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Symmetry properties of the effective action for high-energy scattering in QCD

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We study the effective action describing high-energy scattering processes in the multi-Regge limit of QCD, which should provide the starting point for a new attempt to overcome the limitations of the leading logarithmic and the eikonal approximations. The action can be obtained via simple graphical rules or by integrating in the QCD functional integral over momentum modes of gluon and quark fields that do not appear explicitly as scattering or exchanged particles in the considered processes. The supersymmetry is used to obtain the terms in the action involving quark fields from the pure gluonic ones. We observe Weizsäcker-Williams-type relations between terms describing scattering and production of particles.

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I. INTRODUCTION

In experiments at collider energies kinematical regions become accessible that are characterized by two momentum scales, both much larger than the typical hadronic scale μ. Processes dominated by this kinematics are called semihard [1]. The main features of such processes are calculable perturbatively. Unlike in usual hard processes, large contributions having typically a logarithm of the large scales from each loop change the result qualitatively compared to the Born contributions.

If the largest scale determines the scattering energy and the second scale determines the typical transverse momentum or the momentum transfer, then the semihard kinematical region falls into the Regge region. This situation we encounter in deep-inelastic scattering at small values of the Bjorken variable x. Here we have for the scattering energy squared $s \approx -Q^2/x$, where Q^μ is the momentum transferred by the photon, and therefore we have

$$s \gg -Q^2 \gg \mu^2 . \tag{1.1}$$

The low-x region of structure functions becomes important in hard processes at higher energies. Minijet production in hadronic collisions provides a further example of semihard Regge processes. Clearly here the separation of the perturbative contribution is more difficult compared to the first example. Nevertheless at increasing energies the semihard features should show up more and more clearly, for example, in selected final states with a large rapidity gap between two jets, as proposed in [2].

The perturbative Regge asymptotics has been investigated in the leading logarithmic approximation (LLA) [3]. The resulting amplitudes do not satisfy the s-channel unitarity constraints, and in particular a powerlike increasing contribution to the total cross section is obtained. In the structure functions this appears as a fairly strong increase at small x; i.e., the gluon density becomes so large that the concept of quasifree partons is no longer applicable.

The eikonal approximation developed by Cheng and Wu [4] for Abelian gauge theory ensures s-channel unitarity (but it does not ensure unitarity in subenergy channels). The generalization to the non-Abelian case encounters difficulties, which are understood from the point of view of the LLA. At high energy the scattering is dominated by the exchange of Reggeized gluons with multiparticle intermediate states in the s channel (perturbative QCD Pomeron). In the Abelian case the scattering amplitude with photon quantum numbers in the t channel gets no corrections in the LLA. In QED multiparticle s-channel states become important only on the level of the $e^4 \ln s$ approximation (e^+e^- pair production) [5].

Therefore there is no eikonal scheme relying only on the elastic Born amplitude in the non-Abelian case. An eikonal scheme starting from the Pomeron is used in some models. However, this does not provide an approximation to QCD Regge asymptotics, because the interactions of the QCD Pomerons are not negligible. A Pomeron eikonal approach to the problem of low-x asymptotics of structure functions has been developed in [1].

High-energy scattering in the Regge regime has been studied in gravity in order to understand quantum gravity and strings [6] (in accordance with graviton Reggeization). The eikonal scheme works here, but it neglects the multigraviton intermediate states, which contribute in the LLA because of the graviton self-coupling [8]. As in non-Abelian gauge theories the conventional eikonal scheme has to be replaced by a unitarization scheme based on the LLA results [7].

Including multiparticle production the unitarity conditions become more involved. Unitarity has to be obeyed in all subenergy channels of the inelastic amplitudes [9,10]. The dominating contribution in the LLA comes from multiparticle states obeying the conditions of multi-Regge kinematics (see Fig. 1):

$$\begin{aligned} &s=(p_A+p_B)^2=2(p_Ap_B)\ , \\ &s_i=(k_i+k_{i-1})^2=2(k_ik_{i-1}),\quad i=1,\dots,n+1\ , \\ &k_0\equiv p_{A'},\quad k_{n+1}\equiv p_{B'},\quad k_i=q_{i+1}-q_i\ , \\ &s\gg s_1\sim s_2\sim\cdots\sim s_{n+1} \\ &\quad\gg|q_1^2|\sim|q_2^2|\sim\cdots\sim|q_{n+1}^2|\ , \\ &s_1s_2\cdots s_{n+1}=s\prod_{i=1}^{n}(-k_{\perp i}^2)\ , \\ &(k_1p_A)\ll(k_2p_A)\ll\cdots\ll(k_np_A)\ , \\ &(k_1p_B)\gg(k_2p_B)\gg\cdots\gg(k_np_B)\ , \end{aligned} \tag{1.2}$$

where $k_\perp^\mu$ is defined by the decomposition

$$k^\mu=\frac{(p_Ak)}{(p_Ap_B)}p_B^\mu+\frac{(p_Bk)}{(p_Ap_B)}p_A^\mu+k_\perp^\mu\ . \tag{1.3}$$

In the LLA the elastic amplitude is dominated by the exchange of two Reggeized gluons in the color singlet states interacting by the exchange of gluons in s channel [3]. To obey s-channel unitarity, contributions from the exchange of more than two Reggeized gluons have to be included. These contributions are higher-order corrections from the point of view of the LLA. We keep the condition of the s-channel multiparticle states being in multi-Regge kinematics. In this sense we would like to include only a minimal set of higher-order corrections to the LLA. Contributions from two or more particles being in non-multi-Regge kinematics are not included in the first step. They lead to corrections to the scattering and production vertices considered below and to new vertices. A program to investigate such corrections has been started in [11].

The equations for the contribution of the exchange of more than two Reggeized gluons turn out to be fairly complicated [12]. Moreover, we cannot restrict ourselves to a small number of Reggeized gluons. Also the representation of the problem in the form of a Reggeon calculus is not very helpful because of the complicated multi-Reggeon interactions. As a new idea the effective action approach was proposed [13].

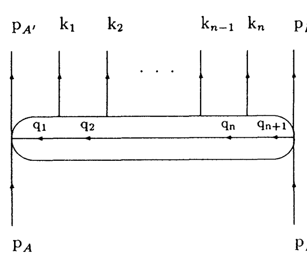

FIG. 1. The general inelastic process in the multi-Regge kinematics.

The effective action describes the scattering and production of particles in the multi-Regge kinematics. In particular, the tree approximation of the multiparticle amplitude reproduces the leading contribution from the sum of tree diagrams of the QCD. Compared to original QCD this formulation is significantly simpler since it does not involve fields with Lorentz and spinorial structure. This is related to the fact that in the LLA the helicities of scattered particles are conserved.

In the considered multi-Regge kinematics there is a natural separation of the longitudinal and transverse directions with respect to the scattering axis. The structure of the effective Lagrangian reflects this separation and, in particular, one can achieve its form emphasizing the scale and conformal symmetries separately in the longitudinal and transverse subspaces. The conformal symmetry in the impact space turned out to be a useful tool for investigating the partial waves for the QCD pomeron and the odderon [14,15]. This gives us reason to expect that this effective action can be transformed (in the impact space) to a two-dimensional model exhibiting full conformal symmetry and will permit us to apply the powerful methods developed for the two-dimensional conformal models.

Reference [13] dealt with pure gluondynamics and with quantum gravity. In a previous paper [22] the effective action for full QCD including quarks was obtained directly from the original QCD action by eliminating with the help of equations of motion certain field modes. In the present paper we discuss in detail some aspects of the derivation and symmetry properties of the effective action.

In the next section we derive from tree graphs the effective vertices for scattering and production of quarks and gluons. By an appropriate choice of the quark wave functions and the gluon polarization vectors these vertices can be represented in a simple form. In order to write down the effective action with these vertices one has to choose fields describing the scattering and exchanged particles. Our choice of fields is based on symmetry arguments discussed in the following sections.

We show that the effective action can be obtained directly from the original QCD action by separating the momentum modes of the fields according to the multi-Regge kinematics and integrating out the modes of highly virtual momenta. The fields appearing in the effective action are now defined in terms of certain modes of the original gauge and fermion fields. In Sec. III, we outline the derivation under a slightly different aspect compared to the paper [22]. We treat mainly the gluon case and discuss shortly the generalization to quarks. In Sec. IV, we use supersymmetry arguments to obtain the fermionic terms out of the gluonic ones.

In Sec. V, we write the fermionic terms in such a way that the action shows a clear separation of longitudinal and transverse subspaces. A definite behavior with respect to rotations and dilatations in both subspaces can be attributed to the fields. We give the supersymmetry transformation relating gluonic and fermionic terms.

Recently Verlinde and Verlinde [16] proposed an effective action for high-energy scattering in gauge theories based on elegant geometrical arguments. We comment on the relation to out multi-Regge effective action (see also [22]).

II. THE EFFECTIVE VERTICES

In this section we derive the effective vertices and graphical rules, which reproduce in a most economic way the leading contribution in the multi-Regge kinematics (1.2) to multiparticle tree amplitudes. Then the effective action is written down in such a way that it reproduces these rules.

We start from tree-level helicity $2\rightarrow2$ and $2\rightarrow3$ amplitudes. It is essential to choose wave functions for gluon and quark helicity states in a convenient way. The results for the tree amplitudes allow one to read off the graphical rules. For writing the effective action we have to introduce fields, the asymptotic states of which correspond to the helicity state wave functions.

In what follows we decompose all the momenta according to Eq. (1.3). In a similar way we also decompose the gluon wave function (for notational simplicity the color indices are suppressed) $\varepsilon^{\mu}(k,\lambda)$ with the helicity λ:

$$\varepsilon^{\mu}(k,\lambda)=\frac{(p_A\varepsilon)}{(p_Ap_B)}p_B^{\mu}+\frac{(p_B\varepsilon)}{(p_Ap_B)}p_A^{\mu}+\epsilon_{\perp}^{\mu}(k,\lambda). \tag{2.1}$$

Since in the multi-Regge kinematics we have a separation of the longitudinal and the transverse components it is very convenient to introduce the light-cone variables and the complex transverse coordinates:

$$\begin{aligned}
&k_{\pm}=k_0\pm k_3,\quad k=k_{\perp}^1+ik_{\perp}^2,\quad k^*=k_{\perp}^1-ik_{\perp}^2\ ,\\
&\epsilon_{\pm}(k,\lambda)=\varepsilon_0(k,\lambda)\pm\varepsilon_3(k,\lambda)\ ,\\
&\epsilon(k,\lambda)=\epsilon_{\perp}^1(k,\lambda)+i\epsilon_{\perp}^2(k,\lambda)\ ,\\
&\partial_{\pm}=\tfrac{1}{2}(\partial_0\pm\partial_3),\quad \partial=\tfrac{1}{2}(\partial_1-i\partial_2),\quad \partial^*=(\partial)^*\ ,\\
&\partial_-x_+=\partial_+x_-=\partial x=\partial^*x^*=1\ .
\end{aligned} \tag{2.2}$$

We choose the incoming particles momenta as $p_A^{\mu}=(\sqrt{s}/2)(1,0,0,1)$ and $p_B^{\mu}=(\sqrt{s}/2)(1,0,0,-1)$.

