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# Heavy quark decomposition of the $S$ matrix and its relation to the pinch technique

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We propose a decomposition of the $S$ matrix into individually gauge invariant subamplitudes, which are kinematically akin to propagators, vertices, boxes, etc. This decomposition is obtained by considering limits of the $S$ matrix when some or all of the external particles have masses larger than any other physical scale. We show at the one-loop level that the effective gluon self-energy so defined is physically equivalent to the corresponding gauge-independent self-energy obtained in the framework of the pinch technique. The generalization of this procedure to arbitrary gluonic $n$-point functions is briefly discussed.

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The pinch technique (PT) [1] is an algorithm that allows the construction of modified gauge invariant (GI) $n$-point functions, through the order by order rearrangement of the Feynman graphs contributing to a certain physical and therefore ostensibly GI amplitude (such as an $S$-matrix element, or a Wilson loop). Even though the most recent applications of the PT are inspired by standard model phenomenology [2–7], it was originally introduced in the context of QCD, as a first step toward the construction of Schwinger-Dyson equations, which would respect the crucial property of gauge invariance, even in their one-loop dressed truncated version [8,9]. The simplest example that demonstrates how the PT works is the gluon two-point function (propagator). Consider the $S$-matrix element $T$ for an elastic scattering process such as $q_1\bar{q}_2 \to q_1\bar{q}_2$, where $q_1$ and $q_2$ are two on-shell test quarks with masses $m_1$ and $m_2$. To any order in perturbation theory $T$ is independent of the gauge-fixing parameter $\xi$. On the other hand, as an explicit calculation shows, the conventionally defined proper self-energy [collectively depicted in Fig. 1(a)] depends on $\xi$. At the one-loop level this dependence is canceled by contributions from other graphs, such as 1(b), 1(c), 1(d), and 1(e) which, at first glance, do not seem to be propagatorlike. That this cancellation must occur and can be employed to define a GI self-energy is evident from the decomposition

$$T(s,t,m_1,m_2) = T_0(t,\xi) + T_1(t,m_1,\xi) + T_2(t,m_2,\xi) + T_3(s,t,m_1,m_2,\xi) \ , \qquad (1)$$

where the function $T_0(t,\xi)$ depends kinematically only on the Mandelstam variable $t = -(\hat{p}_1 - p_1)^2 = -q^2$, and not on $s = (p_1+p_2)^2$ or on the external masses. Typically, self-energy, vertex, and box diagrams contribute to $T_0$, $T_1$ and $T_2$, and $T_3$, respectively. Such contributions are $\xi$ dependent, in general. However, as the sum $T(s,t,m_1,m_2)$ is GI, it is easy to show that Eq. (1) can be recast in the form

$$T(s,t,m_1,m_2) = \hat{T}_0(t) + \hat{T}_1(t,m_1) + \hat{T}_2(t,m_2) + \hat{T}_3(s,t,m_1,m_2) \ , \qquad (2)$$

where the $\hat{T}_i$ $(i = 0,1,2,3)$ are *individually* $\xi$ independent. The propagatorlike parts Figs. 1(f), 1(g), 1(h), and 1(i), stemming from graphs 1(b), 1(c), 1(d), and 1(e), respectively, enforce the gauge independence of $T_0(t)$, and are called "pinch parts." They emerge every time a gluon propagator or an elementary three-gluon vertex contributes a longitudinal $k_\mu$ to the original graph's numerator. The action of such a term is to trigger an elementary

