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Equivalence theorem and probing the electroweak symmetry-breaking sector

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We examine the Lorentz noninvariance ambiguity in longitudinal weak-boson scatterings and the precise conditions for the validity of the equivalence theorem (ET). *Safe* Lorentz frames for applying the ET are defined, and the intrinsic connection between the longitudinal weak-boson scatterings and probing the symmetry-breaking sector is analyzed. A universal precise formulation of the ET is presented for both the standard model and the chiral Lagrangian formulated electroweak theories. It is shown that in electroweak theories with a strongly interacting symmetry-breaking sector, the longitudinal weak-boson scattering amplitude *under proper conditions* can be replaced by the corresponding Goldstone-boson scattering amplitude in which all the internal weak-boson lines and fermion loops are ignored.

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I. INTRODUCTION

The electroweak gauge symmetry is spontaneously broken. As a consequence of absorbing the corresponding spin-0 would-be Goldstone bosons (GB's), the spin-1 weak bosons acquire masses and their longitudinal components V_L^a $(= W_L^\pm, Z_L^0)$ become physical degrees of freedom. While the transverse components V_T^a $(= W_T^\pm, Z_T^0)$ are irrelevant to the symmetry-breaking (SB) mechanism, the interactions of the longitudinal weak bosons (V_L^a's) are expected to be sensitive to probing the SB sector.

Technically, the electroweak equivalence theorem (ET) is used to give a quantitative relation between the V_L amplitude and the corresponding GB amplitude in the high energy region ($E \gg M_W$), as shown in Refs. [1–7]. The most rigorous relation between these two amplitudes (including all the possible multiplicative and additive factors) is given by a general identity, Eq. (1) or (2) in this paper, derived at the level of the Lehmann-Symanzik-Zimmermann (LSZ) reduced S-matrix elements[1] [5]. Based upon this identity we derive the precise formulation of the ET which is given in this paper as the ensemble of Eqs. (10), (10a), and (10b). By this formulation we show that the ET is not just a technical tool in calculating a V_L amplitude using a GB amplitude; it has an even more profound physical content for being able to discriminate processes which are insensitive to probing the electroweak SB sector.[2]

We know that the physical V_L amplitude can be measured by experiments and the GB amplitude, though not directly measurable, carries information about the SB sector. Hence, physically, the ET as a bridge tells us how the V_L-scattering experiments probe the SB sector, while technically, it replaces the calculation of the V_L amplitude by a much simpler calculation of the scalar GB amplitude in certain energy regime where their difference can be safely ignored.[3] The formulation of the ET in the standard model (SM) and in the chiral-Lagrangian-formulated electroweak theories (CLEWT's) has been recently given in Refs. [4–6], where the quantization effects and problems related to the renormalization scheme and

*Mailing address.

[1] Similar identities without the multiplicative factor $C_{\rm mod}^a$ were given in the early literature [1,2]. The appearance of the factor $C_{\rm mod}^a$ has been revealed in Refs. [3–7]. Here we shall adopt the form of the identity generally derived in Ref. [5]. Other related forms may be found in Refs. [3–7].

[2] To our knowledge, this point of view has not been given in the previous literature [1–7].

[3] This is an essential simplification since the V_L amplitude is even much more involved than the V_T amplitude due to the nontrivial cancellations of large E-power factors from the longitudinal polarization vectors in the high energy region. This fact was first revealed by Chanowitz and Gaillard [2].

the gauge-parameter dependence have been systematically studied.[4]

There is, however, another important problem in this subject which has not yet been carefully examined. It is about the Lorentz noninvariance ambiguity in the V_L-scattering amplitudes. We noticed that the spin-0 GB's (and thus the GB amplitudes) are invariant under the proper Lorentz transformation, but both the longitudinal and the transverse components of the *spin-1 massive* weak bosons (and thus their scattering amplitudes) are Lorentz noninvariant (LNI). After a Lorentz transformation, the longitudinal component may mix with the transverse components, and hence the original V_L amplitude will become a mixture of longitudinal and transverse amplitudes. Undoubtedly, one can even Lorentz transform a longitudinal component into a pure transverse one.[5] Thus a conceptual and fundamental question arises: How can we use the LNI V_L amplitudes to probe the electroweak SB sector of which the physical mechanism should clearly be independent of the choices of the Lorentz frames? In this paper, starting from a careful examination of this problem, we construct a universal precise formulation of the ET which shows that the V_L amplitudes can probe the electroweak SB sector unambiguously as long as certain general conditions, as in Eqs. (10a) and (10b), are satisfied.

Generally speaking, the *replacement* between the V_L amplitude and the GB amplitude (with possible multiplicative factors) is LNI unless the LNI part in the V_L amplitude can be ignored. This LNI part has the *same origin* as the transverse amplitudes because they can mix or turn into each other under Lorentz transformations. Hence, the physically important and interesting object is the *Lorentz-invariant* (LI) *part of the V_L amplitude.* When we use the GB amplitude to predict the physical V_L amplitude measured by experiments, it does not distinguish the difference between the experimental results from different Lorentz frames. Thus, by estimating the LNI part in the V_L amplitude we can determine the accuracy and the validity region of our quantitative predictions for the physical V_L amplitudes based on the ET. We emphasize that the content of the precise formulation of the ET is more than just a technical tool for simplifying the calculations of the V_L amplitude. The importance of the ET is first because it provides a conceptual connection between the would-be Goldstone-boson amplitudes directly related to the SB mechanism and the experimentally measurable longitudinal weak-boson amplitudes. Second, as a technical tool, it may simplify the calculation of the V_L amplitude, which, however, can always be directly calculated in spite of its complexity. Hence the most important task is to find out the conditions under which the LNI part of the V_L amplitude can be safely ignored and the LI part becomes dominant in the experimentally measured V_L amplitudes so that the physical V_L scatterings can be used to sensitively and unambiguously probe the electroweak SB sector.

[4]A study of the ET in the CLEWT using a nonlinear gauge quantization procedure was recently done in Ref. [7].

[5]This can be done by, for example, first boosting V_L to its rest frame and then boosting it in a direction perpendicular to the first boost.

II. AVOIDANCE OF LORENTZ NONINVARIANCE AMBIGUITY AND THE UNIVERSAL PRECISE FORMULATION OF THE ET

Let us start from the Ward-Takahashi identity derived in Refs. [2–5]:

$$\langle 0\,|F_0^{a_1}(k_1)\,F_0^{a_2}(k_2)\cdots F_0^{a_n}(k_n)\,\Phi_\alpha|\,0\rangle = 0 \ ,$$

in which F_0^a is the bare gauge-fixing function and Φ_α denotes other possible physical in or out states.[6] After a rigorous LSZ reduction for the external F^a lines, we derived in Ref. [5] the following general identity for the renormalized S-matrix elements:

$$\begin{aligned}
&T[V_L^{a_1},\dots,V_L^{a_n};\Phi_\alpha] = T[\bar{Q}^{a_1},\dots,\bar{Q}^{a_n};\Phi_\alpha],\\
&\bar{Q}^a \equiv -iC^a_{\rm mod}\pi^a + v^a \ , \qquad v^a \equiv v^\mu V^a_\mu \ ,\\
&v^\mu \equiv \epsilon^\mu_L - k^\mu/M_W = O(M_W/E) \ , \qquad (1)
\end{aligned}$$

