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Aspects of four-jet production in polarized proton-proton collisions

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We examine the intrinsic spin dependence of the dominant $gg \to gggg$ subprocess contribution to four-jet production in polarized proton-proton collisions using helicity amplitude techniques. We find that the partonic level, longitudinal spin-spin asymmetry $\hat{a}_{LL}$ is intrinsically large in the kinematic regions probed in experiments detecting four isolated jets. Such events may provide another qualitative or semiquantitative test of the spin structure of QCD in planned polarized pp collisions at BNL RHIC.

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The prospects for a comprehensive program of polarized proton-proton collisions at collider energies at the BNL Relativistic Heavy Ion Collider (RHIC) [1,2], culminating in the approved experiment R5 [3], has motivated a large number of studies of the spin dependence of many standard model processes and their sensitivity to polarized parton distributions. Many processes which already have been well studied, both theoretically and experimentally, at existing unpolarized hadron colliders, have been reexamined in the context of a physics program dedicated to the extraction of the spin-dependent quark, antiquark, and gluon distributions and tests of the spin dependence of the basic hard scattering processes of QCD and the electroweak sector.

Familiar processes such as direct photon production [4–6] and Drell-Yan lepton pair production [6,7] (including W and Z production [6,8]) are known to be sensitive to the longitudinally polarized gluon and sea-quark content of the proton, respectively. (We will henceforth take polarized to mean longitudinal polarization; transverse polarization effects will also be studied at RHIC but will only be briefly discussed here.) Calculations of the radiative corrections to the spin-dependent cross sections for these processes have even appeared [9,10] and confirm the general conclusions of leading-order results. Jet production has also been extensively studied at lowest order [5,11] and the technology for the efficient calculation of the next to leading order (NLO) corrections to helicity amplitudes for jet production is now well known [12] although a detailed analysis of the spin-dependent radiative corrections to jet cross sections has not yet been performed. Taken together, these processes already provide the basis for a substantial experimental program using the two planned RHIC detectors [3], STAR and PHENIX, which are complementary in their physics capabilities.

A wide variety of other processes has also been studied at leading order in the context of the RHIC spin program including heavy quark production [13,14], quarkonium production in different kinematic regions [15–17], three-jet production [18], and double-photon ($\gamma\gamma$) production [19]. All of these processes have been studied experimentally, to varying degrees, in collider energy unpolarized pp or $p\bar{p}$ collisions and seem to agree reasonably well with theoretical expectations. Some other processes whose spin dependence has also been discussed, such as two-jet plus direct photon [18] and $\psi + \gamma$ [20], have yet to be measured but may be detectable for the first time given the large luminosity and energies (up to 2×10^{32} $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ and $\sqrt{s}$=500 GeV) possible at RHIC and the nature of the proposed detectors.

Just as with single inclusive jet, dijet, and three-jet production, four-jet production has been observed at hadron colliders ranging from the CERN Intersecting Storage Rings (ISR) [21,22] ($\sqrt{s}$=63 GeV) to the CERN Super Proton Synchrotron (S$p\bar{p}$S) [23] ($\sqrt{s}$=630 GeV) and most recently at the Fermilab Tevatron [24] ($\sqrt{s}$ =1.8 TeV). In the last two cases, comparisons of data to leading-order QCD predictions for various shape and angular varibles have been made and reasonable agreement is found. In addition to providing another test of standard QCD, this process has the additional feature that it has also been widely discussed as a possible arena in which to study double-parton scattering [25,26], an effect which is expected to appear at sufficiently high energies.

Because the $2\to4$ subprocesses leading to four-jet production are proportional to α_S^4, changes in the value of α_S used and uncertainties in the choice of Q^2 scale (i.e., lack of knowledge of the next order QCD corrections) make the prediction of the absolute rates difficult. This, coupled with the lower rates compared to two- and three-jet production and the increasing difficulty of unambiguously defining n isolated jets, makes this process, at present, a semiquantitative or qualitative test of QCD. Because rapid progress is being made in the calculation of the NLO matrix elements for two- and three-jet production [12] with hope held out for the eventual evaluation

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of the necessary radiative corrections for the 2→4 subprocesses as well [27], this situation may well change in the not-too-distant future.