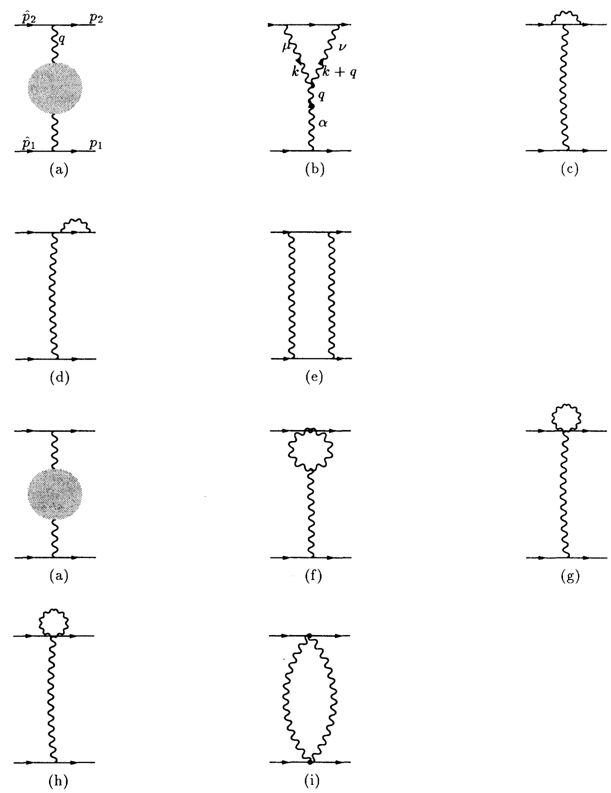


FIG. 1. (a)–(e) are some of the QCD contributions to the $S$ matrix for four-fermion processes. Graphs (f), (g), (h), and (i) are the pinch parts of (b), (c), (d), and (e), respectively. When added to the usual self-energy graphs (a), they give rise to the $\xi$-independent amplitude $\hat{T}_0(t)$. The mirror image graphs corresponding to (b), (c), and (d), as well as the crossed box graph are not shown. Graph (a) contains contributions from virtual fermions, gluons, and Faddeev-Popov ghosts. Solid (wavy) lines represent fermions (gluons).

 

Ward identity of the form $\not k = (\not p + \not k - m) - (\not p - m)$ when it gets contracted with a $\gamma$ matrix. The first term removes the internal fermion propagator (e.g., it produces a "pinch"), whereas the second vanishes on shell. From the GI functions $\hat{T}_i$ $(i = 1, 2, 3)$ one may now extract a GI effective gluon ($G$) self-energy $\hat{\Pi}_{\mu\nu}(q)$, GI $Gq_i\bar{q}_i$ vertices $\hat{\Gamma}^{(i)}_\mu$, and a GI box $\hat{B}$, in the following way:

$$\begin{aligned}
\hat{T}_0 &= g^2 \bar{u}_1 \gamma^\mu u_1 \left[\left(\frac{1}{q^2}\right) \hat{\Pi}_{\mu\nu}(q) \left(\frac{1}{q^2}\right)\right] \bar{u}_2 \gamma^\nu u_2 \ , \\
\hat{T}_1 &= g^2 \bar{u}_1 \hat{\Gamma}^{(1)}_\nu u_1 \left(\frac{1}{q^2}\right) \bar{u}_2 \gamma^\nu u_2 \ , \\
\hat{T}_2 &= g^2 \bar{u}_1 \gamma^\mu u_1 \left(\frac{1}{q^2}\right) \bar{u}_2 \hat{\Gamma}^{(2)}_\nu u_2 \ , \\
\hat{T}_3 &= \hat{B} \ ,
\end{aligned} \tag{3}$$

where $u_i$ are the external spinors and $g$ is the gauge coupling. Since all quantities with carets in the above formula are GI, their explicit form may be calculated using any value of the gauge-fixing parameter $\xi$, as long as one properly identifies and allots all relevant pinch contributions. The choice $\xi = 1$ simplifies the calculations significantly, since it eliminates the longitudinal part of the gluon propagator. Therefore, for $\xi = 1$ the pinch contributions originate only from momenta carried by the elementary three-gluon vertex of graph 1(b) (and its mirror graph, not shown). The one-loop expressions of $\hat{\Pi}_{\mu\nu}(q)$ and $\hat{\Gamma}^{(i)}_\mu$ are given by [2,9]

$$\hat{\Pi}_{\mu\nu}(q) = \Pi^{(\xi=1)}_{\mu\nu}(q) + t_{\mu\nu}\Pi^P(q) \tag{4}$$

