Raising the unification scale in supersymmetry

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In the minimal supersymmetric standard model, the three gauge couplings appear to unify at a mass scale near 2×10^{16} GeV. We investigate the possibility that intermediate scale particle thresholds modify the running couplings so as to increase the unification scale. By requiring consistency of this scenario, we derive some constraints on the particle content and locations of the intermediate thresholds. There are remarkably few acceptable solutions with a single cleanly defined intermediate scale far below the unification scale.

I. INTRODUCTION

Data from the CERN e^+e^- collider suggest that with $N = 1$ supersymmetry [1] at low energy (~ 1 TeV), the three gauge couplings of the standard model converge to unify [2] at one scale $M_X \approx 2 \times 10^{16}$ GeV. This apparent unification is predicted on two assumptions. One is that the weak hypercharge coupling is normalized to its unification into a higher rank Lie group, such as SU(5), SO(10), or E_6. The second is the absence of intermediate thresholds between 1 TeV and M_X. This apparent unification of couplings may be regarded as a "prediction" of the low-energy value of $\sin^2\theta_W$ given the measured value of the strong-coupling constant, and is a tantalizing hint of a unifying structure, such as superstring theory supersymmetric grand-unified theory.

While it is clear that the three gauge couplings have a much better chance to unify with low-energy supersymmetry than without, it may be premature to unequivocally announce their unification, and this simple picture may have to be modified. The main reasons are the large experimental uncertainties in the value of the QCD coupling constant and ignorance of the detailed structure of the supersymmetric thresholds.

Thus it may be that the gauge couplings do not exactly unify at M_X. In that case, we may want to alter this simple picture by adding at least one intermediate threshold between the supersymmetry (SUSY) scale and the "unification" scale at M_X. The question of interest is whether the couplings can then be made to unify at a larger scale after introduction of the new intermediate threshold(s), caused by particles with vectorlike electroweak quantum numbers. These modify the running of the gauge couplings above the intermediate thresholds to achieve true unification at the scale M_U, which we take to be larger than M_X. Be requiring consistency of this scenario, we can derive constraints on the particles at the intermediate thresholds and relations between the intermediate scales M_X and M_U.

There are several reasons to pursue this line of inquiry.

One is that intermediate mass scales appear in many extensions of the minimal supersymmetric standard model (MSSM), such as those which incorporate a light invisible axion [3] or massive neutrinos through the seesaw mechanism [4]. Another is to explain the near-zero values of many of the Yukawa matrix elements through mixing the known particles with vectorlike particles. These particles may appear at intermediate thresholds.

Our primary motivation, however, is superstring theory which indicates that the unification energy should be more than 1 order of magnitude above M_X. The effective low-energy theories generated by superstrings contain, in addition to the three chiral families, many vectorlike particles, incomplete remnants of **27** and **$\overline{27}$** representations of E_6. These vectorlike particles have electroweak singlet masses, assumed to be, in the absence of any special mechanism, of the order of the highest scale around, in this case the Planck mass. However, these theories have a larger invariance group than that of the MSSM, and must develop intermediate thresholds *below* the string scale to break the invariance group to that of the MSSM. This is typically achieved by flat directions in the potential.

If the true scale of gauge coupling unification is higher than the apparent unification scale because of intermediate scale thresholds as assumed here, one may view the "success" of gauge coupling unification as just an accident. We are implicitly taking the point of view that it is not *completely* accidental, and that it is still possible to understand gauge coupling unification through calculable perturbative means. We therefore assume that the three gauge couplings remain perturbative up to the unification scale M_U, and that the reason behind the raising of the unification scale is not some artifact of, e.g., stringy threshold effects, but is really due to the presence of intermediate scale thresholds. We also assume that the normalization of weak hypercharge is indeed the standard one appropriate for unification with SU(2)$_L$ and SU(3)$_c$ into a simple gauge group. (Ref. [5] explores the possibility of different normalizations of the hypercharge as a means of raising the unification scale.)

This paper is organized as follows. In Sec. II, we de-

velop the formalism for unification of couplings with one intermediate scale threshold, and then for several intermediate thresholds. In Sec. III, we discuss the effects of various possibilities for the new particles at the intermediate scale(s), including both new chiral superfields and new gauge vector superfields. In Sec. IV, we discuss the results for one intermediate scale with raised unification. Here we find tight constraints on the particle content and location of the intermediate scale. Section V deals with results for more than one intermediate scale, using as an example a particular three-family superstring model.

II. ONE-LOOP EQUATIONS WITH NEW THRESHOLDS

Let us begin by recalling some salient facts about the running of the gauge couplings. Since we will be comparing the running of gauge couplings with an intermediate scale to the "template" case of the MSSM, it will be sufficient to use one-loop renormalization-group equations only. The three gauge couplings run with scale according to

$$\alpha_i^{-1}(t) = \alpha_i^{-1}(t_X) + \frac{b_i}{2\pi}(t - t_X) , \qquad (2.1)$$

where

$$\alpha_i(t) = \frac{g_i^2(t)}{4\pi} ,$$

are the couplings for the three gauge groups, $i = 1, 2, 3$ for $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$, respectively. The scale is given by

$$t = \ln(\mu/\mu_0) ,$$

where μ_0 is an arbitrary reference energy, and

$$t_X = \ln(M_X/\mu_0) ,$$

is the unification scale. For $N = 1$ supersymmetry we have

$$b_i = 3c_{\text{adjoint}} - \sum_r c_r , \qquad (2.2)$$

where the c_r's are the Dynkin indices of the representations, and the sum is over the left-handed multiplets. The hypercharge is normalized so that

