Can disoriented chiral condensates form? A dynamical perspective

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We address the issue of whether a region of disoriented chiral condensate (DCC), in which the chiral condensate has components along the pion directions, can form. We consider a system going through the chiral phase transition via a quench, in which relaxation of the high temperature phase to the low temperature one occurs rapidly (within a time scale of order ~ 1 fm/c). We use a density matrix based formalism that takes both thermal and quantum fluctuations into account nonperturbatively to argue that if the O(4) linear σ model is the correct way to model the situation in QCD, then it is very unlikely, at least in the Hartree approximation, that a large (> 10 fm) DCC region will form. Typical sizes of such regions are ~ 1–2 fm and the density of pions in such regions is at most of order $\sim 0.2/\text{fm}^3$. We end with some speculations on how large DCC regions may be formed.

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I. INTRODUCTION

The proposition has been put forth recently that regions of misaligned chiral vacuum, or disoriented chiral condensates (DCC), might form in either ultrahigh-energy or heavy nuclei collisions [1–4]. If so, then this would be a striking probe of the QCD phase transition. It might also help explain [5, 6] the so-called Centauro and anti-Centauro events observed in high-energy cosmic ray experiments [7].

How can we tell, from a theoretical standpoint, whether or not we should expect a DCC to form? Clearly, investigating QCD directly is out of the question for now; the technology required to compute the evolution of the relevant order parameters directly from QCD is still lacking, though we can use lattice calculations for hints about some aspects of the QCD phase transition. What we need then is a model that encodes the relevant aspects of QCD in a faithful manner, yet is easier to calculate with than QCD itself.

Wilczek and Rajagopal [8] have argued that the O(4) linear σ model is such a model. It lies within the same *static* universality class as QCD with *two* massless quarks, which is a fair approximation to the world at temperatures and energies below Λ_{QCD}. Thus work done on the $n = 4$ Heisenberg ferromagnet can be used to understand various static quantities arising at the chiral phase transition.

One conclusion from Ref. [8] was that, if as the critical temperature for the chiral phase transition was approached from above the system remained in thermal equilibrium, then it was very unlikely that a large DCC region, with its concomitant biased pion emission, would form.

The point was that the correlation length $\xi = m_\pi^{-1}$ did *not* get large compared to the T_c^{-1}. A more quantitative criterion involving the comparison of the energy in a correlation volume just below T_c with the $T = 0$ pion mass (so as to find the number of pions in a correlation volume) supports the conclusion that as long as the system can equilibrate, no large regions of DCC will form.

The only option left, if we want to form a DCC, is to ensure that the system is far out of equilibrium. This can be achieved by *quenching* the system (although Gavin and Müller [10] claim that *annealing* might also work). What this means is the following. Start with the system in equilibrium at a temperature above T_c. Then suddenly drop the temperature to zero. If the rate at which the temperature drops is much faster than the rate at which the system can adapt to this change, then the state of the system after the quench is such that it is still in the thermal state at the initial temperature. However, the *dynamics* governing the evolution of that initial state is now driven by the $T = 0$ Hamiltonian. The system will then have to relax from the initial state, which is *not* the ground state of the Hamiltonian to the zero temperature ground state. During this time, it is expected that regions in which the order parameter is correlated will grow. We can then hope that the correlation regions will grow to be large enough to contain a large number of pions inside them.

The possibility that the chiral phase transition might occur following a quench in heavy-ion collisions was explored by Wilczek and Rajagopal in Ref. [9]. They argue there that long-wavelength fluctuations in the pion fields can develop after the quench occurs. Modes with wave numbers k smaller than some critical wave number $k_{\rm crit}$ will be unstable and regions in which the pion field is correlated will grow in spatial extent for a period of time. The essence of this mechanism is that the pions are the would-be Goldstone particles of spontaneous chiral symmetry breaking. In the absence of quark masses, the pions would be exactly massless when the $\boldsymbol{\pi}$ and σ fields are in their ground state. However, during the quench, the σ field is displaced from its zero temperature minimum so that the required cancellation between the negative bare mass^2 term in the Lagrangian and the mass^2 induced through the pion interactions with the σ condensate does not occur. This then allows some of the pion momentum modes to propagate as if they had a *negative* mass^2, thus causing exponential growth in these modes. In Ref. [9], the *classical* σ model was simulated. The correlation functions were taken as spatial averages, and the expectations for the growth of various pion field momentum modes were borne out. More recently however, Gavin, Gocksch, and Pisarski [11] have concluded that the strongly coupled linear σ model *does not* produce large correlated domains of pions. These authors also performed a numerical simulation of the *classical* equations of motion.

The mechanism advocated by Wilczek and Rajagopal, the growth of unstable long-wavelength modes leading to large correlations (domains), is similar to the mechanism of spinodal decomposition in field theory [12, 13]. However, there are already hints from previous work [12, 13] that both *quantum and thermal* effects may be important in determining the growth of correlation regions in a field theory. The process of phase separation via the formation and growth of correlated domains is similar to the process of spinodal decomposition in condensed matter systems [14]. Below the critical temperature, a band of long-wavelength modes become unstable and grow [12, 14, 15]. This growth is manifest in the equal-time spatial Fourier transform of the two-point correlation function. This process is well known in classical statistical mechanics [14] and has already been studied within a nonperturbative framework in scalar quantum field theories [12, 15].

The growth of the unstable long-wavelength modes is responsible for large quantum (and thermal) fluctuations, and leads to profuse particle production via parametric amplification [12, 16].

The rest of the paper is devoted to a systematic, nonperturbative analysis of the growth of fluctuations, the onset of long-range correlations and the ensuing production of pions after a rapid (in time scales of the order of few fm/c) cooling into the unstable phase below the critical temperature.

In the next section, we will develop the formalism necessary to take the quantum and thermal fluctuations of the fields into account. Having done that, we turn to the actual numerical solution of the equations we find, and use these solutions to calculate the equal time two-point correlation function for the pions. It will be clear after doing this, that in the case in which the system is either quenched or relaxed from a high temperature phase whose temperature is larger than the critical temperature, the correlation regions are never large enough for the correlations in pions to be observed. We strengthen this conclusion by computing the number of pions in the correlation volume. We then end with some speculations concerning possible ways in which a large DCC region might form.

II. THE O(4) σ MODEL OUT OF EQUILIBRIUM

Our strategy is as follows. We will use the techniques developed by us previously [16] and use the functional Schrödinger representation, in which the time evolution of the system is represented by the time evolution of its *density matrix.*

The next step is to evolve the density matrix in time from this initial state via the quantum Liouville equation:

$$i\hbar\frac{\partial\rho(t)}{\partial t}=[H,\rho(t)], \tag{1}$$

where H is the Hamiltonian of the system *after* the quench. Using this density matrix, we can, at least in principle, evaluate the equal-time correlation function for the pion fields, and observe its growth with time.

Let us now implement this procedure. We start with the σ model Lagrangian density

$$\mathcal{L}=\frac{1}{2}\partial_\mu\mathbf{\Phi}\cdot\partial^\mu\mathbf{\Phi}-V(\sigma,\boldsymbol{\pi}), \tag{2}$$

$$V(\sigma,\boldsymbol{\pi})=\frac{1}{2}m^2(t)\mathbf{\Phi}\cdot\mathbf{\Phi}+\lambda(\mathbf{\Phi}\cdot\mathbf{\Phi})^2-h\sigma, \tag{3}$$

where $\mathbf{\Phi}$ is an O($N+1$) vector, $\mathbf{\Phi}=(\sigma,\boldsymbol{\pi})$, and $\boldsymbol{\pi}$ represents the N pions.

The linear σ model is a low-energy effective theory for an $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$ (up and down quarks) strongly interacting theory. It may be obtained as a Landau-Ginzburg effective theory from a Nambu–Jona-Lasinio model [17]. In fact, Bedaque and Das [18] have studied a quench starting from an $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$ Nambu–Jona-Lasinio model.

We have parametrized the dynamics of the cooling down process in terms of a time-dependent mass term. We can use this to describe the phenomenology of either a sudden quench where the mass^2 changes sign instantaneously or that of a relaxational process in which the mass^2 changes sign on a time scale determined by the dynamics. In a heavy ion collision, we expect this relaxation time scale to be of the order of $\tau\sim 0.5-1$ fm/c. The term $h\sigma$ accounts for the explicit breaking of chiral symmetry due to the (small) quark masses. We leave N arbitrary for now, though at the end we will take $N=3$.

Our first order of business is to identify the correct order parameter for the phase transition and then to obtain its equation of motion. Let us define the fluctuation field operator $\chi(\mathbf{x},t)$ as

$$\sigma=\phi(t)+\chi(\mathbf{x},t), \tag{4}$$

with $\phi(t)$ a c-number field defined by

$$\phi(t) = \frac{1}{\Omega}\int d^3x \langle\sigma(\mathbf{x})\rangle = \frac{1}{\Omega}\int d^3x \frac{\mathrm{Tr}[\rho(t)\sigma(\mathbf{x})]}{\mathrm{Tr}\rho(t)}. \tag{5}$$

Here Ω is the spatial volume in which we enclose the system. The fluctuation field $\chi(\mathbf{x},t)$ is defined so that (i) $\langle\chi(\mathbf{x},t)\rangle = 0$, and (ii) $\dot{\chi}(\mathbf{x},t) = -\dot{\phi}(t)$. Making use of the Liouville equation for the density matrix, we arrive at the equations

$$\dot{\phi}(t) = p(t) = \frac{1}{\Omega}\int d^3x \ \langle\Pi_\sigma(\mathbf{x})\rangle, \tag{6}$$

$$\dot{p}(t) = -\frac{1}{\Omega}\int d^3x \left\langle \frac{\delta V(\sigma,\boldsymbol{\pi})}{\delta\sigma(\mathbf{x})}\right\rangle, \tag{7}$$

where $\Pi_\sigma(\mathbf{x})$ is the canonical momentum conjugate to $\sigma(\mathbf{x})$.

