

## Scaling of Aharonov-Bohm couplings and the dynamical vacuum in gauge theories

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Recent results on the vacuum polarization induced by a thin string of magnetic flux lead us to suggest an analogue of the Copenhagen “flux spaghetti” QCD vacuum as a possible mechanism for avoiding the divergence of perturbative QED, thus permitting a consistent completion of the full, nonperturbative theory. The mechanism appears to operate for spinor, but not scalar, QED.

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Perturbative quantum electrodynamics (QED) is known to produce a divergence of the charge-charge coupling  $\alpha$  at high mass or small length scales, and there is an important open question whether the full, nonperturbative theory exists. We have been inspired by recent results on vacuum currents induced by a thin string of magnetic flux in spinor (or scalar) QED to examine a picture of the vacuum structure which suggests that for the spinor case the theory may exist after all. By this we mean that there may be a consistent if not unique extrapolation into the strong-coupling domain. Likely this would entail the appearance of new degrees of freedom, as occurs in the long-distance domain of quantum chromodynamics, where quarks and gluons are replaced by color-neutral baryons and mesons. A successful extrapolation of QED would be unprecedented, since the familiar pattern is one in which phenomena at short distance scales are found to underlie those at longer distances, with the latter insensitive to many details of the former. Nevertheless, QED provides what may be the first arena in which such an occurrence is at least conceivable, perhaps explaining why there has long been fascination with strong-coupling QED [1].

The first step in our approach is to consider a reorganized perturbation theory for a different running coupling—not the usual coupling of two charges, but the coupling of an electric charge to a line of magnetic flux. We call this, for obvious reasons, an Aharonov-Bohm (AB) coupling [2]. This new coupling might give better guidance than the old because, while it does grow stronger at small distances, to one-loop order in  $\alpha$  it does not diverge. An attractive aspect, at least for cal-

culational convenience, is that the AB coupling always has zero engineering dimension, not just in the case of four spacetime dimensions as with the traditional charge-charge coupling. This simplifies the “dimensional book-keeping,” and facilitates comparison of the behavior in spacetimes of different dimensions.

Let us review the recent results as a background for our proposal. Vacuum currents circulate around an arbitrarily thin flux string [3–6], generating additional magnetic flux in the region outside the string. Serbryanyi [3] calculated, to lowest nontrivial order in  $\alpha$  but all orders in the flux, the induced current for scalar electrons. For spinors the results are qualitatively different, and were obtained by Górnicki [4] (see also [6]). Below we recount the main features of these induced currents, emphasizing the difference between scalar and spinor QED in  $3+1$  and  $2+1$  dimensions. This leads us to define a charge-flux  $\beta$  function which reproduces the results of the conventional charge-charge  $\beta$  function for small flux, and which suggests that QED might possess a “flux spaghetti” vacuum at scales where its coupling becomes strong (short distance, high energy), reminiscent of the Copenhagen picture for quantum chromodynamics (QCD) at long distance scales [7]. At first sight this exacerbates the consistency problem, since now it becomes necessary to find a mechanism which not only arrests the growth of  $\alpha$  at small distances but also is able to support strong fluxes in tubes of small radius. We shall be seeking evidence for such a mechanism, and arguing that we find it in spinor QED.

The fermionic-induced current may be constructed by computing single-particle currents from the exact solutions of the Dirac equation with a classical background electromagnetic field, and then summing the contributions of these currents for all negative energy (i.e., occupied) states. In this approximation one is treating the fermions as quantized fields while ignoring the fluctuations of the gauge field, which explains why the calculation is exact to lowest nontrivial order in  $\alpha$  but all orders