$$b_1 = -\frac{3}{20} \sum_r Y_r^2 ,$$

corresponding to the electric charge

$$Q = I_3 + \frac{Y}{2} .$$

For the three families and two Higgs doublets of chiral superfields in the minimal supersymmetric standard model, we have

$$b_1 = -\frac{33}{5}, \quad b_2 = -1, \quad b_3 = 3 .$$

We start with the trajectories for α_1 and α_2 since their

values at low energies are known with the greatest accuracy. We define t_X as the scale at which these two appear to meet in the MSSM:

$$\alpha_X^{-1} \equiv \alpha_1^{-1}(t_X) = \alpha_2^{-1}(t_X) .$$

The extrapolated data, with $N = 1$ supersymmetry around 1 TeV, show that $\alpha_X^{-1} \approx 24.5$, with $M_X \approx 2 \times 10^{16}$ GeV. However, we do not assume precisely the same value for $\alpha_3(t_X)$ at that scale, since we are assuming that the "unification" at M_X is only apparent; rather we set

$$\alpha_X^{-1} = \alpha_3^{-1}(t_X) + \Delta ,$$

introducing the parameter Δ which parametrizes our ignorance about $\alpha_3(M_Z)$, our ignorance about the precise location of the SUSY thresholds, and our neglect of two-loop effects. The present uncertainties indicate that

$$|\Delta| \leq 1.5 , \qquad (2.3)$$

using the most conservative estimate. We contrast this situation by noting that without low-energy supersymmetry, the same parameters have the values $\alpha_X^{-1} \approx 42$, $M_X \approx 10^{13}$ GeV, and $\Delta \approx 5$.

A. Case of one intermediate threshold

Assume first only one intermediate threshold above the supersymmetric thresholds, at the scale

$$t_I = \ln(M_I/\mu_0), \quad t_I < t_X .$$

The previous equations are still valid as long as we are below the intermediate threshold, that is

$$\alpha_i^{-1}(t) = \alpha_X^{-1} + \frac{b_i}{2\pi}(t - t_X) \quad (i = 1, 2) ,$$
$$\alpha_3^{-1} = \alpha_X^{-1} - \Delta + \frac{b_3}{2\pi}(t - t_X) , \qquad (2.4)$$

for $t \leq t_I$. At the intermediate threshold $t = t_I$, new vectorlike particles with electroweak singlet masses at M_I alter the b_i coefficients to new values

$$b_i \to b_i - \delta_i, \quad i = 1, 2, 3 ,$$

with all δ_i positive as long as the matter is made up of chiral superfields. We assume that their effect is to push the true unification scale to the new value t_U with $t_U > t_X$. Thus, above the intermediate threshold, all three gauge couplings must satisfy

$$\alpha_i^{-1}(t) = \alpha_U^{-1} + \frac{1}{2\pi}(b_i - \delta_i)(t - t_U), \quad t_I \leq t \leq t_U ,$$

where there is only one coupling at unification, α_U.

We have thus two ways of writing the equations for the gauge couplings below the intermediate threshold; one is given by (2.4), the other by

$$\alpha_i^{-1}(t) = \alpha_U^{-1} + \frac{b_i}{2\pi}(t - t_I)$$

$$+ \frac{1}{2\pi}(b_i - \delta_i)(t_I - t_U), \quad i = 1, 2, 3 . \quad (2.5)$$

Comparison of the two yields the three consistency equations

$$b_i - \delta_i \left(\frac{t_U - t_I}{t_U - t_X} \right) = \frac{2\pi}{t_U - t_X}(\alpha_U^{-1} - \alpha_X^{-1}), \quad i = 1, 2 ,$$

$$(2.6)$$

$$b_3 - \delta_3 \left(\frac{t_U - t_I}{t_U - t_X} \right) = \frac{2\pi}{t_U - t_X}(\alpha_U^{-1} - \alpha_X^{-1} + \Delta) .$$

By subtracting the first two, we obtain the constraint

$$\frac{28}{5} - (\delta_2 - \delta_1) \left(\frac{t_U - t_I}{t_u - t_X} \right) = 0 , \quad (2.7)$$

which indicates that $\delta_2 - \delta_1$ must be positive. The difference between the second and the third equations in (2.6) yields

$$4 - \frac{2\pi\Delta}{t_U - t_X} - (\delta_3 - \delta_2) \left(\frac{t_U - t_I}{t_U - t_X} \right) = 0 . \quad (2.8)$$

The remaining equation yields the value of the gauge coupling at unification

$$\alpha_U^{-1} = \alpha_X^{-1} - \frac{1}{2\pi}[\delta_2(t_U - t_I) + t_U - t_X] . \quad (2.9)$$

With only nonexotic matter at the intermediate threshold, the combinations

$$q \equiv \delta_3 - \delta_2 \quad \text{and} \quad \tfrac{2}{5}r \equiv \delta_2 - \delta_1 ,$$

are integers. Then (2.7) and (2.8) can be rewritten as

$$\frac{r}{14} = \frac{t_U - t_X}{t_U - t_I} \quad (2.10)$$

and

$$\frac{1}{4} = \frac{t_U - t_X - \pi\Delta/2}{t_U - t_I} . \quad (2.11)$$

It may be profitable to consider an elementary geometric derivation of (2.10) and (2.11). Consider the evolution of the inverse gauge couplings, which meet at a scale t_X, and assume that they both change directions at a lower scale t_I, to meet at the larger scale t_U, as shown in Fig. 1.