The derivative of the potential in the equation for $\dot{\pi}(t)$ is to be evaluated at $\sigma = \phi(t) + \chi(\mathbf{x},t)$. These equations can be combined into a single one describing the evolution of the order parameter $\phi(t)$:

$$\ddot{\phi}(t) + \frac{1}{\Omega}\int d^3x \left\langle \frac{\delta V(\sigma,\boldsymbol{\pi})}{\delta\sigma(\mathbf{x})}\right\rangle\Bigg|_{\sigma=\phi(t)+\chi(\mathbf{x},t)} = 0. \tag{8}$$

To proceed further we have to determine the density matrix. Since the Liouville equation is first order in time we need only specify $\rho(t=0)$. At this stage we could proceed to a perturbative description of the dynamics (in a loop expansion).

However, as we learned previously in a similar situation [12, 13], the nonequilibrium dynamics of the phase transition cannot be studied within perturbation theory.

Furthermore, since the quartic coupling of the linear σ model λ must be large ($\lambda \approx 4-5$ so as to reproduce the value of $f_\pi \approx 95$ MeV with a "σ mass" ≈ 600 MeV), the linear σ model is a *strongly* coupled theory, and any type of perturbative expansion will clearly be unreliable. Thus, following our previous work [12, 16] and the work of Rajagopal and Wilczek [9] and Pisarski [11] we invoke a Hartree approximation.

In the presence of a vacuum expectation value, the Hartree factorization is somewhat subtle. We will make a series of *assumptions* that we feel are quite reasonable but which, of course, may fail to hold under some circumstances and for which we do not have an *a priori* justification. These are the following: (i) no cross correlations between the pions and the σ field, and (ii) that the two-point correlation functions of the pions are diagonal in isospin space, where by isospin we now refer to the unbroken O(N) ($N=3$) symmetry under which the pions transform as a triplet. These assumptions lead to the following Hartree factorization of the nonlinear terms in the Hamiltonian:

$$\chi^4 \to 6\langle\chi^2\rangle\chi^2 + \text{const}, \tag{9}$$

$$\chi^3 \to 3\langle\chi^2\rangle\chi, \tag{10}$$

$$(\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2 \to \left(2+\frac{4}{N}\right)\langle\boldsymbol{\pi}^2\rangle\boldsymbol{\pi}^2 + \text{const}, \tag{11}$$

$$\boldsymbol{\pi}^2\chi^2 \to \boldsymbol{\pi}^2\langle\chi^2\rangle + \langle\boldsymbol{\pi}^2\rangle\chi^2, \tag{12}$$

$$\boldsymbol{\pi}^2\chi \to \langle\boldsymbol{\pi}^2\rangle\chi, \tag{13}$$

where by "constant" we mean the operator-independent expectation values of the composite operators. Although these will be present as operator-independent terms in the Hamiltonian, they are c-number terms and will not enter in the time evolution of the density matrix.

It can be checked that when $\phi = 0$ one obtains the O($N+1$)-invariant Hartree factorization.

In this approximation the resulting Hamiltonian is quadratic, with a linear term in χ:

$$H_H(t) = \int d^3x \left\{ \frac{\Pi_\chi^2}{2} + \frac{\boldsymbol{\Pi}_\pi^2}{2} + \frac{(\nabla\chi)^2}{2} + \frac{(\nabla\boldsymbol{\pi})^2}{2} + \chi\mathcal{V}^1(t) + \frac{\mathcal{M}_\chi^2(t)}{2}\chi^2 + \frac{\mathcal{M}_\pi^2(t)}{2}\boldsymbol{\pi}^2 \right\}. \tag{14}$$

Here Π_χ, $\boldsymbol{\Pi}_\pi$ are the canonical momenta conjugate to $\chi(\mathbf{x})$, $\boldsymbol{\pi}(\mathbf{x})$, respectively, and $\mathcal{V}^1$ is recognized as the derivative of the Hartree "effective potential" [19, 20] with respect to ϕ (it is the derivative of the nongradient terms of the effective action [12, 16, 21]).

In the absence of an explicit symmetry-breaking term, the Goldstone theorem requires the existence of massless pions, $\mathcal{M}_\pi = 0$ whenever $\mathcal{V}^1$ for $\phi \neq 0$. However, this is *not* the case within our approximation scheme as it stands.

This situation can be easily remedied, however, by noting that the Hartree approximation becomes exact in the large N limit. In this limit, $\langle\boldsymbol{\pi}^2\rangle = O(N)$, $\langle\chi^2\rangle = O(1)$, $\phi^2 = O(N)$. Thus we will approximate further by neglecting the $O(1/N)$ terms in the formal large N limit. This further truncation ensures that the Ward identities are satisfied. We now obtain

$$\mathcal{V}^1(t) = \phi(t)\left[m^2(t) + 4\lambda\phi^2(t) + 4\lambda\langle\boldsymbol{\pi}^2\rangle(t)\right] - h, \tag{15}$$

$$\mathcal{M}_\pi^2(t) = m^2(t) + 4\lambda\phi^2(t) + 4\lambda\langle\boldsymbol{\pi}^2\rangle(t), \tag{16}$$

$$\mathcal{M}_\chi^2(t) = m^2(t) + 12\lambda\phi^2(t) + 4\lambda\langle\boldsymbol{\pi}^2\rangle(t). \tag{17}$$

The Hamiltonian is now quadratic with time-dependent self-consistent masses, and Goldstone's Ward identities are satisfied.

Since the evolution Hamiltonian is quadratic in this approximation, we propose in the Hartree approximation a Gaussian density matrix in terms of the Hartree-Fock states. As a consequence of our assumption of no cross correlation between χ and $\boldsymbol{\pi}$, the density matrix factorizes as

$$\rho(t) = \rho_\chi(t)\otimes\rho_\pi(t). \tag{18}$$

In the Schrödinger representation the density matrix is most easily written down by making use of spatial translational invariance to decompose the fluctuation fields $\chi(\mathbf{x},t)$ and $\boldsymbol{\pi}(\mathbf{x},t)$ into spatial Fourier modes:

$$\chi(\mathbf{x},t)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{k}}\chi_{\mathbf{k}}(t)\exp(-i\mathbf{k}\cdot\mathbf{x}), \tag{19}$$

$$\boldsymbol{\pi}(\mathbf{x})=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{k}}\boldsymbol{\pi}_{\mathbf{k}}\exp(-i\mathbf{k}\cdot\mathbf{x}), \tag{20}$$

where we recall that $\chi(\mathbf{x},t)=\sigma(\mathbf{x})-\phi(t)$. We can now use these Fourier modes as the basis in which to write the density matrices for the σ and the pions. We will use the Gaussian *Ansätze*

$$\rho_\chi[\chi,\tilde{\chi},t]=\prod_{\mathbf{k}}\mathcal{N}_{\chi,k}(t)\exp\left\{-\left[\frac{A_{\chi,k}(t)}{2\hbar}\chi_k(t)\chi_{-k}(t)+\frac{A^*_{\chi,k}(t)}{2\hbar}\tilde{\chi}_k(t)\tilde{\chi}_{-k}(t)\right.\right.$$
$$\left.\left.+\frac{B_{\chi,k}(t)}{\hbar}\chi_k(t)\tilde{\chi}_{-k}(t)\right]+\frac{i}{\hbar}p_{\chi,k}(t)\left[\chi_{-k}(t)-\tilde{\chi}_{-k}(t)\right]\right\}, \tag{21}$$

$$\rho_\pi[\boldsymbol{\pi},\tilde{\boldsymbol{\pi}},t]=\prod_{\mathbf{k}}\mathcal{N}_{\pi,k}(t)\exp\left\{-\left[\frac{A_{\pi,k}(t)}{2\hbar}\boldsymbol{\pi}_k\cdot\boldsymbol{\pi}_{-k}+\frac{A^*_{\pi,k}(t)}{2\hbar}\tilde{\boldsymbol{\pi}}_k\cdot\tilde{\boldsymbol{\pi}}_{-k}+\frac{B_{\pi,k}(t)}{\hbar}\boldsymbol{\pi}_k\cdot\tilde{\boldsymbol{\pi}}_{-k}\right]\right\}. \tag{22}$$

The assumption of isospin invariance implies that the kernels $A_{\pi,k}$, $B_{\pi,k}$ transform as isospin singlets, since these kernels give the two-point correlation functions. Furthermore, Hermiticity of the density matrix requires that the mixing kernel B be real. The lack of a linear term in the pion density matrix will become clear below.