The straightforward method to obtain the effective action uses the explicit form of the helicity wave functions. In the case of gluons these wave functions $\varepsilon^{\mu}(k,\lambda)$ can be written down in the axial gauge where the gauge-fixing vector is either the incoming momentum p_A (L gauge) or the incoming momentum p_B (R gauge), as shown in Fig. 1. Moreover they can be parametrized by their transverse components $\epsilon_{\perp}^{\mu}(k,\lambda)$ (see [13]). Then the gauge transformation of the transverse components $\epsilon_{\perp}^{\mu}(k,\lambda)$ of the gluon wave function,

$$\epsilon_{\perp L}^{\mu}(k,\lambda)=\epsilon_{\perp R}^{\mu}(k,\lambda)-2\frac{(k_{\perp}\epsilon_{\perp_R})}{k_{\perp}^2}k_{\perp}^{\mu}\ , \tag{2.3}$$

written down in the complex number notation takes the form of the phase transformation

$$\epsilon_L(k,\lambda)=-\frac{k}{k^*}[\epsilon_R(k,\lambda)]^* \tag{2.4}$$

(the subscripts L,R denote the L or R gauges, respectively). The gluon helicity states[1] have the following: (a) in the L gauge,

$$\begin{aligned}
&\varepsilon_L^{\mu}(k,+)=e_L(k)\left\{\frac{1}{\sqrt{2}}(\delta_1^{\mu}-i\delta_2^{\mu})+\frac{k^*}{\sqrt{2s}\,k_+}p_A^{\mu}\right\}\ ,\\
&\varepsilon_L^{\mu}(k,-)=\{\varepsilon_L^{\mu}(k,+)\}^*\ ;
\end{aligned} \tag{2.5}$$

(b) in the R gauge,

$$\begin{aligned}
&\varepsilon_R^{\mu}(k,-)=e_R(k)\left\{\frac{1}{\sqrt{2}}(\delta_1^{\mu}-i\delta_2^{\mu})+\frac{k^*}{\sqrt{2s}\,k_-}p_B^{\mu}\right\}\ ,\\
&\varepsilon_R^{\mu}(k,+)=\{\varepsilon_R^{\mu}(k,-)\}^*\ .
\end{aligned} \tag{2.6}$$

The helicity states given by Eqs. (2.5) and (2.6) are determined up to the phase factors $e_L(k)$ and $e_R(k)$. As a consequence of Eq. (2.4) they are subject to the gauge transformation

$$e_L(k)=-\frac{k}{k^*}[e_R(k)]^*\ . \tag{2.7}$$

We proceed in the similar way in the case of quark fields. The quark helicity states $u(k,\lambda)$ have the following forms (again the color indices are suppressed): (a) in the L gauge,

$$\begin{aligned}
&u_L(k,+)=\frac{\chi_L(k)}{\sqrt{2k_+}}\begin{bmatrix}\phi_L(k,+)\\ \phi_L(k,+)\end{bmatrix},\quad \phi_L(k,+)=\begin{bmatrix}k^*\\ k_+\end{bmatrix}\ ,\\
&u_L(k,-)=C[\bar{u}_L(k,+)]^T
\end{aligned} \tag{2.8}$$

(C is the charge conjugation matrix); (b) in the R gauge,

$$\begin{aligned}
&u_R(k,-)=\frac{\chi_R(k)}{\sqrt{2k_-}}\begin{bmatrix}\phi_R(k,-)\\ -\phi_R(k,-)\end{bmatrix}\ ,\\
&\phi_R(k,-)=-\begin{bmatrix}-k^*\\ k_-\end{bmatrix}\ ,\\
&u_R(k,+)=C[\bar{u}_R(k,-)]^T\ .
\end{aligned} \tag{2.9}$$

In Eqs. (2.8) and (2.9) the functions $\chi_L(k)$ and $\chi_R(k)$ are arbitrary phase functions modulo which the helicity states are determined [compare Eqs. (2.5) and (2.6)]. Since the helicity wave functions u_L and u_R are physically equivalent [i.e., $u_L(k,\lambda)=u_R(k,\lambda)$] we get

$$\chi_L(k)=\left[\frac{k}{k^*}\right]^{1/2}[\chi_R(k)]^*\ , \tag{2.10}$$

[1]The helicity state in the axial gauge defined by the vector n^{μ} ($nk\neq0$) is an eigenvector of the rotation generator belonging to the little group simultaneous of both vectors k^{μ} and n^{μ}.

which is the relation analogous to the gauge condition (2.7).

Consider now as an example the tree approximation of the gluon production in the quark gluon scattering. For definiteness we take the case that all particles have the same helicities $\lambda=+$. As known from Refs. [3,17] the result, being a sum of many Feynman graphs, can be written in the factorized form as shown in Fig. 2 (where gluons and fermions are denoted by wavy and solid lines, respectively). Using the form of helicity states introduced above and the complex number notation we obtain

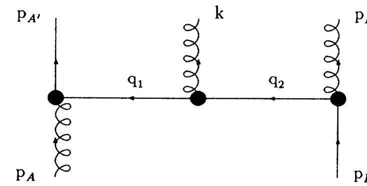

FIG. 2. The gluon production in the gluon-fermion scattering.

$$[g t^{A}_{A' n_1} e_R(p_A)\chi_R(p_{A'})\sqrt{2p_{A'-}}]\frac{1}{q_1^*}\left[g\sqrt{2}t^{k}_{n_1 n_2} q_1^* \frac{e_L(k)^*}{k^*}\right]\frac{1}{q_2^*}[g t^{B'}_{n_2 B}\sqrt{2p_{B+}}\chi_L(p_B)e_L(p_{B'})^*] , \tag{2.11}$$

where the matrices t^a_{bc} are the color group generators of the quark representation. Expression (2.11) is a product of the effective vertices corresponding to the scattering processes appearing on both ends of Fig. 2 and the gluon production vertex. They are connected by the propagators of the helicity + fermionic particles $(1/q^*)$.

We present also for further reference the expression describing the helicity − gluon production in two-fermion scattering (both having the helicities +). The tree-scattering amplitude (see Fig. 3) has the form

$$[g\sqrt{2}t^{n_1}_{A'A}\chi_R(p_{A'})\chi_R(p_A)^*\tfrac{1}{2}(p_{A-}+p_{A'-})]\frac{1}{q_1 q_1^*}\left[ig\sqrt{2}f^{k n_1 n_2}q_1 q_2^* \frac{e_R(k)^*}{k^*}\right]$$
$$\times\frac{1}{q_2 q_2^*}[g\sqrt{2}t^{n_2}_{B'B}\chi_L(p_{B'})^*\chi_L(p_B)\tfrac{1}{2}(p_{B+}+p_{B'+})] , \tag{2.12}$$

where f^{abc} are the color group structure constants. There appears in the large square brackets the effective vertex describing the gluon production from the t-channel gluon line [13].

Another example is provided by the helicity + fermion production in the quark gluon scattering (again all gluons have the helicities +) as is shown in Fig. 4. The tree approximation scattering amplitude has the form

$$[-ig\sqrt{2}f^{n_2 A' A}e_R(p_A)^* e_R(p_{A'})\tfrac{1}{2}(p_{A-}+p_{A'-})]\frac{1}{q_1 q_1^*}$$
$$\times\left[-g\sqrt{2}t^{n_1}_{k n_2}q_1^*\sqrt{k_+}\frac{\chi_L^*(k)}{k^*}\right]\frac{1}{q_2^*}[g t^{B'}_{n_2 B}\sqrt{2p_{B+}}\chi_L(p_B)e_L(p_{B'})^*] . \tag{2.13}$$

Again from Eq. (2.13) one can read off the corresponding effective vertices and the propagators.

As these examples show, despite the color indices there are no other indices. This is an essential simplification in comparison with the original formulation presented in Refs. [3,17]. Appendix B collects Feynman rules from which one can write down the corresponding expression for arbitrary process under consideration.

We want to write the effective action which reproduces the above tree formulas. For this the wave functions e_L, e_R, χ_L, χ_R have to be related to the wave functions of fields appearing in the effective action. These relations are

$$\phi^*(k)=2\sqrt{2}\frac{e_L(k)}{k},\quad \phi(k)=-2\sqrt{2}\frac{e_R(k)}{k} ,$$
$$\chi_+(k)=\sqrt{2k_+}\frac{\chi_L^*(k)}{k^*},\quad \chi_-(k)=\sqrt{2k_-}\frac{\chi_R^*(k)}{k^*} , \tag{2.14}$$
$$\chi_+^*(k)=\sqrt{2k_+}\frac{\chi_L(k)}{k},\quad \chi_-^*(k)=\sqrt{2k_-}\frac{\chi_R(k)}{k} .$$

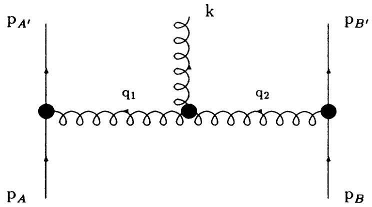

FIG. 3. The gluon production in the gluon-gluon scattering.

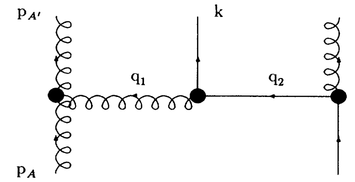

FIG. 4. The fermion production in the gluon-fermion scattering.

The Coulomb interaction in the t channel is carried by real $A_\pm$ fields in the case of gluon and in the case of fermion by complex $a_\pm, a_\pm^*$ fields [22]. The gluonic fields $A_\pm$ should not be identified with the light-cone components of a gluonic four-potential (see Sec. III).

The effective action reproducing the effective vertices and propagators discussed above is given by

$$S = S_k + S_{s-} + S_{s+} + S_p \ , \tag{2.15}$$

where

$$\begin{aligned} S_k &= S_{ks} + S_{kp} \ , \\ S_{ks} &= \int d^4x \left[-\tfrac{1}{2}(\partial^*\phi^a)\Box(\partial\phi^{a*}) + i\bar{\chi}_+^{\alpha*}\Box\partial\chi_-^\alpha + i\bar{\chi}_+^\alpha\Box\partial^*\chi_-^{\alpha*} \right] , \\ S_{kp} &= \int d^4x \left[-2A_+^a\partial\partial^* A_-^a - i\bar{a}_+^\alpha\partial^* a_-^{\alpha*} - i\bar{a}_+^{\alpha*}\partial a_-^\alpha - i\bar{a}_-^{\alpha*}\partial a_+^\alpha - i\bar{a}_-^\alpha\partial^* a_+^{\alpha*} \right] , \end{aligned} \tag{2.16}$$

$$\begin{aligned} S_{s-} = -g\int d^4x \Bigg[& \frac{i}{2}(\partial_-\partial^*\phi)T^a(\partial\phi^*)A_+^a - \frac{i}{2}(\partial^*\phi)T^a(\partial_-\partial\phi^*)A_+^a - [\bar{\chi}_+^* t^a(\overleftrightarrow{\partial}_-\partial\chi_-)]A_+^a - [\bar{\chi}_+ t^a(\overleftrightarrow{\partial}_1\partial^*\chi_-^*)]A_+^a \\ & - \bar{a}_+^* t^a(\partial_-\chi_+)(\partial\phi^{a*}) - \bar{a}_+ t^a(\partial_-\chi_+^*)(\partial^*\phi^a) - (\partial_-\bar{\chi}_+^*)t^a a_+(\partial\phi^{a*}) - (\partial_-\bar{\chi}_+)t^a a_+^*(\partial^*\phi^a) \Bigg] , \end{aligned} \tag{2.17}$$

$$\begin{aligned} S_{s+} = -g\int d^4x \Bigg[& \frac{i}{2}(\partial_+\partial^*\phi^*)T^a(\partial\phi)A_-^a - \frac{i}{2}(\partial^*\phi^*)T^a(\partial_+\partial\phi)A_-^a - [\bar{\chi}_-^* t^a(\overleftrightarrow{\partial}_+\partial\chi_+)]A_-^a \\ & - [\bar{\chi}_- t^a(\overleftrightarrow{\partial}_+\partial^*\chi_+^*)]A_-^a - \bar{a}_-^* t^a(\partial_+\chi_-)(\partial\phi^a) - \bar{a}_- t^a(\partial_+\chi_-^*)(\partial^*\phi^{a*}) \\ & - (\partial_+\bar{\chi}_-^*)t^a a_-(\partial\phi^a) - (\partial_+\bar{\chi}_-)t^a a_-^*(\partial^*\phi^{a*}) \Bigg] , \end{aligned} \tag{2.18}$$

$$\begin{aligned} S_p = g\int d^4x \big(& \phi^a(\partial A_-)T^a(\partial^* A_+) - \phi^{a*}(\partial^* A_-)T^a(\partial A_+) \\ & - \frac{i}{2}\{\phi^a[-\bar{a}_+^* t^a(\partial a_-) + (\partial^*\bar{a}_+)t^a a_-^*] + \phi^{a*}[\bar{a}_+ t^a(\partial^* a_-^*) - (\partial\bar{a}_+^*)t^a a_-] \\ & \quad + \phi^a[-(\partial\bar{a}_-^*)t^a a_+ + \bar{a}_- t^a(\partial^* a_+^*)] + \phi^{a*}[(\partial^*\bar{a}_-)t^a a_+^* - \bar{a}_-^* t^a(\partial a_+)]\} \\ & \quad + i[\bar{\chi}_- t^a a_-^*(\partial^* A_+^a) - \bar{\chi}_-^* t^a a_-(\partial A_+^a) + \bar{a}_- t^a\chi_-^*(\partial^* A_+^a) - \bar{a}_-^* t^a\chi_-(\partial A_+^a) \\ & \quad + \bar{a}_+ t^a\chi_+^*(\partial^* A_-^a) - \bar{a}_+^* t^a\chi_+(\partial A_-^a) + \bar{\chi}_+ t^a a_+^*(\partial^* A_-^a) - \bar{\chi}_+^* t^a a_+(\partial A^a)]\big) \ . \end{aligned} \tag{2.19}$$

The bar over fermionic fields denotes complex conjugation (only for the Majorana particles we have $\bar{\chi} = \chi^*$, $\bar{a} = a^*$). The indices a and α refer to the adjoint representation of the gauge group and the quark representation, respectively. In brackets bilinear in gluon fields the trace in the adjoint representation is understood. T^a are the matrices representing the generators of the adjoint representation:

$$(T^a)_{bc} = -if^{abc}, \quad (\mathcal{A}_1 T^a \mathcal{A}_2) = -if^{abc}\mathcal{A}_1^b\mathcal{A}_2^c \ .$$

In brackets bilinear in the quark fields the sum over the gauge group indices of the quark representation t^a and over flavors is understood. The operator $\Box = 4(\partial_+\partial_- - \partial\partial^*)$, $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$.