with $t_{\mu\nu} = q^2 g_{\mu\nu} - q_\mu q_\nu$, and

$$\begin{aligned}
\Pi^P(q) &= -2ic_a g^2 \int_n \frac{1}{k^2(k+q)^2} \\
&= 2c_a \left(\frac{g^2}{16\pi^2}\right) \left[C_{\rm UV} - \ln\left(\frac{-q^2}{\mu^2}\right) + 2\right] \ ,
\end{aligned} \tag{5}$$

where $C_{\rm UV} = 2/\epsilon - \gamma + \ln(4\pi)$, $\epsilon = n - 4$, $\gamma = 0.577\ldots$ is the Euler constant, and $\int_n \equiv \int d^n k/(2\pi)^n$ in the dimensionally regularized loop integral. Similarly, for the vertex we have

$$\begin{aligned}
[\hat{\Gamma}_\mu]^{(i)} = ig^2 &\left[\left(\frac{c_a}{2}\right) \int_n \frac{\gamma^\rho S_i(p_i - k)\gamma^\sigma \Gamma^F_{\rho\sigma\mu}}{k^2(k+q)^2} + \left(\frac{c_a}{2} - c_f\right) \int_n \frac{\gamma^\rho S_i(\hat{p}_i - k)\gamma_\mu S_i(p_i - k)\gamma_\rho}{k^2}\right] \\
&+ \left[\gamma^\mu \frac{1}{\not p_i - m_i} \hat{\Sigma}_i(p) + \hat{\Sigma}_i(\hat{p}_i) \frac{1}{\hat{\not p} - m_1} \gamma^\mu\right] \ ,
\end{aligned} \tag{6}$$

where $\Gamma^F_{\mu\nu\alpha} = 2q_\mu g_{\nu\alpha} - 2q_\nu g_{\nu\alpha} - (2k+q)_\alpha g_{\mu\nu}$ [10], $c_f$ is the Casimir eigenvalue of the fermion representation, $\hat{p}_i = p_i + q$, and

$$\hat{\Sigma}_i(p) = g^2 c_f \int_n \frac{1}{k^2} \gamma_\sigma \frac{1}{\not k + \not p - m_i} \gamma^\sigma = \Sigma^{(\xi=1)}_i(p) \tag{7}$$

is the one-loop GI quark propagator, derived in [11].

In principle, this procedure can be generalized to an arbitrary $n$-point function. In particular, the GI three- and four-point functions $\hat{\Gamma}_{\mu\nu\alpha}$ and $\hat{\Gamma}_{\mu\nu\alpha\beta}$ have been derived in [9] and [12]. The Green's functions obtained via the PT, in addition to being GI, are endowed with several characteristics properties. Most noticeably, the gluon $n$-point functions computed thus far [$\hat{\Pi}_{\mu\nu} = t_{\mu\nu}\hat{\Pi}$, $\hat{\Gamma}_{\mu\nu\alpha}$, $\hat{\Gamma}_{\mu\nu\alpha\beta}$ $(n = 2, 3, 4)$] satisfy the following simple QED-like Ward identities:

$$\begin{aligned}
q_1^\mu \hat{\Gamma}_{\mu\nu\alpha}(q_1, q_2, q_3) &= t_{\nu\alpha}(q_2)\hat{d}^{-1}(q_2) - t_{\nu\alpha}(q_3)\hat{d}^{-1}(q_3) \ , \\
q_1^\mu \hat{\Gamma}^{abcd}_{\mu\nu\alpha\beta} &= f_{abp}\hat{\Gamma}^{cdp}_{\nu\alpha\beta}(q_1 + q_2, q_3, q_4) + \text{c.p.} \ ,
\end{aligned} \tag{8}$$

where $\hat{d}^{-1}(q) = q^2 - \hat{\Pi}(q)$, $f^{abc}$ are the structure constants of the gauge group, and the abbreviation c.p. in the right-hand side (RHS) stands for "cyclic permutations." In addition, the gluon-quark vertices $[\hat{\Gamma}^a_\mu]^{(i)}$ of Eq. (6) are ultraviolet finite.