The Liouville equation is most conveniently solved in the Schrödinger representation, in which

$$\Pi_\chi(\mathbf{x})=-i\hbar\frac{\delta}{\delta\chi}\ ,\ \Pi^j_\pi(\mathbf{x})=-i\hbar\frac{\delta}{\delta\pi_j},$$

$$i\hbar\frac{\partial\rho(t)}{\partial t}=\Big(H[\Pi_\chi,\boldsymbol{\Pi}_\pi,\chi,\boldsymbol{\pi};t]-H[\tilde{\Pi}_\chi,\tilde{\boldsymbol{\Pi}}_\pi,\tilde{\chi},\tilde{\boldsymbol{\pi}};t]\Big)\rho(t). \tag{23}$$

Comparing the terms quadratic, linear, and independent of the fields (χ ; $\boldsymbol{\pi}$), we obtain the following set of differential equations for the coefficients and the expectation value:

$$i\frac{\dot{\mathcal{N}}_{\chi,k}}{\mathcal{N}_{\chi,k}}=\frac{1}{2}(A_{\chi,k}-A^*_{\chi,k}), \tag{24}$$

$$i\dot{A}_{\chi,k}=\left[A^2_{\chi,k}-B^2_{\chi,k}-\omega^2_{\chi,k}(t)\right], \tag{25}$$

$$i\dot{B}_{\chi,k}=B_{\chi,k}\left(A_{\chi,k}-A^*_{\chi,k}\right), \tag{26}$$

$$\omega^2_{\chi,k}(t)=k^2+\mathcal{M}^2_\chi(t), \tag{27}$$

$$\ddot{\phi}+m^2(t)\phi+4\lambda\phi^3+4\lambda\phi\langle\boldsymbol{\pi}^2(\mathbf{x},t)\rangle-h=0, \tag{28}$$

$$i\frac{\dot{\mathcal{N}}_{\pi,k}}{\mathcal{N}_{\pi,k}}=\frac{1}{2}(A_{\chi,k}-A^*_{\chi,k}), \tag{29}$$

$$i\dot{A}_{\pi,k}=\left[A^2_{\pi,k}-B^2_{\pi,k}-\omega^2_{\pi,k}(t)\right], \tag{30}$$

$$i\dot{B}_{\pi,k}=B_{\pi,k}\left(A_{\pi,k}-A^*_{\pi,k}\right), \tag{31}$$

$$\omega^2_{\pi,k}(t)=k^2+\mathcal{M}^2_\pi(t). \tag{32}$$

The lack of a linear term in (22) is a consequence of a lack of a linear term in $\boldsymbol{\pi}$ in the Hartree Hamiltonian, as the symmetry has been specified to be broken along the σ direction.

To completely solve for the time evolution, we must specify the initial conditions. We will *assume* that at an initial time ($t=0$) the system is in *local thermodynamic equilibrium* at an initial temperature T, which we take to be higher than the critical temperature,$T_c\approx$ 200 MeV, where we use the phenomenological couplings and masses to obtain T_c.

This assumption thus describes the situation in a high-energy collision in which the central rapidity region is at a temperature larger than critical, and thus in the symmetric phase, and such that the phase transition occurs via the rapid cooling that occurs when the region in the high temperature phase expands along the beam axis.

The assumption of local thermodynamic equilibrium for the Hartree-Fock states determines the initial values of the kernels and the expectation value of the σ field and its canonical momentum:

$$A_{\chi,k}(t=0)=\omega_{\chi,k}(0)\coth[\beta\hbar\omega_{\chi,k}(0)], \tag{33}$$

$$B_{\chi,k}(t=0)=-\frac{\omega_{\chi,k}(0)}{\sinh[\beta\hbar\omega_{\chi,k}(0)]}, \tag{34}$$

$$A_{\pi,k}(t=0)=\omega_{\pi,k}(0)\coth[\beta\hbar\omega_{\pi,k}(0)], \tag{35}$$

$$B_{\pi,k}(t=0)=-\frac{\omega_{\pi,k}(0)}{\sinh[\beta\hbar\omega_{\pi,k}(0)]} \tag{36}$$

$$\phi(t=0)=\phi_0\ ;\ \dot{\phi}(t=0)=0, \tag{37}$$

with $\beta=1/k_BT$. We have (arbitrarily) assumed that the expectation value of the canonical momentum conjugate to the σ field is zero in the initial equilibrium ensemble. These initial conditions dictate the following *Ansätze* for the real and imaginary parts of the kernels $A_{\pi,k}(t)$, $A_{\chi,k}(t)$ in terms of complex functions $\mathcal{A}_{\pi,k}(t)=\mathcal{A}_{R;\pi,k}(t)+i\mathcal{A}_{I;\pi,k}(t)$ and $\mathcal{A}_{\chi,k}(t)=\mathcal{A}_{R;\chi,k}(t)+i\mathcal{A}_{I;\chi,k}(t)$ [16]:

$$A_{R;\pi,k}(t)=\mathcal{A}_{R;\pi,k}(t)\coth[\beta\hbar\omega_{\pi,k}(0)], \tag{38}$$

$$B_{\pi,k}(t) = -\frac{\mathcal{A}_{R;\pi,k}(t)}{\sinh[\beta\hbar\omega_{\pi,k}(0)]}, \tag{39}$$

$$A_{I;\pi,k}(t) = \mathcal{A}_{I;\pi,k}(t). \tag{40}$$

The differential equation for the complex function $\mathcal{A}$ can be cast in a more familiar form by a change of variables

$$\mathcal{A}_{\pi,k}(t) = -i\frac{\dot{\Psi}_{\pi,k}(t)}{\Psi_{\pi,k}(t)}, \tag{41}$$

with $\Psi_{\pi,k}$ obeying the following Schrödinger-like differential equation, and boundary conditions:

$$\left[\frac{d^2}{dt^2} + \omega^2_{\pi,k}(t)\right]\Psi_{\pi,k}(t) = 0, \tag{42}$$

$$\Psi_{\pi,k}(t=0) = \frac{1}{\sqrt{\omega_{\pi,k}(0)}},$$
$$\dot{\Psi}_{\pi,k}(t=0) = i\sqrt{\omega_{\pi,k}(0)}. \tag{43}$$

Since in this approximation the dynamics for the pions and σ fields decouple, we will only concentrate on the solution for the pion fields; the effective time-dependent frequencies for the σ fields are completely determined by the evolution of the pion correlation functions. In terms of these functions we finally find

$$\langle \boldsymbol{\pi}_k(t)\cdot\boldsymbol{\pi}_{-k}(t)\rangle = \frac{N\hbar}{2} \mid \Psi_{\pi,k}(t) \mid^2 \coth\left[\frac{\hbar\omega_{\pi,k}(0)}{2k_BT}\right]. \tag{44}$$

In terms of this two-point correlation function and recognizing that the $\Psi_{\pi,k}(t)$ only depends on k^2, we obtain the important correlations

$$\langle \boldsymbol{\pi}^2(\mathbf{x},t)\rangle = \frac{N\hbar}{4\pi^2}\int dk k^2 \mid \Psi_{\pi,k}(t) \mid^2 \coth\left[\frac{\hbar\omega_{\pi,k}(0)}{2k_BT}\right], \tag{45}$$

$$\langle \boldsymbol{\pi}(\mathbf{x},t)\cdot\boldsymbol{\pi}(\mathbf{0},t)\rangle = \frac{N\hbar}{4\pi^2}\int dk k\frac{\sin(kx)}{x} \mid \Psi_{\pi,k}(t) \mid^2 \times \coth\left[\frac{\hbar\omega_{\pi,k}(0)}{2k_BT}\right]. \tag{46}$$

The presence of the temperature-dependent function in the above expressions encodes the finite temperature correlations of the initial state. The set of equations (28) and (42), with the above boundary conditions, completely determine the nonequilibrium dynamics in the Hartree-Fock approximation. We will provide a numerical analysis of these equations in the next section.

