} ,$$
$$\rho_1 = -\frac{q_e^2 + q_m^2}{2m} , \quad \psi_1 = 0 . \tag{14}$$

We pass now to evaluate the second order corrections, which satisfy the equations

$$\begin{aligned}
[r(r-2m)\phi_2']' &= 2\frac{r-2m}{r}\rho_1\phi_1' - \frac{48m^2}{r^4}\left(\phi_1 + \frac{\rho_1}{r}\right) \\
&\quad + 2(q_m^2 + q_e^2)\frac{\phi_1}{r^2} + 4(q_m^2 - q_e^2)\frac{\rho_1}{r^3} \\
&\quad + 2q_e q_m\frac{\chi_1}{r^2} + \frac{r(r-2m)}{2}\chi_1'^2 , \\
[r(r-2m)\chi_2']' &= 2\frac{r-2m}{r}\rho_1\chi_1' + 4q_m^2\frac{\chi_1}{r^2} \\
&\quad + 8q_m q_e\left(\frac{\phi_1}{r^2} - 2\frac{\rho_1}{r^3}\right) , \\
\rho_2'' &= -r\phi_1'^2 + \frac{8m}{r^2}\phi_1'' - \tfrac{1}{4}r\chi_1'^2 , \\
[(r-2m)\psi_2]' &= -\frac{r-2m}{r}\rho_2'' - \frac{r-m}{r}\rho_2' - \frac{m}{r^2}\rho_2 \\
&\quad + \frac{2m}{r^3}\rho_1^2 - (q_e^2 - q_m^2)\frac{\phi_1}{r^2} + 2(q_m^2 + q_e^2)\frac{\rho_1}{r^3} \\
&\quad + 4m\left(\frac{r-2m}{r^2}\phi_1'' - 2\frac{r-3m}{r^3}\phi_1'\right) \\
&\quad - q_e q_m\frac{\chi_1}{r^2} .
\end{aligned} \tag{15}$$

The asymptotically flat solutions of (15), with boundary conditions such that m is the physical mass of the black hole, are given by

$$\begin{aligned}
\phi_2 &= -\left[\frac{73 - 45q_m^2}{60m^3 r} + \frac{73 - 15q_m^2 + 30q_e^2 - 15q_m^4 - 60q_m^2 q_e^2 + 15q_e^4}{60m^2 r^2}\right. \\
&\quad \left. + \frac{73 + 45q_e^2}{45mr^3} + \frac{73 + 5q_m^2 + 50q_e^2}{30r^4} + \frac{112m}{75r^5} + \frac{8m^2}{9r^6}\right] , \\
\chi_2 &= q_e q_m\left[\frac{3}{2m^3 r} + \frac{3 - 2(q_m^2 + q_e^2}{2m^2 r^2} + \frac{2}{3mr^3} + \frac{1}{3r^4}\right] , \\
\rho_2 &= -\left[\frac{4 - 4(q_m^2 - q_e^2) + (q_m^2 + q_e^2)^2}{8m^2 r} + \frac{2 - q_m^2 + q_e^2}{3mr^2}\right. \\
&\quad \left. + \frac{7 - 3(q_m^2 - q_e^2)}{3r^3} + \frac{16m}{5r^4} + \frac{24m^2}{5r^5}\right] , \\
(r-2m)\psi_2 &= -\left[\frac{1 - 2(q_m^2 - q_e^2)}{3mr^2} - \frac{11 - 5(q_m^2 - q_e)^2}{3r^3}\right. \\
&\quad \left. + \frac{[2 - 50(q_m^2 - q_e^2)]m}{15r^4} + \frac{272m^3}{15r^6}\right]
\end{aligned} \tag{16}$$

and hence, up to order α^2,

$$\begin{aligned}
R^2 &\sim r^2 - \alpha\frac{q_m^2 + q_e^2}{m}r - \alpha^2\frac{1 + q_e^2 - q_m^2}{m^2} + O\left(\frac{1}{r}\right) , \\
\lambda^2 &\sim 1 - \frac{2m}{r} - \frac{1 + 2q_e^2 - 2q_m^2}{3mr^3}\alpha^2 + O\left(\frac{1}{r^4}\right) , \\
\Phi &\sim -\left[\frac{2 + q_e^2 - q_m^2}{2m}\alpha + \frac{73 - 45q_m^2}{60m^3}\alpha^2\right]\left(\frac{1}{r}\right) + O\left(\frac{1}{r^2}\right) , \\
a &\sim -q_e q_m\left[\left(\frac{2\alpha}{m} + \frac{3\alpha^2}{m^2}\right)\frac{1}{r} + \frac{5 + 6q_e^2 - 2q_m^2}{2m^2 r^2}\alpha^3\right. \\
&\quad \left. + O\left(\frac{1}{r^2}\right)\right] .
\end{aligned} \tag{17}$$

