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ϵ'/ϵ and anomalous gauge boson couplings

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We study ϵ'/ϵ in the standard model and ϵ'/ϵ due to anomalous $WW\gamma$ and WWZ interactions as a function of the top quark mass. In the standard model, ϵ'/ϵ is in the range 10^{-3}–10^{-4} for the central value of the top quark mass reported by the CDF. The anomalous gauge couplings can have large contributions to the CP-violating $I = 2$ amplitude in $K \to \pi\pi$. Within the allowed regions for the anomalous gauge couplings, ϵ'/ϵ can be dramatically different from the standard model prediction.

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I. INTRODUCTION

The $SU(2)_L\times U(1)_Y$ standard model (SM) of electroweak interactions is in very good agreement with present experimental data. The experimental data from the CERN e^+e^- collider LEP and SLAC Linear Collider (SLC) and the theoretical predictions in the SM for gauge-fermion couplings agree at the 1% level or better [1]. However, one of the most direct consequences of the SM, the self-interaction of the gauge particles, the W, Z, and photon, characteristic of non-Abelian gauge theories, has not been directly tested. It is important to study these self-interactions to establish whether the weak bosons are gauge particles with interactions predicted by the SM, gauge particles of some extensions of the SM which predict different interactions at loop levels, or even nongauge particles whose self-interactions at low energies are described by effective interactions.

Large uncertainties are introduced into studies of physics beyond the SM due to our lack of knowledge of the top quark mass m_t. D0 has put the lower bound on m_t to be 131 GeV [2]. The Collider Defector at Fermilab (CDF) Collaboration has announced evidence for the existence of the top quark with a mass of $174 \pm 10^{+13}_{-12}$ GeV [3]. If confirmed, this information will allow us to make better predictions of new physics beyond the SM. In this paper we show how the information from the CDF about the top quark mass helps the study of the effect of anomalous gauge couplings on the CP-violating parameter ϵ'/ϵ in comparison with the SM prediction.

In general there will be more gauge boson self-interaction terms than the tree level SM predicts. The most general WWV interactions with the W boson on shell, invariant under $U(1)_{em}$, can be parametrized as [4]

$$\begin{aligned} L_V = -ig_V \Bigg[& \kappa^V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda^V}{M_W^2} W^+_{\sigma\rho} W^{-\rho\delta} V_\delta{}^\sigma + \tilde{\kappa}^V W^+_\mu W^-_\nu \tilde{V}^{\mu\nu} + \frac{\tilde{\lambda}^V}{M_W^2} W^+_{\sigma\rho} W^{-\rho\delta} \tilde{V}_\delta{}^\sigma \\ & + g_1^V (W^{+\mu\nu} W^-_\mu - W^+_\mu W^{-\mu\nu}) V_\nu + g_4^V W^+_\mu W^-_\nu (\partial^\mu V^\nu + \partial^\nu V^\mu) \\ & + g_5^V \epsilon_{\mu\nu\alpha\beta} (W^{+\mu} \partial^\alpha W^{-\nu} - \partial^\alpha W^{+\mu} W^{-\nu}) V^\beta \Bigg] , \end{aligned} \tag{1}$$

where $W^{\pm\mu}$ are the W boson fields; V can be the γ or Z fields; $W_{\mu\nu}$ and $V_{\mu\nu}$ are the W and V field strengths, respectively; and $\tilde{V}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}V^{\alpha\beta}$. The terms proportional to κ, λ, and $g^Z_{1,5}$ are CP conserving and $\tilde{\kappa}$, $\tilde{\lambda}$, and g_4^Z are CP violating. For $V = \gamma$, $g_V = e$, and for $V = Z$, $g_V = g\cos\theta_W$. g_1^γ defines the W boson charge; one can always set it to 1. In the SM at the tree level, $\kappa^V = 1$, $g_1^Z = 1$, and all other couplings in Eq. (1) are zero. $\Delta\kappa^V = \kappa^V - 1$, $\Delta g_1^Z = g_1^Z - 1$, $\tilde{\kappa}^V$, $\tilde{\lambda}^V$, g_4^V, and g_5^V are called the anomalous gauge boson couplings.