Motivated by the large rates possible for many hadron processes at the high luminosity polarized *pp* RHIC facility, in this Brief Report we briefly discuss the spin dependence of the dominant QCD subprocess contributing to such four-jet production, namely $gg \to gggg$; the analysis presented here thus extends and complements the discussion of the spin dependence of three-jet production given in Ref. [18]. While the Collider Detector at Fermilab (CDF) [24] Collaboration has presented evidence for a small (∼5%) contribution from double-parton scattering events at $\sqrt{s}$=1800 GeV, the UA2 [23] data at a lower energy (below the maximum RHIC energy) are consistent with no such contribution; it is therefore unlikely that double-parton scattering will play any role in multijet production at RHIC, and we will discuss only the spin dependence of 2→4 processes.

We follow the analysis of four-jet production at Tevatron energies as given in Ref. [26] with some minor differences. We consider only the dominant $gg \to gggg$ subprocess and because we are interested in the spin dependence we use the exact expressions for the matrix elements for this process instead of relying on any approximation technique; specifically, we use the compact expressions for the necessary helicity amplitudes in Refs. [28,29]. In the calculation of the appropriate cross sections, we modify the cuts of Ref. [26] Set 2 (motivated by the smaller RHIC energy of $\sqrt{s}$=500 GeV) and insist that $p_T > 15$ GeV and $|\eta| \leq 0.8$ for each jet and that $\cos(\theta_{ij}) \leq 0.643$ for each jet pair. Finally, we insist that there is a minimum total transverse energy of $E_T \geq 70$ GeV. For this leading-order calculation, we use the updated parton distributions of Duke-Owens (in this case, only the gluon distribution of Set I is required) of Ref. [30]. For definiteness, we use $Q^2 = \hat{s}$ in the distributions and running coupling, and we discuss below the sensitivity of the results to this choice.

The resulting differential cross section as a function of transverse momentum, $d\sigma/dp_T$ versus p_T (with four entries per event), is shown as the dot-dashed curve in Fig. 1 and has a very similar shape as the corresponding plot [Fig. 2(a)] in Ref. [26]. The spin-dependent cross sections

$$\frac{d\Delta\sigma}{dp_T} \equiv \frac{1}{2}\left(\frac{d\sigma(++)}{dp_T} - \frac{d\sigma(+-)}{dp_T}\right), \tag{1}$$

where (++), (+−) refers to the helicities of the incident protons, are also shown in Fig. 1 for two choices of the polarized gluon distribution. The spin-dependent gluon distribution, conventionally written as $\Delta G(x,Q^2) \equiv G_+(x,Q^2) - G_-(x,Q^2)$ [where + (−) refers to parton helicity in the same (opposite) direction as the parent proton helicity], is assumed, for simplicity, to have the form $\Delta G(x,Q^2) = x^\alpha G(x,Q^2)$. The two choices α=1 (solid curve) and α=0.25 (dashed curve) correspond to an integrated gluon contribution $\Delta G = \int \Delta G(x,Q^2)dx$ equal to 0.5 and 4.5, respectively, and thus bracket the expectations for a "normal" to "large" gluonic contribution to the total proton spin.

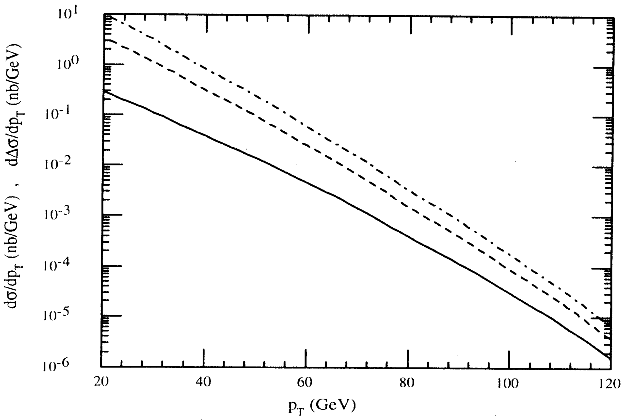

FIG. 1. The solid and dashed curves show the spin-dependent, differential cross section, $d\Delta\sigma/dp_T$ (nb/GeV) versus p_T (GeV), for four-jet production from the $gg \to gggg$ subprocess with the cuts discussed in the text; the solid (dashed) curves correspond to a polarized gluon distribution given by $\Delta G(x,Q^2) = x^\alpha G(x,Q^2)$ with $\alpha = 1(0.25)$. The dot-dashed curve is the unpolarized differential cross section $d\sigma/dp_T$ (nb/GeV) versus p_T(GeV) with the same assumptions.