We stress again that corrections of order α^2 or higher to the action, as given, for example, in [8], do not contribute to our calculations before order α^3, so that they can be consistently neglected in our approximation.

From the solutions (17) it follows that the black hole has a dilatonic and axionic charge given respectively at order α by $D = (2 + q_e^2 - q_m^3)\alpha/2m$ and $A = (2q_e q_m)\alpha/m$; these are not, however, independent parameters, but are functions of q_e, q_m, and m, in accordance with the weak form of the no-hair conjecture [9]. The metric has a singularity at $r = r_- \sim \alpha(q_e^2 + q_m^2)/m$ and a horizon at $r = r_+ \sim 2m\{1 - [1 - 2(q_e^2 + q_m^2)^2]/\alpha^2 12m^2\}$. We notice that, when $r_- > r_+$, the singularity is naked. This regime is, however, out of the range of our approximations.

The temperature of the black hole can be readily obtained by requiring the regularity of the Euclidean section and is given by [4]

$$\begin{aligned}
T^{-1} \equiv \beta &\sim 8\pi m\left[1 - \alpha^2\left(\frac{\tilde{\psi}_2(2m)}{m} - 2\tilde{\psi}_2'(2m)\right)\right] \\
&= 8\pi m\left(1 - \alpha^2\frac{73 - 45(q_m^2 - q_e^2)}{120m^4}\right) .
\end{aligned} \tag{18}$$

This should be compared with the value $T = (8\pi m)^{-1}$ found in [2] when the order α' corrections to the action are neglected: the temperature is no longer independent of the electric and magnetic charge. However, while for a black hole carrying only magnetic charge the temperature has a maximum and then goes to zero for a finite value of m [4], the temperature of a purely electric black hole is at this order of approximation a monotonic decreasing function of the mass and therefore no massive remnant is to be expected in the last case. It is also amusing to notice that for $q_e = q_m$ the temperature coincides with that of an uncharged black hole.

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APPENDIX

We show here that the perturbative result in the absence of higher order corrections agrees with the exact solution found in [2]. If one neglects the Gauss-Bonnet term, the field equations are, at first order,

$$\begin{aligned}
[r(r-2m)\phi_1']' &= \frac{q_e^2 - q_m^2}{r^2} , \\
[r(r-2m)\chi_1']' &= 4\frac{q_e q_m}{r^2} , \\
\rho_1'' &= 0 , \\
[(r-2m)\psi_1]' &= -\frac{m}{r^2}\rho_1 - \frac{q_e^2 + q_m^2}{2r^2} ,
\end{aligned} \tag{A1}$$

whose solutions are

$$\begin{aligned}
\phi_1 &= \frac{q_m^2 - q_e^2}{2mr} , \quad \chi_1 = -\frac{q_e q_m}{mr} , \\
\rho_1 &= -\frac{q_e^2 + q_m^2}{2m} , \quad \psi_1 = 0 .
\end{aligned} \tag{A2}$$

At second order, the field equations become

$$\begin{aligned}
[r(r-2m)\phi_2']' &= 2\frac{r-2m}{r}\rho_1\phi_1' + 2(q_m^2 + q_e^2)\frac{\phi_1}{r^2} \\
&\quad + 4(q_m^2 - q_e^2)\frac{\rho_1}{r^3} + 2q_e q_m \frac{\chi_1}{r^2} \\
&\quad + \tfrac{1}{2}r(r-2m)\chi_1'^2 , \\
[r(r-2m)\chi_2']' &= 2\frac{r-2m}{r}\rho_1\chi_1' + 4q_m^2\frac{\chi_1}{r^2} \\
&\quad + 8q_m q_e\left(\frac{\phi_1}{r^2} - 2\frac{\rho_1}{r^3}\right) , \\
\rho_2'' &= -r\phi_1'^2 - \tfrac{1}{4}r\chi'^2 , \\
[(r-2m)\psi_2]' &= -\frac{r-2m}{2}\rho_2'' - \frac{r-m}{r}\rho_2' - \frac{m}{r^2}\rho_2 + \frac{2m}{r^3}\rho_1^2 \\
&\quad - (q_e^2 - q_m^2)\frac{\phi_1}{r^2} + 2(q_m^2 + q_e^2)\frac{\rho_1}{r^3} .
\end{aligned} \tag{A3}$$