The ratios of the slopes of the lines above t_I satisfy, for α_1^{-1} and α_2^{-1},

$$\frac{b_2 - b_1}{b_2' - b_1'} = \frac{OB}{OB'} = \frac{t_X - t_I}{t_U - t_I} , \quad (2.12)$$

from which (2.10) follows. We can apply the same technique to the evolution of α_2 and α_3 (including the near miss at M_X parametrized by Δ) to obtain (2.11).

We may think of the intermediate threshold as a "lens" which refocuses the lines $\alpha_1^{-1}(t)$, $\alpha_2^{-1}(t)$, and $\alpha_3^{-1}(t)$ so
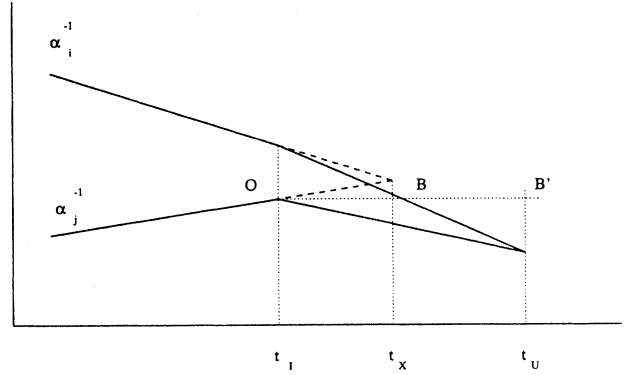

FIG. 1. The evolution of two inverse gauge couplings that actually meet at a scale t_U.

that they meet at t_U rather than t_X. If $q > 4$, the intermediate threshold acts as a divergent lens, and the two lines for α_2^{-1} and α_3^{-1} never intersect. If $q = 4$, the same two lines are parallel and again never meet. Thus we must have $q < 4$ for the two curves to intersect beyond t_I. In addition, q cannot be negative or Δ would be too large. This is easy to understand, since $q < 0$ corresponds to a strongly focusing lens which would make α_2 and α_3 meet at a *lower* scale than they would in the MSSM. To avoid having α_2 and α_3 meet prematurely, Δ would have to be large and positive when $q < 0$. To see this, note that we can write

$$\Delta = \frac{1}{2\pi}[4(t_U - t_X) - q(t_U - t_I)] . \quad (2.13)$$

So, for instance, if $q = -1$, we find that even in the case of small hierarchies $M_X/M_I = 10$ and $M_U/M_X = 10$, one has $\Delta = 2.2$, which correspond to a larger error than the experimental uncertainties on α_3 warrant. For more substantial hierarchies, or for more negative values of q, the situation becomes rapidly even worse. Thus it is sufficient to consider only the four cases $q = 0, 1, 2, 3$. Similarly, from (2.10) we find that if $r \geq 14$, the α_1^{-1} and α_2^{-1} lines will never meet, while if $r < 0$, they will meet prematurely, implying a lowered scale of unification . If $r = 0$, the unification scale is not raised and $M_U = M_X$. Thus we have $0 < r < 14$.

The scale of true unification can be extracted from (2.11) and (2.10) in terms of M_X, M_I, and the parameters q, Δ, and r, respectively:

$$M_U = M_X \left(\frac{M_X}{M_I} \right)^{q/(4-q)} e^{2\pi\Delta/(4-q)} , \quad (2.14)$$

$$M_U = M_X \left(\frac{M_X}{M_I} \right)^{r/(14-r)} . \quad (2.15)$$

Taken together, these imply

$$q = \frac{2}{7}r - \frac{2\pi\Delta}{t_U - t_I} , \quad (2.16)$$

or equivalently

$$\Delta = \frac{1}{\pi}(t_U - t_I)\left(\frac{r}{7} - \frac{q}{2}\right) . \tag{2.17}$$

If $\Delta > 0$, then r must be a positive integer in the range $\frac{7}{2}q < r < 14$. On the other hand, if Δ is negative, we have $0 < r < 7q/2$. The special case $\Delta = 0$ yields a nontrivial result only when $2r = 7q$. In that case, for nonexotic matter, the only solution is $q = 2$, $r = 7$, and from (2.14) or (2.15), M_X is the geometric mean between M_I and M_U. This corresponds to the seemingly perverse case of the gauge couplings unifying both with and without the intermediate threshold. For nonzero Δ, the hierarchies of scales are summarized by the two equations

$$\frac{M_X}{M_I} = \exp\left[\frac{\pi\Delta(14 - r)}{2r - 7q}\right] , \tag{2.18}$$

$$\frac{M_U}{M_X} = \exp\left[\frac{\pi r\Delta}{2r - 7q}\right] . \tag{2.19}$$

Another constraint which should be taken into account is that our equations are meaningless if the gauge couplings become too large. It is difficult to say exactly how large is too large, but if we arbitrarily require that $\alpha_U^{-1} \geq 2$, then given the numerical value $\alpha_X^{-1} \approx 25$, from (2.9) we obtain (safely neglecting $t_U - t_X$)

$$\delta_2(t_U - t_I) < 145 . \tag{2.20}$$

B. Multiple intermediate thresholds

So far we have assumed only one intermediate threshold between 1 TeV and M_X, but as previously discussed, this may not be a realistic assumption. More generally, suppose there are N distinct intermediate mass scales M_{Ia} $(a = 1, \ldots, N)$ between 1 TeV and the unification scale. At each of these N thresholds, δ_{1a}, δ_{2a}, and δ_{3a} are the decreases in slope of the running inverse gauge couplings. One may then use the master formula (2.12) iteratively to build the corresponding equations. The results are

$$t_U - t_X = \frac{1}{4}\sum_{a=1}^{N} q_a(t_U - t_{Ia}) + \frac{\pi\Delta}{2}$$

and

$$t_U - t_X = \frac{1}{14}\sum_{a=1}^{N} r_a(t_U - t_{Ia}) ,$$

where $q_a = \delta_{3a} - \delta_{2a}$ and $r_a = 5(\delta_{2a} - \delta_{1a})/2$ for each of the N thresholds. Now requiring $t_U - t_X > 0$ constrains the particle content. One may view this case as one of multiple lenses, some divergent, some convergent.