There have been many experimental and theoretical studies of the anomalous gauge boson couplings. Collider experiments at high energies have put constraints on some of these couplings [5,6]. It has been shown that rare decays can provide important constraints [7–9,11]. In Refs. [9,10] using the recent data from CLEO on $b \to s\gamma$ [12] and data on $K_L \to \mu^+\mu^-$ [13], constraints comparable or better than those obtained in collider physics were obtained. Rare B decays may provide more stringent constraints [11]. The constraints from rare decays are better than those obtained from the $g - 2$ of the muon [14]. In the literature the most stringent con-

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straints on the anomalous gauge boson couplings are from oblique corrections to the precision electroweak experiments [15]. The anomalous gauge coupling contributions to the oblique corrections are sometimes quadraticly or even quarticly divergent. Care must be taken when evaluating these contributions. Strictly, one should return to the underlying theories to remove the quartic and quadratic divergences [15]. For purely phenomenlogical studies, we think the constraints from direct W pair productions [5,6] and rare decays [9,10] (the divergences here are at most logarithmic) are more reliable. For the CP-violating anomalous coupling, the best constraints are from neutron and electron electric dipole moments [16].

In obtaining the bounds on the anomalous gauge boson couplings, most of the analyses assumed only one coupling is different from the SM tree level predictions. A real underlying theory would produce more than just a single anomalous coupling. If the analyses were carried out including all anomalous couplings simultaneously, the bounds would be much weaker. It is nevertheless interesting to find out if, when these stringent bounds are applied, there are still large effects on other processes. In this paper we will show that there can be still large effects on ϵ'/ϵ from the anomalous $WW\gamma$ and WWZ couplings.

The parameter ϵ'/ϵ is a very important quantity to study. It measures direct CP violation in $K \to \pi\pi$. Experimental measurements are not conclusive at this stage [17]:

$$\mathrm{Re}(\epsilon'/\epsilon) = \begin{cases} (23 \pm 6.5) \times 10^{-4} & \text{NA31,} \\ (7.4 \pm 6.0) \times 10^{-4} & \text{E731.} \end{cases} \tag{2}$$

While the result of NA31 clearly indicates a nonzero ϵ'/ϵ, the value of E731 is compatible with zero. However, the two results are consistent at the two standard deviation level. The SM prediction for ϵ'/ϵ depends on the value of the top quark mass. It has been shown that for a small top quark mass, the most important contributions to ϵ'/ϵ are from the strong penguin diagram and isospin breaking due to quark masses. For a large top quark mass, the electroweak penguin diagrams also become important [18,19]. In fact the sign of ϵ'/ϵ may change for m_t larger than 220 GeV. If the top quark mass is indeed about 174 GeV as reported by CDF, the electroweak penguin contribution will not cancel the other contributions completely. The predicted value for ϵ'/ϵ is about 10^{-3}–10^{-4} which will be within the reach of future experiments. We will then be able to find out if there are other contributions to ϵ'/ϵ. This illustrates the importance of knowing the m_t in determining the physics beyond the SM.

The anomalous gauge interactions are purely electroweak, and so their contributions to ϵ'/ϵ will not affect the strong penguin diagram but may have significant effects on the electroweak penguin diagrams. We will show that anomalous gauge couplings can change the result dramatically.

II. NEUTRAL FLAVOR-CHANGING EFFECTIVE HAMILTONIAN

The effective Hamiltonian H_{eff} for flavor-changing neutral currents with $\Delta F = 1$, at the one-loop level, is given by

$$H_{\text{eff}} = H_{\text{SM}} + H_{\text{AGC}} , \tag{3}$$

where H_{SM} is the SM contribution which can be found in Ref. [20], and H_{AGC} contains the contributions from anomalous gauge couplings. It is given by