After a lengthy but straightforward calculation, one obtains

$$\begin{aligned}
\phi_2 &= \frac{q_e^4 - 4q_e^2 q_m^2 - q_m^4}{4m^2r^2} , \quad \chi_2 = -q_e q_m \frac{q_e^2 + q_m^2}{m^2 r^2} , \\
\rho_2 &= -\frac{(q_e^2 + q_m^2)^2}{8m^2 r} , \quad \psi_2 = 0 .
\end{aligned} \tag{A4}$$

Substituting the results in (10) and (11) yields

$$\lambda^2 = 1 - \frac{2m}{r} , \quad R^2 = r^2 - \alpha\frac{q_e^2 + q_m^2}{m} r ,$$

$$F_{01} = \frac{q_e}{r^2} , \quad a = -\alpha\frac{2q_e q_m}{mr^2}\left(1 + \alpha\frac{3q_e^2 - q_m^2}{2mr}\right) + \cdots ,$$

$$\Phi = -\left(\alpha\frac{q_e^2 - q_m^2}{2mr} + \alpha^2\frac{q_e^4 - 4q_e^2 q_m^2 - q_m^4}{4m^2r^2}\right) + \cdots .$$

As expected, these results coincide with the expansion in α of the exact solution found in (3):

$$\begin{aligned}
ds^2 &= -\left(1 - \frac{2m}{r}\right)dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 \\
&\quad + r\left(r - \frac{\alpha(q_e^2 + q_m^2)}{2m}\right)d\Omega^2 ,
\end{aligned}$$

$$\Phi = -\tfrac{1}{2}\ln\left(\frac{mr[mr - \alpha(q_e^2 - q_m^2)]}{(mr)^2 - 2\alpha q_e^2 mr + \alpha^2 q_e^2(q_e^2 + q_m^2)}\right) ,$$

$$a = -q_e q_m \frac{2mr - \alpha(q_e^2 + q_m^2)}{(mr)^2 - 2\alpha q_e^2 mr + \alpha^2 q_e^2(q_e^2 + q_m^2)} ,$$

$$F_{01} = \frac{q_e}{r^2} , \quad F_{23} = \frac{q_m}{r(r - \alpha(q_e + q_m)/2m)} .$$

[1] D. Garfinkle, G. T. Horowitz, and A. Strominger, Phys. Rev. D **43**, 3140 (1991).
[2] A. Shapere, S. Trivedi, and F. Wilczek, Mod. Phys. Lett. A **6**, 2677 (1991).
[3] A. Sen, Nucl. Phys. **B404**, 109 (1993).
[4] S. Mignemi and N. R. Stewart, Phys. Rev. D **47**, 5259 (1993).
[5] J. H. Schwarz, in *String Theory, Quantum Gravity and the Unification of the Fundamental Interactions*, Proceedings of the Workshop, Rome, Italy, 1992, edited by M. Bianchi *et al.* (World Scientific, Singapore, 1993), p. 503.
[6] B. A. Campbell, N. Kaloper, and K. A. Olive, Phys. Lett. B **285**, 199 (1992).
[7] B. A. Campbell, M. J. Duncan, N. Kaloper, and K. A. Olive, Nucl. Phys. **B351**, 778 (1991).
[8] D. J. Gross and J. H. Sloan, Nucl. Phys. **B291**, 41 (1987); see also K. Sfetsos and A. A. Tseytlin, Phys. Rev. D **49**, 2933 (1994), and references therein.
[9] S. Coleman, J. Preskill, and F. Wilczek, Mod. Phys. Lett. A **6**, 2353 (1991).