where π^a's are GB fields.[7] (In this paper, we use W to denote either $W^\pm$ or Z, and E is the energy of the W boson, unless specified otherwise.) The finite constant modification factor $C^a_{\rm mod}$ has been systematically studied in the literature [3–7] and is proved to be renormalization-scheme and gauge-parameter dependent.[8] In general, $C^a_{\rm mod}$ is not unity and the difference $C^a_{\rm mod} - 1$ comes from loop contributions [3–7]. A convenient renormalization scheme, scheme II, was constructed in Refs. [4–6] so that the modification factors $C^a_{\rm mod}$ in both the SM and the CLEWT are exactly unity, and the application of the ET is greatly simplified. It has also been shown that $C^a_{\rm mod}-1 = O((g^2,\lambda)/16\pi^2)$ for the SM with a light Higgs boson [3–5], and $C^a_{\rm mod}-1 = O(g^2/16\pi^2)$ for both the heavy Higgs SM [3–5] and the CLEWT [6], provided that the GB wave function renormalization constant Z_{π^a} is subtracted at a scale $\mu = O(M_W)$ and the physical mass pole of weak-boson propagator is determined by the on-shell scheme.

The identity in (1) can be rewritten as

$$T[V_L^{a_1},\dots,V_L^{a_n};\Phi_\alpha] = C\cdot T[-i\pi^{a_1},\dots,-i\pi^{a_n};\Phi_\alpha] + B, \qquad (2)$$

where

[6]The subscript α denotes possible Lorentz indices.

[7]Here, the π^a field by definition has an opposite sign to that in Ref. [5]. Consequently, the coefficient of π^a in $\bar{Q}^a$ is $-i$ instead of $+i$.

[8]The $C^a_{\rm mod}$ factor has also been examined for the U(1) Higgs theory in Refs. [4,5,8].

$$C \equiv C^{a_1}_{\rm mod} \cdots C^{a_n}_{\rm mod} \ , \tag{2a}$$

$$B \equiv B[v, -i\pi; \Phi_\alpha]$$
$$\equiv \sum_{l=1}^{n} (C^{a_{l+1}}_{\rm mod} \cdots C^{a_n}_{\rm mod} T[v^{a_1}, \ldots, v^{a_l}, -i\pi^{a_{l+1}}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + \text{permutations of } v\text{'s and } \pi\text{'s}) \ . \tag{2b}$$

Hereafter we shall use the shorthand notation $T[V_L; \Phi_\alpha]$ and $T[-i\pi; \Phi_\alpha]$ for $T[V_L^{a_1}, \ldots, V_L^{a_n}; \Phi_\alpha]$ and $T[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha]$, respectively. Under Lorentz transformations, the amplitude of spin-0 scalar particles is invariant. If Φ_α, in (2), contains no field or only external physical scalar field(s) and/or photons, then from (2) the Lorentz-noninvariant V_L amplitude can be decomposed into two parts. The first part is $C \cdot T[-i\pi; \Phi_\alpha]$ which is Lorentz invariant (LI), and the second part is the v_μ-suppressed B term which is Lorentz noninvariant (LNI) because of the external spin-1 massive vector field(s). Such a decomposition clearly shows the essential difference between the V_L amplitude and the V_T amplitude since the former contains a Lorentz-invariant GB amplitude which is the intrinsic source causing a large-V_L amplitude in the case of strongly coupled SB sector. We note that only the LI part of the V_L amplitude is sensitive to probing the SB sector, while its LNI part contains a significant *Lorentz-frame-dependent* B term and therefore cannot be sensitive to the electroweak SB mechanism.

Strictly speaking, when Φ_α contains field(s) such as V_T's and fermions, the GB amplitude is not exactly LI due to nontrivial Lorentz transformations of Φ_α. The change of the GB amplitude due to Lorentz transformations of Φ_α may not be small when compared with the GB amplitude itself. For instance, if Φ_α contains a V_T field, this change can be of the same order of magnitude as the GB amplitude itself because after a Lorentz transformation the mixed GB amplitude (with one external V_T replaced by V_L) is only suppressed by $O(M_W/E)$ [see the 2nd relation in Eq. (7)], and this suppression factor is largely compensated by the enhancement factor $O(E/M_W)$ arising from the polarization vector of the resulting V_L. For a fermion field in Φ_α, it is easy to see that this change is always $O(m_f/E)$ suppressed because this change vanishes in the $m_f/E \to 0$ limit. (Here, m_f and E are the mass and energy of the fermion, respectively.) Since the basic properties of the physical mechanism of the electroweak SB sector are clearly independent of Lorentz frames, this LNI GB amplitude [due to the LNI Φ_α field(s)] would be less sensitive to probing the SB mechanism.[9] In the case of strongly coupled SB sector, the extra V_T('s) and/or fermion field(s) in Φ_α make the leading contribution of the GB amplitude contain more pure gauge couplings and/or Yukawa couplings (of the SM fermions) and lower E-power dependence. Taking the CLEWT as an example, we easily see that only the pure scalar vertices contain the largest E-power dependence, while all other vertices containing gauge bosons and/or fermions involve less derivatives and more gauge and/or Yukawa couplings. Therefore, in each order of perturbative expansion, the GB amplitude containing the extra V_T('s) and/or fermion field(s) in Φ_α is at least $O(M_W/E)$ or $O(m_f/E)$ suppressed relative to the pure GB amplitude (containing no external V_T and/or fermion fields).[10]

Despite the fact that Φ_α might contain some LNI contributions, it will not cause the longitudinal-transverse ambiguity in replacing a longitudinal weak-boson line in the V_L amplitude by a corresponding Goldstone-boson line in the GB amplitude as long as the LNI B term can be safely ignored. Thus, we have to find the conditions under which the B term in (2) is negligibly small compared with the $C \cdot T[-i\pi; \Phi_\alpha]$ term. *Such conditions can be conveniently found from (2) by estimating the magnitude of the B term from the analysis of the Lorentz transformation of the V_L amplitude.* To estimate the B term, we first examine how the V_L amplitude transforms under Lorentz transformations.[11] Without loss of generality, let us consider a Lorentz boost with velocity β_0 along the $\hat{x}$ direction (from $oxyzt$ frame to $o'x'y'z't'$ frame) for an external longitudinal boson V_L (and also an external transverse boson) with momentum $k^\mu = (E, 0, 0, k)$ in $oxyzt$ frame:[12]

$$\begin{array}{ll}
\text{in } oxyzt \text{ frame} & \text{in } o'x'y'z't' \text{ frame} \\
k^\mu = (E,0,0,k) \ , & k'^\mu = (\gamma_0 E, -\beta_0\gamma_0 E, 0, k) \ , \\
\epsilon_L^\mu = \frac{1}{M_W}(k,0,0,E) \ , & {\epsilon'_{(L)}}^\mu = \frac{1}{M_W}(\gamma_0 k, -\beta_0\gamma_0 k, 0, E) \ , \\
\epsilon_{T_1}^\mu = (0,1,0,0) \ , & {\epsilon'_{(T_1)}}^\mu = (-\gamma_0\beta_0, \gamma_0, 0, 0) \ , \\
\epsilon_{T_2}^\mu = (0,0,1,0) \ , & {\epsilon'_{(T_2)}}^\mu = (0,0,1,0) \ .
\end{array} \tag{3}$$

The three new polarization vectors in the $o'x'y'z't'$ frame are defined as

[9]One exception is the top-condensate SM [9] in which the top quark Yukawa coupling is related to the Higgs boson self-couplings. For $m_t = O(M_W)$, this model must predict a light Higgs boson which can be detected through processes other than the V_L scatterings.