In the next sections we present the symmetry arguments which justify the choice of fields given by Eq. (2.14) and lead to the effective action (2.15).

III. EFFECTIVE ACTION FROM THE QCD FUNCTIONAL INTEGRAL

In the previous section the effective action was obtained by analyzing the leading contributions of tree Feynman graphs. On the other hand, effective action is conventionally understood as a result of some fields appearing in the path-integral representation being integrated out. The aim of the present section is to show that also the effective action (2.15) can be obtained in such a way, i.e., by integrating over certain modes in the original QCD action. This integration procedure differs in some aspect from those which one encounters most frequently. In the usual situation one integrates over all modes of the fields which do not appear in the initial and final states. In our case, because of the multi-Regge kinematics (1.2), the outgoing particles are close to one of the incoming momenta p_A or p_B. The virtual particles transferring the momentum through t channel carry small longitudinal momenta. Because of this separation the integration over modes which are absent in the resulting effective action is more sophisticated than in the more familiar cases (Heisenberg-Euler electrodynamics or the integration over heavy fields). A similar asymmetric separation of modes appears in the derivation of the effective action for the high-temperature limit.

We consider the high-energy scattering of gluons and quarks of peripheral type, i.e., with momentum transfer of order $\mu_\perp$, where $s \gg \mu_\perp^2 \gg \mu^2$. In the leading contribution essentially all produced particles are in momentum space either close to the incoming particle p_A or close to p_B.

Let us start the discussion with the pure gluonic case. We shall concentrate first on the particles flying approximately in the direction of the incoming momentum p_A and treat the other incoming gluon as an external source. In the light-cone axial gauge $A_-^a=0$ the gluonic part of the QCD action expressed in terms of the light-cone components $A_\pm^a$ and the transverse components $A_\perp^{a\sigma} \equiv A^{a\sigma}$ [Eq. (2.1)] takes the form

$$S_g=\int d^4x \left[\frac{1}{2} A_\sigma^a \Box A^{a\sigma} + \frac{1}{2}(\partial_- A_+^a + \partial_\sigma A^{a\sigma})^2 - g f^{abc}(A_\sigma^a \partial_- A^{b\sigma}) A_+^c - g f^{abc} A_\sigma^b A_\rho^c \partial^\sigma A^{a\rho} - \frac{g^2}{4} f^{abc} f^{ab'c'} A_\sigma^b A_\rho^c A^{b'\sigma} A^{c'\rho} \right] . \tag{3.1}$$

We define

$$A_+^{\prime a} = A_+^a + \partial_-^{-1} \partial_\sigma A^{a\sigma} \tag{3.2}$$

and integrate over this field combination. Simulating the effect of the particles flying close to p_B by an external source, the modes of A'_+ with small k_- could be used to describe this source.

Performing the Gaussian integration over $A_+^{\prime a}$ we obtain

$$S_1=\int d^4x \left[\frac{1}{2} A_\sigma^a \Box A^{a\sigma} + g f^{abc}(A_\sigma^a \partial_- A^{b\sigma}) \frac{1}{\partial_-} \partial_\rho A^{c\rho} + \frac{g^2}{2} f^{abc} f^{ab'c'} (A_\sigma^b \partial_- A^{c\sigma}) \frac{1}{\partial_-^2} (A_\rho^{b'} \partial_- A^{c'\rho}) - g f^{abc} A_\sigma^b A_\rho^c \partial^\sigma A^{a\rho} - \frac{g^2}{4} f^{abc} f^{ab'c'} A_\sigma^b A_\rho^c A^{b'\sigma} A^{c'\rho} \right] . \tag{3.3}$$

The transverse components A_σ are the physical gluon fields. We use the complex notation for transverse vectors and derivatives introduced above to write the action in the form

$$S_1=\int d^4x \left\{ -\frac{1}{2} A^a \Box A^{a*} - i\frac{g}{2} [A T^a \overleftrightarrow{\partial}_- A^*] \cdot \partial_-^{-1} (\partial A^a + \partial^* A^{a*}) - \frac{g^2}{8} [(A T^a \overleftrightarrow{\partial}_- A^*)] \partial_-^{-2} [A T^a \overleftrightarrow{\partial}_- A^*] + i\frac{g}{2} [A T^a A^*](\partial A - \partial^* A^*) - \frac{g^2}{8} [A T^a A^*][A T^a A^*] \right\} . \tag{3.4}$$

We divide the transverse gauge fields A^a into parts with respect to the corresponding momentum k:

$$A^a \to A^a + A_1^a + \mathcal{A}^a . \tag{3.5}$$

$\mathcal{A}$ represents the modes where

$$k_+ k_- \ll k k^* \sim \mu_\perp^2 . \tag{3.6}$$

This is the typical momentum range for exchanged particles in the peripheral scattering.

A_1 represents the modes where

$$k_+ k_- \gg k k^* \sim \mu_\perp^2 . \tag{3.7}$$

These modes will be integrated out in the next step. We integrate generally over all modes with $|k^2| \gg \mu_\perp^2$. Formally our approximation does not include the effects of a running coupling. They are not associated with logarithms of the large energy s. However, from this it becomes clear that in the resulting action the coupling is to be understood as renormalized at a scale of order $\mu_\perp$, which we assume to be large compared to the hadronic scale μ.

The original notation A is kept for the modes, where

$$k_+ k_- \sim k k^* \sim \mu_\perp^2 . \tag{3.8}$$

This is the typical momentum range for scattering and produced particles going in the s-channel direction. Moreover, the dominant contribution comes from s-channel intermediate states with particles strongly ordered and well separated in longitudinal momenta according to the multi-Regge condition (1.2) and close to mass shell: i.e.,

$$|k_+ k_- - k k^*| \ll \mu_\perp^2 . \tag{3.9}$$

The effect of pairs of particles not being in the multi-Regge configuration as well as the effect of particles being off shell (in addition to the effects obtained by integrating over A_1 below) go beyond our approximation. A systematic study of such corrections has been started in [11].

Now we analyze the terms resulting from the action (3.4) after substituting the decomposition (3.5) with respect to momentum modes.

The kinetic term decomposes into three parts. By definition the propagation between different modes is small. In the kinetic term of $\mathcal{A}$ the transverse derivatives and in the one for A_1 the longitudinal derivatives dominate

$$\int d^4x\{-2A_1^a\partial_+\partial_-A_1^{a*}-\tfrac{1}{2}A^a\Box A^{a*}+2\mathcal{A}^a\partial\partial^*\mathcal{A}^{a*}\} . \tag{3.10}$$

We concentrate now on the second term in (3.4). Its dominant contribution comes from the configuration, where the field, on which ∂_- acts, has large k_- and the field on which ∂_-^{-1} acts has small k_-. From this we understand that the form of this term with respect to ∂_- leads naturally to the ordering of k_- and to the multi-Regge kinematics (1.2).

Consider first the case if both fields entering the current $(AT^a\overleftrightarrow{\partial}_-A^*)$ are not of type $\mathcal{A}$, i.e., describe scattering particles. If also the third field is not of type $\mathcal{A}$, then the resulting term describes particle production by bremsstrahlung [Fig. 5(a)]:

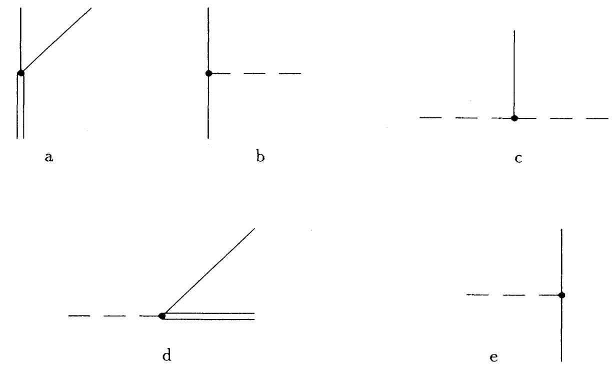

FIG. 5. Vertices resulting from the second term in the action (3.4) by separating fields modes of types $\mathcal{A}$ (dashed line), A (solid line), and A_1 (double line). We imagine the graphs as parts of graph for the multiparticle amplitude in Fig. 1. The s channel goes vertically and the t channel horizontally. Lines going more to the right carry smaller k_-.

$$-\frac{ig}{2}[(A+A_1)T^a\partial_-(A+A_1)^*+(A+A_1)^*T^a\partial_-(A+A_1)]\partial_-^{-1}[\partial(A^a+A_1^a)+\partial^*(A^a+A_1^a)^*] . \tag{3.11}$$

If the third field is of type $\mathcal{A}$ then the corresponding term describes peripheral scattering of particles close to p_A. We introduce [22]

$$\mathcal{A}_+=-\frac{2}{\partial_-}\partial\mathcal{A},\quad \mathcal{A}_+^*=-\frac{2}{\partial_-}\partial^*\mathcal{A}^* \tag{3.12}$$

and write the contribution as [Fig. 5(b)]

$$\frac{ig}{4}[(A+A_1)T^a\partial_-(A+A_1)^*+(A+A_1)^*T^a\partial_-(A+A_1)](\mathcal{A}_+^a+\mathcal{A}_+^{a*}) . \tag{3.13}$$

Consider now the case where two fields are of type $\mathcal{A}$. Defining [22]

$$\partial^*\mathcal{A}_-^*=\partial_-\mathcal{A},\quad \partial\mathcal{A}_-=\partial_-\mathcal{A}^* , \tag{3.14}$$

we have the contribution [Fig. 5(c)]

$$\frac{ig}{2}[(A_1+A)T^a\partial\mathcal{A}_-+(A+A_1)^*T^a\partial^*\mathcal{A}_-^*](\mathcal{A}_+^a+\mathcal{A}_+^{a*}) \tag{3.15}$$

describing particle production. Typical $\mathcal{A}_-$ carries a large longitudinal momentum component k_-, which is of the same order as the one carried by A. The corresponding momentum component carried by $\mathcal{A}_+$ is much smaller. The multi-Regge chain with strongly ordered momentum components k_- appears by iterating this vertex. Because of the mass shell condition (3.9) for fields of type A the multi-Regge kinematics (1.2) holds. The definition of $\mathcal{A}_\pm$ takes into account that the large momentum denominators have to cancel successively against corresponding numerators. Using this notation we keep in mind that the $\mathcal{A}_+$ and $\mathcal{A}_-^*$ are not independent.

There are more contributions from the second term in (3.4) where the field entering with the derivative ∂_- is of type $\mathcal{A}$:

$$ig\left\{[\partial\mathcal{A}_-T^a(A+A_1)+\partial^*\mathcal{A}_-^*T^a(A+A_1)^*]\partial_-^{-1}(\partial A^a+\partial^*A^{a*})\right.$$
$$\left.+\frac{1}{2}\left[\left(\frac{\partial}{\partial_-}\mathcal{A}_-\right)T^aA+\left(\frac{\partial^*}{\partial_-}\mathcal{A}_-^*\right)T^aA^*\right](\partial A^a+\partial^*A^{a*})\right\} . \tag{3.16}$$

If the k_- momentum component of the last field is much smaller than those of the other fields, then the first term contributes to the production of bremsstrahlung type, Fig. 5(d), and the second gives a small contribution. Here $(A+A_1)$ is to be replaced just by A_1, because the corresponding momentum obeys (3.7).

In the other case, if the k_- component of the last field in (3.16) is of the same order as the one of A_-, also the second field in the curly brackets describes a quasireal gluon (3.9) [replace $(A+A_1)$ by A]. The result contributes to scattering of particles close to p_B, with $\mathcal{A}_-$ describing the interaction with particles closer to p_A [Fig. 5(e)]. Then the second term is important and produces an unpleasant singularity in the multi-Regge limit, where for the exchanged particles k_- is much less than the transverse momentum k. We return to the last case in a moment and show that the singularity cancels.

Consider now the bremsstrahlung contribution of (3.16). The large longitudinal momentum in the denominator of the A_1 propagator cancels, if the adjacent vertex radiates bremsstrahlung. The corresponding contributions to particle production are of the same order as the one by the vertex (3.15) and adding them the latter vertex turns into the effective vertex. Clearly only one A_1 propagator is compensated by one bremsstrahlung vertex. Contributions with more A_1 propagators are suppressed by powers of large subenergies in the multi-Regge kinematics. Therefore, we integrate over A_1 just by picking up the bremsstrahlung contributions and including them into the nearest effective production vertex. In this way we are led to consider the contributions to particle production shown in Fig. 6.