These multiple thresholds act like one effective lens, which leads us to recast these equations by choosing a single effective intermediate scale $\bar{t}_I$ which should reflect the "average" of the individual thresholds in some sense. The choice of $\bar{t}_I$ is to some extent arbitrary, as long as $\bar{t}_I < t_X < t_U$, and indeed the appropriate choice for a

definition of $\bar{t}_I$ depends on the particular example being studied. Then one defines

$$\bar{\delta}_i = \sum_{a=1}^{N} \delta_{ia}\left(\frac{t_U - t_{Ia}}{t_U - \bar{t}_I}\right) , \tag{2.21}$$

in which each δ_{ia} is weighted more (less) when the corresponding intermediate scale t_{Ia} is lower (higher) than $\bar{t}_I$. In terms of

$$\bar{q} \equiv \sum_{a=1}^{N} q_a\left(\frac{t_U - t_{Ia}}{t_U - \bar{t}_I}\right) ,$$

$$\bar{r} \equiv \sum_{a=1}^{N} r_a\left(\frac{t_U - t_{Ia}}{t_U - \bar{t}_I}\right) , \tag{2.22}$$

we obtain

$$\frac{\bar{q}}{4} = \frac{t_U - t_X - \pi\Delta/2}{t_U - \bar{t}_I} \tag{2.23}$$

and

$$\frac{\bar{r}}{14} = \frac{t_U - t_X}{t_U - \bar{t}_I} . \tag{2.24}$$

The gauge coupling at unification is

$$\alpha_U^{-1} = \alpha_X^{-1} - \frac{1}{2\pi}[\bar{\delta}_2(t_U - \bar{t}_I) + t_U - t_X] . \tag{2.25}$$

Note that the above equations have the same form as in the case of a single intermediate threshold, but with "averaged" quantities $\bar{t}_I$, $\bar{q}$, $\bar{r}$, etc. In fact, one still has the constraints

$$0 \leq \bar{q} < 4 , \tag{2.26}$$

$$0 < \bar{r} < 14 , \tag{2.27}$$

from requiring that the coupling constants unify, but not too early. The main difference is that $\bar{q}$, $\bar{r}$, and $\bar{\delta}_2$ need not be integers. Each of the equations (2.8)–(2.20) derived in the case of a single intermediate threshold now hold with t_I, δ_i, q, r replaced by $\bar{t}_I$, $\bar{\delta}_i$, $\bar{q}$, $\bar{r}$.

III. PARTICLES AT THE INTERMEDIATE THRESHOLD(S)

In order to analyze each case in detail, it is convenient to list the possible representations of the new particles that generate the intermediate thresholds, and compute their δ_i coefficients. For the purposes of this paper, we focus on low-energy theories that could have originated from superstring theories. Thus we restrict ourselves to representations contained in **27**, **$\overline{27}$**, and **78** representations of E$_6$, under the decomposition

$$E_6 \subset SU(2)_L \times SU(3)_c \times U(1)_Y .$$

In some string compactifications, specifically with higher level Kac-Moody, chiral multiplets transforming as the

adjoint can survive [6], with their remnants appearing in the low energy theory. The results are summarized in Table I.

The last three representations in Table I appear only in the adjoint of E_6. We note that for all these representations, $t(\delta_2 - \delta_1)$ is even and r is an integer. More generally, it can be shown that all representations for which $t(\delta_2 - \delta_1)$ is odd necessarily describe leptons with half-integer electric charges, or quarks which yield bound states with noninteger charges. It follows from Table I that

$$q = -n_1 + n_3 + n_4 - n_5 - n_6 - 2n_7 + 3n_8 \ ,$$

$$r = n_1 - 3n_2 - 4n_3 - n_4 + 7n_5 - 5n_6 + 5n_7 \ ,$$

where there are n_i vectorlike representations at the intermediate threshold. In the superstring compactification scenario, the vectorlike representations come from the fundamental of E_6. If there are no chiral superfield remnants of the adjoint $n_6 = n_7 = n_8 = 0$. We see from the above that the quantity $q + r$ must be a multiple of 3:

$$q + r = 6n_5 - 3n_2 - 3n_3 - 6n_6 + 3n_7 + 3n_8 \ .$$

We should also take into account the possibility that the gauge group is enlarged above the intermediate scale(s). In such a case, it is possible to identify the coupling constants for the enlarged gauge group and run the new gauge couplings up to the high scale. However, it is not really necessary to do so. Instead, one can simply follow the running of the three low-energy gauge couplings even though they are embedded within the gauge couplings are properly normalized for unification into a simple gauge group, one can take into account the effects of gauge bosons and gauginos living at the intermediate scales by simple step functions in the β functions. Therefore we generalize our analysis to include possible vectorlike remnants of a single vector supermultiplet adjoint of E_6. Table II is exactly the same as the first 6 rows of Table I, except that the entries now appear multiplied by the factor -3, in accordance with the formula (2.2) for the b_i, since they belong to the vector supermultiplet.