[10]The heaviest known external fermions are (anti)top quarks. Thus $O(m_f/E) \le O(m_t/E) = O(M_W/E)$.

[11]We thank Lay Nam Chang for enlightening discussions on this point.

[12]Equivalently, one can study the Lorentz transformation relation of the spin-1 helicity amplitudes by using the spin-rotation matrices as shown in Ref. [10]. But, here, for the purpose of order of magnitude estimate, it is more convenient to study the Lorentz transformations of the longitudinal polarization vector ϵ_L^μ and the transverse polarization vector ϵ_T^μ.

$$\epsilon_L'^{\mu} = \frac{1}{M_W}(a, -\beta_0\gamma_0^2 E^2/a, 0, \gamma_0 Ek/a) \ , \quad \epsilon_{T_1}'^{\mu} = (0, k/a, 0, \beta_0\gamma_0 E/a) \ , \quad \epsilon_{T_2}'^{\mu} = (0,0,1,0) \ , \tag{4}$$

where $a \equiv \sqrt{(k^2+\beta_0^2\gamma_0^2E^2)}$, $\gamma_0 = 1/\sqrt{1-\beta_0^2}$, and $k'\cdot\epsilon_\lambda' = 0$, for $\lambda = L, T_1, T_2$. After a little algebra, we get

$$\begin{aligned}
&\epsilon_{(L)}'^{\mu} - \epsilon_L'^{\mu} = b_L \epsilon_L'^{\mu} + \sum_{j=1}^{2} b_{T_j}\epsilon'^{\mu}_{T_j} \ , \quad \epsilon'^{\mu}_{(T_i)} - \epsilon'^{\mu}_{T_i} = \sum_{j=1}^{2} h_{i,T_j}\epsilon'^{\mu}_{T_j} + h_{i,L}\epsilon'^{\mu}_L \ , \\
&b_L = \gamma_0 k/a - 1 \ , \quad b_{T_1} = \beta_0\gamma_0 M_W/a \ , \quad b_{T_2} = 0 \ , \\
&h_{1,T_1} = \gamma_0 k/a - 1 \ , \quad h_{1,T_2} = 0 \ , \quad h_{i,L} = -\beta_0\gamma_0 M_W/a \ , \quad h_{2,T_j} = h_{2,L} = 0 \ .
\end{aligned} \tag{5}$$

Hence, for high energy scattering $E \sim k \gg M_W$, we generally have

$$b_L = O(M_W^2/E^2) \ , \quad b_{T_j} \le O(M_W/E) \ ; \quad h_{i,T_j} \le O(M_W^2/E^2) \ , \quad h_{i,L} = O(M_W/E) \ , \tag{6}$$

where we have taken $\gamma_0 \gtrsim O(1)$. Thus, for a boosted external weak-boson field,

$$\begin{aligned}
V_{(L)}^{a}{}' &= \epsilon_{(L)}^{\mu}{}' V_\mu^{a\prime} = \left[1 + O\left(\frac{M_W^2}{E^2}\right)\right] V_L^{a\prime} + \sum_{j=1}^{2} O\left(\frac{M_W}{E}\right) V_{T_j}^{a}{}' \ , \\
V_{(T_i)}^{a}{}' &= \epsilon_{(T_i)}^{\mu}{}' V_\mu^{a\prime} = \sum_{j=1}^{2}\left[1 + O\left(\frac{M_W^2}{E^2}\right)\right] V_{T_j}^{a}{}' + O\left(\frac{M_W}{E}\right) V_L^{a\prime} \ .
\end{aligned} \tag{7}$$

Now, consider the variation $\Delta B \equiv B[(v)', -i\pi; \Phi'_{(\alpha)}] - B[v', -i\pi; \Phi'_\alpha]$, which is the difference between the boosted amplitude $B[(v)', -i\pi; \Phi'_{(\alpha)}]$ and the corresponding amplitude $B[v', -i\pi; \Phi'_\alpha]$ defined in the $o'x'y'z't'$ frame. Since the LNI B term does not contain LI spin-0 scalar subset which is the only intrinsic source that may cause the V_L amplitude to be large, the variation ΔB should be of the same order of magnitude as B term itself, i.e.,

$$O(\Delta B) = O(B[(v)', -i\pi; \Phi'_{(\alpha)}]) = O(B[v', -i\pi; \Phi'_\alpha]) = O(B[v, -i\pi; \Phi_\alpha]) \ .$$

Thus we can estimate B by estimating ΔB. From (2) and (7),

$$\begin{aligned}
\Delta B &\equiv B[(v)', -i\pi; \Phi'_{(\alpha)}] - B[v', -i\pi; \Phi'_\alpha] = T[V'_{(L)}; \Phi'_{(\alpha)}] - T[V'_L; \Phi'_\alpha] - C\cdot T[-i\pi; \Phi'_{(\alpha)} - \Phi'_\alpha] \\
&\equiv T[V'_L + \Delta V'_L; \Phi'_\alpha + \Delta\Phi'_\alpha] - T[V'_L; \Phi'_\alpha] - C\cdot T[-i\pi; \Delta\Phi'_\alpha] \\
&= T[\Delta V'_L; \Phi'_\alpha] + (T[\Delta V'_L; \Delta\Phi'_\alpha] + B[v', -i\pi; \Delta\Phi'_\alpha]) \quad \text{[cf. (2)]} \\
&= O(T[\Delta V'_L; \Phi'_\alpha]) \\
&= O\left(\frac{M_W^2}{E_j^2}\right) T[V_L'^{a_1}, \ldots, V_L'^{a_n}; \Phi'_\alpha] + O\left(\frac{M_W}{E_j}\right) T[V_{T_j}'^{a_{r_1}}, V_L'^{a_{r_2}}, \ldots, V_L'^{a_{r_n}}; \Phi'_\alpha] \qquad \text{[cf. (7)]} \\
&= O\left(\frac{M_W^2}{E_j^2}\right) C\cdot T[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + O\left(\frac{M_W}{E_j}\right) C'\cdot T[V_{T_j}^{a_{r_1}}, -i\pi^{a_{r_2}}, \ldots, -i\pi^{a_{r_n}}; \Phi_\alpha] \qquad \text{[cf. (2)]} \\
C &= C_{\rm mod}^{a_1}\cdots C_{\rm mod}^{a_n} \ , \qquad C' = C_{\rm mod}^{a_{r_2}}\cdots C_{mod}^{a_{r_n}} \ .
\end{aligned} \tag{8}$$

Here, in estimating the order of magnitude of ΔB, we have ignored $T[\Delta V'_L; \Delta\Phi'_\alpha]$ and $B[v', -i\pi, \Delta\Phi'_\alpha]$, which vanish when Φ_α contains no field or only scalar(s) and/or photon(s), and can be at most of the same order of magnitude as the B term itself. For the same reason, we have also neglected the LNI parts generated from replacing $V_{T_j}'^{a_{r_1}}$ and Φ'_α by $V_{T_j}{}^{a_{r_1}}$ and Φ_α in the last step of (8). Let E_j be the energy of the jth external longitudinal weak boson. We can thus estimate the order of magnitude of B from (8) by making the M_W/E_j expansion when $E_j \sim k_j \gg M_W$. Then,[13]

$$\begin{aligned}
B &= \sum_{l=1}^{n}(C_{\rm mod}^{a_{l+1}}\cdots C_{\rm mod}^{a_n} T[v^{a_1}, \cdots, v^{a_l}, -i\pi^{a_{l+1}}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + \text{permutations of } v\text{'s and } \pi\text{'s}) \\
&= O\left(\frac{M_W^2}{E_j^2}\right) C\cdot T[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + O\left(\frac{M_W}{E_j}\right) C'\cdot T[V_{T_j}^{a_{r_1}}, -i\pi^{a_{r_2}}, \ldots, -i\pi^{a_{r_n}}; \Phi_\alpha] \ .
\end{aligned} \tag{9}$$

[13]As we know, this is the first time that the order of magnitude of the B term is explicitly given in a general form.