Regardless of any such properties, however, an ambiguity is associated with the construction of Green's functions via the PT. It is obvious, for instance, that, after a GI gluon self-energy and gluon-quark vertex has been constructed via the PT, one still has the freedom to add an arbitrary term of the form $(q^2 g_{\mu\nu} - q_\mu q_\nu)f(q^2)$ to the self-energy, and subtract it from the vertex. As long as the function $f(q^2)$ is GI, such an operation satisfies the criterion of individual gauge invariance for the self-energy and vertex, respects their Ward identities, and preserves the uniqueness of the $S$ matrix. It is therefore desirable to have a physical prescription which eliminates this ambiguity. To accomplish this, we propose to use an alternative, physically motivated prescription for extracting GI subamplitudes of the $S$ matrix that are kinematically akin to self-energies, vertices, and boxes. In this framework, the effective gluon self-energy is defined to be the limit of the $S$ matrix as both external fermion masses $m_1$ and $m_2$ are taken to be larger than any other mass scale in the process (they are, however, comparable to each other, e.g., $m_1 \approx m_2$). This heavy quark limit of the $S$ matrix was previously exploited in [13] to define a gauge-independent QCD $\beta$ function, and was extended to the case of massive gauge field in [14]. To one loop this limit coincides with the static quark-antiquark potential for very massive quarks [15,16], and, as we will show, is physically equivalent to the PT result of Eq. (5). We also propose an extension of this idea to the case where only some of the external fermion fields are very heavy — this allows us to define gauge invariant subamplitudes of the $S$ matrix, which are kinematically akin to vertices and boxes.

Any reasonable effective propagator should only depend on the momentum transfer $t$ and not on kinematical details such as masses or total momentum $s$ of the incoming or outgoing particles. Similarly, any viable definition of a $Gq_i\bar{q}_i$ vertex should only depend on $t$ and the quark mass $m_i$ [17] but no other kinematical details. This reasoning can obviously be generalized to higher $n$-point functions. Motivated by these observations, we propose to define propagator and vertexlike subamplitudes by taking appropriate kinematical limits of the $S$ matrix. For the simple case of a four-quark on-shell amplitude $T$ (Fig. 1) we define the three limits

$$\begin{aligned} L_0(t) &= T(s,t,m_1=M,m_2=M) \ , \\ L_1(t,m_1) &= T(s,t,m_1,m_2=M) \ , \\ L_2(t,m_2) &= T(s,t,m_1=M,m_2) \ , \end{aligned} \tag{9}$$

where the mass $M$ is assumed to be larger than any other mass scale appearing in the process, except for any cutoffs introduced in intermediate calculations in order to regularize ultraviolet divergences. Note, however, that, since the external particles are on shell, $s=(p_1+p_2)^2 \geq (m_1+m_2)^2$ is also of the order of $M^2$, in any of these limits [18]. Each of the above quantities is GI, since it corresponds to a particular limit of the GI $S$ matrix element $T$. They can be systematically computed by expanding the $S$ matrix in powers of $(\mu_0/M)$, where $\mu_0$ is any of the remaining mass scales. The limits considered above correspond to well-defined physical situations. $L_0$, for example, is the dominant contribution to the $S$ matrix when the momentum transfer $t$ is considerably smaller than the masses of all the scattered particles, e.g., $t=-q^2 \ll m_1, m_2$.

We can define the linear combinations

$$\begin{aligned} \tilde{T}_0(t) &= L_0 \ , \\ \tilde{T}_i(t,m_i) &= (L_i - L_0) \quad (i=1,2) \ , \\ \tilde{T}_3(s,t,m_1,m_2) &= T(s,t,m_1,m_2) - L_0 \\ &\quad -[(L_1-L_0)+L_2-L_0)]. \end{aligned} \tag{10}$$

We have thus arrived at a decomposition of the $S$ matrix into individually GI and kinematically distinct subamplitudes, which we can identify as effective self-energy $\tilde{T}_0(t)$, vertices $\tilde{T}_1(t,m_1)$ and $\tilde{T}_2(t,m_2)$, and boxes $\tilde{T}_3(s,t,m_1,m_2)$. Clearly, the sum of these subamplitudes is the original $S$ matrix: e.g.,

$$\begin{aligned} T(s,t,m_1,m_2) &= \tilde{T}_0(t) + \tilde{T}_1(t,m_1) + \tilde{T}_2(t,m_2) \\ &\quad + \tilde{T}_3(s,t,m_1,m_2) \ . \end{aligned} \tag{11}$$