TABLE I. Contributions from chiral multiplets.

| Representation | Chiral supermultiplets | | | | | |
	δ_1	δ_2	δ_3	q	r	No.
$(\mathbf{2},\mathbf{1}^c)_{-1} + c$	$\frac{3}{5}$	1	0	-1	1	n_1
$(\mathbf{1},\mathbf{1}^c)_2 + c$	$\frac{6}{5}$	0	0	0	-3	n_2
$(\mathbf{1},\bar{\mathbf{3}}^c)_{-4/3} + c$	$\frac{8}{5}$	0	1	1	-4	n_3
$(\mathbf{1},\bar{\mathbf{3}}^c)_{2/3} + c$	$\frac{2}{5}$	0	1	1	-1	n_4
$(\mathbf{2},\mathbf{3}^c)_{1/3} + c$	$\frac{1}{5}$	3	2	-1	7	n_5
$(\mathbf{2},\mathbf{3}^c)_{-5/3} + c$	5	3	2	-1	-5	n_6
$(\mathbf{3},\mathbf{1}^c)_0$	0	2	0	-2	5	n_7
$(\mathbf{1},\mathbf{8}^c)_0$	0	0	3	3	0	n_8

TABLE II. Contributions from vector supermultiplets.

| Representation | Vector supermultiplets | | | | | |
	δ_1	δ_2	δ_3	q	r	No.
$(\mathbf{2},\mathbf{1}^c)_{-1} + c$	$-\frac{9}{5}$	-3	0	3	-3	$N_1(\leq 1)$
$(\mathbf{1},\mathbf{1}^c)_2 + c$	$-\frac{18}{5}$	0	0	0	9	$N_2(\leq 2)$
$(\mathbf{1},\bar{\mathbf{3}}^c)_{-4/3} + c$	$-\frac{24}{5}$	0	-3	-3	12	$N_3(\leq 2)$
$(\mathbf{1},\bar{\mathbf{3}}^c)_{2/3} + c$	$-\frac{6}{5}$	0	-3	-3	3	$N_4(\leq 1)$
$(\mathbf{2},\mathbf{3}^c)_{1/3} + c$	$-\frac{3}{5}$	-9	-6	3	-21	$N_5(\leq 2)$
$(\mathbf{2},\mathbf{3}^c)_{-5/3} + c$	-15	-9	-6	3	15	$N_6(\leq 1)$

The numbers in parentheses reflect the multiplicity of the representation in a single adjoint of E_6. The adjoint also contains five singlets with no hypercharge, as well as the triplet which contains the SU(2) gauge and gaugino fields and the color octet of gluons and gluinos, which are already contained in the MSSM.

In order to account for the representations already present in the Wess-Zumino multiplets, we simply have to replace n_i, by $n_i' = n_i - 3N_i$. Thus, the previous formulas still apply, with the difference that the n_i' can now be negative.

For each choice of possible subgroups of E_6 as a gauge group above M_I, we can write down (up to several inequivalent embeddings) the nonzero. N_i's corresponding to the gauge bosons which get mass at M_I. Note that we must only consider gauge groups with $N_5 = N_6 = 0$, because otherwise SU(2)$_L$ and SU(3)$_c$ would necessarily be unified at M_I, which is in conflict with the fact that they have different couplings at that scale. (The corresponding gauge bosons surely could not have intermediate scale masses in any case, because of proton decay bounds). So, we list the possibilities according to rank.

Rank 4.

Case 0 :   SU(2)$_L$ × SU(3)$_c$ × U(1),   all $N_i = 0$.

Rank 5.

Case 1 :   SU(2)$_L$ × SU(3)$_c$ × SU(2) × U(1),
            (a) All $N_i = 0$ or (b) $N_2 = 1$.

Case 2 :   SU(3)$_L$ × SU(3)$_c$ × U(1), $N_1 = 1$.

Case 3 :   SU(4)$_c$ × U(1),   (a) $N_3 = 1$ or (b) $N_4 = 1$.

Case 4 :   SU(4)$_c$ × SU(2), $N_3 = 1$, $N_2 = 1$.

Rank 6.

Case 5 :   SU(2)$_L$ × SU(3)$_c$ × U(1), $N_2 = 2$.

Case 6 :   SU(3)$_L$ × SU(3)$_c$ × SU(2) × U(1),
            (a) $N_1 = 1$, $N_2 = 1$ or (b) $N_1 - 1$.

Case 7 :   SU(3)$_L$ × SU(3)$_c$ × SU(3), $N_1 = 1$, $N_2 = 2$.

Case 8 :   SU(2)$_L$ × SU(4)$_c$ × SU(2) × U(1), $N_4 = 1$.

Case 9 :   SU(2)$_L$ × SU(5)$_c$ × U(1),
            (a) $N_2 = 1$, $N_3 = 1$, $N_4 = 1$ or (b) $N_3 = 2$.

Case 10 :   SU(2)$_L$ × SU(6)$_c$, $N_2 = 2$, $N_3 = 2$, $N_4 = 1$.

In each of cases 1, 3, 6, and 9, there are inequivalent

embeddings of the standard model gauge group, resulting in two different possibilities for the N_i. There are also acceptable subgroups of E_6 obtained by adding U(1) factors to the ran 4 and 5 possibilities listed above. The extra U(1) factors do not contribute to the N_i, and do not affect the one-loop renormalization-group equations for the gauge couplings.

If the gauge group above the intermediate scale is larger than the standard model's, there must appear at the same intermediate threshold chiral superfields containing standard model singlets, to break the gauge group. In particular models, one must check for their presence and that the order parameters do not produce unacceptable R-parity violation (or baryon number violation in models which have alternative discrete symmetries).