$$\begin{aligned} H_{\text{AGC}} = \frac{G_F}{2\sqrt{2}\pi} \sum_i V_{iq} V_{iq'}^* \Bigg[& \frac{e}{8\pi} G(x_i)_A \bar{q}' [m_{q'}(1-\gamma_5) + m_q(1+\gamma_5)] \sigma_{\mu\nu} q F^{\mu\nu} \\ & + \alpha_{\text{em}} Q_f H(x_i)_A \bar{q}' \gamma_\mu (1-\gamma_5) q \bar{f} \gamma^\mu f \\ & + \alpha_{\text{em}} \cot^2\theta_W F(x_i)_A \bar{q}' \gamma_\mu (1-\gamma_5) q \bar{f} \gamma^\mu \left(Q_f \sin^2\theta_W - T^3 \frac{1-\gamma_5}{2} \right) f \Bigg] , \end{aligned} \tag{4}$$

with $x_i = m_i^2/m_W^2$, and

$$\begin{aligned} G(x)_A &= -(\Delta\kappa + i\tilde{\kappa}) \left(\frac{x}{(1-x)^2} + \frac{x^2(3-x)}{2(1-x)^3} \ln x \right) \\ &\quad -(\lambda + i\tilde{\lambda}) \left(\frac{x(1+x)}{2(1-x)^2} + \frac{x^2}{(1-x)^3} \ln x \right) , \\ H(x)_A &= \Delta\kappa \frac{x}{4} \ln \frac{\Lambda^2}{m_W^2} + \lambda \left(\frac{x(1-3x)}{2(1-x)^2} - \frac{x^3}{(1-x)^3} \ln x \right) , \\ F(x)_A &= -\Delta g_1^Z \frac{3}{2} x \ln \frac{\Lambda^2}{m_W^2} + g_5^Z \left(\frac{3x}{1-x} \right. \\ &\quad \left. + \frac{3x^2}{(1-x)^2} \ln x \right) , \end{aligned} \tag{5}$$

where Λ is the cutoff scale.

The four-fermion contact terms in Eq. (4) are generated by photon exchange involving an anapole γqq interaction. The γqq interaction generated by the G_A term in this equation is a noncontact interaction. These terms are physically distinct, and including both of them involves no double counting. For terms which are divergent in the loop integral, we have just kept the leading term proportional to $\ln(\Lambda^2/m_W^2)$. We used the unitary gauge in our calculations. Our first term in H_A does not agree with Ref. [7] where the author obtained a cutoff-independent result. The term in H_A proportional to $\Delta\kappa$ is similar to the term in the SM with $\kappa = 1$. In the R_ξ gauge, this term is gauge dependent [20]. In the unitary gauge this term diverges. This gauge-dependent divergent term is canceled by terms from "box" and Z exchanges in physical processes. In our case because the coupling $\Delta\kappa$ is anomalous, there are no terms coming from box and Z exchange to cancel the divergence and the cutoff dependence remains.

The Hamiltonian in Eq. (3) is the lowest nonvanishing order contribution to the flavor-changing neutral current. It has been show that QCD corrections are important in the SM [18,19,21]. QCD corrections should be included in phenomenological analyses. To this end, we carry out the leading logarithmic QCD correction to the weak effective Hamiltonian. We will use the notation in Ref. [19]. The effective Hamiltonian at the energy scale μ relevant to us can be written as

$$H_{\rm eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \sum_i C_i(\mu)Q_i(\mu) , \tag{6}$$

where $i = 1,..., 10$ and

$$C_i(\mu) = z_i(\mu) + \tau\tilde{y}_i(\mu) , \quad \tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^* . \tag{7}$$