The sum of the first two terms (in both cases) has the same color structure as the third term. In momentum space the third graph is in both cases proportional to

$$g\frac{2}{|q|^2}\left[\frac{q^*}{2}A+\frac{q}{2}A^*\right] .$$

The sum of the first two bremsstrahlung terms yields correspondingly the momentum factors

$$-g\frac{p_-}{(p+k)^2}\frac{1}{k_-}(k^*A+kA^*)=-g\left[\frac{1}{k}A+\frac{1}{k^*}A^*\right] \tag{3.17}$$

which can be related to the effective bremsstrahlung vertex

$$-\frac{ig}{2}(\partial\partial^*\mathcal{A}_-^a)\left[\frac{1}{\partial^*}A\right]T^a(\mathcal{A}_+ +\mathcal{A}_+^*)+\text{c.c.} \tag{3.18}$$

We have used (3.7) for the A_1 propagator, allowing the approximation $(p+k)^2\approx p_-k_+$, and the mass-shell condition (3.9) for the momentum k of the produced gluon. The resulting contribution correspond to replacing the particle production vertex (3.14) by the following effective vertex being the sum of expressions (3.14) and (3.18):

$$-\frac{ig}{2}\{[(\partial^{*-1}A)T^a(\partial\mathcal{A}_-)]\partial^*(\mathcal{A}_+^a+\mathcal{A}_+^{a*})+[(\partial^{-1}A^*)T^a(\partial^*\mathcal{A}_-^*)]\partial(\mathcal{A}_+^a+\mathcal{A}_+^{a*})\} . \tag{3.19}$$

There is no contribution from bremsstrahlung to the production of particles belonging with respect to longitudinal momentum ordering (1.2) not to the adjacent vertex. This would take more than one A_1 propagator and is suppressed as discussed above.

This is also the reason why the third term in (3.4) does not contribute to particle production. This term gives no essential contribution with one or more of the involved fields being of the type A_1. Therefore, it was possible to do the A_1 integral without mentioning about this term.

When in the third term of Eq. (3.4) two fields are of the type $\mathcal{A}$, this results in terms of the form

$$[AT^a(\partial\mathcal{A}_-)+A^*T^a(\partial^*\mathcal{A}_-^*)]\partial_-^{-2}[AT^a(\partial_-^2\partial^{*-1}\mathcal{A}_+^*)+A^*T^a(\partial_-^2\partial^{-1}\mathcal{A}_+)]$$

or (3.20)

$$[AT^a(\partial\mathcal{A}_-)+A^*T^a(\partial^*\mathcal{A}_-^*)]\partial_-^{-2}[(\partial_-A)T^a(\partial_-\partial^{*-1}\mathcal{A}_+^*)+(\partial_-A^*)T^a(\partial_-\partial^{-1}\mathcal{A}_+)] .$$

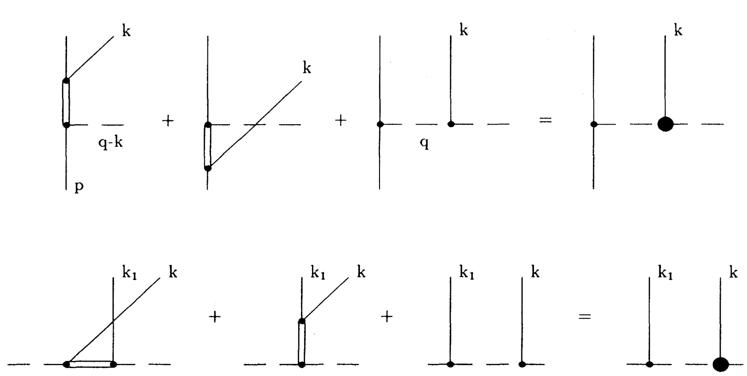

FIG. 6. Diagrams contributing to the effective production vertex.

In order to compensate the denominator in the first case the particle corresponding to $\mathcal{A}_+$ has to have a large k_- component compared to the one of A in the second set of square brackets. Then the propagator of $\mathcal{A}_+$ would have a large longitudinal part, i.e., the modes are in fact of type A_1 rather than of type $\mathcal{A}$. Therefore, the first term in (3.20) gives a negligible contribution. In the second case the large denominator should be canceled by the momenta of both fields in the second set of square brackets. But only one of them can have a large k_- component.

We see that the only non-negligible contribution of the third term in (3.4) arises in the case if all fields are of type A (corresponding to elastic scattering) or in the case if one field is $\mathcal{A}_-$ and the others are of type A (corresponding to the "right end" of a multi-Regge chain) (Fig. 7). Both contributions are represented by the vertex

$$\frac{ig}{2}(\mathcal{A}^a_- + \mathcal{A}^{a*}_-)\partial\partial^*\partial_-^{-2}[AT^a\overleftrightarrow{\partial}_- A^*] \; . \tag{3.21}$$

Indeed, the integral over $\mathcal{A}_\pm$ with the kinetic term (3.10) and the vertices (3.13) or (3.15) (omitting A_1 in those expressions) and (3.21) reproduce the mentioned contributions of the third term in (3.4). The vertex (3.21) can be understood as describing the scattering of produced particles close in momenta to the right incoming particle with momentum p_B. There are more contributions of this type, Fig. 5(e), arising in particular from the second term in (3.4), if one of the fields is $\mathcal{A}_-$. Such a contribution is contained in (3.16) as discussed above, if the second field ($A + A_1$) in the curly brackets is of type A. Another contribution is contained in (3.13): we express the field of type $\mathcal{A}$ as $\mathcal{A}_-$ (3.14) instead of $\mathcal{A}_+$ and arrive at

$$-\frac{ig}{2}[\partial_-^{-2}\partial\partial^*(\mathcal{A}^a_- + \mathcal{A}^{a*}_-)][AT^a\overleftrightarrow{\partial}_- A^*] \; . \tag{3.22}$$

This contribution cancels the one of (3.21) from the first term in (3.4). Therefore, we are left with the contribution from (3.16). Omitting A_1 we rewrite it up to the total derivatives as

$$-ig\{\mathcal{A}^a_-\partial[AT^a\partial_-^{-1}(\partial A + \partial^* A^*)] + \tfrac{1}{2}(\partial_-^{-1}\mathcal{A}^a_-)\partial[AT^a(\partial A + \partial^* A^*)]\} + \text{c.c.} \tag{3.23}$$

Integrating over A_1 we took into account up to now the particular contribution (Fig. 6), if in one of the adjacent vertices there is a type-$\mathcal{A}$ field. There are corresponding contributions from the case, if this type-$\mathcal{A}$ field is replaced by a type A one and also if both adjacent vertices have a type-$\mathcal{A}$ field. They result in vertices, which are readily obtained from the bremsstrahlung part (3.18) of the effective vertex (3.19).

In the first case by substituting $\mathcal{A}_+$ by A via (3.12),

$$-ig\tfrac{1}{2}(\partial^{*-1}AT^a\partial\partial^*\mathcal{A}_-)\partial_-^{-1}(\partial A^a + \partial^* A^{a*}) + \text{c.c.} \tag{3.24}$$

and in the second case by replacing A by $\mathcal{A}_+$,

$$-\frac{ig}{4}\frac{1}{2}[\partial_-(\partial\partial^*)^{-1}\mathcal{A}_+]T^a(\partial\partial^*\mathcal{A}_-)(\mathcal{A}^a_+ + \mathcal{A}^{a*}_+) + \text{c.c.} \tag{3.25}$$

There is an additional factor $\frac{1}{2}$ since now two fields are of the same type. Indeed, the first two graphs in Fig. 6 are equivalent, if the solid and dashed lines are not distinguished.

The fourth term in (3.4) does not involve longitudinal derivatives and may seem to be unimportant in the considered asymptotics. Nevertheless, it gives rise to singular contributions compensating the ones in (3.23) (compare [22]). Indeed, its contribution in the case if one of the fields is $\mathcal{A}_-$ is given by

$$\frac{ig}{2}[(\partial^*\partial_-^{-1}\mathcal{A}^*_-)T^a A^* + AT^a(\partial\partial_-^{-1}\mathcal{A}_-)](\partial A^a - \partial^* A^{a*}) \tag{3.26}$$

and can be represented up to total derivatives as

$$\frac{ig}{2}(\partial_-^{-1}\mathcal{A}^a_-)\partial[AT^a(\partial A - \partial^* A^*)] + \text{c.c.} \tag{3.27}$$

We sum the contributions (3.23), (3.24), and (3.27) and obtain using (3.9) the scattering vertex for particles close to p_B:

$$\frac{ig}{2}A^a_-\left[\left\{\frac{\partial}{\partial^*}A\right\}T^a\left\{\partial_+\frac{\partial^*}{\partial}A^*\right\}\right] + \text{c.c.} \tag{3.28}$$

In Eq. (3.28) we denoted by A_- the field $\mathcal{A}_-$ restricted to the real values. We observe, that in the dominating interaction terms, $\mathcal{A}_+$ and $\mathcal{A}^*_+$ enter always as the sum. Therefore, it is consistent to relax the constraint relating $\mathcal{A}_+$ and $\mathcal{A}^*_-$ and to consider $\mathcal{A}_+$ and $\mathcal{A}_-$ as real and independent (denoted by A_+ and A_-, respectively). This has also been used to obtain (3.28).

In the gauge $A_- = 0$ with the momentum p_B of the right incoming particle as the gauge vector (R gauge) the current of the scattering particles close to p_a has a simple form (3.13),

$$\tilde{j}^a_- = i(AT^a\overleftrightarrow{\partial}_- A^*) \; , \tag{3.29}$$

whereas the current of scattering particles close to p_B is obtained from a sum of many contributions as (3.28):

$$\tilde{j}^a_+ = i(A_L T^a\overleftrightarrow{\partial}_+ A^*_L) \; . \tag{3.30}$$

A_L represents the transverse components of the gauge potential in the L gauge, $A_+ = 0$, related to $A = A_R$ by

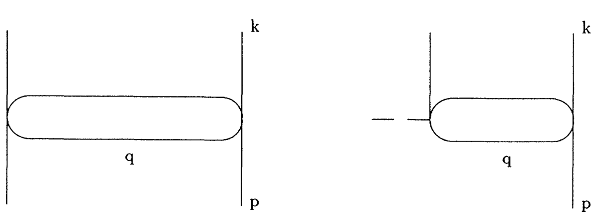

FIG. 7. Contribution from the third term of the action (3.4). The blob represents the nonlocal vertex involving ∂_-^{-2}.

$$\partial A_L = -\partial^* A^* \ . \tag{3.31}$$

The result (3.30) was to be expected, because of the parity symmetry interchanging incoming particles A and B. The effective vertex (3.19) is symmetric under this transformation because of (3.31). Parity symmetry of the scattering terms determines the form (3.30) of $\tilde{j}^a_+$ if $\tilde{j}^a_-$ (3.29) is given.

Let us discuss the argument to derive (3.31) and return to the gauge field action (3.1) before the integration over A'_+. Any correlation function with $\partial_\mu A^\mu$ inserted is small $O(g)$ due to the form of integral over A'_+. So in both the R gauge ($A_- = 0$) and the L gauge ($A_+ = 0$) we have correspondingly ($A_R = A$)

$$\partial_\mu A_R^{a\mu} = O(g), \quad \partial_\mu A_L^{a\mu} = O(g) \ . \tag{3.32}$$

From the gauge transformation relating A_R and A_L we have the following relation between the transverse components written in complex notation:

$$\partial A_L^a = \partial A_R^a + \partial\partial^* \omega^a + O(g) \ . \tag{3.33}$$

Here $\omega^a = -\partial_-^{-1} A^a_{L-} = \partial_+^{-1} A^a_{R+}$ is the parameter of the gauge transformation. Together with (3.32) this leads to (3.31) up to $O(g)$. This relation corresponds to the one between the polarization vectors (2.4). It suggests to introduce a complex scalar field ϕ^a replacing the transverse components of the gauge potential in describing the scattering gluons [13]:

$$A^a = A_R^a = i\partial^* \phi^a, \quad A_L^{a*} = i\partial^* \phi^{a*} \ . \tag{3.34}$$

We have shown that the dominating contributions of the interaction terms in (3.4) can be represented by the effective gluon production vertex (3.18) and by vertices involving the currents $\tilde{j}_-$ (3.30) and $\tilde{j}_+$ (3.29) of scattering gluons close to p_A and p_B, respectively:

$$\begin{aligned}
S_g^{\text{eff}} &= S_{ksg} + S_{kpg} + S_{sg} + S_{pg} \ , \\
S_{ksg} &= -\frac{1}{2} \int d^4x \ A^a \Box A^{a*} \ , \\
S_{kpg} &= -2 \int d^4x \ A^a_+ \partial\partial^* A^a_- \ , \\
S_{sg} &= \frac{g}{2} \int d^4x \{ \tilde{j}^a_- A^a_+ + \tilde{j}^a_+ A^a_- \} \ , \\
S_{pg} &= ig \int d^4x \{ J^{a*}(\partial^{*-1} A^a) - J^a(\partial^{-1} A^{a*}) \} \ , \\
J^{a*} &= \partial^* A_+ T^a \partial A_- \ .
\end{aligned} \tag{3.35}$$

Note that because we consider the fields $A_\pm$ as being independent ones the coefficient in S_{kpg} differs from one

which we get by substitution of definitions (3.12) and (3.14) into (3.10) [22]. The appearance of A_L in $\tilde{j}_+$ is essential, because this guarantees symmetry under interchanging the incoming particles p_A and p_B. The symmetry of the production term holds because of (3.31). Starting the analysis with particles close to p_B and working in L gauge $A_+ = 0$ we obtain the same result.