We emphasize that the condition $E_j \sim k_j \gg M_W \quad (j=1,2,\ldots,n)$ *for each external longitudinal weak boson* is *necessary* in making the M_W/E_j expansion and ensuring the B term (and its Lorentz variation) to be much smaller than $C \cdot T[-i\pi;\Phi_\alpha]$, as shown in (2). If the energy of one of the $V_L^{a_j}$'s is low, say, $E_j \sim k_j = O(M_W)$, then a Lorentz transformation may cause large variations in the V_L amplitude and the Lorentz-frame-dependent B term can be as large as $C \cdot T[-i\pi;\Phi_\alpha]$, even in the cases where the total energy of the scattering has already been much larger than M_W.

In conclusion, we give the general and precise formulation of the ET,[14]

$$T[V_L^{a_1},\ldots,V_L^{a_n};\Phi_\alpha] = C \cdot T[-i\pi^{a_1},\ldots,-i\pi^{a_n};\Phi_\alpha] + O(M_W/E_j \text{ suppressed}), \tag{10}$$

and, from Eqs. (2b) and (9), the conditions for ignoring the LNI and M_W/E_j-suppressed B term on the right-hand side (RHS) of (10) are

$$E_j \sim k_j \gg M_W \qquad (j=1,2,\ldots,n) \ , \tag{10a}$$

$$B \ll C \cdot T[-i\pi^{a_1},\ldots,-i\pi^{a_n};\Phi_\alpha] \ . \tag{10b}$$

Before going into detailed discussions, we first point out several important features contained in the above formulation. First, the second term on the RHS of (10), i.e., the B term, as emphasized is only $O(M_W/E_j\text{—suppressed})$ relative to the leading contributions in $C \cdot T[-i\pi;\Phi_\alpha]$, and therefore is *not necessarily* of the $O(M_W/E_j)$ in magnitude. As clearly shown in (9), the magnitude of the B term explicitly depends on the size of the amplitudes $T[-i\pi^{a_1},\ldots,-i\pi^{a_n};\Phi_\alpha]$ and $T[V_{T_j}^{a_{r_1}},-i\pi^{a_{r_2}},\ldots,-i\pi^{a_{r_n}};\Phi_\alpha]$. Consequently, *the B term itself can be either larger or smaller than* $O(M_W/E_j)$. For example, as we shall prove in the following, the largest B term in the CLEWT is of $O(g^2)$; cf. Eq. (17). Second, the actual suppression factor in the B term is M_W/E_j instead of $M_W/\sqrt{s}$ as appeared in some current literature. ($\sqrt{s}$ is the total center-of-mass energy of the scattering.) So condition (10a) is usually stronger than $\sqrt{s} \gg M_W$. The existence of the condition (10b) for the CLEWT has been recently pointed out in Refs. [6,7]. Here we emphasize that (10b) generally exists for *any perturbation expansion*, not only for the chiral perturbation expansion, but also for the usual loop expansion (adopted in the SM) and the large-$\mathcal{N}$ expansion, etc.[15] This will be examined in detail later. Third, the *equivalence* theorem is about the "equivalence" between the V_L amplitude and the GB amplitude (not the GB amplitude plus the B term). Therefore it is important to give explicit conditions, i.e., (10a) and (10b), under which the M_W/E_j-suppressed B term in (10) can be ignored to establish the equivalence between the V_L amplitude and the GB amplitude. It is clear that one can technically improve the prediction of the V_L amplitude from the RHS of (10) by including the complicated B term (or part of B) [11], but this is not an improvement of the equivalence between the V_L and the GB amplitudes. As noted in our above discussion, the LNI B term has the same origin as the transverse amplitudes and is thus insensitive to probing the electroweak SB sector. More specifically, even for the CLEWT with strongly coupled SB sector, the largest B term is of $O(g^2)$ [cf. Eq.(17) or (21)], which depends only on the electroweak gauge coupling and is not sensitive to the interactions responsible for the electroweak symmetry breaking. (The same conclusion holds for the leading amplitudes of pure transverse gauge boson scatterings.) Therefore, for the longitudinal weak-boson scattering processes to be sensitive to the electroweak SB sector, conditions (10a) and (10b) must be satisfied such that the scalar GB amplitudes can *dominate* the contributions to the V_L amplitudes. *This physical content is essentially independent of how to technically compute V_L amplitudes.*

Let us further analyze the important implications of Eqs. (10a) and (10b) in detail. First, we note that condition (10a) defines the *safe* Lorentz frames for the precise formulation and the application of the ET. As we pointed out, a longitudinal weak boson can turn into a mixture of longitudinal and transverse states under Lorentz transformations while the scalar Goldstone boson is invariant. This implies that (10) cannot hold in all Lorentz frames. To resolve this longitudinal-transverse ambiguity, a set of *safe* Lorentz frame has to be defined such that for *each* external V_L particle $E_j \gg M_W$.[16] This means that V_L *is sensitive to probing the* SB *sector only in the sufficiently high energy region where the V_L, originally coming from "eating" the* GB, *mainly behaves like the* GB, *and the effects of its mixing with the transverse components are always M_W/E_j or $(M_W/E_j)^2$ suppressed and negligibly small.* If we change this high energy property by making Lorentz transformations such that $M_W/E_j = O(1)$, this longitudinal-transverse ambiguity can no longer be ignored and the LNI part of $T[V_L;\Phi_\alpha]$ will be of the same order of magnitude as the LI part of $T[-i\pi;\Phi_\alpha]$ [cf. (9)].

[14]Here we still generally keep the modification C factor in the ET. The exact simplification of the C factor as unity has been given before for both the SM [4,5] and the CLEWT [6].

[15]This general fact, as we know, has not been revealed before.

[16]Here we do not take the *unphysical limit* as $M_W(= gf_\pi/2) \to 0$, which requires either the gauge coupling $g=0$, implying no Higgs mechanism and the disappearance of physical longitudinal component of the W boson, or the vacuum expectation value $f_\pi = 0$, in contradiction with the nonvanishing physical Fermi scale and the presence of the electroweak symmetry breaking. Such limits are actually *unnecessary* for the precise formulation of the ET.