IV. RESULTS: ONE INTERMEDIATE THRESHOLD

In the case of only one threshold, one can combine the results of the previous two sections to enumerate the possibilities for raising the unification scale. In Sec. II, we found that $q = 0, 1, 2, 3$ and that r is an integer between 0 and 14, and from Sec. III we found that $q + r$ is a multiple of 3. Of special interest, perhaps, are the cases for which M_X/M_I is large, so that the different scales are cleanly separated and may be definitely associated with different physics. For example, if M_I is to be associated with an invisible axion scale, we expect $M_I \sim 10^{10 \pm 2}$ GeV, so that $M_X/M_I = 10^{6 \mp 2}$. If we want the hierarchy M_X/M_I to be large, without having $|\Delta|$ be too large or giving M_U outside of the correct range between M_X and the Planck scale, there are tight restrictions which we now discuss, classified in terms of the value of q.

When $q = 0$, the unification scale does not depend on r. It is given by

$$\frac{M_U}{M_X} = \exp\left[\frac{\pi\Delta}{2}\right] , \qquad (4.1)$$

while the allowed values of r are multiples of three, $r = 3, 6, 9, 12$, corresponding to

$$\frac{M_X}{M_I} = \exp\left[\frac{\pi\Delta}{2}\left(\frac{11}{3}, \frac{4}{3}, \frac{5}{9}, \frac{1}{6}\right)\right] , \qquad (4.2)$$

respectively. Clearly, Δ must be positive in order to raise the unification scale in this case, with larger values of Δ corresponding to more substantial hierarchies in M_U/M_X and M_X/M_I. However, note that the hierarchy M_X/M_I is severely limited unless $r = 3$, and even then M_X/M_I cannot exceed 6×10^3 for $\Delta < 1.5$. In the cases $r = 6, 9, 12$, M_X/M_I cannot be large.

For $q = 1$, the unification scale is given by

$$M_U = \frac{M_X^{4/3}}{M_I^{1/3}} e^{2\pi\Delta/3} , \qquad (4.3)$$

independent of r. The possible values of r are 2, 5, 8, and 11, and the results for Δ in terms of the hierarchies

TABLE III. Sample hierarchies for $q = 1$.

r	2	5	8	11
$\Delta/(t_U - t_X)$	−0.48	0.19	0.36	0.43
$\Delta/(t_X - t_I)$	−0.08	0.11	0.48	1.6

$t_U - t_X$ and $t_X - t_I$ from (2.18) and (2.19) are given in Table III.

From Table III and (4.3), we can see that the hierarchy M_X/M_I can be very large if $r = 2$ or 5. The case $r = 2$ can accommodate intermediate scales as low as 10^8 GeV for Δ negative, and $r = 5$ can give M_I as low as 10^{10} GeV, for Δ positive. The case $r = 8$ does not allow M_X/M_I to be larger than about 20, because otherwise we see from Table III that Δ would be larger than allowed by the experimental constraint (2.3). The case $r = 11$ does not allow M_X/M_I to be large enough to be meaningful at all.

For $q = 2$, the unification scale is given by

$$M_U = \frac{M_X^2}{M_I} e^{\pi\Delta} . \qquad (4.4)$$

The possible values of r are 1, 4, 7, 10, and 13, and the results for Δ are given in Table IV.

Clearly, in the case $r = 13$ there can be no appreciable hierarchy in M_X/M_I because of the constraint (2.3) on Δ. In the case $r = 10$, the constraint on Δ implies that M_X/M_I can be at most 20 or so. The case $r = 1$ can give M_X/M_I as large as 200, but then does not allow M_U to be significantly larger than M_X. In the case $r = 7$, Δ must be zero, as we have already noted, and from (4.4), the hierarchy M_X/M_I must be less than 10^3 in order that M_U not exceed the Planck scale. The remaining case $r = 4$ can allow M_X/M_I to be as large as about 3×10^3, but no larger, because otherwise we see from Table IV that Δ would be too negative.

For $q = 3$, the unification scale is given by

$$M_U = \frac{M_X^4}{M_I^3} e^{2\pi\Delta} . \qquad (4.5)$$

The possible values of r are 3, 6, 9, and 12, and the results for Δ are given in Table V.

Clearly there is no way to get even an order-of-magnitude hierarchy in M_X/M_I in the case $r = 12$, because otherwise from (4.5), M_U would exceed the Planck scale since Δ is positive. The other cases have negative Δ, and therefore can accommodate a slightly larger hierarchy; for $p = 3, 6, 9$, one can have M_X/M_I as large as 30, 70, and 50, respectively, without having Δ be too negative or exceeding the Planck scale bound on M_U.

To summarize the preceding results, there are remarkably few cases in which one can have a large hierarchy of scales M_X/M_I. Only in the cases $q = 1$, $r = 2$ and

TABLE IV. Sample hierarchies for $q = 2$.

r	1	4	7	10	13
$\Delta/(t_U - t_X)$	−3.8	−0.48	0	0.19	0.29
$\Delta/(t_X - t_I)$	−0.29	− 0.19	0	0.48	3.8

TABLE V. Sample hierarchies for $q = 3$.

r	3	6	9	12
$\Delta/(t_U - t_X)$	-1.6	-0.48	-0.11	0.08
$\Delta/(t_X - t_I)$	-0.43	-0.36	-0.19	0.48

$q = 1$, $r = 5$ can one hope to have $M_X/M_I \geq 10^4$. These appear to be the only acceptable cases if one wishes to associate M_I with an invisible axion scale (or anything else below 10^{12} GeV). The cases $q = 0$, $r = 3$, and $q = 2$, $r = 4$ and $q = 2$, $r = 7$ can give hierarchies which are roughly in the range $M_X/M_I \sim 10^3$. All of the other cases give smaller upper limits for M_X/M_I.