The coefficients C_i satisfy the renormalization group equation to the first order in α_s and $\alpha_{\rm em}$:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right)\mathbf{C}(\mu) = \frac{1}{2\pi}[\alpha_s(\mu)\gamma^{(s)T} + \alpha_{\rm em}(\mu)\gamma^T]\mathbf{C}(\mu) , \tag{8}$$

where $\gamma^{(s)}$ and γ are the anomalous dimension matrices which were obtained in Ref. [22]. The Wilson coefficients at the scale μ is obtained by first calculating the coefficients at the scale m_W and then using the renormalization group to evolve down to the scale μ. In general, z_i and $\tilde{y}_i$ will evolve differently due to different threshold effects. In our calculation we will use experimental values for the CP-conserving amplitudes and calculate the CP-violating amplitudes. The CP-violating amplitudes are proportional to $\tilde{y}_i{\rm Im}(\tau)$. So we only need to calculate the Wilson coefficients $\tilde{y}_i$. The four-quark operators are defined as

$$\begin{aligned}
Q_1 &= \bar{s}\gamma_\mu(1-\gamma_5)d\bar{u}\gamma^\mu(1-\gamma_5)u , \quad Q_2 = \bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)d , \\
Q_3 &= \bar{s}\gamma_\mu(1-\gamma_5)d\sum_q \bar{q}\gamma^\mu(1-\gamma_5)q , \quad Q_4 = \sum_q \bar{s}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)d , \\
Q_5 &= \bar{s}\gamma_\mu(1-\gamma_5)d\sum_q \bar{q}\gamma^\mu(1+\gamma_5)q , \quad Q_6 = -2\bar{s}(1+\gamma_5)q\bar{q}(1-\gamma_5)d , \\
Q_7 &= \frac{3}{2}\bar{s}\gamma_\mu(1-\gamma_5)d\sum_q Q_q\bar{q}\gamma^\mu(1+\gamma_5)q , \quad Q_8 = -3\sum_q Q_q\bar{s}(1+\gamma_5)q\bar{q}(1-\gamma_5)d , \\
Q_9 &= \frac{3}{2}\bar{s}\gamma_\mu(1-\gamma_5)d\sum_q Q_q\bar{q}\gamma^\mu(1-\gamma_5)q , \quad Q_{10} = \frac{3}{2}\sum_q Q_q\bar{s}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)d .
\end{aligned} \tag{9}$$

Among these operators there are only seven linearly independent ones. We use $Q_{1,2,3,5,6,7,8}$ as the independent operators. The corresponding coefficients $y_{1,2,3,5,6,7,8}$ are given by

$$\begin{aligned}
y_1 &= \tilde{y}_1 - \tilde{y}_4 + \frac{3}{2}\tilde{y}_9 + \frac{1}{2}\tilde{y}_{10} , \quad y_2 = \tilde{y}_2 + \tilde{y}_4 + \tilde{y}_{10} , \\
y_3 &= \tilde{y}_3 + \tilde{y}_4 - \frac{1}{2}\tilde{y}_9 - \frac{1}{2}\tilde{y}_{10} , \quad y_i = \tilde{y}_i , i = 5,6,7,8 .
\end{aligned} \tag{10}$$

The boundary conditions at m_W for the Wilson coefficients in the SM can be found in Ref. [18–20] which depend on the top quark mass. We will not display them here. When the anomalous gauge coupling contributions are included, due to the new contributions, the boundary conditions at m_W for the Wilson coefficients are different from the SM. The new contributions will change $\tilde{y}_{3,7,9}$. From Eq. (4) we obtain the anomalous gauge boson coupling contributions to the Wilson coefficients at the m_W scale:

$$\begin{aligned}
\tilde{y}_3(m_W) &= -\frac{\alpha_{\rm em}}{24\pi}\cot^2\theta_W F_A(x_t) , \\
\tilde{y}_7(m_W) &= -\frac{\alpha_{\rm em}}{6\pi}[H_A(x_t) + \sin^2\theta_W\cot^2\theta_W F_A(x_t)] , \\
\tilde{y}_9(m_W) &= -\frac{\alpha_{\rm em}}{6\pi}[H_A(x_t) - \cos^2\theta_W\cot^2\theta_W F_A(x_t)] .
\end{aligned} \tag{11}$$

The other coefficients are not changed from those of the SM. Note that the new contributions to the effective Hamiltonian depend only on $\Delta\kappa^\gamma$, λ^γ, Δg_1^Z, and g_5^Z. Contributions from the other anomalous couplings are suppressed by factors such as $O(m_{d,s}^2, m_K^2)/m_W^2$. We give in Tables I and II the values for y_i as a function of m_t and the anomalous couplings.