We stress that the fields appearing in the result (3.35) are just different modes of one and the same gluon field A. It is obtained just by rewriting (3.4) and integrating over modes with large longitudinal momentum components (3.7) with approximations keeping the dominant contributions to high-energy peripheral scattering.

We did not include the vertex (3.25) involving three particle of type $\mathcal{A}$ and its counterpart obtained by interchanging indices $+$ and $-$. These vertices do not contribute to tree amplitudes, and therefore they did not arise in the graphical approach in Sec. II. But they give rise to Reggeization of exchanged particles as well as s-channel intermediate states with virtual scattering particles. The sum of both contributions to Reggeization is independent of the parameter $\mu_\perp$ introduced to separate A_1 and A. If we do not restrict the virualness of the scattered particles A between vertices involving $\mathcal{A}_\pm$ then the latter contribution yields the complete Reggeization. Under this assumption vertices of the type (3.25) do not appear in the effective action. We plan to discuss this point involving the bootstrap relations in a separate publication.

By introducing ϕ^a (3.34) we resolve the constraint relating $A = A_R$ and A_L and obtain (3.35) with the substitutions

$$\begin{aligned}
S_{ksg} &= -\frac{1}{2} \int d^4x \partial\phi^a \Box \partial^* \phi^{a*} \ , \\
S_{pg} &= -g \int d^4x \{ J^{a*} \phi^a + J^a \phi^{a*} \} \ , \\
\tilde{j}^a_- &= i [\partial^* \phi T^a \partial_- \partial\phi^* + \partial\phi^* T^a \partial_- \partial^* \phi] \ , \\
\tilde{j}^a_+ &= i [\partial^* \phi^* T^a \partial_+ \partial\phi + \partial\phi T^a \partial_+ \partial^* \phi^*] \ .
\end{aligned} \tag{3.36}$$

Now the analysis can be generalized to the case with quarks included. We decompose the quark field ψ^a into light-cone components (see Appendix A).

$$\psi^a = \psi^a_- + \psi^a_+, \quad \psi^a_- = \tfrac{1}{4}\gamma_+\gamma_-\psi^a, \quad \psi^a_+ = \tfrac{1}{4}\gamma_-\gamma_+\psi^a \ . \tag{3.37}$$

We choose the gauge $A^a_- = 0$ and obtain, for the fermionic terms of the QCD action,

$$\begin{aligned}
i\bar{\psi}\gamma^\mu(\partial_\mu - igt^a A^a_\mu)\psi = {} & i\bar{\psi}'_+\gamma_+\partial_-\psi'_+ + \frac{i}{4}\bar{\psi}_-\gamma_-\frac{\Box}{\partial_-}\psi_- + \frac{g}{2}(\bar{\psi}_- t^a \gamma_- \psi_-) A^a_+ + g(\bar{\psi}'_+ t^a \gamma_\sigma \psi_- + \bar{\psi}_- t^a \gamma_\sigma \psi'_+) A^{a\sigma} \\
& - \frac{g}{4} A^{a\sigma} \left[\left[\frac{\partial^\rho}{\partial_-} \bar{\psi}_- \right] \gamma_\rho \gamma_- \gamma_\sigma t^a \psi_- + \bar{\psi}_- \gamma_\sigma \partial_-^{-1} \gamma_- \hat{\partial}_\perp t^a \psi_- \right] \ .
\end{aligned} \tag{3.38}$$

We introduced

$$\psi'^{a}_{+}=\psi^{a}_{+}+\partial_{-}^{-1}\frac{\gamma_{-}}{4}\hat{\partial}_{\perp}\psi^{a}_{-} \tag{3.39}$$

and used the notation

$$\Box=4\partial_{+}\partial_{-}+\Box_{\perp},\quad \Box_{\perp}=\partial_{\sigma}\partial^{\sigma},\quad \hat{\partial}_{\perp}=\gamma_{\sigma}\partial^{\sigma}\ . \tag{3.40}$$

We assumed also the summation over the fermion color indices which are suppressed for simplicity of notation.

Now we see that ψ'_{+} plays a role analogous to A'_{+} above. Performing the Gaussian integration over ψ'_{+} and A'_{+} we are left with A and ψ_{-}. Analogous to (3.5) we decompose ψ_{-} into modes corresponding to the kinematic regions (3.7), (3.9), and (3.6), respectively:

$$\psi_{-}\rightarrow\psi_{1-}+\psi_{-}+\tilde{\psi}_{-}\ . \tag{3.41}$$

Integration over A_1 and ψ_{1-} leads to effective vertices of gluon and fermion production. Analogous to $\mathcal{A}_{+}$ we introduce

$$\tilde{\psi}_{+}=-\partial_{-}^{-1}\frac{\gamma_{-}}{4}\hat{\partial}_{\perp}\tilde{\psi}_{-},\quad \bar{\tilde{\psi}}_{+}=-\frac{1}{4}\left[\frac{\partial^{\rho}}{\partial_{-}}\bar{\tilde{\psi}}_{-}\right]\gamma_{\rho}\gamma_{-} \tag{3.42}$$

and the analogon of $\mathcal{A}_{-}$ is $\tilde{\psi}_{-}$ itself.

After the integration over ψ'_{+}, A'_{+} with (3.35) the fermionic kinetic term becomes

$$\int d^4x\left\{\frac{i}{4}\bar{\psi}_{1-}\gamma_{-}\partial_{+}\psi_{1-}+\frac{i}{4}\bar{\psi}_{-}\gamma_{-}\frac{\Box}{\partial_{-}}\psi_{-}\right.$$
$$\left.+i\bar{\tilde{\psi}}_{+}\hat{\partial}_{\perp}\tilde{\psi}_{-}+i\bar{\tilde{\psi}}_{-}\hat{\partial}_{\perp}\tilde{\psi}_{+}\right\}\ . \tag{3.43}$$

Despite the common origin [see Eq. (3.42)], the fields $\tilde{\psi}_{+},\tilde{\psi}_{-}$ in Eq. (3.43) are treated as being independent ones. This again leads to the coefficients in front of terms involving these fields being different than the coefficients which follow from the substitution of Eqs. (3.41) and (3.42) into (3.38) [22].

It is convenient to decompose the fermion fields with respect to a basis of Majorana spinors u_{ij} $(i,j=+,-)$ (see Appendix A):

$$\psi_{-}=2i[(\partial\chi_{-})u_{-+}-(\partial^{*}\chi^{*}_{-})u_{--}]\ ,$$
$$\psi_{+}=2i[(\partial\chi_{+})u_{++}-(\partial^{*}\chi^{*}_{+})u_{+-}]\ ,$$
$$\tilde{\psi}_{-}=a^{*}_{-}u_{-+}+a_{-}u_{--},\quad \tilde{\psi}_{+}=a^{*}_{+}u_{++}+a_{+}u_{+-}\ , \tag{3.44}$$
$$\bar{\psi}_{-}=-2i[(\partial^{*}\bar{\chi}_{-})\bar{u}_{-+}-(\partial\bar{\chi}^{*}_{-})\bar{u}_{--}]\ ,$$
$$\bar{\psi}_{+}=-2i[(\partial^{*}\bar{\chi}_{+})\bar{u}_{++}-(\partial\bar{\chi}^{*}_{+})\bar{u}_{+-}]\ ,$$
$$\bar{\tilde{\psi}}_{-}=\bar{a}^{*}_{-}\bar{u}_{-+}+\bar{a}_{-}\bar{u}_{--},\quad \bar{\tilde{\psi}}_{+}=\bar{a}^{*}_{+}\bar{u}_{++}+\bar{a}_{+}\bar{u}_{+-}\ .$$

The scattering quarks are now described by the complex-valued fields χ_{-} and χ^{*}_{-}. In the case of Dirac fermions they are not related to each other by complex conjugation, but such a relation holds in the case of Majorana fermions.

Starting the analysis with particles close to p_B and working in the L gauge $A_{+}=0$ we would describe the scattering fermions instead by ψ_{+} with components χ_{+} and χ^{*}_{+}. Their relation to χ_{-} and χ^{*}_{-} is obtained in an analogous way as we obtained the relation (3.31) between $A_R=A$ and A_L. Temporarily we write ψ_R for the field in the R gauge, $A_{-}=0$, and ψ_L for the field in the L gauge, $A_{+}=0$. The form of the integral over ψ'_{+} in the R gauge or the analogon ψ'_{-} in the L gauge leads to the relation

$$\begin{aligned}&\psi_{-}\psi_{R+}+\tfrac{1}{4}\gamma_{-}\hat{\partial}_{\perp}\psi_{R-}=O(g)\ ,\\&\partial_{+}\psi_{L-}+\tfrac{1}{4}\gamma_{+}\hat{\partial}_{\perp}\psi_{L+}=O(g)\ .\end{aligned} \tag{3.45}$$

The gauge transformation relating the two descriptions yields $\psi_L=\psi_R+O(g)$ and this implies

$$\partial_{-}\chi_{+}=-\partial^{*}\chi^{*}_{-},\quad \partial_{+}\chi_{-}=-\partial^{*}\chi^{*}_{+}\ . \tag{3.46}$$

The complex conjugation of Eq. (3.46) leads to the relations between fields with a bar $(\bar{\chi},\bar{\chi}^{*})$.

The modes of type $\mathcal{A}$ in the components $a_{\pm},a^{*}_{\pm}$ (also $\bar{a}_{\pm},\bar{a}^{*}_{\pm}$) describe the exchanged fermions. Originally they are not independent since there is a relation between fields with index plus and those with index minus from (3.42). As we already mentioned above, this constraint is relaxed [22] and we consider $a_{\pm},a^{*}_{\pm}$ and those fields with a bar as being independent ones. Only for Majorana particle we have $\bar{a}=a^{*}$ and $\bar{a}^{*}=a$. Similar to the case of $\mathcal{A}_{\pm}$ this turns out to be consistent and correctly accounts for the independent degrees of freedom.

Instead of repeating in detail the analysis of the interaction terms in the case of fermions included we show in the next section how supersymmetry can be used to reconstruct the fermionic terms from the known gluonic ones. For this we temporarily restrict ourselves to one Majorana field and change its gauge group representation to the adjoint one $(t^a\rightarrow T^a)$.

IV. RECONSTRUCTION BY SUPERSYMMETRY RELATIONS

We use the supersymmetry in Yang-Mills theory with a Majorana fermion in the adjoint representation. The action has a form

$$S_{\rm YM}=\int d^4x\left\{-\frac{1}{4}F^{a}_{\mu\nu}F^{a\mu\nu}+\frac{i}{2}\bar{\psi}^{a}(\hat{\partial}_{ab}-igT^{c}_{ab}\hat{A}^{c})\psi^{b}\right\}\ , \tag{4.1}$$

where the spinor ψ^a obeys the Majorana condition

$$\psi^{a}=C\bar{\psi}^{aT}\quad\text{and}\quad (T^{a})_{bc}=(-i)f^{abc}\ .$$

Equation (4.1) is invariant under the transformation

$$\begin{aligned}&\delta A^{a}_{\mu}=2i\bar{\alpha}\gamma_{\mu}\psi^{a}=-2i\bar{\psi}^{a}\gamma_{\mu}\alpha\ ,\\&\delta\psi^{a}=\sigma_{\mu\nu}F^{a\mu\nu}\alpha\ ,\\&\sigma_{\mu\nu}=\tfrac{1}{2}[\gamma_{\mu},\gamma_{\nu}]\ ,\end{aligned} \tag{4.2}$$

but the algebra of transformations (4.2) does not close on translations without introducing auxiliary fields [18].