The condition (10a) is actually quite strong. Naively one may expect that requiring the total center-of-mass (c.m.) energy $E_{\rm c.m.} \gg M_W$ can always guarantee the equivalence of the V_L amplitude and the GB amplitude. However, we shall show as follows that even in the SM, there are counterexamples to this weaker condition in which only $E_{\rm c.m.} \gg M_W$ is satisfied but (10a) is violated. Subsequently, Eq. (10) does not hold. To illustrate this point, we consider the scattering process $Z_L + H \to Z_L + H$, where H is the SM Higgs particle. In the c.m. frame of $Z_L H$, the exact tree-level Z_L and GB amplitudes are

$$
\begin{aligned}
T[Z_LH \to Z_LH] &= ig^2\left[\frac{p^2(1-\cos\theta)-M_Z^2\cos\theta}{2M_Z^2}\,\frac{t+2m_H^2}{t-m_H^2} + [p^2(1-\cos\theta)-M_Z^2\cos\theta]\right.\\
&\quad \left.\times\left(\frac{1}{u-M_Z^2}+\frac{1}{s-M_Z^2}\right) - \frac{p^2}{M_Z^2}\left(\frac{(\cos\theta\sqrt{p^2+M_Z^2}+\sqrt{p^2+m_H^2})^2}{u-M_Z^2}+\frac{s}{s-M_Z^2}\right)\right] ,\\
T[\pi^0H \to \pi^0H] &= i\left[-\frac{m_H^2}{f_\pi^2}\,\frac{t+2m_H^2}{t-m_H^2} - \frac{m_H^4}{f_\pi^2}\left(\frac{1}{u-M_Z^2}+\frac{1}{s-M_Z^2}\right) + \frac{g^2}{4}\left(\frac{s-t}{u-M_Z^2}+\frac{u-t}{s-M_Z^2}\right)\right] ,
\end{aligned}
\tag{11}
$$

where p is the c.m. momentum, θ is the scattering angle, and s, t, u are the Mandelstam variables. We consider two typical high energy limits: $E_{\rm c.m.} \gg m_H \sim M_Z$ and $E_{\rm c.m.} > m_H \gg M_Z$, where $E_{\rm c.m.} = \sqrt{s}$ is the total energy. In the first case, the energy of the Z boson, $E_Z \sim p \gg M_Z$ so that our new condition (10a) is satisfied; while in the second case $E_Z \sim p \sim O(M_Z)$, which violates the (10a). In both cases the conventional condition $E_{\rm c.m.} \gg M_Z$ is satisfied.

(i) For the first case $E_{\rm c.m.} \gg m_H \sim M_Z$, which implies $E_Z \sim p \gg M_Z$, Eq. (11) gives

$$
\begin{aligned}
T[Z_LH \to Z_LH] &= -i\left[\frac{m_H^2}{f_\pi^2}+\frac{g^2}{4}\,\frac{3+\cos^2\theta}{1+\cos\theta}\right] + O(g^2M_Z^2/p^2, \lambda m_H^2/p^2) ,\\
T[\pi^0H \to \pi^0H] &= -i\left[\frac{m_H^2}{f_\pi^2}+\frac{g^2}{4}\,\frac{3+\cos^2\theta}{1+\cos\theta}\right] + O(g^2M_Z^2/p^2, \lambda m_H^2/p^2) ,\\
T[Z_LH \to Z_LH] &= T[i\pi^0H \to -i\pi^0H] + O(g^2M_Z^2/p^2, \lambda m_H^2/p^2) .
\end{aligned}
\tag{12}
$$

Thus, the V_L amplitude is equivalent to the GB amplitude, and can be used to probe the SB sector. In this case, the c.m. frame is a *safe* frame in applying the ET.

(ii) For the second case[17] $E_{\rm c.m.} > m_H \gg M_Z$, which implies $E_Z \sim p = O(M_Z)$, Eq. (11) gives

$$
\begin{aligned}
T[Z_LH \to Z_LH] &= i4\frac{(p^2+M_Z^2)\cos\theta - 3p^2}{f_\pi^2} + O(p/m_H, M_Z/m_H) ,\\
T[\pi^0H \to \pi^0H] &= i2\frac{-2p^2(1-\cos\theta)+M_Z^2}{f_\pi^2} + O(p/m_H, M_Z/m_H) ,\\
T[Z_LH \to Z_LH] - T[i\pi^0H \to -i\pi^0H] &= i2\frac{-4p^2+M_Z^2(2\cos\theta-1)}{f_\pi^2} + O(p/m_H, M_Z/m_H) .
\end{aligned}
\tag{13}
$$

As shown in the above equations, the difference between the V_L amplitude and the GB amplitude has the same size as the V_L amplitude itself. Thus, the V_L amplitude is not equivalent to the GB amplitude. The c.m. frame in this case is therefore *not a safe* frame for applying the ET because in this frame our condition (10a) is violated.

Next, we examine condition (10b) for ignoring the LNI B term, which is the sum of all the v_μ-suppressed terms in (2). Based upon the order of magnitude estimate of the B term given in Eq.(9), we can further express the (10b) as

$$
O\left(\frac{M_W^2}{E_j^2}\right) T[-i\pi^{a_1},\ldots,-i\pi^{a_n};\Phi_\alpha] + O\left(\frac{M_W}{E_j}\right) T[V_{T_j}^{a_{r_1}}, -i\pi^{a_{r_2}},\ldots,-i\pi^{a_{r_n}};\Phi_\alpha] \ll T[-i\pi^{a_1},\ldots,-i\pi^{a_n};\Phi_\alpha] . \tag{14}
$$

[17]For example, $E_{\rm c.m.} = 1$ TeV, $m_H = 800$ GeV.

Here we have dropped the factor $1/C^{a_{r_1}}_{\rm mod}$ in the second term on the LHS since we can always adopt the scheme II of Refs. [4–6] to make $C^{a}_{\rm mod} \equiv 1$. Even in some other schemes as described in the paragraph just below Eq. (1), $C_{\rm mod} - 1$ is of $O((g^2,\lambda)/16\pi^2)$ and $O(g^2/16\pi^2)$ for the light Higgs SM and the heavy Higgs SM (or the CLEWT), respectively, so that $1/C^{a_{r_1}}_{\rm mod}$ will not affect the order of magnitude estimate on the LHS of (14) since only the leading terms are relevant. Condition (14) shows that after ignoring the B term, we only need to keep in the GB amplitude the contributions that satisfy the condition in (14). If we further make a perturbative expansion on the GB amplitude, (14) would then constrain the smallest term to be included in the GB amplitude for a fixed energy or the lowest energy required to calculate the GB amplitude to a desired accuracy.