The above decomposition of the $S$ matrix into individually GI and kinematically distinct subamplitudes relies on a procedure different from the PT. The question that naturally arises is how the individual terms of Eqs. (2) and (11) are related. As we will show by an explicit one-loop calculation, $\hat{T}_0$ of Eq. (2) and $\tilde{T}_0$ of Eq. (10) are related as follows:

$$\tilde{T}_0(t) = \hat{T}_0(t) + g^2 \bar{u}_1 \gamma^\mu u_1 \left(\frac{1}{q^2}\right) [C t_{\mu\nu}] \left(\frac{1}{q^2}\right) \bar{u}_2 \gamma^\nu u_2 \ . \tag{12}$$

In Eq. (12) $C$ is a GI finite numerical constant. Thus the GI self-energy $\tilde{\Pi}_{\mu\nu}(q)$ extracted from $\tilde{T}_0(t)$ and the $\hat{T}_0$ obtained from Eq. (2) satisfy

$$\tilde{\Pi}_{\mu\nu}(q) = \hat{\Pi}_{\mu\nu}(q) + C t_{\mu\nu} \ . \tag{13}$$

Clearly, the term proportional to $C$ in the RHS of Eq. (13) can be removed by a finite counterterm, or, equivalently, absorbed in the final normalization of the $S$ matrix.

We now proceed to derive relation Eq. (13) by calculating the large-mass limit of the graphs (a)–(e), shown in Fig. 1. We give the result for each graph separately, in order to compare their large-mass limit to their contribution within the PT framework. It turns out that the pinch parts of individual graphs do *not* coincide with their large-mass limit, and that the result of Eq. (13) emerges only after a delicate cancellation between all graphs.

To compute the leading one-loop contribution to $\tilde{T}_0(t)$ (or equivalently $L_0$) we evaluate the $S$ matrix in the limit $m_1 \approx m_2 \to M$, where $M \gg -q^2$ [19]. For simplicity we consider elastic scattering, so that $q^2 < 0$. As a consequence, there are no imaginary parts in the Feynman graphs. We define the Euclidean momentum $Q^2 = -q^2 > 0$. Throughout the calculation we use dimensional regularization, where the UV cutoff is set by the usual pole $1/\epsilon$. In addition, the 't Hooft mass $\mu$ has to be introduced. The infrared divergences are regulated by introducing an infrared gluon mass $\lambda$ in the intermediate calculations [20].

We then compute all one-loop Feynman graphs contributing to the process, neglecting terms proportional to any of the ratios $(Q/M)$, $(\lambda/M)$, and $(\mu/M)$ (or higher powers of such ratios), and retaining only logarithmic and constant terms. We emphasize that the above expansion is carried out *after* the integration over the loop momenta has been performed in dimensional regularization. Effectively this means that $M$ is always much smaller than the cutoff $\Lambda$ [e.g., $1/\epsilon \to \ln(\Lambda/\mu) \gg \ln(M/\mu)$]. In this calculation all choices for $\xi$ are equivalent, since the $S$-matrix element is $\xi$ independent; we choose $\xi=1$, because in this gauge the pinch contributions of all graphs except (b) vanish. This facilitates the comparison between the PT and the large-mass limit, at the level of individual graphs.