We restrict ourselves to the subset of transformations (4.2) by choosing the parameter α in the form (see Appendix A)

$$\alpha=\alpha_+ u_{+-}+\alpha_+^* u_{++} \tag{4.3}$$

with α_+ being a Grassmanian complex number. In this case the algebra of transformation (4.2) restricted by condition (4.3) closes on the translations (∂_-) as is required by supersymmetry [19]. Moreover, this subset of transformations leaves A_-^a invariant and therefore it is appropriate for working in the R gauge $A_-^a=0$.

Under this subset of transformations (4.2) the transverse gauge fields A_σ^a and the fermion field projection ψ_- transform into each other: i.e.,

$$\begin{aligned}\delta A_\sigma^a&=2i\bar{\alpha}\gamma_\sigma\psi_-^a\ ,\\ \delta\psi_-^a&=-2\partial_-(\gamma^\sigma A_\sigma^a)\gamma_+\alpha\ .\end{aligned} \tag{4.4}$$

Above, in Sec. III, we have introduced the fields ϕ^a and χ_-^a [Eqs. (3.34) and (3.44)] replacing the transverse components A_σ^a of the gauge fields and the spinor field ψ_-^a. The transformation law of fields ϕ^a and χ_-^a read

$$\begin{aligned}&\delta\phi^a=4i\alpha_+\chi_-^{a*},\quad \delta\phi^{a*}=4i\alpha_+^*\chi_-^a\ ,\\ &\delta\chi_-^a=-2(\partial_-\phi^{a*})\alpha_+,\quad \delta\chi_-^{a*}=-2(\partial_-\phi^a)\alpha_+^*\ .\end{aligned} \tag{4.5}$$

Disregarding contributions $O(g)$ we find that fields $A_+'^a$ and $\psi_+'^A$ [Eqs. (3.2) and (3.39)] also transform into each other

$$\begin{aligned}\delta A'_+&=2i\bar{\alpha}\gamma_+\psi'_+\ ,\\ \delta\psi'_+&=-2(\partial_- A'_+)\alpha\ .\end{aligned} \tag{4.6}$$

These fields are integrated out and we have separated the modes of the remaining fields A_σ and ψ_- with respect to kinematics. For the modes of type A, related to the scattering particles close to p_A, we have the supersymmetry transformation given above (4.5). For the modes of type $\mathcal{A}$ the transformation follow also from (4.4) by their definition in terms of A_σ,ψ:

$$\begin{aligned}&A_\pm=\tfrac{1}{2}[\mathcal{A}_\pm+\mathcal{A}_\pm^*]\ ,\\ &\delta A_+=2i[\alpha_+^* a_+ +\alpha_+ a_+^*]\ ,\\ &\delta A_-=-i\left[\alpha_+^*\left[\frac{\partial_-}{\partial}a_-^*\right]+\alpha_+\left[\frac{\partial_-}{\partial^*}a_-\right]\right]\ ,\\ &\delta a_+^*=-2(\partial_- A_+)\alpha_+^*\ ,\\ &\delta a_-^*=4\alpha_+(\partial A_-)\ .\end{aligned} \tag{4.7}$$

We repeat that in Eq. (4.7) the field $A_\pm$ are defined by the transverse part of gluon potential and should not be confused with the corresponding fields in original QCD Lagrangian (3.1). Similar as in (4.5) the remaining transformation follow by complex conjugation.

The kinetic term of the modes of type A has the form

$$\begin{aligned}S_{ks}&=\int d^4x\left[\frac{1}{2}A_\sigma^a\Box A^{a\sigma}+\frac{i}{8}\bar{\psi}_-^a\gamma_-\frac{1}{\partial_-}\Box\psi^a\right]\\ &=\int d^4x\left[-\frac{1}{2}(\partial^*\phi^a)\Box(\partial\phi^{a*})\right.\\ &\qquad\left.+i(\partial\chi_-^a)\frac{1}{\partial_-}\Box(\partial^*\chi_-^{a*})\right]\ .\end{aligned} \tag{4.8}$$

It is invariant under transformations (4.5).

The minimal coupling $\partial_+\delta_{bc}\to\partial_+\delta_{bc}-(ig/2)A_+^aT_{bc}^a$ in Eq. (4.8) leads to the interaction term [compare (3.36)]

$$\frac{g}{2}\int d^4x(j_-^a+j_-^{a*})A_+^a \tag{4.9}$$

with the current j_-^a given by

$$j_-^a=-i(\partial_-\partial^*\phi)T^a(\partial\phi^*)+2(\partial\chi_-)T^a(\partial^*\chi_-^*)\ . \tag{4.10}$$

Now, applying the transformations (4.5) to the current j_-^a (j_-^{a*}) we obtain its superpartner v_-^{a*} (v_-^a):

$$\begin{aligned}&\delta j_-^a=2\alpha_+\partial_- v_-^{a*},\quad \delta v_-^{a*}=-4i\alpha_+^* j_-^a\ ,\\ &v_-^{a*}=-2(\partial\phi^*)T^a(\partial^*\chi_-^*)\ .\end{aligned} \tag{4.11}$$

The interaction terms of the scattered particles are obtained by constructing the invariant S_{s-} with respect to the transformations (4.5) and (4.7) and involving the currents j_-^a, v_-^a and fields A_+^a, a_+^a. It is given by the formula

$$S_{s-}=\frac{g}{2}\int d^4x[(j_-^a+j_-^{a*})A_+^a+v_-^{a*}a_+^a-v_-^a a_+^{a*}]\ . \tag{4.12}$$

Under the transformations (4.5) and (4.7) the integrand of (4.12) changes by the total ∂_- derivative (we use here that in our kinematics $\partial_- a_+$ is small).

In terms of component fields the action (4.12) takes the form

$$\begin{aligned}S_{s-}=-g\int d^4x&\left[\frac{i}{2}(\partial_-\partial^*\phi)T^a(\partial\phi^*)A_+^{a*}\right.\\ &-(\partial\chi_-)T^a(\partial^*\chi_-^*)A_+^{a*}\\ &\left.-(\partial^*\chi_-^*)T^a(\partial\phi^*)a_+^a+\text{H.c.}\right]\ .\end{aligned} \tag{4.13}$$

Supersymmetry connects also the terms in the effective action describing production of gluons and fermions. We have seen that only one combination of the gluon fields $(\partial\mathcal{A}+\partial^*\mathcal{A}^*)\sim A_\pm$ transfers the interaction between scattering particles. However, there are fermion exchanges of two types, $a_\pm$ and $a_\pm^*$, distinguished by the flow of helicity. Therefore, the reconstruction of fermionic terms related to particle exchanges and production is not straightforward if we start from the leading gluonic terms. The considered supersymmetry subgroup produces leading fermionic terms both from leading and nonleading gluonic ones. There is a simple supersymmetry relation between the exchanged gluons $A_\pm$ and the exchanged fermions with one helicity only. It is different from relations (4.7) derived for $A_\pm, a_\pm, a^*_\pm$ from their definitions.

The corresponding transformations leave the kinetic term of the exchanged fields invariant if only one of the two fermionic terms is kept [see Eqs. (3.43) and (3.44)]:

$$\tilde{S}_{kp} = \int d^4x \{ -2A^a_+ \partial\partial^* A^a_- - i a^{a*}_+ \partial^* a^{a*}_- \} . \quad (4.14)$$

The transformations are

$$\tilde{\delta} A_+ = 2i\alpha_+ a^*_+ , \quad \tilde{\delta} A_- = -2i\alpha_+ \frac{\partial_-}{\partial} a^*_- ,$$
$$\tilde{\delta} a^*_+ = -4\alpha^*_+ (\partial_- A_+), \quad \tilde{\delta} a^*_- = 4\alpha_+ (\partial A_-) . \quad (4.15)$$

We write the gluonic production vertex as $-g(J^a \phi^{a*} + J^{a*} \phi^a)$ and we extend the form of the effective current J^a in such a way, that its variation under (4.15) is a total ∂_- derivative

$$J^{a*} = -\left[(\partial A_-) T^a (\partial^* A_+) - \frac{i}{2} (\partial^* a^*_+) T^a a^*_- \right] . \quad (4.16)$$

We obtain the multiplet of effective currents

$$\tilde{\delta} J^{a*} = 2\alpha^*_+ (\partial_- v^a) ,$$
$$\tilde{\delta} v^a = -4i\alpha_+ J^{a*} , \quad (4.17)$$

where

$$v^a = -i a^*_- T^a (\partial^* A_+) . \quad (4.18)$$

From Eqs. (4.5) and (4.17) we see that

$$-g \int d^4x (\phi^a J^{a*} - \chi^{a*}_- v^a) \quad (4.19)$$

is invariant.

We write down this invariant explicitly and we add to it analogous terms corresponding to the other fermion helicity:

$$S_{pg} + S_{p-} = -g \int d^4x \left[\phi^{a*} \left[(\partial^* A_-) T^a (\partial A_+) - \frac{i}{2} a_- T^a (\partial a_+) \right] + i\chi^a_- a_- T^a (\partial A_+) + \text{H.c.} \right] . \quad (4.20)$$

Analogously the kinetic term for exchanged particles is obtained from (4.14) by adding the contribution of the other fermion helicity. It follows directly from (3.43) and (3.44) that

$$S_{kp} = \int d^4x \{ -2A^a_+ \partial\partial^* A^a_- - i a^{a*}_+ \partial^* a^{a*}_- - i a^a_+ \partial a^a_- \} . \quad (4.21)$$

As we have seen the supersymmetry (4.3) and (4.4) allows one to easily reconstruct the scattering and production terms involving fermions if we restrict ourselves to particles close to p_A. Fermion exchange over a large rapidity interval is suppressed and of the same order as small contributions neglected in course of the derivation above. The easiest way to reconstruct the missing fermionic terms for particles close to p_B is to repeat the procedure from the beginning, i.e., concentrating on particles close to p_B and working in R gauge $A_+ = 0$. The results are analogous with the substitutions of indices $+ \leftrightarrow -$ and $\phi \leftrightarrow \phi^*$. To obtain the full effective action with one Majorana fermion we have to add to the gluonic action (3.36) the fermionic kinetic terms from (4.8) and (4.21), the fermionic interaction terms from (4.13) and (4.20) and further fermionic interaction terms S_{s+} and S_{p+} obtained from the latter S_{s-} and S_{p-} by the substitutions $+ \leftrightarrow -$, $\phi \leftrightarrow \phi^*$:

$$S = S_{ks} + S_{kp} + S_{s+} + S_{s-} + S_{pg} + S_{p+} + S_{p-} . \quad (4.22)$$

The result (4.22) can be extended easily to the case of Dirac fermion using definitions (3.44). In this way we obtain the result given in (2.15).

V. SYMMETRIC FORM OF THE EFFECTIVE ACTION

The resulting form of the effective action (4.22) still has some unpleasant features. There is an inverse derivative ∂_-^{-1} in the kinetic term of the χ_- field and this term is not symmetric under $+ \leftrightarrow -$. On the other hand, χ_- and χ_+ are not independent, see (3.46).

The separation of longitudinal and transverse dimensions in the peripheral high-energy scattering should be reflected by the form of the action. Separating modes according to kinematics we have introduced different fields for exchanged and scattering particles. Each term in (4.22) is Lorentz and scale symmetric. We would like to have these symmetries separately in the longitudinal and transverse subspaces. The gluonic terms (3.35) and (3.36) obey this symmetry property, except the kinetic term for ϕ, where it holds only after replacing $\Box$ by $4\partial_+\partial_-$.

Trying to assign separate longitudinal and transverse scale dimensions to all fields, we arrive at an unsatisfactory conclusion for fermions: The supersymmetry parameter α_+ carries dimension $-\frac{1}{2}$, and this is clearly a longitudinal dimension,

$$\dim_\alpha \alpha_+ = \begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} .$$

From this we have

$$\dim_\alpha \phi = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \dim_\alpha \mathcal{A}_\pm = \begin{bmatrix} 1 \\ 0 \end{bmatrix} ,$$
$$\dim_\alpha \chi_\pm = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}, \quad \dim_\alpha a_+ = \begin{bmatrix} \frac{3}{2} \\ 0 \end{bmatrix}, \quad \dim_\alpha a_- = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} . \quad (5.1)$$

This assignment works only for those fermionic terms reconstructed by the α_+ supersymmetry (4.3) and (4.4), i.e., S_{s-}, S_{p-}. It is asymmetric under $+ \leftrightarrow -$.