A. Pion production

In the Hartree approximation, the Hamiltonian is quadratic, and the fields can be expanded in terms of creation and annihilation of Hartree-Fock states

$$\boldsymbol{\pi}_k(t) = \frac{1}{\sqrt{2}}\left[\mathbf{a}_k\Psi^\dagger_{\pi,k}(t) + \mathbf{a}^\dagger_{-k}\Psi_{\pi,k}(t)\right]. \tag{47}$$

The creation $\mathbf{a}^\dagger_k$ and annihilation $\mathbf{a}_k$ operators are *independent of time* in the Heisenberg picture and the mode functions $\Psi_{\pi,k}(t)$ are the solutions to the Hartree equations (42) which are the Heisenberg equations of motion in this approximation. The boundary conditions (43) correspond to positive frequency particles for $t \leq 0$. The creation and annihilation operators may be written in terms of the Heisenberg fields (47) and their canonical momenta. Passing on to the Schrödinger picture at time $t = 0$, we can relate the Schrödinger picture operators at time t to those at time $t = 0$ via a Bogoliubov transformation:

$$\mathbf{a}_k(t) = \mathcal{F}_{+,k}(t)\mathbf{a}_k(0) + \mathcal{F}_{-,k}(t)\mathbf{a}^\dagger_{-k}(0), \tag{48}$$

with

$$\mid \mathcal{F}_{+,k}(t) \mid^2 = \frac{1}{4}\frac{\mid \Psi_{\pi,k}(t) \mid^2}{\mid \Psi_{\pi,k}(0) \mid^2} \times\left[1 + \frac{\mid \dot{\Psi}_{\pi,k}(t) \mid^2}{\omega^2_{\pi,k}(0) \mid \Psi_{\pi,k}(t) \mid^2}\right] + \frac{1}{2},$$
$$\mid \mathcal{F}_{+,k}(t) \mid^2 - \mid \mathcal{F}_{-,k}(t) \mid^2 = 1. \tag{49}$$

At any time t the expectation value of the number operator for pions (in each k mode) is

$$\langle N_{\pi,k}(t)\rangle = \frac{\mathrm{Tr}\mathbf{a}^\dagger_{\pi,k}(t)\cdot\mathbf{a}_{\pi,k}(t)\rho(0)}{\mathrm{Tr}\rho(0)} = \frac{\mathrm{Tr}\mathbf{a}^\dagger_{\pi,k}(0)\cdot\mathbf{a}_{\pi,k}(0)\rho(t)}{\mathrm{Tr}\rho(t)}. \tag{50}$$

After some straightforward algebra we find

$$\langle N_{\pi,k}\rangle(t) = (2 \mid \mathcal{F}_{+,k}(t,t_o) \mid^2 - 1)\langle N_{\pi,k}\rangle(0) + \left(\mid \mathcal{F}_{+,k}(t,t_o) \mid^2 - 1\right). \tag{51}$$

The first term represents the "induced" and the second term the "spontaneous" particle production. In this approximation, particle production is a consequence of parametric amplification. The Hartree-Fock states are examples of squeezed states, and the density matrix is a "squeezed" density matrix. The squeeze parameter (the ratio of the kernels at a time t to those at time $t = 0$) is time dependent and determines the time evolution of the states and density matrix. The relation between squeezed states and pion production has been advanced by Kogan [22] although not in the context of an initial thermal density matrix.

Thus far we have established the formalism to study the nonequilibrium evolution during the phase transition. A question of interpretation must be clarified before proceeding further. Our description, in terms of a statistical density matrix, describes an isospin-invariant mixed state, and thus does not prefer one isospin direction over another. A real experiment will furnish one realization of all the available states mixed in the density matrix. However, if the pion correlation functions become long ranged (as a statistical average) it is clear that in a particular realization, at least one isospin component is becoming correlated over large distances; thus, it is in this statistical sense that our results should be understood.

This concludes our discussion of the formalism we will use to study the nonequilibrium evolution of the pion system. We now turn to a numerical analysis of the problem.

III. NUMERICAL ANALYSIS

The phenomenological set of parameters that define the linear σ model as an effective low-energy theory are (we will be somewhat cavalier about the precise value of these parameters as we are interested in the more robust features of the pion correlations)

$$M_\sigma \approx 600 \text{ MeV}, \quad f_\pi \approx 95 \text{ MeV}, \quad \lambda \approx 4.5,$$
$$h \approx (120 \text{ MeV})^3, \quad T_c \approx 200 \text{ MeV}. \qquad (52)$$

The above value of the critical temperature differs somewhat from the lattice estimates ($T_c \approx 150$ MeV), and our definition of λ differs by a factor four from that given elsewhere [9, 11].

The first thing to notice is that this is a *strongly* coupled theory, and unlike our previous studies of the dynamics of phase transitions [12, 13] we expect the relevant time scales to be much *shorter* than in weakly coupled theories.

We must also notice that the linear σ model is an effective low-energy *cutoff* theory. There are two physically important factors that limit the value of the cutoff. (i) This effective theory neglects the influence of the nucleons, and in the Hartree approximation, the vector resonances are missed. These two features imply a cutoff of the order of about 2 GeV. (ii) The second issue is that of the triviality bound. Assuming that the value of the coupling is determined at energies of the order of M_σ, its very large value implies that the cutoff should not be much larger than about 4–5 GeV, since otherwise the theory will be dangerously close to the Landau pole. From the technical standpoint this is a more important issue since in order to write the renormalized equations of motion we need the ratio between bare and renormalized couplings. In the Hartree approximation this is the "wave function renormalization constant" for the composite operator $\boldsymbol{\pi}^2$.

Thus we use a cutoff $\Lambda = 2$ GeV. The issue of the cutoff is an important one since $\langle \boldsymbol{\pi}^2 \rangle$ requires renormalization, and in principle we should write down renormalized equations of motion. In the limit when the cutoff is taken to infinity the resulting evolution should be insensitive to the cutoff. However, the chosen cutoff is not very much larger than other scales in the problem and the "renormalized" equations will yield solutions that are cutoff sensitive. However, this sensitivity will manifest itself on distance scales of the order of 0.1 fm or smaller, and we are interested in detecting correlations over many fm. The *size* of the correlated regions and the time scales for their growth will be determined by the long wavelength unstable modes [12] (see below), and thus should be fairly insensitive to the momentum scales near the cutoff. The short distance features of the correlation functions, such as, e.g., the *amplitude* of the fluctuations, will, however, be rather sensitive to the cutoff.

The most severe ultraviolet divergence in the composite operator $\boldsymbol{\pi}^2$ is proportional to Λ^2. This divergence is usually handled by a subtraction. We will subtract this term (including the temperature factors) in a renormalization of the mass at $t = 0$. Thus

$$m_B^2(t=0) + 4\lambda \langle \boldsymbol{\pi}^2 \rangle (t=0;T) = m_R^2(t=0;T), \qquad (53)$$

where we made explicit the temperature dependence of $\boldsymbol{\pi}^2$ and $m_R^2(t=0;T)$. Furthermore we will parametrize the time-dependent mass term as

$$m_R^2(t) = \frac{M_\sigma^2}{2}\left[\frac{T^2}{T_c^2}\exp\left(-2\frac{t}{t_r}\right) - 1\right]\Theta(t) + \frac{M_\sigma^2}{2}\left[\frac{T^2}{T_c^2} - 1\right]\Theta(-t). \qquad (54)$$

This parametrization incorporates the dynamics of the expansion and cooling processes in the plasma in a phenomenological way. It allows for the system to cool down with an effective temperature given by

$$T_{\text{eff}}(t) = T\exp\left[-\frac{t}{t_r}\right], \qquad (55)$$

where T is the initial value of the temperature in the central rapidity region, and t_r is a relaxation time. This parametrization also allows us to study a "quench" corresponding to the limiting case $t_r = 0$.

It is convenient to introduce the natural scale fm $^{-1} \approx$ 200 MeV $= M_F$ and define the dimensionless variables

$$\phi(t) = M_F f(t) \;\; , \;\; \Psi_{\pi,k}(t) = \frac{\psi_q(\tau)}{\sqrt{M_F}},$$
$$k = M_F q \;\; , \;\; t = \frac{\tau}{M_F} \;\; , \;\; t_r = \frac{\tau_r}{M_F} \;\; , \;\; x = \frac{z}{M_F}. \qquad (56)$$

In these units

$$\frac{\Lambda}{M_F} = 10 \;\; , \;\; \frac{M_\sigma}{M_F} = 3 \;\; , \;\; H = \frac{h}{M_F^3} \approx 0.22, \qquad (57)$$

$$\frac{\omega_{\pi,k}^2(0)}{M_F^2} = W_q^2 = q^2 + \frac{9}{2}\left[\frac{T^2}{T_c^2} - 1\right] + 4\lambda f^2(0). \qquad (58)$$

Thus we have to solve simultaneously the Hartree set of equations:

$$\frac{d^2 f}{d\tau^2} + \frac{9}{2} f \left[\frac{T^2}{T_c^2}\exp\left(-2\frac{\tau}{\tau_r}\right) - 1\right] + 4\lambda f^3 + 4f\lambda\Sigma(0,\tau) - H = 0, \qquad (59)$$

$$\left\{\frac{d^2}{d\tau^2} + q^2 + \frac{9}{2}\left[\frac{T^2}{T_c^2}\exp\left(-2\frac{\tau}{\tau_r}\right) - 1\right] + 4\lambda f^2(\tau) + 4\lambda\Sigma(0,\tau)\right\}\psi_q(\tau) = 0, \qquad (60)$$

$$\Sigma(z,\tau) = \langle \boldsymbol{\pi}(\mathbf{x},t)\cdot\boldsymbol{\pi}(\mathbf{0},t)\rangle/M_F^2 = \frac{3}{4\pi^2}\int_0^{10} dq q \frac{\sin(qz)}{z}(\mid \psi_q(\tau)\mid^2 - \mid \psi_q(0)\mid^2) \times \coth\left[\frac{W_q}{10T(\mathrm{GeV})}\right], \quad (61)$$

with the boundary conditions

$$f(0) = f_0 \ , \ \frac{df(0)}{d\tau} = 0, \quad (62)$$

$$\psi_q(0) = \frac{1}{\sqrt{W_q}} \ , \ \frac{d\psi_q}{d\tau}(0) = i\sqrt{W_q}. \quad (63)$$

Finally, once we find the Hartree mode functions, we can compute the total pion density as a function of time:

$$\frac{N_\pi(t)}{\Omega} = \frac{\overline{N}_\pi(\tau)}{(\mathrm{fm})^3} = \frac{1}{2\pi^2 \ \mathrm{fm}^{\ 3}}\int_0^{10} dq \ q^2 \langle N_{\pi,q}(\tau)\rangle, \quad (64)$$

with $\langle N_{\pi,q}(\tau)\rangle$ given by (51) in terms of the dimensionless variables.