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in flux. Consider the static flux  $F$  to be confined in an infinitely long zero-radius flux tube, and choose the scalar potential  $A_0 = 0$ .  $CP$  invariance implies that the induced charge density  $\langle j^0 \rangle$  vanishes in  $3+1$  dimensions, while the induced current  $\langle \vec{j} \rangle$  is in general nonvanishing. Translational and rotational symmetries enable us to write  $\langle \vec{j} \rangle = j(r)\hat{\phi}$  in cylindrical coordinates,  $r$  being the distance from the flux tube in the  $z = 0$  plane. The dependence of the current  $j(r)$  on the flux  $F$  is given by [4, 6]

$$j_f^F(r) = \text{sgn}(F) j_f^\delta(r), \quad (1)$$

with

$$\begin{aligned} F &\equiv \text{sgn}(F)(N + \delta), \\ N &\in \mathcal{Z}_+ \cup \{0\}, \\ 0 &\leq \delta < 1, \end{aligned} \quad (2)$$

and

$$j_f^\delta(r) = -\frac{e \sin(\delta\pi)}{\pi^3 r^3} \int_0^\infty dt t \exp\left[-t - \frac{(mr)^2}{2t}\right] K_\delta(t). \quad (3)$$

Here  $m$  is the mass of the fermion of charge  $e$ ,  $K_\delta(t)$  is the modified Bessel function, and the subscript  $f$  in (1) and (3) refers to fermions.

It is apparent from (3) that for a fixed flux the induced current is a monotonically decreasing function of  $r$  and vanishes exponentially for large  $r$ . The variation of the current with  $F$  is sketched in Fig. 1. A number of features should be noted. First, the current vanishes at integer values of flux, just as the AB effect does [2, 8]. Second, for  $F > 0$  the current does not change sign and is periodic under the shift  $F \rightarrow F + N$ ,  $N \in \mathcal{Z}_+$  [4, 6]. Last, the current is antisymmetric about  $F = 0$  as required by charge conjugation. Thus the direction of the induced current is always such as to produce a flux opposing the applied one. This means that one may deduce the sign of the confined flux by looking at the induced current in the region outside the flux tube.

The current in (3) behaves like  $e/r^3$  near the origin,

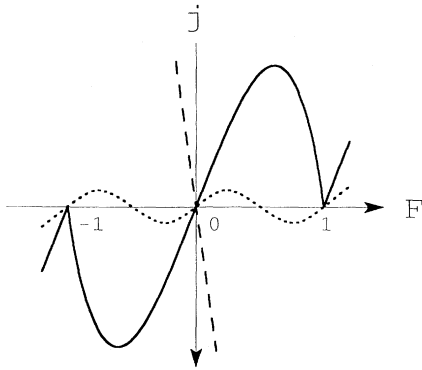


FIG. 1. The  $F$  dependence of the induced fermionic current.  $j_f^F(r)$  (solid line), scalar current  $j_s^F(r)$  (dotted line), and spin-1 current (dashed line) at a fixed nonzero value of  $r$ . Positive current is screening, that is, induces flux opposed to the applied flux in the string.

so that the induced flux is logarithmically divergent. We shall come back to this point shortly. For the analogous problem in  $2+1$  dimensions [5, 6], with massive fermions which break parity, one has in addition to  $\langle \vec{j} \rangle$  a nonvanishing  $\langle j^0 \rangle$  and an induced angular momentum. However, unlike  $\langle \vec{j} \rangle$ , the induced charge and angular momentum are nonvanishing even for integer  $F$  and are not periodic in  $F$  because they receive contributions from threshold (energy =  $\pm m$ ) states that do not contribute to  $\langle \vec{j} \rangle$ . In a further distinction, logarithmic divergence of the induced flux for small radius does not occur in  $2+1$  dimensions, a fact linked with the superrenormalizability of this theory.

For scalar QED the current is [3, 4]

$$\begin{aligned} j_s^F(r) &= \text{sgn}(F) j_s^\delta(r), \\ j_s^\delta(r) &= \frac{1}{4} [j_f^{(1-\delta)}(r) - j_f^\delta(r)], \end{aligned} \quad (4)$$

where  $F$ ,  $\delta$ , and  $j_f(r)$  are given by Eqs. (1)–(3) and the subscript  $s$  in (4) refers to scalars. The scalar current vanishes at half-integer values of flux in addition to the integer ones, as shown in Fig. 1. From (3) and (4) we see that for  $F > 0$  (the results for  $F < 0$  follow by charge conjugation), the current is not of fixed sign but rather opposes the applied flux for  $0 < F < 1/2$  while reinforcing it for  $1/2 < F < 1$ , the pattern repeating with period 1 for  $F > 1$ . Therefore, unlike the spinor case, for fundamental charged scalars the induced current outside the solenoid does not reveal the sign of the flux in the solenoid. The scalar and spinor currents differ because the interaction of a spinless charged particle with a thin flux tube is a pure AB effect [2], while for the spinors an attractive magnetic moment interaction permits penetration of a low-energy electron to the interior of the tube [8–12].