However, the kinetic terms of type-$\mathcal{A}$ fields and the interaction terms describing gluon production are symmetric under $+ \leftrightarrow -$ and $\phi \leftrightarrow \phi^*$. These terms also allow a symmetric assignment of scaling dimensions:

$$\dim_\beta \chi_\pm = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}, \quad \dim_\beta a_\pm = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix} . \tag{5.2}$$

The gluon fields have $\dim_\alpha = \dim_\beta$. We see that also fermion production terms $S_{p-} + S_{p+}$ involving both $a_\pm$ and $\chi_\pm$ are compatible with the assignment $\dim_\beta$. It is possible to achieve this also for the remaining terms by applying relation (3.46) connecting χ_+ and χ_-. It is clearly not compatible with definite transverse and longitudinal dimensions of $\chi_\pm$. At first glance this procedure seems to be not unique. There are two ways to rewrite the kinetic and the scattering terms of χ_- compatible with (5.2). However, it is reasonable to include from each of these terms both possible results. The effective action for gluons and for one Majorana fermion in the adjoint representation then takes the form

$$
\begin{aligned}
S_{ks} &= \int d^4x \left\{ -\frac{1}{2}(\partial^*\phi^a)\Box\partial\phi^{a*} + \frac{i}{2}\chi_+^a\Box\partial\chi_-^a + \frac{i}{2}\chi_+^{a*}\Box\partial^*\chi_-^{a*} \right\} , \\
S_{kp} &= \int d^4x [-2A_+^a\partial\partial^* A_-^a - ia_+^a\partial a_-^a - ia_+^{a*}\partial^* a_-^{a*}] , \\
S_{s-} &= -g\int d^4x \left\{ \frac{i}{2}[(\partial_-\partial^*\phi)T^a(\partial\phi^*)A_+^a + (\partial_-\partial\phi^*)T^a(\partial^*\phi)A_+^a] + A_+^a(\partial_-\chi_+)T^a(\partial\chi_-) \right. \\
&\qquad \left. + A_+^a(\partial^*\chi_-^*)T^a(\partial_-\chi_+^*) - (\partial\phi^{a*})(\partial_-\chi_+)T^a a_+ - (\partial^*\phi^a)(\partial_-\chi_+^*)T^a a_+^* \right\} , \\
S_{s+} &= -g\int d^4x \left\{ \frac{i}{2}[(\partial_+\partial^*\phi^*)T^a(\partial\phi)A_-^a + (\partial_+\partial\phi)T^a(\partial^*\phi^*)A_-^a] + A_-^a(\partial_+\chi_-)T^a(\partial\chi_+) \right. \\
&\qquad \left. + A_-^a(\partial^*\chi_+^*)T^a(\partial_+\chi_-^*) - (\partial\phi^a)(\partial_+\chi_-)T^a a_- - (\partial^*\phi^{a*})(\partial_+\chi_-^*)T^a a_-^* \right\} , \\
S_p &= g\int d^4x \left[\phi^a(\partial A_-)T^a(\partial^* A_+) - \phi^{a*}(\partial^* A_-)T^a(\partial A_+) \right. \\
&\qquad - \frac{i}{2}\{ \phi^a(-a_+T^a(\partial a_-) + (\partial^* a_+^*)T^a a_-^*) + \phi^{a*}[a_+^* T^a(\partial^* a_-^*) - (\partial a_+)T^a a_-]\} \\
&\qquad \left. + i[\chi_-^* T^a a_-^*(\partial^* A_+^a) - \chi_- T^a a_-(\partial A_+^a) + \chi_+^* T^a a_+^*(\partial^* A_-^a) - \chi_+ T^a a_+(\partial A^a)] \right] .
\end{aligned} \tag{5.3}
$$

Relation (3.46) has to be considered as an additional constraint. The symmetry properties in the subspaces hold for the kinetic term in Eq. (5.3) only after replacing $\Box$ by $4\partial_+\partial_-$. The behavior of the fields under Lorentz transformations in longitudinal and rotations in transverse subspaces is as follows. ϕ is scalar in both subspaces. $A_\pm$ are vector components in longitudinal but scalar in transverse subspaces. $\chi_\pm$ and $a_\pm$ behave as two-dimensional spinors in both longitudinal and transverse subspaces, where the indices $\pm$ refer to the longitudinal space and the presence of absence of the sign of complex conjugation ($*$) indicates the behavior under transverse rotations.

The new form of the effective action, in particular the kinetic term in Eq. (5.3), leads us to a new supersymmetry Δ. After removing from (5.3) all terms involving $a_\pm^*$ the remaining expression is invariant under the following transformation acting on $A_\pm, \phi, \phi^*, \chi_\pm, \chi_\pm^*, a_\pm$:

$$
\begin{aligned}
&\Delta\mathcal{A}_\pm = \frac{i}{\sqrt{2}}\beta_\pm a_\pm, \quad \Delta a_\pm = \sqrt{2}\beta_\mp\partial^*\mathcal{A}_\pm , \\
&\Delta\phi = i\sqrt{2}\beta_-\chi_+, \quad \Delta\phi^* = i\sqrt{2}\beta_+\chi_- , \\
&\Delta\chi_- = \frac{1}{\sqrt{2}}\beta_-\partial^*\phi^*, \quad \Delta\chi_+ = \frac{1}{\sqrt{2}}\beta_+\partial^*\phi .
\end{aligned} \tag{5.4}
$$

The transformations of $\chi_\pm^*$ follow from relation (3.46). It is convenient first to rewrite all the terms with $\chi_\pm^*$ by using (3.46) into terms involving $\chi_\pm$ before checking the invariance of the action (5.3) under transformations (5.4).

The supersymmetry algebra closes on transverse translations:

$$
\begin{aligned}
&\Delta = \beta_-\Delta_+ + \beta_+\Delta_- , \\
&\Delta_+^2 = \Delta_-^2 = 0, \quad \Delta_+\Delta_- + \Delta_-\Delta_+ = i\partial^* .
\end{aligned} \tag{5.5}
$$

There is also the analogous supersymmetry Δ^* under which (5.3) is invariant after removing the terms involving $a_\pm$. The corresponding transformations laws are obtained from (5.4) by formal complex conjugation.

The new transverse supersymmetry Δ allows one to reconstruct the terms involving $\chi_\pm, a_\pm$ from the pure gluonic ones. The terms involving $\chi_\pm^*, a_\pm^*$ are reconstructed using Δ^* or simply by complex conjugation. Compared to the original longitudinal supersymmetry the transverse supersymmetry refers to the final form of the effective action.

In the present form the symmetries discussed above are to be considered merely as relations between interaction terms. We have to keep in mind the relation (3.46) between χ_+ and χ_- and also that the transverse parts of the kinetic terms violates the separate scaling symmetries.

We can replace the fermionic kinetic terms in (5.3) which are of second order in time by terms involving just a first-order time derivative:

$$S_{ks}^{\chi}=4i\int d^4x\{(\partial^*\chi_+^{a*})(\partial_-\partial\chi_+^a)+(\partial^*\chi_-^{a*})(\partial_+\partial\chi_-^a)+(\partial^*\chi_+^{a*})(\partial\partial^*\chi_-^{a*})+(\partial^*\chi_-^{a*})(\partial\partial^*\chi_+^{a*})\} \ . \tag{5.6}$$

This is just the representation of the original free Majorana action in components $\partial^*\chi_\pm^*$ and $\partial\chi_\pm$. After this replacement the constraint (3.46) can be dropped. χ_+ and χ_- are now independent. Together with their complex conjugates they represent the field variables and conjugate momenta of a Majorana fermion. The symmetry properties discussed above do not apply to this form of the fermion kinetic terms.

VI. THE WEIZSÄCKER-WILLIAMS-TYPE RELATIONS

Our derivation of the effective action is based on separating modes related to scattering and exchanged particles. Correspondingly we have obtained vertices describing scattering of particles and effective vertices describing particle production. It turns out that between the scattering and production part of the effective action there are relations which we call Weizsäcker-Williams-(WW-) type relations, which to each term in the production part attributes in a definite way a term in the scattering part. These symmetry relations are the traces of the common origin of fields of type A $(\phi,\chi_\pm)$ and of type $\mathcal{A}(A_\pm,a_\pm)$ as certain modes of one and the same gluon or quark field. They hold both for the cases of Dirac and Majorana fermions.

Let us start with the term in S_p (2.19) describing the gluon production from the t-channel gluonic line:

$$g\int d^4x\,[\phi^a(\partial A_-)T^a(\partial^* A_+)-\phi^{a*}(\partial^* A_-)T^a(\partial A_+)] \ . \tag{6.1}$$

Separately, each term in (6.1) corresponds to a definite helicity produced gluon.

Let us concentrate for definiteness on the term in (6.1) involving ϕ field. Consider the case that the field A_- represents only "soft" modes in the sense that

$$\partial\partial^* A_-\to 0 \ . \tag{6.2}$$

Then the integration by part in the considered term of (6.1) leads to

$$-g\int d^4x\,(\partial^*\phi)T^a(\partial A_-)A_+^a \ . \tag{6.3}$$

Now we apply the following substitution rule to the "soft" field A_-:

$$A_-\Longrightarrow -i\partial_-\phi^* \tag{6.4}$$

which leads to the formula

$$ig\int d^4x\,(\partial^*\phi)T^a(\partial_-\partial\phi^*)A_+^a \ . \tag{6.5}$$

Expression (6.5) coincides with the gluonic term in the scattering part S_{s-} (2.17) (we use the fact that the longitudinal momentum component k_- of A_+ is small). The substitution rule (6.4) implies the "softness" of the field ϕ^* in (6.5). Also, let us note that because the A_- field is a real one, the "soft" ϕ^* field is pure imaginary.

The analogous procedure can be applied in the case when the A_+ field in the first term of (6.1) is the "soft" one:

$$\partial\partial^* A_+\to 0 \ . \tag{6.6}$$

The integration by part and use of the substitution rule

$$A_+\Longrightarrow i\partial_+\phi^* \tag{6.7}$$

leads to

$$ig\int d^4x\,A_-^a(\partial\phi)T^a(\partial_+\partial^*\phi^*) \ , \tag{6.8}$$

i.e., we recover the gluonic term in the scattering part S_{s+} (2.18), with the ϕ^* field being a "soft" one.

The similar considerations performed in the case of the term in (6.1) involving the ϕ^* field lead to the substitution rules for the "soft" A_- or A_+ fields;

$$\begin{aligned} A_-&\Longrightarrow i\partial_-\phi \ , \\ A_+&\Longrightarrow -i\partial_+\phi \ , \end{aligned} \tag{6.9}$$

and to expressions (6.5) and (6.8) involving the "soft" ϕ field.

The WW substitution rules (6.4), (6.7), and (6.9) can be understood from the definitions of ϕ (3.24) and $\mathcal{A}_\pm$ (3.12) and (3.14) in terms of the original gluonic field.

Similar WW-type relations hold between vertices involving fermionic fields. Let us consider the gluon production from the t-channel fermion line [see (2.19)]:

$$-\frac{ig}{2}\int d^4x\{\phi^a[(\partial^*\bar{a}_+)t^a a_-^*]-\bar{a}_+^*t^a(\partial a_-)+\phi^{a*}[\bar{a}_+t^a(\partial^* a_-^*)-(\partial\bar{a}_+^*)t^a a_-]\} \ . \tag{6.10}$$

We assume that $\bar{a}_+^*$ and $\bar{a}_+$ are "soft": i.e.,

$$\partial^*\bar{a}_+\to 0 \ \text{ and } \ \partial\bar{a}_+^*\to 0 \ . \tag{6.11}$$

Then the integration by parts in (6.10) and use of the substitution rules

$$\bar{a}_+^*\Longrightarrow -2i\partial^*\bar{\chi}_+, \quad \bar{a}_+\Longrightarrow 2i\partial\bar{\chi}_+^* \tag{6.12}$$

lead to the expression

$$-g\int d^4x\{(\partial\phi^a)(\partial^*\bar{\chi}_+)t^a a_- + (\partial^*\phi^{a*})(\partial\bar{\chi}_+^*)t^a a_-^*\}\ , \tag{6.13}$$

i.e., we recover [see (3.46)] the two last vertices in S_{s+} (2.18).

If we assume that in the expressions (6.10) a_- and a_-^* are the "soft" fields, i.e.,

$$\partial a_- \to 0,\ \ \partial^* a_-^* \to 0\ , \tag{6.14}$$

then with the use of the substitution rules

$$a_-^* \Longrightarrow 2i\partial\chi_-,\ \ a_- \Longrightarrow -2i\partial^*\chi_-^* \tag{6.15}$$

and relations (3.46) and (3.47) we recover the vertices in the third line of S_{s-} (2.17).

As a final example let us consider the following part of the fermion production vertices in S_p (2.19):

$$-ig\int d^4x\{\bar{\chi}_-^* t^a a_-(\partial A_+^a) - \bar{\chi}_- t^a a_-^*(\partial^* A_+^a)\}\ . \tag{6.16}$$

Restricting the a_-, a_-^* fields to the "soft" modes [see (6.14)] and using the rules (6.15) we obtain from (6.16) the expression

$$2g\int d^4x\ A_+^a[(\partial\bar{\chi}_-^*)t^a(\partial^*\chi_-^*) + (\partial^*\bar{\chi}_-)t^a(\partial\chi_-)] = -2g\int d^4x\ A_+^a[(\partial_-\bar{\chi}_+)t^a(\partial^*\chi_-^*) + (\partial_-\bar{\chi}_+^*)t^a(\partial\chi_-)]\ , \tag{6.17}$$

i.e., we recover the second line of S_{s-} (2.17).