The mechanism of domain formation and growth is the fast time evolution of the unstable modes [12].

Let us consider first the case of $H = 0$. Then $f = 0$ is a fixed point of the evolution equation for f and corresponds to cooling down from the symmetric (disoriented) phase in the absence of explicit symmetry-breaking perturbations. Let us consider the simpler situation of a quench ($\tau_r = 0$). The equation for the Hartree mode functions (60) shows that for $q^2 < 9/2$ the corresponding

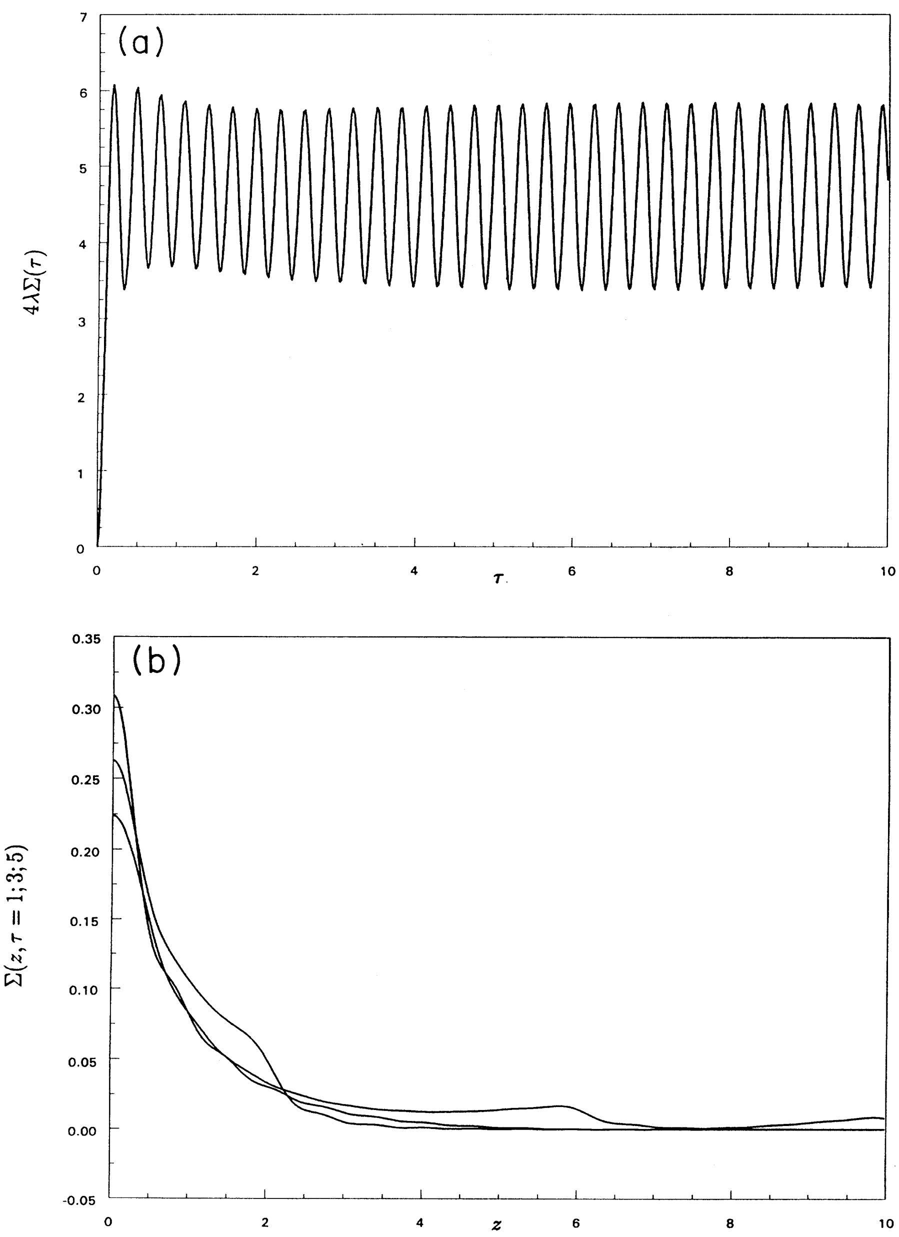

FIG. 1. (a) $4\lambda\Sigma(\tau)$ vs τ. $T_c = 200$ MeV; $T = 250$ MeV; $\tau_r = 0$; $f(0) = 0$; $H = 0$; $\lambda = 4.5$. (b) $\Sigma(z,\tau = 1;3;5)$ vs z for the same values of the parameters as in Fig. 1(a); larger values of time correspond to larger amplitudes at the origin. (c) $\overline{N}_\pi(\tau)$ vs τ for the same values of the parameters as in Fig. 1(a). $\overline{N}_\pi(0) = 0.15$.

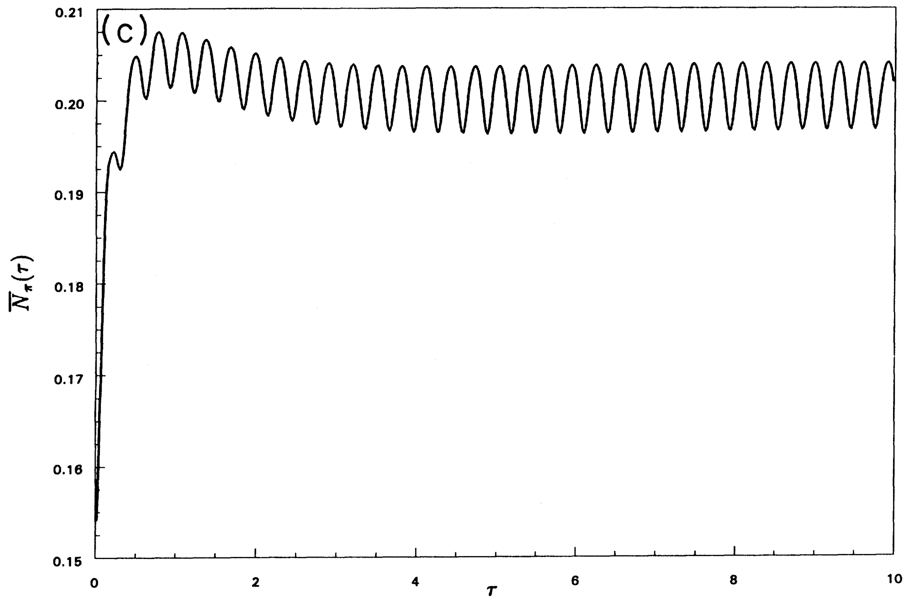

FIG. 1 *(Continued)*.

modes are unstable at early times and grow exponentially.

This growth feeds back on $\Sigma(\tau)$ which begins to grow and tends to overcome the instability. As the unstable fluctuations grow, only longer wavelengths remain unstable, until the time when $4\lambda\Sigma(\tau) \approx 9/2$, at which point no wavelength is unstable. The modes will continue to grow, however, because the derivatives will be fairly large, but since the instabilities will be overcome beyond this time the modes will have an oscillatory behavior.

We expect then that the fluctuations will grow during the time for which there are instabilities. This time scale depends on the value of the coupling; for very small coupling, $\Sigma(\tau)$ will have to grow for a long time before $4\lambda\Sigma(\tau) \approx 9/2$ and the instabilities are shut off. On the other hand, for strong coupling this time scale will be rather small, and domains will not have much time to grow.

It is clear that allowing for a nonzero magnetic field or $f(0) \neq 0$ will help to shut off the instabilities at earlier times, thus making the domains even smaller.

For typical relaxation times ($\approx 1 - 2$ fm/c), domains will not grow too large either because the fast growth of the fluctuations will catch up with the relaxing modes and shut off the instabilities [when $T_{\rm eff}(\tau) < T_c$] on short time scales. Thus we expect that for a nonzero relaxation time $\tau_r \approx$ fm/c, domains will not grow too large either, because the fluctuations will shut off the instabilities [when $T_{\rm eff}(\tau) < T_c$] on shorter time scales (this argument will be confirmed numerically shortly).

We conclude from this analysis that the optimal situation for which large DCC regions can grow corresponds to a quench from the symmetric phase in the absence of a magnetic field.

Figure 1(a) shows $4\lambda\Sigma(\tau)$ vs τ, Fig. 1(b) shows $\Sigma(z, \tau = 1; 3; 5)$ vs z, and Fig. 1(c) shows $\overline{N}_\pi(\tau)$ vs τ for the following values of the parameters: T_c = 200 MeV; $T = 250$ MeV; $\tau_r = 0; f(0) = 0$; $H = 0$; $\lambda = 4.5$ corresponding to a quench from the symmetric phase at a temperature slightly above the critical temperature and no magnetic field.