In the superstring scenario, an estimate of string effects indicates that the scale of string unification should be related to the gauge coupling through the formula [7]

$$M_U \approx 2.5\sqrt{\alpha_U} \times 10^{18} \text{ GeV} . \tag{4.6}$$

Taking $M_X = 10^{16}$ GeV and $\alpha_U^{-1} < \alpha_X^{-1} \approx 25$, Eq. (4.6) implies that contact with the superstring can be made provided that $M_U/M_X > 50$.

As an example, suppose we take $\Delta = 0.82$ with $r = 5$, $q = 1$. Then Eq. (4.8) can be satisfied together with the other constraints by $n_1 = 4$, $n_2 = n_3 = 0$, $n_4 = 6$, and $n_5 = 1$. We find that

$$M_U = 7.5 \times 10^{17} \text{ GeV}, \quad M_I = 4.4 \times 10^{12} \text{ GeV}, \quad \alpha_U^{-1} = 11 .$$

This is one of the solutions with low r for which there is only one intermediate threshold well separated from M_X. It is interesting that most of the solutions with just one intermediate scale threshold do not allow M_X/M_I to be very large.

V. RESULTS: SEVERAL INTERMEDIATE THRESHOLDS

In most superstring theories, the effective low-energy gauge group at the string scale is larger than the standard model gauge group, and it is necessary to have several intermediate scale thresholds. Even if there is only one order parameter associated with the intermediate scale, the masses of the vectorlike particles are related to that order parameter by various dimensionless couplings which are certainly not always close to unity. This will result in some "smearing" of the threshold associated with each order parameter. Thus in a realistic model, the assumption of just one intermediate scale is probably not justified. However, we can still profitably analyze the situation in terms of the averaged quantities $\bar{t}_I$, $\bar{q}$, $\bar{r}$, etc. which were introduced in Sec. II. These quantities summarize the effects of the intermediate scale mass thresholds in terms of a single effective intermediate scale, with the main difference being that $\bar{q}$ and $\bar{r}$ need not be integers.

Let us apply our analysis to the interesting example of the three-family Gepner-Schimmrigk superstring model [8,9]. Below the string scale, the surviving gauge group is the

$$\text{SU(2)}_L \times \text{SU(3)}_c \times \text{SU(3)}_R$$

subgroup of E_6, corresponding to our case 7 (of Sec. III) with $N_1 = 1$ and $N_2 = 2$. This gauge group is subsequently broken to

$$\text{SU(2)}_L \times \text{SU(3)}_c \times \text{SU(2)} \times \text{U(1)} ,$$

corresponding to our case 1(b) with $N_2 = 1$, and then to the standard model gauge group. There are thus at least two *a priori* distinct intermediate scale order parameters associated with each reduction in rank. The chiral superfields which survive below the string scale are classified under the gauge group $\text{SU(3)}_L \times \text{SU(3)}_c \times \text{SU(3)}_R$ as

$$9 \text{ leptons} \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}) ,$$

$$6 \text{ mirror leptons} \sim (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}) ,$$

$$3 \text{ quarks} \sim (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1}) ,$$

$$3 \text{ antiquarks} \sim (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) ,$$

and, unlike most other string models, no mirror quarks $(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})$ or mirror antiquarks $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$.

This particle content includes, besides the chiral superfields for the three families of quarks and leptons and two Higgs doublets of the minimal supersymmetric standard model, chiral superfields corresponding to

$$n_1 = 20, \quad n_2 = 6, \quad n_3 = 0, \quad n_4 = 3, \quad n_5 = 0 .$$

Combining these with the vector superfields, we have a total vectorlike particle content yielding

$$n_1' = 17, \quad n_2' = 0, \quad n_3' = 0, \quad n_4' = 3, \quad n_5' = 0 .$$

Thus, if all of these particles were concentrated at just one intermediate mass scale, we would have

$$\delta_1 = \frac{57}{5}, \quad \delta_2 = 17, \quad \delta_3 = 3 ,$$

giving

$$q_{\text{total}} = -14 \text{ and } r_{\text{total}} = 14 . \tag{5.1}$$

These values lie outside the range established by (2.26) and (2.27). If the particle thresholds affect the gauge coupling unification in a perturbative and meaningful way below M_X, thee must be some smearing, with the "averaged" quantity $\bar{q}$ higher than q_{total} and $\bar{r}$ lower than r_{total}. Otherwise, from the discussion in Sec. II, α_3 and α_2 would meet too early (just above intermediate scale) and α_2 and α_1 would *never* meet. It is clear that to move things in the right direction, the contributions of the δ_{2a} to each of $\bar{q}$ and $\bar{r}$ should be weighted less heavily than those of δ_{1a} and δ_{3a}. This can only occur if the masses of the electrosinglet down-quark vectorlike chiral superfields corresponding to n_4 are smaller than the average effective scale of the other particles. (Note that in this example, $N_3 = N_4 = 0$).