Now, let us assume that in formula (6.16) the field A_+ is a "soft" one [see (6.6)]. Then the substitution rule (6.7) [see also (6.9)] supplemented by the integration by part leads to

$$g\int d^4x\{(\partial\phi^a)(\partial_+\bar{\chi}_-^*)t^a a_- + (\partial^*\phi^{a*})(\partial_+\bar{\chi}_-)t^a a_-^*\} \tag{6.18}$$

coinciding with the last line of S_{s+} (2.18). Also in the fermionic case the WW substitution rules can be understood from the definitions of $\chi_\pm$ and $a_\pm$ (3.44) and (3.46) in terms of the original quark field.

VII. DISCUSSION

The effective action (2.15) describes QCD scattering processes in the multi-Regge regime. It summarizes results about the leading contribution in this region obtained by analyzing graphs using ideas of unitarity, s-channel helicity conservation, and the separation of longitudinal and transverse dimensions. We have derived the effective action from QCD by separating the modes of the original gluon and quark fields according to the multi-Regge kinematics and integrating over modes which do not correspond to scattering or exchanged particles in the considered peripheral process. Supersymmetry has been used to obtain the fermionic terms of the effective action from the pure gluonic ones. This supersymmetry is the subgroup of the one of the supersymmetric Yang-Mills theory, which is associated with the direction of the momentum of an incoming particle. A supersymmetry related to the transverse directions transforms the fermionic and gluonic terms of the final form of this action into each other. The vertices corresponding to scattering and production of particles are connected to each other by Weizsäcker-Williams type relations.

The resulting action reproduces the leading contribution in QCD to high-energy peripheral scattering amplitudes in a most economic way, because most of the negligible contributions in this kinematic region are excluded. Therefore, we have a simple and symmetric structure of the result. The fields describing the scattered and the exchanged gluons and quarks do not have four-dimensional Lorentz or Dirac indices any more. They are attributed a definite behavior under scaling and Lorentz or rotation transformations separately in the longitudinal and transverse subspaces, reflecting the kinematics of the processes where the action applies. The action is symmetric (with a modification in the kinetic term of scattering particles) under scaling and Lorentz transformations separately in both subspaces.

The applicability of the multi-Regge effective action is, of course, restricted to the multi-Regge kinematics in the scattering amplitudes, a condition which has been used in all steps of the derivation. In this regime the action (2.15) reproduces the leading contribution correctly. This is because in the considered asymptotics intermediate states in all subenergy channels obeying multi-Regge kinematics dominate. Contributions from pairs of particles in non-multi-Regge configuration or from loops with particles far off shell lead to corrections to our result [11].

In the dominating kinematics the longitudinal momenta of the exchanged particles are small compared to their transverse momenta. This condition is not included explicitly in the propagators of $A_\pm$ and it may be necessary in some loop integrals to impose it by a cutoff. A modification of the effective vertices would be desirable, which suppresses contributions from outside of the multi-Regge region.

In the s-channel intermediate states the scattering particles can be virtual as long as their momenta squared are small compared to the subenergies. One may fix an upper limit of the virtualness of scattering particles. Then additional triple vertices involving exchange particles (type $\mathcal{A}$) have to be included into the action.

In the approach by Verlinde [16] a scaling argument leads to neglecting completely the contribution of the transverse field strength components to the gauge field action. Different to Verlinde the contributions from the transverse field strength squared are essential for our result. The integration over A'_+ produces the interaction terms from which we obtain finally the particle production vertex. It seems that the approach [16] does not apply to the inelastic amplitudes.

Verlinde established a remarkable connection of their effective action to the two-dimensional Wess-Zumino-Novikov-Witten (WZNW) model. It was shown earlier [20] that constructing amplitudes obeying just elastic unitarity in s and u channels results in a factorizable S matrix coinciding with the results for σ models [21].

The multi-Regge effective action is the first step in a new approach to high-energy peripheral scattering overcoming the deficits of both the eikonal and the leading logarithmic approximations. With this action we would like to calculate QCD amplitudes including the effects of multiparticle intermediate states and satisfying the unitarity conditions in all subenergy channels. The Weizsäcker-Williams-type relations support the expectation that also the t-channel unitarity conditions can be obeyed in this approach.

The separation of longitudinal and transverse subspaces and of scattering and exchanges field suggests to apply functional methods for further simplifications. The exchanged fields ($A_{\pm}, a_{\pm}$) are closely related to reggeons. One should try to find a tractable representation of the effective reggeon field theory. Conformal symmetry in the two-dimensional transverse space is expected to play an important role.

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APPENDIX A

Below we summarize our notation and conventions related to the fermionic sector of the effective action. We introduce four nilpotent matrices $\gamma_{\pm}$ and γ, γ^* according to the formulas:

$$\gamma_{\pm} \equiv \gamma_0 \pm \gamma_3, \quad \gamma \equiv \gamma^1 + i\gamma^2, \quad \gamma^* \equiv \gamma^1 - i\gamma^2 . \tag{A1}$$

They satisfy the relations

$$\begin{aligned} &\{\gamma_+, \gamma_-\} = 4, \quad \{\gamma, \gamma^*\} = -4 , \\ &(\gamma_{\pm})^{\dagger} = \gamma_{\mp}, \quad (\gamma)^{\dagger} = -\gamma^* . \end{aligned} \tag{A2}$$

From the matrices (A1) we built four projection operators

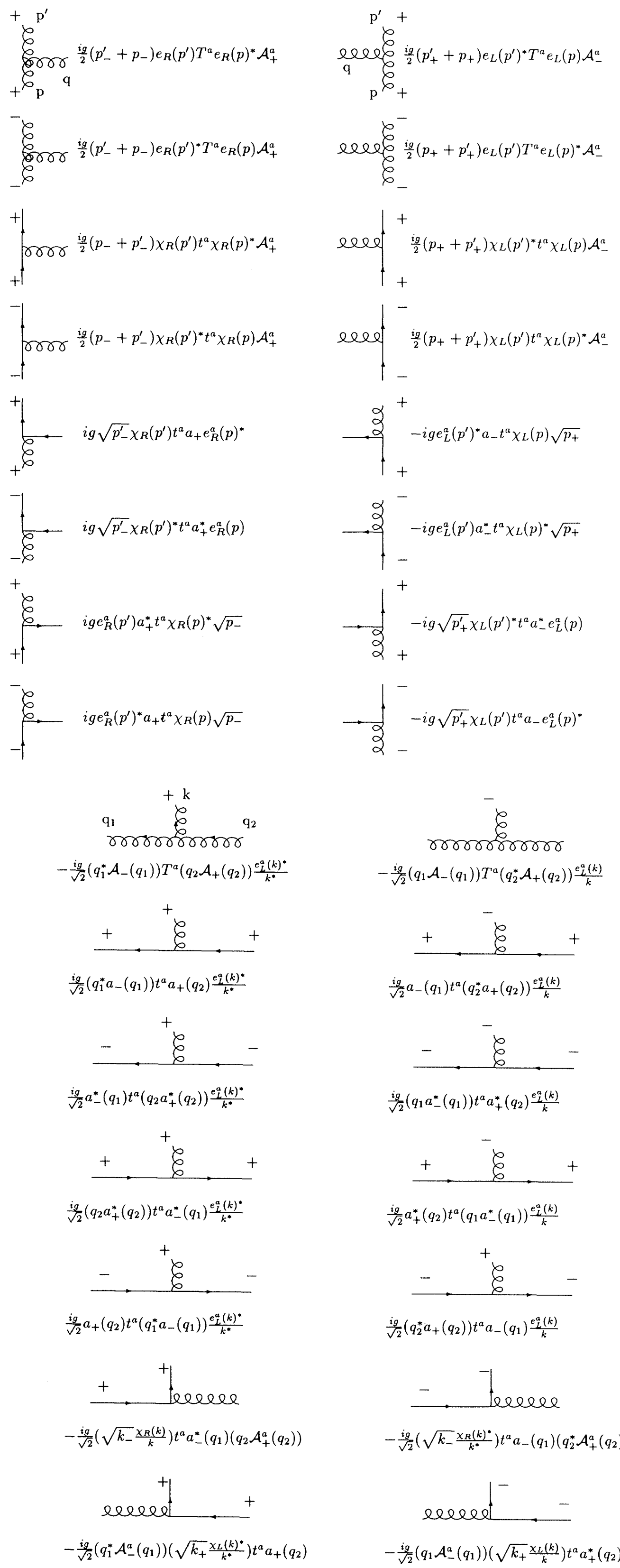
$$\begin{aligned} \Pi_+ &= \tfrac{1}{4}\gamma_-\gamma_+, \quad P_+ = -\tfrac{1}{4}\gamma\gamma^* , \\ \Pi_- &= \tfrac{1}{4}\gamma_+\gamma_-, \quad P_- = -\tfrac{1}{4}\gamma^*\gamma , \end{aligned} \tag{A3}$$

which lead to the decomposition of the unit operator in spinor space:

$$\mathbb{1} = \Pi_+ P_+ + \Pi_+ P_- + \Pi_- P_+ + \Pi_- P_- . \tag{A4}$$

Each term on the right-hand side (RHS) of Eq. (A4) defines the corresponding fermionic subspace on which it

TABLE I. The scattering and production vertices.



projects. We choose in each of these subspaces one basic spinor $u_{ij}(i,j=+\text{ or }-)$ satisfying

$$\Pi_i P_j u_{ij} = u_{ij} \tag{A5}$$

and being the Majorana one

$$u_{i+} = C\bar{u}_{i-}^T \ . \tag{A6}$$

For the basic spinors u_{ij} we assume (a) the normalization conditions

$$\begin{aligned} &\bar{u}_{+j}\gamma_+ u_{+j} = 1 \ , \\ &\bar{u}_{-j}\gamma_- u_{-j} = 1, \quad j=+,- \ ; \end{aligned} \tag{A7}$$

(b) the phase conventions

$$\begin{aligned} &u_{++} = -\tfrac{1}{2}\gamma_- u_{-+}, \quad u_{++} = \tfrac{1}{2}\gamma u_{+-} \ , \\ &u_{+-} = \tfrac{1}{2}\gamma_- u_{--}, \quad u_{+-} = -\tfrac{1}{2}\gamma^* u_{++} \ . \end{aligned} \tag{A8}$$

All other matrix elements between the basic spinors u_{ij} can be calculated with the help of Eqs. (A6)–(A8). With all these ingredients the arbitrary spinor can be decomposed in the basis u_{ij} and this fact leads in particular to Eq. (3.20).

APPENDIX B

We introduce the notation

$$\mathcal{F}[f(k)] = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} f(k) \ .$$

The propagators of t-channel fields are

$$\langle 0|T[\mathcal{A}_+^a(x)\mathcal{A}_-^b(y)]|0\rangle = \mathcal{F}\left[\frac{2i\delta^{ab}}{kk^*}\right] ,$$

$$\langle 0|T[a_+^a(x)\bar{a}_-^{b*}(y)]|0\rangle = \langle 0|T[a_-^a(x)\bar{a}_+^{b*}(y)]|0\rangle = \mathcal{F}\left[\frac{2i\delta^{ab}}{k^*}\right] ,$$

$$\langle 0|T[a_+^{a*}(x)\bar{a}_-^b(y)]|0\rangle = \langle 0|T[a_-^{a*}(x)\bar{a}_+^b(y)]|0\rangle = \mathcal{F}\left[\frac{2i\delta^{ab}}{k}\right] .$$

The propagators of s-channel fields are

$$\langle 0|T[\phi^a(x)\phi^{b*}(y)]|0\rangle = \mathcal{F}\left[\frac{8i\delta^{ab}}{(k^2+i\varepsilon)kk^*}\right] ,$$

$$\langle 0|T[\chi_-^a(x)\bar{\chi}_-^b(y)]|0\rangle = \mathcal{F}\left[\frac{2i\delta^{ab}k_-}{(k^2+i\varepsilon)kk^*}\right] ,$$

$$\langle 0|T[\chi_-^{a*}(x)\bar{\chi}_-^{b*}(y)]|0\rangle = \mathcal{F}\left[\frac{2i\delta^{ab}k_-}{(k^2+i\varepsilon)kk^*}\right] ,$$

etc.

In Table I we present the interaction vertices. The s-channel momenta flow from the bottom of the page to the top, whereas the t-channel momenta flow from the right to the left of the page. The signs + or − denote the corresponding particle helicities.

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