We clearly see that the fluctuations grow to overcome the instability in times ≈ 1 fm, and the domains never get bigger than about ≈ 1.5 fm. Figure 1(c) shows that the number of pions per cubic fermi is about 0.15 at the initial time (equilibrium value) and grows to about 0.2 in times about 1–2 fm after the quench. This pion density is thus consistent with having only a few pions in a pion-size correlation volume.

Figures 2(a–c) show again the same functions but now we let the system relax from an initial temperature $T = T_c$ with a relaxation time $t_r = 1$ fm/c. Notice that now the fluctuations grow less rapidly as they are modulated by the relaxation time, but again on a time scale of the order of a fm/c, they become big enough to shut off the long wavelength instabilities. Figure 2(b) shows $\Sigma(z, \tau = 1; 3; 5; 7)$ vs z. Once again, pions are correlated over distances of the order of a 1 fm. The reason the fluctuations grow so quickly and thus shut off the growth of the unstable modes so quickly is the strongly coupled nature of the theory.

The possibility of long range correlations exists if the initial state is in *equilibrium at the critical temperature*. In this situation there are already long range correlations in the initial state that will remain for some time as the temperature factors enhance the contributions for long wavelength modes since the Boltzmann factor $\approx 1/k$ for long wavelength fluctuations. Figures 3(a–c) show this situation for the values of the parameters $T = T_c = 200$ MeV; $\tau_r = 0$; $f(0) = 0$; $H = 0$; $\lambda = 4.5$. Figure 3(b) shows $\Sigma(z, \tau = 1; 2)$ vs z. In this case the number of pions per cubic fm in the initial state is ≈ 0.12

and reaches a maximum of about 0.17 within times of the order of 1 fm/c. The pions, however, are correlated over distances of about 4–5 fm with a large number of pions per correlation volume ≈ 50. These large correlation volumes are a consequence of the initial long range correlations. This is the situation proposed by Gavin, Gocksch, and Pisarski for the possibility of formation of large domains, as there is a "massless" particle in the initial state.

We believe that this situation is not very likely as the central rapidity region must remain in equilibrium at (or very close to) the critical temperature before the quench occurs.

To contrast this situation with that of a weakly coupled theory, Figs. 4(a–c) show the same functions with the following values of the parameters: $\lambda = 10^{-6}$; $T_c = 200$ MeV; $T = 250$ MeV; $\tau_r = 0$; $f(0) = 0$; $H = 0$. Now the fluctuations are negligible up to times of about 4 fm/c, during which time the correlation functions grow [Fig. 4(b)] and pions become correlated over distances of the order 4–5 fm. As shown in Fig. 4(c) the number of pions per cubic fm becomes enormous, a consequence of a large parametric amplification. In this extremely weakly coupled theory, the situation of a quench from above the critical temperature to almost zero temperature does produce a large number of coherent pions and domains which

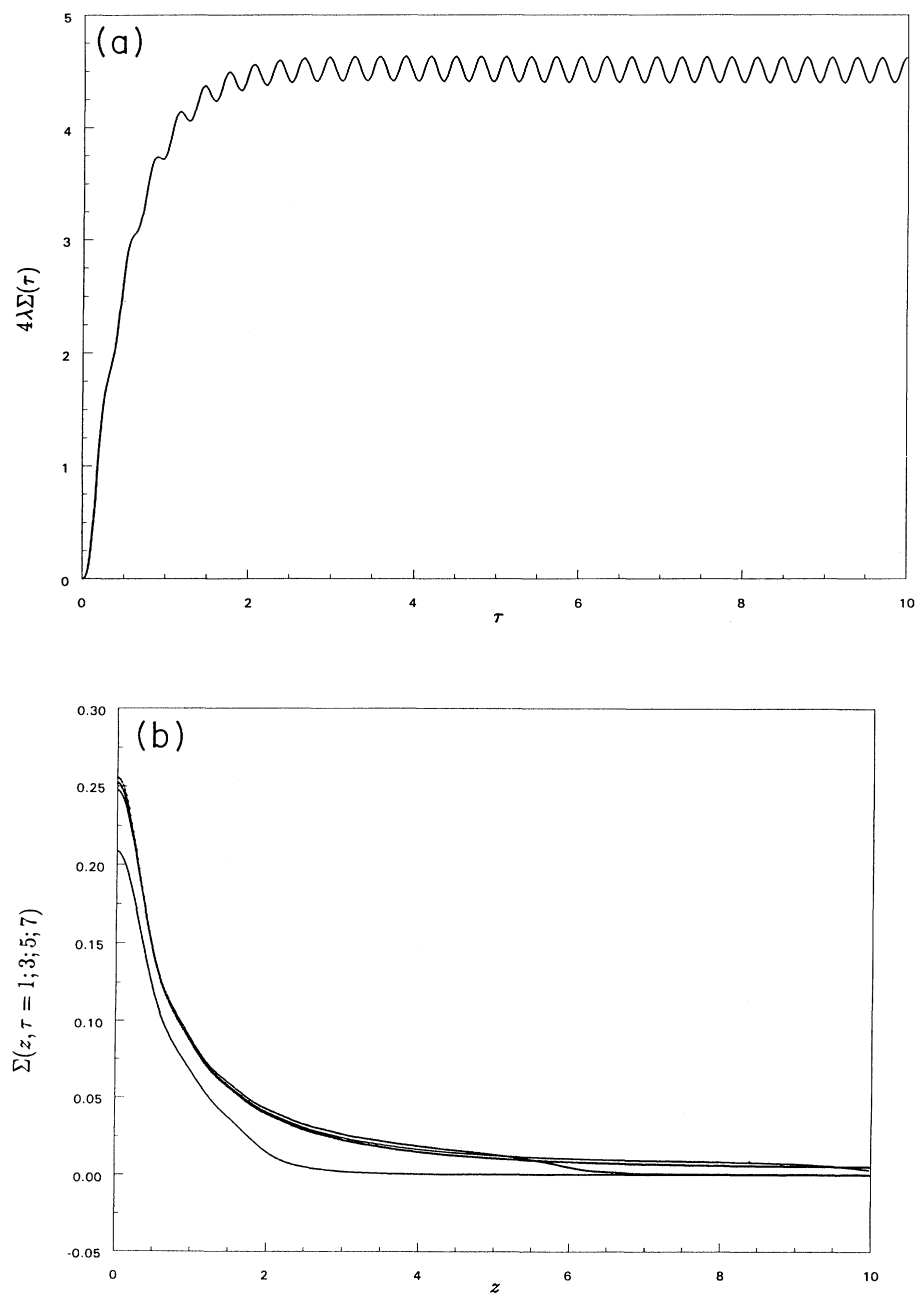

FIG. 2. (a) $4\lambda\Sigma(\tau)$ vs τ. $T = 250$ MeV; $T_c = 200$ MeV; $\tau_r = 1$; $f(0) = 0$; $H = 0$; $\lambda = 4.5$. (b) $\Sigma(z, \tau = 1; 3; 5; 7)$ vs z for the same values of the parameters as in Fig. 2(a); larger values of time correspond to larger amplitudes at the origin. (c) $\overline{N}_\pi(\tau)$ vs τ for the same values of the parameters as in Fig. 2(a). $\overline{N}_\pi(0) = 0.15$.

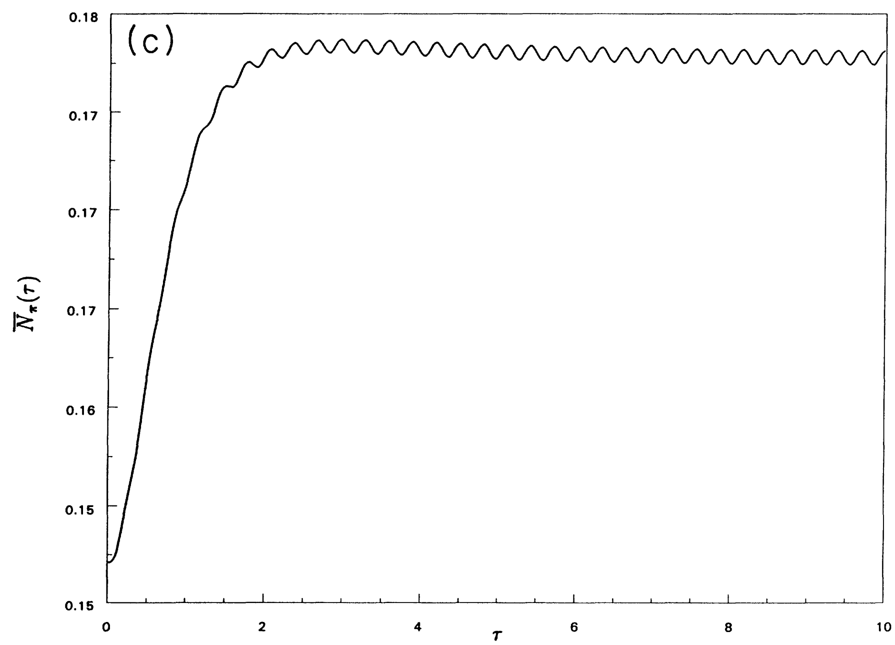

FIG. 2 *(Continued)*.

are much larger than typical pion sizes. This is precisely the situation studied previously [12] within a different context.