Let us denote by $\bar{t}_{n_1}$, $\bar{t}_{n_2}$, and $\bar{t}_{n_4}$ the arithmetic means

of the scales associated with the chiral superfields corresponding to the weak doublet vectorlike leptons, n_1, the weak singlet charged leptons, n_2, and the downlike electroweak singlet quarks, n_4, respectively. Similarly, the arithmetic means of the scales associated with the vector supermultiplets corresponding to N_1 and N_2 are denoted by $\bar{t}_{N_1}$ and $\bar{t}_{N_2}$. Then it is convenient to choose for the effective intermediate scale $\bar{t}_I = \bar{t}_{n_4}$, which is just the scale associated with the effective threshold for α_3. With this choice, one finds

$$\bar{q} = -20 \left(\frac{t_U - \bar{t}_{n_1}}{t_U - \bar{t}_I} \right) + 3 \left(\frac{t_U - \bar{t}_{N_1}}{t_U - \bar{t}_I} \right) + 3 , \quad (5.2)$$

$$\bar{r} = -\bar{q} + 18 \left(\frac{\bar{t}_{n_2} - \bar{t}_{N_2}}{t_U - \bar{t}_I} \right) , \quad (5.3)$$

$$\bar{\delta}_2 = -\bar{q} + 3 . \quad (5.4)$$

Note that $\bar{t}_I$ cannot be larger than $\bar{t}_{N_1}$ or $\bar{t}_{N_2}$, because the vectorlike color triplets can only obtain their masses at or below the scale at which the gauge group is broken down to that of the standard model. Also, two of the vectorlike pairs corresponding to n_1 must have masses at scales below $\bar{t}_{N_{1,2}}$, for the same reason. Since these contribute negatively to the right-hand side (RHS) of (5.2), the net positive contributions to $\bar{q}$ are quite limited. So we see that the only way to obtain $0 < \bar{q} < 4$ is for the vectorlike weak doublet leptons, corresponding to n_1 to be located (on average) well above $\bar{t}_I$. From (5.3), one can also see that the scale $\bar{t}_{n_2}$ associated with the charge lepton chiral superfields must also be located above $\bar{t}_{N_2}$. Finally, we see from (5.4) that if the thresholds are arranged appropriately for gauge coupling unification, then $\bar{\delta}_2$ is automatically not larger than 3, so that the constraint (2.20) from perturbativity of the couplings does not limit the effective intermediate scale $\bar{t}_I$ at all. Another way to see this is to note that the slope of α_3^{-1} can never be negative with this particle content. (Of course, in models with a larger sector of strongly interacting chiral superfields, the requirement of perturbativity can be quite important.)

If some of the chiral superfields have masses located far below M_X, we have seen that some of these must include the color triplet fields corresponding to n_4. This can be understood from the fact that only these color triplets give a positive contribution to $\bar{q}$ among the chiral superfields of the model. One should note, however, that there is a potential embarrassment associated with such light color triplets; they can easily lead to proton decay at unacceptable rates if their masses are below M_X, depending on their couplings to the quark and lepton superfields of the MSSM. This can be avoided if, e.g., one assumes the existence of a discrete symmetry [10] prohibiting some or all of the baryon-number and lepton-number-violating couplings. Actually, the presence of vectorlike down-type quarks below M_X seems to be a fairly general feature of string-type models in which intermediate scale thresholds are used to raise the unification scale: see, for example, [11,12]. One can understand this semiquantitatively by

examining the values of q and r for the chiral supermultiplets in Table I. Only the chiral superfields corresponding to n_3 and n_4 can give a positive contribution to $\bar{q}$. However, the superfields for n_3 (which are innocuous for proton decay) also give a relatively large negative contribution to $\bar{r}$. Since $\bar{q}$ and $\bar{r}$ both must be positive to raise the unification scale, it seems that the color triplet with electric charge $\pm\frac{1}{3}$ corresponding to n_4 must be weighted relatively heavily in the averaged quantities. This is another way of saying that they are relatively light compared to the other chiral superfields which are important in redirecting the running gauge couplings to their new meeting point. Of course, one can always achieve a raised unification scale fairly safely by employing only threshold which are close to M_X. In most superstring models [12], this is almost required, since the large number of strongly interacting chiral superfields would cause the gauge couplings to be nonperturbative if the effective intermediate scale were much lower than about 10^{15} GeV.

VI. CONCLUSION

In this paper, we have examined the possibility that the true unification scale can be raised above its apparent value of 2×10^{16} GeV by calculable perturbative means. It might seem rather surprising that in the MSSM the gauge couplings should appear to be nicely headed for unification at M_X, only to be redirected to a new meeting place at M_U. Indeed, the apparent perverseness of this situation allows us to put some nontrivial constraints on the scenario. In the simplest case of just one cleanly defined intermediate scale, it is striking that the hierarchy M_U/M_I is generally quite limited. In the probably more realistic case of a "smeared" intermediate scale or several intermediate scales, one cannot be s precise because of the vastly increased number of unknown parameters. However, one can still put useful constraints on the placement of the intermediate scales and particles, by writing things in terms of a single effective intermediate threshold. Here too, in most realistic models based on superstrings, there is a tendency for many of the vectorlike particles to be very heavy, based simply on the requirement that the gauge coupling remain perturbative and thus calculable in principle at high energies. Even in models like the one considered in Sec. V, in which the absence of a larger number of vectorlike strongly interacting particles causes perturbativity to be easily maintained, one finds that it is difficult to raise the unification scale consistently with intermediate scales much below M_X. If one insists on having some chiral superfields at relatively low intermediate scales, we find that generally these chiral superfields include color triplets with electric charge $\pm\frac{1}{3}$, which may be dangerous for proton decay without assuming some extra symmetry.

The difficulty in obtaining examples in which a raised unification scale is achieved due to a relatively low intermediate scale corresponds to our intuition that it would be surprising if the unification of gauge couplings were totally accidental. The lower the intermediate scale(s) are, the more we must regard the apparent success of the

unification of gauge couplings as just a perverse accident. On the other hand, if there are intermediate scale thresholds which are only slightly below the unification scale, then the near perfect unification of couplings should be regarded as partly, but certainly not completely, accidental. This scenario seems to be the one preferred by superstring models.

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