TABLE I. y_i as a function of m_t in the SM at $\mu = 1$ GeV for $\Lambda_4 = 0.25$ GeV, $m_b = 5$ GeV, $m_c = 1.35$ GeV.

m_t (GeV)	140	165	180	200	240
y_1	0.041	0.039	0.038	0.037	0.033
y_2	-0.049	-0.049	-0.048	-0.048	-0.047
y_3	-0.020	-0.019	-0.019	-0.018	-0.017
y_5	0.012	0.012	0.012	0.013	0.013
y_6	-0.091	-0.092	-0.093	-0.093	-0.094
$y_7/\alpha_{\rm em}$	-0.003	0.029	0.051	0.083	0.155
$y_8/\alpha_{\rm em}$	0.081	0.121	0.149	0.188	0.278

TABLE II. y_i as a function of m_t and the anomalous gauge couplings at $\mu = 1$ GeV for $\Lambda_4 = 0.25$ GeV, $m_b = 5$ GeV, and $m_c = 1.35$ GeV, and the cutoff $\Lambda = 1$ TeV.

	m_t (GeV)	140	165	180	200	240
$\Delta\kappa = 0.2$	$y_7/\alpha_{\rm em}$	-0.036	-0.016	-0.003	0.0165	0.059
	$y_8/\alpha_{\rm em}$	0.039	0.064	0.080	0.104	0.157
$\Delta\kappa = -0.5$	$y_7/\alpha_{\rm em}$	0.078	0.142	0.185	0.248	0.393
	$y_8/\alpha_{\rm em}$	0.184	0.264	0.319	0.398	0.580
$\lambda = 1$	$y_7/\alpha_{\rm em}$	-0.034	-0.008	0.010	0.037	0.099
	$y_8/\alpha_{\rm em}$	0.042	0.074	0.096	0.129	0.207
$\lambda = -3$	$y_7/\alpha_{\rm em}$	0.088	0.141	0.174	0.221	0.321
	$y_8/\alpha_{\rm em}$	0.197	0.263	0.305	0.363	0.489
$\Delta g_1^Z = 0.015$	$y_7/\alpha_{\rm em}$	0.008	0.045	0.070	0.106	0.189
	$y_8/\alpha_{\rm em}$	0.095	0.142	0.173	0.218	0.321
$\Delta g_1^Z = -0.15$	$y_7/\alpha_{\rm em}$	-0.121	-0.134	-0.143	-0.157	-0.190
	$y_8/\alpha_{\rm em}$	-0.066	-0.082	-0.094	-0.111	-0.153
$g_5^Z = 0.9$	$y_7/\alpha_{\rm em}$	-0.094	-0.079	-0.067	-0.048	-0.001
	$y_8/\alpha_{\rm em}$	-0.033	-0.014	0.001	0.025	0.086
$\Delta g_5^Z = -0.15$	$y_7/\alpha_{\rm em}$	0.012	0.047	0.071	0.104	0.180
	$y_8/\alpha_{\rm em}$	0.099	0.144	0.173	0.215	0.310

III. CONTRIBUTIONS TO ϵ'/ϵ

The parameter ϵ'/ϵ is a measure of CP violation in $K_{L,S} \to 2\pi$ decays. It is defined as

$$\frac{\epsilon'}{\epsilon} = i\frac{e^{i(\delta_2-\delta_0)}}{\sqrt{2}(i\xi_0+\bar{\epsilon})}\omega\,(\xi_2-\xi_0)\ , \tag{12}$$

where $\bar{\epsilon} \approx 2.26\times10^{-3}e^{i\pi/4}$ is the CP-violating parameter in $K^0 - \bar{K}^0$, δ_i are the strong rescattering phases, $\omega = |{\rm Re}A_2/{\rm Re}A_0| \approx 1/22$, and $\xi_i = {\rm Im}A_i/{\rm Re}A_i$. Here A_0 and A_2 are the decay amplitudes with $I = 0$ and 2 in the final states, respectively.