It is remarkable that the access of electrons to the interior of a thin flux tube produces sensitivity only to the sign of the flux (in addition to the fractional part). However, as mentioned above, the sensitivity to absolute magnitude of  $F$  for parity-violating QED in  $2+1$  dimensions [5, 6] cautions us that the precise sensitivity in different situations depends very much on the symmetry constraints. Sensitivity to more than the fractional part of the flux may be viewed as a failure of decoupling between high- and low-energy phenomena: When the radius of the tube is arbitrarily small, fermions confined inside would have arbitrarily high energy. Nevertheless, low energy fermions in the partial wave with smallest total kinetic angular momentum and magnetic moment parallel to the flux still penetrate enough to reveal information beyond the AB phase. Thus the interaction between electrons and thin flux tubes characterized by purely magnetic fields provides an example midway between the pure AB case and the general case of distributed magnetic fields.

Let us recast the above discussion in a different form. A powerful concept for understanding the scale dependence of the dynamics in some field theory is the  $\beta$  function, which gives, for example, the dependence on distance  $r$  of the force between two electric charges. If the force is written as

$$\mathbf{f} = \frac{q_1 q_2}{r^2} \hat{r}, \quad (5)$$

we want to know how the product  $q_1 q_2$  changes as we change the length scale by a factor  $\lambda$ . One may find this by computing the induced vacuum charge density generated by one of the charges. On dimensional grounds, the density at small distance scales must go as  $\alpha/r^3$ , so that the change in total charge between two shells of radii  $r_1$  and  $r_2$  is proportional to  $\alpha \ln(r_1/r_2)$ . The result of this by now standard calculation for spinor QED is [13]

$$\beta = \frac{-d \ln(q_1 q_2)}{d \ln \lambda} = 2\alpha/3\pi, \quad (6)$$

where  $\alpha$  is the fine structure constant, and the expression is valid to lowest nontrivial order in  $\alpha$ . The vacuum screens the interaction between the two charges, so that as they move closer together the effective charge product is less screened, and increases logarithmically.

Consider now the scale dependence in the coupling of a charge  $q$  to a line flux  $F$ . At the same (one-loop) order, the answer may be deduced for spinor QED from Eq. (3). At small  $r$  and small  $F$ , the induced current becomes proportional to  $eF/r^3$ . Integrating this current leads to an induced magnetic field proportional to  $eF/r^2$ , so that the induced flux between two shells depends logarithmically on the ratio of the shell radii. The final result for our new charge-flux  $\beta$  function in this small  $F$  regime agrees exactly with the conventional  $\beta$  function, as we would expect because the small flux is just generated by a current of charges, and the coupling of two charges should determine the coupling of a current with a charge. However, a new feature emerges when we consider flux of arbitrary magnitude. Then, as seen in Fig. 1, the charge-flux  $\beta$  function always produces screening, trying to drive the flux to the next smaller integer value as the length scale increases, so that the  $\beta$  function vanishes for any integer  $F$ . Put differently, as the charge and the flux string are brought closer to each other (i.e., as the length scale decreases), the charge-flux product rather than diverging approaches the next higher Aharonov-Bohm quantum value.

We are now in a position to paint our picture of the QED vacuum: A magnetic field fluctuation produced by charged-particle excitations in the vacuum is strong at short-distance scales. Then, as perceived at large distances, the (screened) flux due to this fluctuation approaches an integer, because these are the values required by the zeros of our  $\beta$  function. This suggests that in pure QED the vacuum might be a “spaghetti” of exponentially thin flux strings, each perceived on moderate or large distance scales as carrying very nearly an integer number of flux units. Phrased differently, this assertion becomes almost a tautology. Since charge-charge coupling becomes strong at short distances, so must charge-flux coupling. Thus, finding a mechanism to support such a flux becomes an important new requirement for demonstrating that QED is consistent. We shall return in a little while to arguments that a suitable mechanism indeed exists in spinor QED.