Using the $gg \to gggg$ matrix elements and these cuts, we have calculated differential distributions in the other kinematic variables discussed in Ref. [26], namely $p_{\rm out}$, $\phi_{\rm min}$, and $\cos(\theta^*_{23})$. The transverse momentum out of the plane passing through the beam and the jet of largest p_T, i.e., $p_{\rm out}$, is defined via

$$p_{\rm out} \equiv \frac{1}{2}\sum_i |p^i_{\rm out}| \tag{2}$$

while $\phi_{\rm min}$ is the minimum angle in the transverse plane between the largest p_T jet and the other three jets and finally $\cos(\theta^*_{23})$ is the cosine of the angle between the second and third most energetic jets in the four-jet center of mass. Differential distributions in each of these quantities look very similar to the corresponding plots in Figs. 2(a)–2(d) in Ref. [26].

The measurable spin-spin asymmetry in any observable quantity, defined by

$$A_{LL} = \frac{d\sigma(++) - d\sigma(+-)}{d\sigma(++) + d\sigma(+-)}, \tag{3}$$

is determined by both the partonic level spin dependence of the underlying hard scattering,

$$\hat{a}_{LL} = \frac{d\hat{\sigma}(++) - d\hat{\sigma}(+-)}{d\sigma(++) + d\sigma(+-)}, \tag{4}$$

and the magnitude of the polarized parton distributions since

$$A_{LL}d\sigma = \sum_{i,j}\int dx_a \int dx_b d\hat{\sigma}\hat{a}_{LL}\Delta f_i(x_a,Q^2) \times \Delta f_j(x_b,Q^2). \tag{5}$$

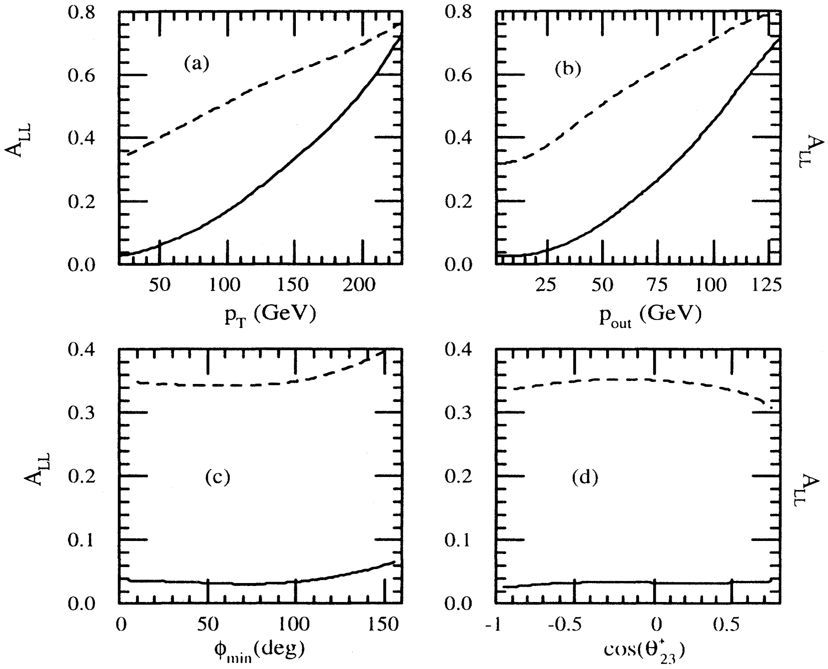

FIG. 2. The spin-spin asymmetry, A_{LL}, in various differential cross sections for four-jet production. The plots correspond to asymmetries in differential distributions for (a) p_T, (b) $p_{\rm out}$, (c) $\phi_{\rm min}$, and (d) $\cos(\theta^*_{23})$. The solid (dashed) curves in each plot correspond to polarized gluon distributions given by $\Delta G(x,Q^2) = x^\alpha G(x,Q^2)$ with α=1(0.25).