In perturbative calculations, we may make loop expansion with the expansion parameter $\hbar$, the momentum expansion with the expansion parameter E/Λ, or the large-$\mathcal{N}$ expansion with the expansion parameter $1/\mathcal{N}$, etc. Practically we can only calculate the amplitude T to a *finite* order in the perturbation expansion, i.e., $T = \sum_{\ell=0}^{N} T_\ell = \sum_{\ell=0}^{N} \bar{T}_\ell \alpha^\ell$, where α denotes the expansion parameter. In perturbative expansion, we have $T_0 > T_1,\ T_2,\dots,T_N$. Let $T_{\rm min}$ be the smallest one in the set $\{T_0,T_1,\dots,T_N\}$. Condition (14) then implies[18]

$$O\left(\frac{M_W^2}{E_j^2}\right) T_0[-i\pi^{a_1},\dots,-i\pi^{a_n};\Phi_\alpha] + O\left(\frac{M_W}{E_j}\right) T_0[V^{a_{r_1}}_{T_j},-i\pi^{a_{r_2}},\dots,-i\pi^{a_{r_n}};\Phi_\alpha] \ll T_{\rm min}[-i\pi^{a_1},\dots,-i\pi^{a_n};\Phi_\alpha]. \tag{15}$$

When $N = 0$, i.e., only the leading order in the expansion is kept, (15) reduces to (10a). Hence, *to leading order in any perturbative expansion, the condition* (10a) *is always sufficient to ensure the smallness of the* B *term.* The extra condition (15) is nontrivial only if higher order contributions are included.[19] This is why in many previous tree-level calculations for the V_L amplitudes the ET was found to work well after condition (10a) is satisfied. Actually, when applying the ET to any perturbation theory, two kinds of expansions have to be considered: One is the expansion in α, the intrinsic expansion parameter of the theory itself; another is the expansion in power of M_W/E_j, as required by the ET [cf. Eq. (10)]. In the first expansion we usually try to include contributions beyond leading order, while in the second expansion we always keep only the leading order term for both the physical and the technical reasons explained above. The condition (15) is required to ensure the M_W/E_j-suppressed B terms from leading order in α to be much smaller than the smallest term $T_{\rm min}[-i\pi;\Phi_\alpha]$ kept in the GB amplitude. If (15) is satisfied—i.e., (10b) is satisfied—the V_L amplitude is equivalent to the GB amplitude. Thus in this case, the V_L amplitude can be given by a much simpler calculation of the GB amplitude. This is the technical aspect of (10). Physically, *the applicability of (10) implies that this* V_L *amplitude is sensitive to probing the SB sector to the accuracy of* $T_{\rm min}[-i\pi;\Phi_\alpha]$. If (15) is not satisfied—i.e., the smallest term kept in the GB amplitude does not dominate the LNI and M_W/E_j-suppressed B term—then (10b) is not satisfied; therefore, (10) is not true. Hence, the V_L amplitude and the GB amplitude are not equivalent, and this V_L-scattering process cannot be sensitive to probing the electroweak SB sector to the accuracy of $T_{\rm min}[-i\pi;\Phi_\alpha]$. In addition to its *technique content* as a tool in simplifying the V_L amplitude calculations, the above formulation of the ET, Eqs. (10), (10a), and (10b), *has a profound physical content in discriminating processes which are insensitive to probing the electroweak SB sector to certain required precision.*

To illustrate the condition (15), we consider two typical examples with $N = 1$, i.e., up to next-to-leading order. They are the high energy $2 \to 2$ pure V_L scatterings predicted in the CLEWT and in the SM with a light Higgs boson ($m_H \ll E$). We shall work in the c.m. frame of V_L-V_L which is a safe Lorentz frame for $M_W \ll E$.

First, we examine (15) in the CLEWT, where the SB sector is nonlinearly realized and strongly interacting. Now T_0 and T_1 are the E^2-level and the E^4-level contributions, respectively. By a direct power counting [13], these scattering amplitudes are found to behave as

$$\begin{aligned} T_0[\pi^a\pi^b \to \pi^c\pi^d] &= O\left(\frac{E^2}{f_\pi^2}\right) \ , \\ T_0[V_T^a\pi^b \to \pi^c\pi^d] &= O\left(g\frac{E}{f_\pi}\right) \ , \\ T_1[\pi^a\pi^b \to \pi^c\pi^d] &= O\left(\frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2}\right) \ , \end{aligned} \tag{16}$$

where $\Lambda \simeq 4\pi f_\pi \simeq 3\,\text{TeV}$ is the cutoff of the CLEWT according to the usual dimensional analysis [14]. The order-of-magnitude estimates in (16) are easy to understand. For the amplitude $T_0[\pi^a\pi^b \to \pi^c\pi^d]$, it is just

[18]For special cases with *both* T_0 amplitudes on the LHS of (15) vanishing, the nontrivial condition is given via replacing the two T_0 amplitudes by corresponding higher order amplitudes of maximum values among $T_1,\dots,T_N$. In this case, (15) simply reduces to (10a) up to next-to-leading order. Explicit examples of such kind are discussed in detail elsewhere.

[19]For example, in the $1/\mathcal{N}$ expansion formalism, some previous studies [12] applied the ET only to leading order so that condition (15) is unnecessary there. The specific form of (15) in the $1/\mathcal{N}$-expansion beyond leading order will be given elsewhere.

the standard low energy theorem result [15], where the dimensionful scale factor in the denominator is f_π^2, not $\Lambda^2 \simeq (4\pi f_\pi)^2$. The amplitude $T_0[V_T^a \pi^b \to \pi^c \pi^d]$ with one external transverse gauge boson can at most be of $O(g\frac{E}{f_\pi})$ because any vertex with only one gauge boson line will contain a factor g and one less partial derivative than the corresponding GB vertex. The next-to-leading order amplitude $T_1[\pi^a \pi^b \to \pi^c \pi^d]$ is well known to be E^2/Λ^2 suppressed relative to the leading order contribution $T_0[\pi^a \pi^b \to \pi^c \pi^d]$ due to the momentum expansion in the CLEWT. Substituting (16) into (15), we find that the largest B term gives

$$B = O(g^2) \ , \tag{17}$$

which also coincides with a previous explicit calculation for the $W_L^+ W_L^- \to Z_L Z_L$ scattering [16]. Thus, the condition (15) for ignoring the B term in the CLEWT is[20] $O(g^2) \ll \frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2}$. After replacing g^2 by $(2M_W/f_\pi)^2$, we obtain

$$\frac{M_W^2}{E^2} \ll \frac{1}{4}\frac{E^2}{\Lambda^2} \qquad \text{or} \qquad (0.7\,\text{TeV}/E)^4 \ll 1 \ . \tag{18}$$

From (18), we see that the higher the energy E is, the better the condition (18) is satisfied. For examples, for $E = 800\,\text{GeV}$, $1\,\text{TeV}$, and $1.3\,\text{TeV}$, Eq. (18) gives $0.56 \ll 1$, $0.23 \ll 1$, and $0.081 \ll 1$, respectively. These numerical results indicate that the ET technically works well if[21] $E \geq 1\,\text{TeV}$. Most importantly, it also tells us that in order to sensitively probe the strongly interacting SB sector, up to the order of E^4, we *must* raise the collider energy far beyond the TeV region so that there will be enough V_L-V_L luminosities in the TeV region for $V_L V_L \to V_L V_L$ scatterings. In this example, we assume that there is no light resonance (defined as a resonance with mass much less than 1 TeV) involved in the pure V_L scattering. Next, let us examine what if there is a resonance, such as a SM Higgs boson, far below 1 TeV.