To separate different contributions to ϵ'/ϵ, we parametrize ϵ'/ϵ as

$$\frac{\epsilon'}{\epsilon} = \left(\frac{\epsilon'}{\epsilon}\right)_6 (1-\bar{\Omega})\ , \tag{13}$$

where $(\epsilon'/\epsilon)_6$ is the contribution from y_6 which is given by

$$\left(\frac{\epsilon'}{\epsilon}\right)_6 = \frac{\omega}{2\epsilon}\frac{G_F}{|A_0|}y_6\langle Q_6\rangle_0{\rm Im}(V_{td}V_{ts}^*)\ . \tag{14}$$

Here $\langle Q_i\rangle_I$ is defined as $\langle Q_i\rangle_I = \langle(\pi\pi)_I|Q_i|K\rangle$. The parameter $\bar{\Omega}$ contains several different contributions:

$$\bar{\Omega} = \Omega_{\eta+\eta'} + \Omega_{\rm EWP} + \Omega_{\rm octet} + \Omega_{27} + \Omega_P\ , \tag{15}$$

where $\Omega_{\eta+\eta'}$ is the contribution due to isospin breaking in the quark masses which is estimated to be in the range 0.2–0.4 [22,23]. We will use $\Omega_{\eta+\eta'} = 0.25$ for illustration. The other contributions are defined as

$$\begin{aligned}\Omega_{\rm EWP} &= \frac{1-\sqrt{2}\omega}{\omega}\frac{y_7\langle Q_7\rangle_2 + y_8\langle Q_8\rangle_2}{y_6\langle Q_6\rangle_0}\ ,\\ \Omega_{\rm octet} &= -\frac{y_1\langle Q_1\rangle_0 + y_2\langle Q_2\rangle_0}{y_6\langle Q_6\rangle_0}\ ,\\ \Omega_{27} &= \frac{1}{\omega}\frac{(y_1+y_2)\langle Q_2\rangle_2}{y_6\langle Q_6\rangle_0}\ ,\\ \Omega_P &= -\frac{y_3\langle Q_3\rangle_0 + y_5\langle Q_5\rangle_0}{y_6\langle Q_6\rangle_0}\ .\end{aligned} \tag{16}$$