We have analyzed numerically many different situations in the strongly coupled case ($\lambda \approx$4–5) including the magnetic field and letting the expectation value of the σ field "roll down," etc., and in *all* of these cases in which the initial temperature is higher than the critical (between 10 and 20 % higher) we find the common feature that the time and spatial scales of correlations are ≈ 1 fm. Thus it seems that within this approach the strongly coupled linear σ model is incapable of generating large domains of correlated pions.

IV. DISCUSSIONS AND CONCLUSIONS

Our study differs in many qualitative and quantitave ways from previous studies. In particular we incorporate both quantum and thermal fluctuations and correlations in the initial state. In previous studies it was argued that because one is interested in long wavelength fluctuations these may be taken as *classical* and the classical evolution equations (with correlations functions replaced by spatial averages) were studied. We think that it is important to quantify why and when the long wavelength fluctuations are classical within the present approximation scheme. This may be seen from the temperature factors in the Hartree propagators. These are typically (incorporating now the appropriate powers of $\hbar$)

$$\hbar \coth \left[\frac{\hbar\omega_k}{2k_BT}\right] .$$

Thus, modes with wavelength k and energies ω_k are classical when $\hbar\omega_k \ll k_BT$ and yield a contribution to the propagator:

$$\hbar \coth \left[\frac{\hbar\omega_k}{2k_BT}\right] \approx 2k_BT/\omega_k$$

(notice the cancellation of the $\hbar$). For long-wavelength components this happens when

$$\frac{M_\sigma^2}{T^2}[T^2/T_c^2 - 1] \ll 1,$$

because the "thermal mass" (squared) for the excitations in the heat bath is $\frac{M_\sigma^2}{2}[T^2/T_c^2-1]$. For the phenomenological values of M_σ and T_c, the "classical" limit is obtained when

$$\left[\frac{T^2}{T_c^2} - 1\right] \ll 0.1, \qquad (65)$$

that is, when the initial state is in *equilibrium* at a temperature that is *extremely* close to the critical temperature. This is the situation that is shown in Figs. 3(a–c) where, indeed, we obtain very large correlated domains that were already present in the initial state after a quench from the critical temperature all the way to zero temperature.

After an energetic collision it seems rather unlikely that the central region will be so close to the critical temperature. If the temperature is higher than critical, in order for the system to cool down to the critical temperature (or very near to it) and to remain in *local thermodynamic equilibrium*, very long relaxation times are needed, as the long wavelength modes are typically critically slowed down during a transition. Long relaxation times will allow the fluctuations to shut off the instabilities as they begin to grow, and the system will lose its long range correlations. This was the original argument that discarded an equilibrium situation as a candidate for large domains. Furthermore, typical heavy ion collisions or high-energy processes will not allow long relaxation times (typically of a few fm/c). Thus we believe that in most generic situations, a classical approximation for the long wavelength modes is not reliable *in the Hartree* approximation. We should make here a very important point. We are *not* saying that large coherent fluctuations cannot be treated semiclassically. They can. What we are asserting with the above analysis is that within the Hartree approximation, long wavelength excitations cannot be treated as classical. The Hartree approximation in the form used by these (and most other) authors *does*

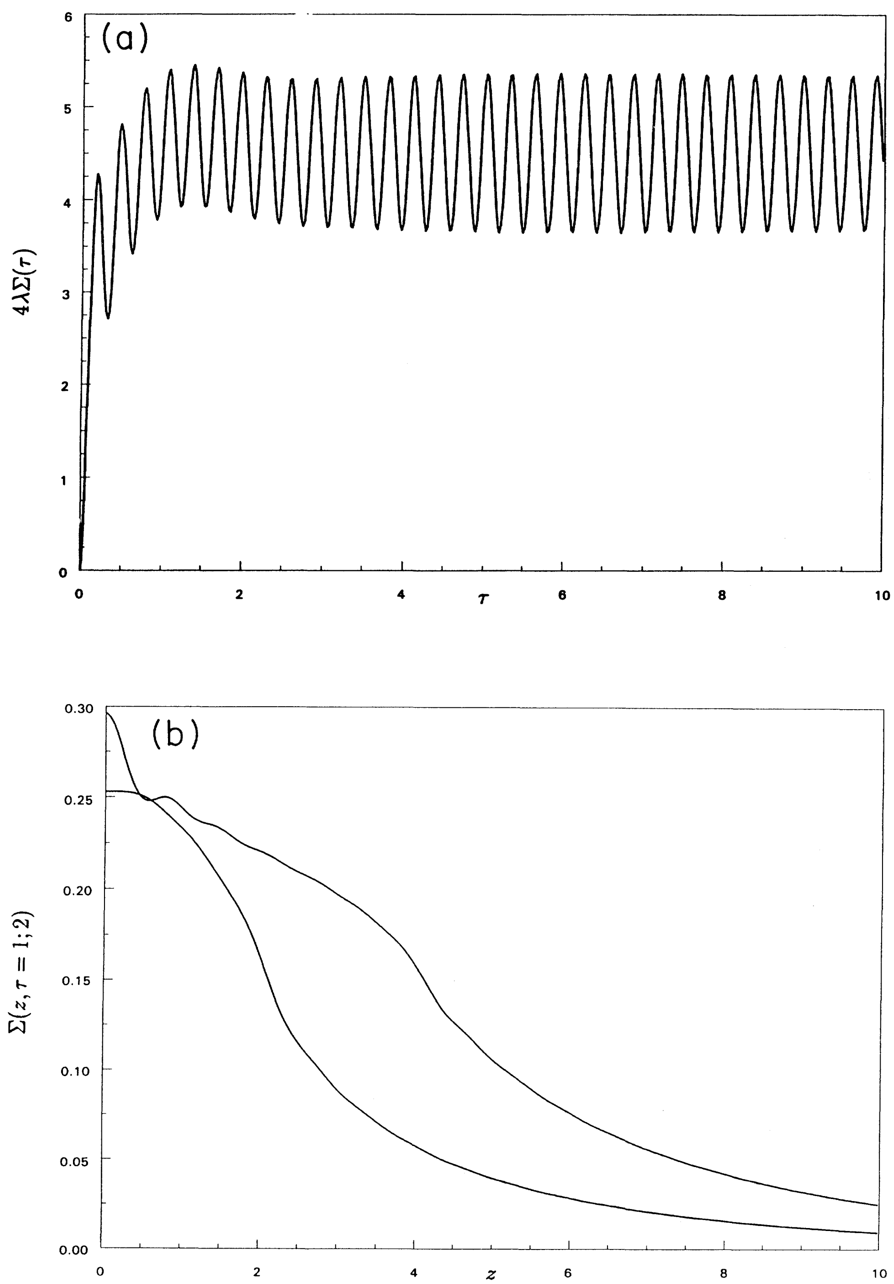

FIG. 3. (a) $4\lambda\Sigma(\tau)$ vs τ. $T = T_c = 200$ MeV; $\tau_r = 0$; $f(0) = 0$; $H = 0$; $\lambda = 4.5$. (b) $\Sigma(z, \tau = 1; 2)$ vs z for the same values of the parameters as in Fig. 3(a); larger values of time correspond to larger amplitudes at the origin. (c) $\overline{N}_\pi(\tau)$ vs τ for the same values of the parameters as in Fig. 3(a). $\overline{N}_\pi(0) = 0.12$.

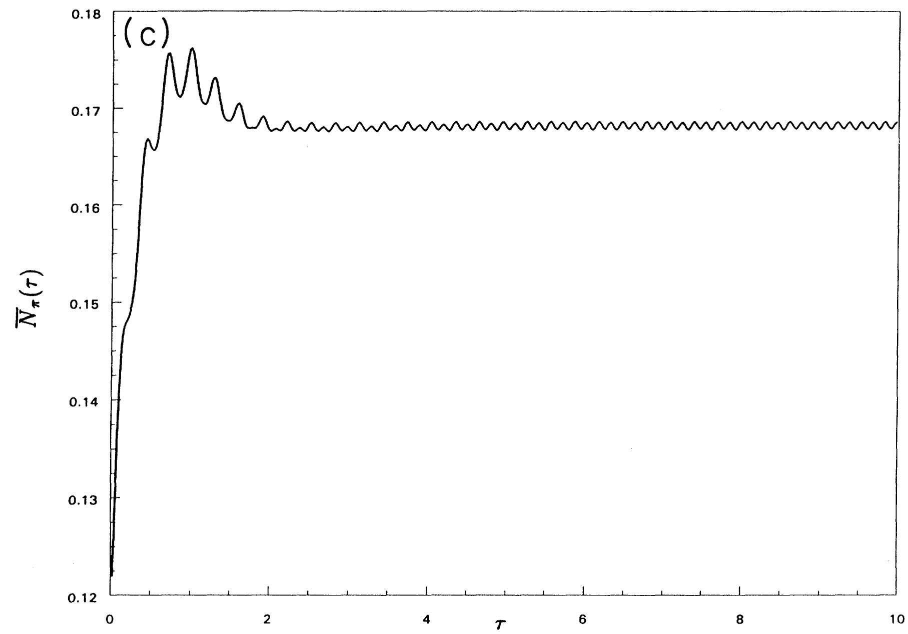

FIG. 3 *(Continued).*

not capture correctly the physics of coherent semiclassical nonperturbative configurations.