In the case of the SM with $m_H, M_W \ll E$, the one-loop level $2 \to 2$ scattering amplitude T_1 is of the order [22]

$$T_1[\pi^{a_1}, \ldots, \pi^{a_4}] = O\left(\frac{g^2, \lambda}{16\pi^2}\right) \ T_0[\pi^{a_1}, \ldots, \pi^{a_4}] \ , \tag{19a}$$

$$T_0[V_{T_j}^{a_{r_1}}, \pi^{a_{r_2}}, \ldots, \pi^{a_{r_4}}] = O\left(\frac{M_W}{E}\right) \ T_0[\pi^{a_1}, \cdots, \pi^{a_4}] \ , \tag{19b}$$

where the factor $1/16\pi^2$ $[= \pi^2/(2\pi)^4]$ is the characteristic of each loop correction.[23] Thus (15), (19a), and (19b) give

$$O\left(\frac{M_W^2}{E^2}\right) \ll O\left(\frac{g^2, \lambda}{2 \cdot 16\pi^2}\right)$$

or

$$\left(\frac{1.4\,\text{TeV}}{O(g, \sqrt{\lambda}) \cdot E}\right)^2 \ll 1 \ , \tag{20}$$

which is a rather strong condition. For $\lambda = 10\,g^2$, i.e., $m_H = \sqrt{2\lambda}\, f_\pi \approx 700\,\text{GeV}$, the condition (20) requires $(0.7 \ \text{TeV}/E)^2 \ll 1$. For $E = 1\,\text{TeV}$, $1.3\,\text{TeV}$, and $2\,\text{TeV}$, Eq. (20) gives $0.49 \ll 1$, $0.29 \ll 1$, and $0.12 \ll 1$, respectively. For $\lambda = g^2$, i.e., $m_H = 225\,\text{GeV}$, Eq. (20) means $(2.2\,\text{TeV}/E)^2 \ll 1$, which requires E be at least a few TeV to probe the SB sector of the SM with a light Higgs boson to the accuracy of including loop corrections in the GB amplitude. This is, however, not a disaster because to probe the SB sector of the SM with a light Higgs boson we would have to search for a light resonance in the region $E_{\text{c.m.}} \sim m_H$. It has been extensively studied in the literature how to detect such a SM Higgs boson resonance through other production mechanisms other than the V_L-V_L fusion process at the CERN LHC (Large Hadron Collider, pp), the NLC (Next Linear Collider, $e^- e^+$), and some photon-photon linear colliders [18, 19]. Because the V_L-V_L scattering amplitude in the SM is unitary, if the SM Higgs boson is not heavy, the V_L-V_L scattering amplitude in the vicinity of 1 TeV can never be large enough to be useful for probing the SB sector of the SM with a light Higgs resonance. Our condition (20) sets the lower limit of the energy range in which the ET can be used to calculate $T[V_L; \Phi_\alpha]$ in terms of $T[-i\pi; \Phi_\alpha]$ to the accuracy of including one-loop corrections in the SM with $m_H \ll E$.

III. ET FOR PURE LONGITUDINAL SCATTERINGS IN PROBING THE STRONGLY COUPLED SB SECTOR

Here we give a further discussion on the precise formulation of the ET for pure longitudinal weak-boson scatterings in the case of a strongly interacting SB sector. We first estimate the largest contribution in the B term, as defined in (2), based upon Eq. (15) and the results from a precise power counting [13]. For both the SM with a heavy Higgs boson, $m_H \gg E$, and the general CLEWT, we find that B is of $O(g^2) f_\pi^{D_T}$, where D_T is the dimension of the scattering amplitude T, and $D_T = 4 - n_e$, for n_e external V_L or GB lines. This is only a direct generalization of our above counting result (17) for the $2 \to 2$ scattering with $n_e = 4$. [For pure longitudinal

[20]This is different from the condition derived in Ref. [7], for example, in which the B term was estimated as $O(M_W/E)$ instead of $O(g^2)$. [See the second inequality in Eq. (27) of the first paper or Eq. (65) of the second paper in Ref. [7].] The authors of Ref. [7] kindly informed us recently that their new analyses (in preparation) agreed with our condition (18).

[21]When the energy E is close to the effective cutoff Λ of the CLEWT, the higher order corrections in the momentum expansion become important and should be included, but it does not necessarily imply a violation of the ET.

[22]Since the $U(1)_{\text{em}}$ gauge coupling e is suppressed by $\sin\theta_W = 0.48$ relative to g, it is sufficient to take g for the order of magnitude estimate.

[23]Equation (19a) also coincides with previous explicit one-loop calculations [17].

weak-boson scatterings, the minimum g dependence in the B term is of $O(g^2)$ because based upon Eq. (9) or the LHS of Eq. (15) the g dependence can arise either from the factor $O(M_W^2/E_j^2)$ (containing a g^2 factor) or from the factor $O(M_W/E_j)$ (containing a g factor) and the additional g factor accompanying with each gauge boson field $V_T^{a_{r_1}}$.] It is easy to see that in the GB amplitudes all tree-level Feynman graphs with internal gauge boson line(s) are at most of $O(g^2)f_\pi^{D_T}$, i.e., of the same order as the largest contribution in B, because one internal gauge boson line will induce an extra g^2 factor from the two vertices attached to it and reduce the E power by a factor of 2 as compared with the tree-level diagrams with only pure GB lines which are of the order $O(\frac{E^2}{f_\pi^2})f_\pi^{D_T}$ as given by the low energy theorem [15]. For higher loops or higher dimensional operators, the graphs with internal gauge-boson line(s) will be suppressed by higher powers of E/Λ. Thus, beyond the tree level, all graphs in the GB amplitudes with internal gauge boson line(s) are at most of[24] $O(g^2\frac{E^2}{\Lambda^2})f_\pi^{D_T}$. Therefore, once we ignore the largest B terms according to the condition (10b) or (15), we should also correspondingly ignore all the GB graphs with internal gauge-boson lines to all orders in the heavy Higgs boson mass expansion or the momentum expansion. Furthermore, fermion fields can only appear in loops in the GB amplitudes; their contributions are at most of $O(y_f^2\frac{E^2}{\Lambda^2})f_\pi^{D_T}$ [13], where $y_f \le y_t = O(g)$ and y_f is the Yukawa coupling of fermion f. (Here we assume all possible non-SM heavy fermions have been integrated out in the CLEWT.) Thus, their contributions should also be ignored once the B term, of $O(g^2)f_\pi^{D_T}$, is ignored.

In conclusion, for pure longitudinal weak-boson scatterings in theories with the strongly interacting SB sector, the ET [Eqs. (10), (10a), and (10b)] can be further simplified as

$$T[V_L^{a_1},\dots,V_L^{a_n}] = \bar{C}\cdot T[-i\pi^{a_1},\dots,-i\pi^{a_n}]|_{g,e,y_f=0} + O(g^2)f_\pi^{D_T},$$
$$E_j \sim k_j \gg M_W \qquad (j=1,2,\dots,n)\ , \tag{21a}$$
$$O(g^2)f_\pi^{D_T} \ll \bar{C}\cdot T[-i\pi^{a_1},\dots,-i\pi^{a_n}]|_{g,e,y_f=0}\ , \tag{21b}$$
$$\bar{C} = \bar{C}^{a_1}_{\rm mod}\cdots\bar{C}^{a_n}_{\rm mod}\ , \qquad \bar{C}^a_{\rm mod} = C^a_{\rm mod}|_{g,e,y_f=0} = \left(\frac{M_a}{M_a^{\rm phys}}\sqrt{\frac{Z_{V^a}}{Z_{\pi^a}}}Z_{M_a}\right)\Bigg|_{g,e,y_f=0}\ , \tag{21c}$$