Such a vacuum could have some interesting properties.

For example, if the approximately integer flux at large distance scales were nonzero, a particle with charge incommensurate to that of the electron would excite the flux spaghetti so that the effective mass of the particle would be raised to a scale characteristic of the flux tube radius. This would be a self-consistent solution, since such massive charged particles would not contribute to the  $\beta$  function in the perturbative regime. Thus, the QED vacuum might produce “spontaneous electric charge quantization,” since incommensurate charges would be allowed in principle, but could not have low mass. Also, one must reconsider the coupling between two point charges in the background of the flux spaghetti. We discuss below how the presence of the spaghetti could damp the otherwise catastrophic growth of the coupling found in conventional perturbation theory.

Let us reiterate our picture of QED at exponentially short-distance scales: The charge-charge coupling increases, and fluctuations become stronger accordingly until the flux spaghetti can be supported. Once the spaghetti is produced, the coupling stops growing. The QED vacuum is then filled with a spaghetti of strong but finite magnetic flux tubes (with all possible velocities, to assure Lorentz invariance), and the coupling is large but finite. Recently one of us [14] considered a different possibility, with all magnetic flux suppressed at short distances. While this is conceivable (i.e., it also may resolve the logarithmic divergence of magnetic flux mentioned earlier), it leaves still open the original question of consistency of purely electric coupling at short distances.

One may ask about the energy cost of producing a spaghetti vacuum for QED. Flux costs energy, and thus the spaghetti vacuum possesses higher electromagnetic field energy than the standard perturbative vacuum. However, in the large-coupling regime the nonperturbative energy functional may prefer to develop flux quanta with the field distributed over a small length scale in the vacuum. A related picture has been studied intensively for  $(2+1)$ -dimensional QED [15].

While we do not know how to carry out a precise analysis in  $3+1$  dimensions, we feel that there are suggestive qualitative indications that the scheme is consistent. First, assume that there are in the vacuum flux tubes (with all possible velocities, as mentioned earlier) possessing a certain, exponentially small radius. Electron wave functions which are spread out over regions large on the scale of this radius will be insensitive to the passage of such a tube, effectively equivalent to a pure gauge transformation. On the other hand, wave functions confined to a region small on this scale will be buffeted by the passage of flux tubes of all velocities, and hence fluctuating fields of unlimited mean-squared strength. This means that the effective mass of the electron will increase rapidly and without limit as the squared four-momentum passes through a critical value corresponding to the inverse-squared radius of the flux tubes, generating a natural cutoff for the electron propagator and assuring consistency of the theory.

Near that cutoff, electrons should have large effective mass, and therefore propagate nonrelativistically. This means that fluctuations in which electron-positron pair

magnetic moments at neighboring sites are lined up will be favored by relatively low action (compared to configurations with random orientation of neighboring magnetic moments), leading immediately to tubes of flux with the appropriate radius. Thus the assumption of flux spaghetti for spinor QED leads to a mechanism generating flux spaghetti.

For scalar QED the charge-flux  $\beta$  function deduced from the small flux limit of (4) agrees with the standard one-loop result [13], which is  $1/4$  the spinor value. For larger flux, the scalar charge-flux  $\beta$  function tries as length scales increase to drive the flux to the nearest integer, with the  $\beta$  function vanishing both at integers and half-integers. Note that our suggested mechanism for self-consistency of the flux spaghetti does not work for scalar QED, since here there are no magnetic moments to line up into tubes of flux. Thus scalar QED might well be inconsistent even if spinor QED were consistent.