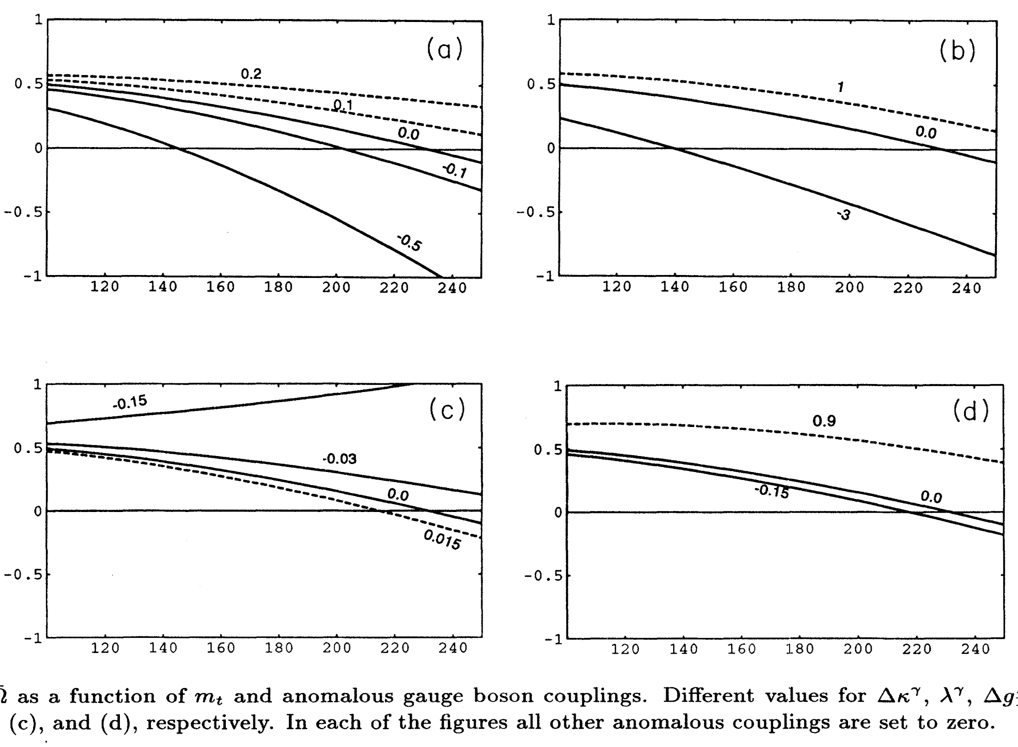

FIG. 1. $1-\bar{\Omega}$ as a function of m_t and anomalous gauge boson couplings. Different values for $\Delta\kappa^\gamma$, λ^γ, Δg_1^Z, and g_5^Z are used in (a), (b), (c), and (d), respectively. In each of the figures all other anomalous couplings are set to zero.

The calculation of the hadronic matrix elements is the most difficult task [18,19,22,24,25]. There is no satisfactory procedure for this calculation at present. We will use the values in Ref. [19] in our tables and figures for illustration, and put our emphasis on the effects of the anomalous couplings. In Fig. 1, we show the dependence of $1-\bar{\Omega}$ as a function of m_t and the anomalous gauge boson couplings.

IV. DISCUSSION

We show in Table I the SM predictions for the Wilson coefficients as a function of top quark mass m_t. In Table II, we show the effects of anomalous couplings on $y_{7,8}$ as functions of m_t and the anomalous couplings. It is clear that the anomalous couplings can dramatically change the SM predictions.

In our numerical analyses of the effects of anomalous coupling on the Wilson coefficients, we will assume that only one anomalous coupling is nonvanishing. As has been mentioned before, this may not be true. We nevertheless carry out the analysis this way to illustrate the effects of anomalous couplings on ϵ'/ϵ. We use some values of the anomalous couplings which are consistent with constraints from rare decays because they are all derived from the effective Hamiltonian in Eq. (3). The constraints from rare decays are top quark mass m_t dependent. Using the recent CLEO bound on $b \to s\gamma$ at the 95% C.L. [12], the anomalous couplings $\Delta\kappa^\gamma$ and λ^γ are constrained to be in the range -2.2–0.35 and -6.7–1.1, respectively, for $m_t = 174$ GeV. For larger m_t, the constraints are more stringent [9]. These constraints are cutoff scale Λ independent. However, the constraints from $K_L \to \mu^+\mu^-$ are cutoff dependent. For $m_t = 174$ GeV the experimental data on $K_L \to \mu^+\mu^-$ constrain Δg_1^Z to be in the range -0.5–0.1 for cutoff scale $\Lambda = 1$ TeV. For larger Λ the constraint is more stringent [10]. g_5^Z is constrained to be in the range 4 to -1. The constraint on g_5^Z is cutoff independent. The specific values for the anomalous couplings are given in Table II. We used values for the anomalous couplings which are also consistent with the constraints from collider physics [5,6].

The anomalous couplings affect all the Wilson coefficients through renormalization. However, the effects on $y_{1,2,3,5,6}$ are less than 5% and can be neglected. The effects on $y_{7,8}$ are large. In Table II, we show the effects of anomalous couplings on $y_{7,8}$ as functions of m_t and the anomalous couplings.

In Fig. 1, we show the dependence of $1-\bar{\Omega}$ as a function of m_t and the anomalous gauge boson couplings. The anomalous gauge boson couplings have a large effect on $\Omega_{\rm EWP}$. The effect on other contributions to Ω can be neglected.