where $\pi_0^a = \sqrt{Z_{\pi^a}}\pi^a$, $V_0^a = \sqrt{Z_{V^a}}V^a$, and $M_{a0} = Z_{M_a}M_a$. π_0^a and V_0^a are bare fields, and $M_a = M_W$ or M_Z. $M_a^{\rm phys}$ denotes the physical mass of the $W^\pm$ or Z^0 boson and is equal to M_a only in the on-shell renormalization scheme [4–6]. We note that in the above equations, the condition $g,e,y_f=0$ is meant to ignore all the gauge coupling or Yukawa-coupling-dependent contributions in the GB amplitudes after replacing M_W and M_Z (or m_f) by the products of g (or y_f) and f_π, because they are at most of the same order as B term. The g^2- and y_f^2-dependent terms in the modification factor $(C^a_{\rm mod}-1)$ come from loop corrections and are at most of $O(\frac{g^2,y_f^2}{16\pi^2}) \le O(g^2\frac{f_\pi^2}{\Lambda^2})$ [3–7]. [Recall that $y_f \le O(g)$.] This modification factor times the largest term in the GB amplitude, of $O(\frac{E^2}{f_\pi^2})f_\pi^{D_T}$, can only be of $O(g^2\frac{E^2}{\Lambda^2})f_\pi^{D_T}$, which is again $\frac{E^2}{\Lambda^2}$ suppressed relative to the B term and should be ignored. Then we find that those complicated Δ_i quantities inside of $C^a_{\rm mod}$, as defined in [4–6], disappear after ignoring all g^2- and y_f^2-dependent terms. So we can make the finite modification C factor *exactly unity* by simply choosing the unphysical wave function renormalization constant Z_{π^a} as

$$Z_{\pi^a} = \left[\left(\frac{M_a}{M_a^{\rm phys}}\right)^2 Z_{V^a}Z_{M_a}^2\right]\Bigg|_{g,e,y_f=0}\ , \quad \text{(scheme III)}$$
$$\bar{C}^a_{\rm mod} = C^a_{\rm mod}|_{g,e,y_f=0} = 1\ . \tag{22}$$

We call the above renormalization prescription scheme III in which all other renormalization conditions can be freely chosen as in any of the standard renormalization schemes.

In the general CLEWT, up to the E^4 level, the pure GB amplitude without internal gauge boson lines can be easily counted as of the form $O(1)\times f_\pi^{D_T}\frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2}$, which is a direct generalization of Eq. (16) from $n_e=4$ to any arbitrary $n_e \ge 4$. Only the one-loop graphs from the E^2-level operator $(f_\pi^2/4)\,{\rm Tr}[(D_\mu U)^\dagger(D^\mu U)]$ and the tree graphs from the E^4-level operators [20], such as $\alpha_1(f_\pi/\Lambda)^2\,[{\rm Tr}(D_\mu U)^\dagger(D^\mu U)]^2$ and $\alpha_2(f_\pi/\Lambda)^2\,[{\rm Tr}(D_\mu U)^\dagger(D_\nu U)]^2$, can contribute to this leading energy behavior. The Feynman diagrams from the other E^4-level operators, such as[25]

$$-ig\alpha_{9L}(f_\pi/\Lambda)^2\,{\rm Tr}[W^{\mu\nu}(D_\mu U)(D_\nu U)^\dagger]\ ,$$
$$-ig'\alpha_{9R}(f_\pi/\Lambda)^2\,{\rm Tr}[B^{\mu\nu}(D_\mu U)(D_\nu U)^\dagger]\ ,$$

[24]For the SM with a heavy Higgs boson, Λ is replaced by m_H. For the CLEWT, Λ is taken to be about $4\pi f_\pi$.

[25]The custodial SU(2)-symmetry-violating operator $(1/8)\Delta\rho f_\pi^2\,[{\rm Tr}(\tau^3U^\dagger D_\mu U)]^2$ can contribute to some pure GB graphs without internal gauge boson lines, whose contributions, however, are at most of $O(\Delta\rho\frac{E^2}{f_\pi^2})f_\pi^{D_T} = O(\frac{m_t^2}{16\pi^2 f_\pi^2}\frac{E^2}{f_\pi^2})f_\pi^{D_T} = O(y_t^2\frac{E^2}{\Lambda^2})f_\pi^{D_T} = O(g^2\frac{E^2}{\Lambda^2})f_\pi^{D_T}$, where y_t is the top quark Yukawa coupling.

and

$$gg'\alpha_{10}(f_\pi/\Lambda)^2 Tr[UB^{\mu\nu}U^\dagger W_{\mu\nu}] ,$$

must contain gauge boson lines and are therefore *not sensitive* to probing the SB sector via longitudinal scatterings. Thus up to E^4 level the condition (21b) gives

$$O(g^2) \ll \frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2} \quad \text{or} \quad \frac{M_W^2}{E^2} \ll \frac{1}{4}\frac{E^2}{\Lambda^2} . \tag{23}$$

We note that the result of (23) holds independent of the number of external lines involved in pure V_L scattering processes. Our condition (18) for a pure $2 \to 2$ V_L scattering is only a special case of (23).

As $E \gtrsim 1$ TeV, Eq. (23) is satisfied. Our above precise formulation of the ET, Eqs. (21) and (21a)–(21c), therefore provides a rigorous theoretical reasoning for justifying many previous applications of the ET in the literature to study the strongly coupled SB sector by ignoring all the internal gauge boson lines in the GB amplitudes. Most importantly, our result (23) shows that *in order to probe strongly coupled* SB *sector from pure longitudinal weak-boson scattering processes with any number of external lines, we must experimentally measure their production rates in the energy region above* 1 TeV.

IV. CONCLUSIONS

We have examined the Lorentz noninvariance ambiguity for longitudinal weak-boson scatterings and derived the precise conditions, Eqs. (10a) and (10b) [or (15)], for the equivalence of the V_L amplitude and the GB amplitude, as shown in (10). After analyzing the intrinsic connection between the ET and the problem of probing the electroweak SB sector, we presented the universal formulation of the ET in Eqs. (10), (10a), and (10b) for both the SM and the general CLEWT. We have also defined the *safe* Lorentz frames in which condition (10a) holds. We gave an explicit example, $Z_LH \to Z_LH$, to show that the center-of-mass frame of this scattering process for a heavy Higgs boson ($M_W \ll m_H < E_{\rm c.m.}$) is not a safe frame because (10a) in this case is not satisfied. Therefore, in the c.m. frame the $Z_LH \to Z_LH$ amplitude cannot be estimated by using the corresponding GB amplitude $\pi^0H \to \pi^0H$, as shown in (13). We note that the above formulation of the ET not only serves as a technical tool in simplifying the V_L-amplitude calculation using the GB amplitude when conditions (10a) and (10b) are satisfied, but, most importantly, *this formulation also discriminates processes which are not sensitive to probing the electroweak SB sector when (10a) or (10b) fails.* Furthermore, the condition in Eq. (15) determines whether the V_L-scattering process of interest is sensitive to probing the SB sector to the desired precision in perturbative calculations. The minimum energy scale required for testing the SB sector (assuming no light resonance present) of the SM and the CLEWT beyond the leading order (up to the E^4 level) were given in (18) or (23). We found that longitudinal weak-boson scatterings can only be sensitive to probing strongly coupled electroweak SB sector in the TeV region, i.e., $E \geq O(1)$ TeV. In this case, for pure longitudinal weak-boson scatterings, the ET takes a very simple form in which the GB amplitude is calculated by ignoring all the internal gauge-boson lines and fermion loops [cf. (21) and (21a–c)]. Here the multiplicative modification factors can be exactly simplified as unity in a very simple renormalization scheme, scheme III [cf. (22)].

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