It would be interesting to duplicate the computations of the charge-flux  $\beta$  function for one more case, that of charged spin-1 particles. While we are unaware of such a computation in the literature, we may guess the qualitative character of the effect from known results (see [16–18] and references therein), as sketched in Fig. 1. Since the usual perturbative  $\beta$  function of such a theory (which in its simplest form is precisely Yang-Mills gauge theory [19]) has opposite sign from that in the scalar and spinor cases, one expects that as length scales grow (rather than shrink) fluxes should increase. This sign reversal of the  $\beta$  function has been described as a paramagnetism of the vacuum [17]. The vacuum for charged scalars is diamagnetic, although as we have seen it actually pushes the flux towards the nearest integer value, whether higher or lower.

The diamagnetism for charged spinors has been explained as a consequence of the Pauli principle [17, 18] but we feel this is only part of the story: The magnetic field attracts electrons with appropriately oriented magnetic moment, thus reducing their effective mass. For particles in the filled negative energy sea this reduction in mass actually raises single-particle energies, and hence increases the vacuum energy. Thus the entire Dirac sea picture, which includes more than the Pauli principle, is needed to understand the diamagnetism. For spin 1 the paramagnetism of single-particle states should translate into a reduction in vacuum energy associated with zero-point oscillations, thus explaining the changed sign of the  $\beta$  function in non-abelian gauge theories: Spin 1 is the first value for which the naive single-particle paramagnetism is realized also for the field theory vacuum.

The charge-flux  $\beta$  function for this case indicates that here also flux spaghetti must be an important aspect of the QCD vacuum. Such a picture was proposed by Nielsen and Olesen [7] on the basis of a closely reasoned and intricate analysis building on the stimulating work of Savvidy [20] and others. The  $\beta$  function approach introduced here gives an alternative way to understand why such a structure should be natural, and the dynamical considerations for spinor QED suggest that gluon magnetic moments could support this structure.

Recent results in numerical lattice gauge theory for

noncompact QED might be relevant here. Efforts to use this theory to build a base for continuum QED are hampered by the fact that one would expect new degrees of freedom to be excited in the strong-coupling domain (degrees of freedom which from the conventional viewpoint would be called “fundamental,” since they would be found at small distance scales), but without knowing what they are one cannot incorporate them into the lattice theory. There has been a fierce debate about whether strong-coupling noncompact lattice spinor QED does [21] or does not [22] imply a nontrivial continuum theory. However, the most appropriate use of the lattice calculations might well be to suggest what new degrees of freedom could appear at strong coupling. In this connection, noncontroversial aspects may be relevant: In the strong-lattice-coupling regime there is chiral symmetry breaking (electron mass of the same order as the lattice scale), strong alignment between fermion magnetic moment and magnetic field (such as we propose), and the appearance of magnetic monopoles [21]. The monopoles had not occurred to us before, but upon reflection seem a possible corollary to the other phenomena: At short-distance scales where we suggest that electron degrees of freedom become latent, perhaps latent monopoles are on the same footing. This opens the possibility that at high energies the classical electric-magnetic duality rotation symmetry [23] is restored for the full quantum system [24], leading to a unique fixed point for the electromagnetic coupling  $\alpha = 1/2$ . This is larger than the naive or perturbative critical value  $\alpha = 1/4\pi$  [25].

By drawing attention to flux tubes the new description has identified a possibly important feature of the vacuum on scales where the perturbative coupling becomes strong. This suggests qualitatively similar behaviors for the QED vacuum at small distance scales and the QCD vacuum at large distance scales. Our more detailed if still crude considerations indicate that there is a dynamical mechanism to support flux tubes in spinor QED, which therefore may join QCD as a consistent theory in the sense of possessing a natural extrapolation to the strong-coupling regime. It is tempting to identify these flux tubes with the strings of string theory, suggesting that a string structure might be deduced by extrapolation from the physics of lower energies in a wide class of (nonasymptotically free) gauge theories. In contrast to the possibility of extrapolation of spinor QED, scalar QED instead may resemble  $\phi^4$  theory, which is believed to become consistent only if new short-distance degrees of freedom are added “by hand.” If all this be so, then it may show at a deeper level than before that the electron’s intrinsic magnetic coupling is essential to the completeness of QED. There remains the formidable task of developing a systematic calculational scheme which could put these qualitative ideas on a sound footing.

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