Using the value $\langle Q_6\rangle_0 = -0.255$ GeV^3 for $m_s = 0.175$ GeV, we have

$$\left(\frac{\epsilon'}{\epsilon}\right)_6 \approx 8\mathrm{Im}(V_{td}V_{ts}^*) \ . \tag{17}$$

Here we have used $y_6 \approx -0.09$. Using information from CP violation in K-$\bar{K}$ mixing and data from B-$\bar{B}$ mixings [13], the allowed range for $\mathrm{Im}(V_{td}V_{ts}^*)$ is constrained to be in the region 3×10^{-4}–0.5×10^{-4} for m_t varying from 100 GeV to 250 GeV. We see that ϵ'/ϵ in the SM is between 10^{-3} and -3×10^{-4}. There is a strong dependence on the top quark mass m_t. For the hadronic matrix elements used here, ϵ'/ϵ changes sign at about 230 GeV in the SM as mentioned before. If the top quark mass is determined, the uncertainties for ϵ'/ϵ will be greatly reduced. One still needs to use the measured physical top quark mass properly. The top quark mass in the loop calculation to the one-loop level runs with momentum in principle. In all the calculations in the literature, the top quark mass is fixed as a constant. To the leading order, this approximation is good, in that the q^2 dependence represents higher order effects. We believe that the appropriate mass to use in the one-loop calculation is the running mass $m_t(m_t)$, which is the mass appearing in the renormalized Lagrangian. The relation between pole mass and running mass is discussed in Ref. [26]. A physical top quark mass of 174 GeV corresponds to a running mass about 165 GeV. The variation in the results as one changes from pole mass to running mass can be taken as an indication of the sensitivity of the results of this calculation to higher order effects. The Wilson coefficients $y_{7,8}$ can change as much as 20% using pole mass and running mass. We display our results for $m_t = 165$ GeV as an example. For $m_t = 165$ GeV, we find that $1-\bar{\Omega} = 0.3$ and $Im(V_{td}V_{ts}^*)$ is in the range 2×10^{-4}–0.5×10^{-4}. Therefore ϵ'/ϵ is in the range 5×10^{-4}–10^{-4} which will soon be accessible to experiments at CERN and Fermilab.

There are, of course, uncertainties due to our poor understanding of the hadronic matrix elements, and due to errors in the QCD scale Λ_4 for four-flavor effective quarks. In Ref. [25], using a different set of hadronic matrix elements, it is found that the value for $1-\bar{\Omega}$ can vary a factor of 2. It has recently been shown that the next-to-leading order QCD corrections [27] can reduce the ϵ'/ϵ about 10–20%. The uncertainty in Λ_4 is $\pm30\%$. In the above analysis, we have neglected contributions from a gluon dipole penguin operator of the form $\bar{q}\sigma_{\mu\nu}\lambda^a(1-\gamma_5)qG_a^{\mu\nu}$. It has been shown that to leading order in chiral perturbation theory, this contribution vanishes [28]. Higher order chiral perturbation calculations indicate that this contribution may enhance the value for ϵ'/ϵ by about 10% for $m_t = 165$ GeV [28,29]. When taking into account all the effects mentioned, we conclude that for $m_t = 165$ GeV, ϵ'/ϵ is in the range 10^{-3}–10^{-4}.

From Fig. 1 it can be easily seen that the anomalous gauge couplings can change the result dramatically. ϵ'/ϵ can be much larger than the SM prediction and the value of m_t where the sign change of ϵ'/ϵ occurs can be significantly shifted. The change of sign for ϵ'/ϵ can occur for m_t as small as 120 GeV for allowed values for the anomalous gauge couplings. Future measurements on ϵ'/ϵ will certainly provide useful information about the anomalous gauge couplings. In Fig. 1, we also show $1-\bar{\Omega}$ with the anomalous couplings set to ±0.1 for $\Delta\kappa^\gamma$ and Δg_1^Z. We see that even with such small anomalous couplings, the effects on ϵ'/ϵ are still sizable. If we use the same bounds for λ^γ and g_5^Z, the contributions are small (less than 5%).

We conclude that in the SM, ϵ'/ϵ is predicted to be in the range 10^{-3}–10^{-4} for $m_t = 165$ GeV. The predicted values are within the reach of future experiments. There can be large effects from the anomalous gauge boson interactions on ϵ'/ϵ, and hence measurement of ϵ'/ϵ can provide useful information about the anomalous gauge boson couplings.

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