Thus although the most promising situation, within the model under investigation, is a quench from the critical temperature (or very close to it) down to zero temperature, it is our impression that this scenario is physically highly unlikely.

There is another very tantalizing possibility for the formation of large correlated pion domains within the *linear* σ model with $h \neq 0$ and that is via the creation of a critical droplet that will complete the phase transition (first order in this case) via the process of thermal activation over a "free energy" barrier. The small magnetic field (resulting from the small up and down quark masses) introduces a small metastability [23]. The classical equations of motion allow for a solution in which the pion field is zero everywhere and a droplet in the σ field [this is the O(3) symmetric bounce responsible for thermal activation in scalar metastable theories in three dimensions [24]]. Using a spherically symmetric *Ansatz* for a sigma droplet of radius R,

$$\sigma_{cl}(r) \approx f_\pi \tanh[M_\sigma(r-R)], \tag{66}$$

and assuming, for the sake of argument, that the thin-wall approximation is reliable, we obtain an approximate form for the energy of the droplet:

$$E \approx 4\pi R^2 f_\pi^2 M_\sigma - \frac{4\pi}{3} R^3 h f_\pi . \tag{67}$$

The critical radius is thus (this approximation is clearly reliable only as an order-of-magnitude estimate)

$$R_c \approx 3-5 \text{ fm} .$$

Typical free energy barriers are thus of the order of 500–600 MeV. By considering the fluctuations of the pions around this configuration, it is conceivable (although we cannot provide a more convincing argument at this stage) that the unstable mode of the droplet (dilation) that makes the droplet grow to complete the phase transition via thermal activation, produces a large amount of correlated pions. This scenario, however, requires supercooling (the false vacuum to be trapped) which again requires long relaxation times (again unlikely for strong coupling).

As argued above, this possibility cannot be studied via a Hartree approximation which only provides a (select) resummation of the perturbative expansion and is probably reliable only for short times, before nonperturbative configurations truly develop.

Thus we conclude that although our analysis provides a negative answer to the question of the possibility of large correlated domains near the chiral phase transition, these results are valid only within the Hartree approximation of the linear σ model. There are several conceivable possibilities that would have to be studied thoroughly before any conclusions are reached: (i) perhaps the linear σ model is not a good candidate for studying the *dynamics* of the chiral phase transition (although it describes the universality class for the static properties); (ii) there are large coherent field configurations (droplets) that are not captured in the Hartree approximation. This possibility is rather likely and is closer to the scenario envisaged by Bjorken, Kowalski, and Taylor [5]. These semiclassical coherent configurations may be responsible for large regions of correlated pions. An important ingredient in this latter case must be a deeper understanding of the dynamical (relaxation) time scales, for which a deeper understanding of the underlying strongly interacting theory is needed. A particularly relevant question is whether such a strongly coupled theory can yield enough supercooling so as to produce such a configuration.

We believe that these two possibilities must be studied further to give an unequivocal answer to the question of large correlated domains. We are currently studying the second possibility in more detail.

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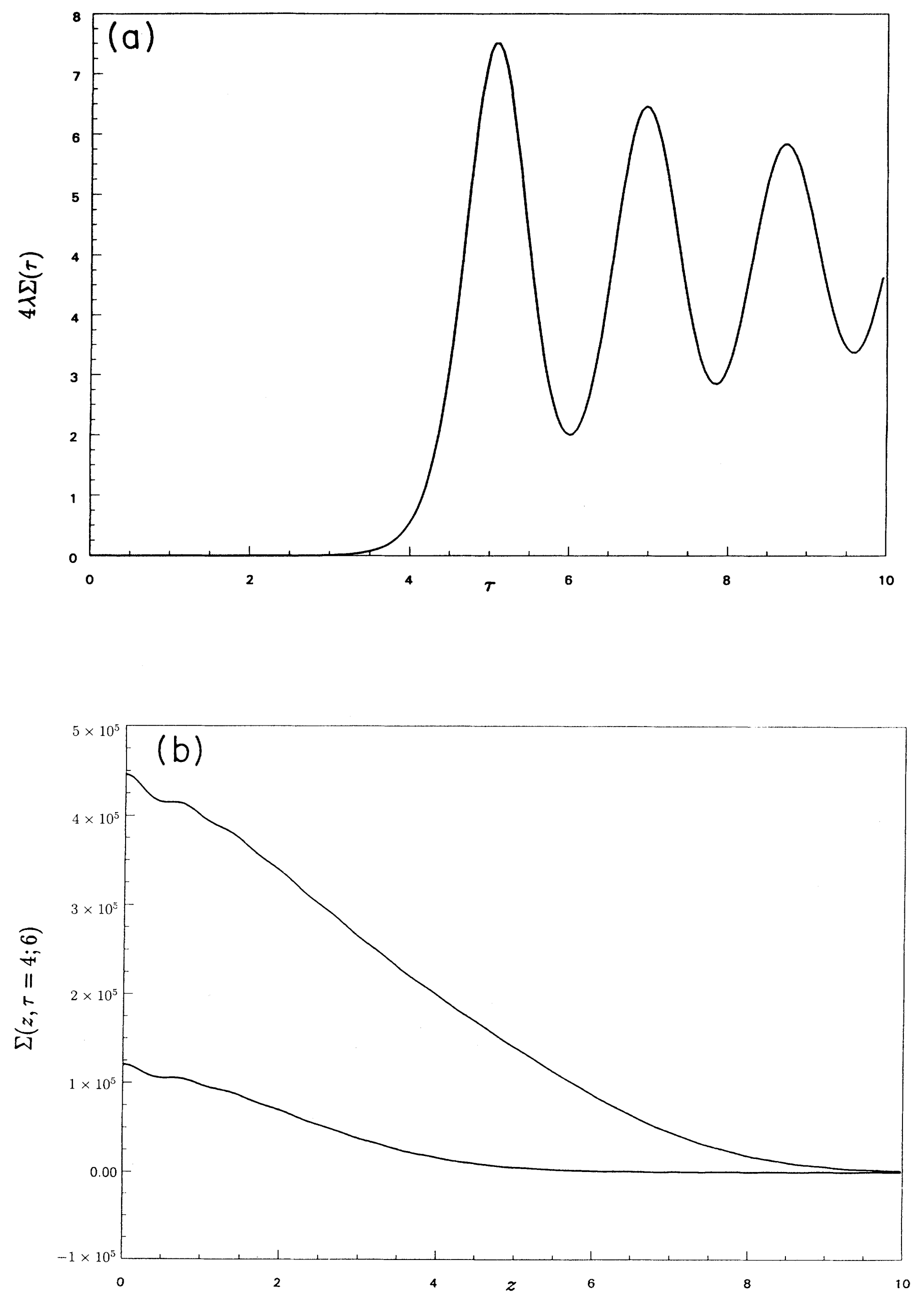

FIG. 4. (a) $4\lambda\Sigma(\tau)$ vs τ. $T_c = 200$ MeV; $T = 250$ MeV; $\tau_r = 0$; $f(0) = 0$; $H = 0$; $\lambda = 10^{-6}$. (b) $\Sigma(z, \tau = 4; 6)$ vs z for the same values of the parameters as in Fig. 4(a); larger values of time correspond to larger amplitudes at the origin. (c) $\overline{N}_\pi(\tau)$ vs τ for the same values of the parameters as in Fig. 4(a). $\overline{N}_\pi(0) = 0.15$.

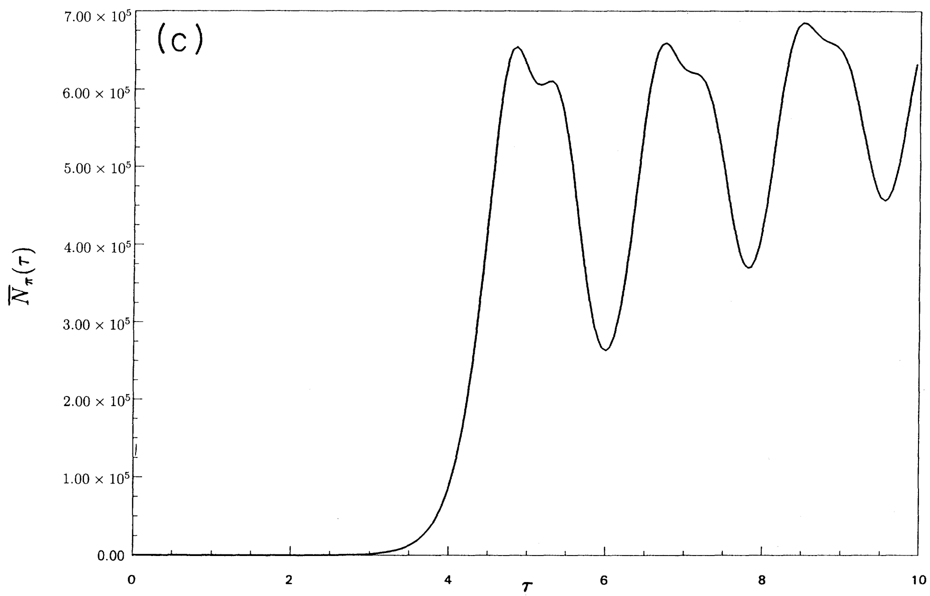

FIG. 4 *(Continued).*

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