Using the polarized parton distributions mentioned above and the exact $2g{\to}4g$ matrix elements, we find the observable asymmetries for the four variables p_T, $p_{\rm out}$, $\phi_{\rm min}$, and $\cos(\theta^*_{23})$ shown in Figs. 2(a)–2(d). The increasing value of A_{LL} with larger values of p_T [Fig. 2(a)] and $p_{\rm out}$ [Fig. 2(b)] is reminiscent of similar effects in two-jet [5,11] and three-jet [18] production and simply reflects the increase in gluon polarization for large x (where, of course, the cross sections are very small). Figures 2(a) and 2(b) also show that the intrinsic spin-spin asymmetry of the hard-scattering subprocess, $\hat{a}_{LL}$, is large; we estimate that the average value of the partonic level spin-spin asymmetry in the configurations measured in four-jet production is roughly $\langle\hat{a}_{LL}\rangle \sim 0.8$ which is even larger than the corresponding value of $\langle\hat{a}_{LL}\rangle \sim 0.7$ found for the $gg{\to}ggg$ contribution to three-jet production. We have also checked explicitly that when one gluon is allowed to be soft the resulting partonic level spin-spin asymmetries reduce to those for three-jet production derived in Ref. [18]. On the other hand, the angular varibles we have studied show little variation for a given set of polarized distribution functions as seen in Figs. 2(c) and 2(d); this is also consistent with earlier results on spin dependence in three-jet events. These results reaffirm the fact observed in the case of two- and three-jet production, namely, that once one is in the kinematic regime required for the isolation of jets, the intrinsic longitudinal spin-spin asymmetries for gluon initiated processes is almost maximally large and varies little with the relative orientation of the jets.

Even though the predictions for the absolute rates of four-jet production are extremely sensitive to the choice of Q^2 used in the calculation, the fact that many distributions of angular and shape variables agree well with leading order (LO) predictions implies that ratios of cross sections may well be less sensitive to changes in the perturbative parameters. In the context of spin measurements in three-jet production, for example, it has been noted [18] that varying the value of Q^2 in the range $Q^2 = 4\hat{s}$ to $Q^2 = \hat{s}/9$ gives rise to a variation in the differential cross section of order 300%, while predicted spin-spin asymmetries are hardly changed at all.

To examine the sensitivity of our analysis to such variations, we have performed a similar test by varying Q^2 from the nominal value of $Q^2 = \hat{s}$ used above. Using $Q^2 = 4\hat{s}(\hat{s}/4)$, we find that the differential cross sections are decreased (increased) by a factor of 2 consistently over the range of p_T which is quite similar to the results of Ref. [26]; with the same variation in Q^2, however, the spin-spin asymmetries in all variables we consider are changed by less than 5%. We therefore expect that spin-spin asymmetries will be more sensitive to the nature of the polarized gluon distributions (the exponent α, for example, in our simple parametrization) than to uncertainties arising from a lack of knowledge of the NLO corrections; this is certainly consistent with the cases where the NLO calculations have been performed [9,10].

One can also consider transverse spin effects in jet physics but the situation is far less diverse. The quarks and antiquarks (and not the gluons) carry the "transversity" [31] so that only qq or $q\bar{q}$ initiated processes in transversely polarized pp collisions need be considered. It has been known for some time [32] that the partonic level transverse spin-spin asymmetries, $\hat{a}_{TT}$, for the relevant $2{\to}2$ processes vanish in the case of unlike-quark scattering via $qq'{\to}qq'$ and are much smaller (roughly a factor of 10) than the corresponding longitudinal asymmetry ($\hat{a}_{LL}$) for like-quark scattering via $qq{\to}qq$ due to a color factor. An identical pattern is seen in the transverse spin-spin asymmetries for three-jet production via $qq'{\to}qq'g$ and $qq{\to}qqg$ [33]. We expect a similar situation in the four-jet case for the processes $qq'{\to}qq'gg$, $qq{\to}qqgg$, and

$qq' \rightarrow qq'qq$. Since these processes only form a significant part of the four-jet cross section at the very largest values of p_T where the cross sections are unmeasurably small, we expect no significant transverse spin dependence in such processes.

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