The most involved parts of the calculation are the box diagrams. It is important to recognize that both the direct and the crossed graph must be appropriately combined in order to obtain the correct color structure. It is also interesting to notice that the expressions that survive the large-$M$ limit are of non-Abelian nature only, namely, proportional to $c_a$. If we call $B_{\rm dir}$ the total contribution of the direct graph and $B_{\rm cr}$ the respective contribution from the crossed, we have that $B_{\rm dir} = (R_aR_b)_1(R_aR_b)_2 S_{\rm dir}$ and $B_{\rm cr} = (R_aR_b)_1(R_bR_a)_2 S_{\rm cr}$ where $S_{\rm dir}$ and $S_{\rm cr}$ are the remainders of the boxes, after the color structure has been factored out. The important step is to show that in the large-$M$ limit we have $S_{\rm dir} = -S_{\rm cr}$. Thus, the total box contribution $\hat{B}$ becomes

$$\tilde{B} = (R_a R_b)_1 [R_a, R_b]_2 S_{\text{dir}} = \tfrac{1}{2} c_a (R_c)_1 (R_c)_2 S_{\text{dir}} \; . \tag{14}$$

The results for the individual Feynman graphs are (we omit external spinors and an overall factor of $g^2/Q^2$)

$$[(\text{a})] = \Pi_{\mu\nu}^{(\xi=1)} \; ,$$

$$[(\text{b}) + (\text{b})_{\text{mirror}}] = \frac{g^2}{16\pi^2} c_a \left[3C_{\text{UV}} + 4 + 3 \ln\left(\frac{\mu^2}{M^2}\right)\right] g_{\mu\nu} + \cdots \; ,$$

$$[(\text{c}) + (\text{c})_{\text{mirror}}] = \frac{g^2}{16\pi^2} (2c_f - c_a) \left[C_{\text{UV}} + 4 + 2 \ln\left(\frac{\lambda^2}{\mu^2}\right) + 3 \ln\left(\frac{\mu^2}{M^2}\right)\right] g_{\mu\nu} + \cdots \; , \tag{15}$$

$$[(\text{d}) + (\text{d})_{\text{mirror}}] = -2 \frac{g^2}{16\pi^2} c_f \left[C_{\text{UV}} + 4 + 2 \ln\left(\frac{\lambda^2}{\mu^2}\right) + 3 \ln\left(\frac{\mu^2}{M^2}\right)\right] g_{\mu\nu} + \cdots \; ,$$

$$[(\text{e}) + (\text{e})_{\text{cr}}] = 2 \frac{g^2}{16\pi^2} c_a \ln\left(\frac{\lambda^2}{Q^2}\right) g_{\mu\nu} + \cdots \; ,$$

where the ellipses denote terms of order $O(1/M)$ or higher. We notice that the sum of all vertex and box graphs listed in Eq. (15) is equal to $2(g^2 c_a/16\pi^2)$ $[C_{\text{UV}} - \ln(Q^2/\mu^2)]$, which is, up to a physically irrelevant constant, the pinch contribution to self-energy, given in Eq. (5). The total contribution to $\tilde{T}_0$ reads

$$\begin{aligned}
\tilde{T}_0 &= g^2 \bar{u}_1 \gamma^\mu u_1 \left(\frac{1}{q^2}\right) \left\{ \Pi_{\mu\nu}^{(\xi=1)} + 2 \left(\frac{g^2 c_a}{16\pi^2}\right) \left[C_{\text{UV}} - \ln\left(\frac{-q^2}{\mu^2}\right)\right] t_{\mu\nu} \left(\frac{1}{q^2}\right) \right\} \bar{u}_2 \gamma^\nu u_2 \\
&= g^2 \bar{u}_1 \gamma^\mu u_1 \left(\frac{1}{q^2}\right) \left[\Pi_{\mu\nu}^{(\xi=1)} + \Pi^P(q) - 4 \left(\frac{g^2 c_a}{16\pi^2}\right) t_{\mu\nu}\right] \left(\frac{1}{q^2}\right) \bar{u}_2 \gamma^\nu u_2 \\
&= \hat{T}_0 - g^2 \bar{u}_1 \gamma^\mu u_1 \left(\frac{1}{q^2}\right) \left[4 \left(\frac{g^2 c_a}{16\pi^2}\right) t_{\mu\nu}\right] \left(\frac{1}{q^2}\right) \bar{u}_2 \gamma^\nu u_2 \; ,
\end{aligned} \tag{16}$$

which is the advertised result in the first line of Eq. (12), with $C = -4(g^2 c_a/16\pi^2)$. The first relation of Eq. (13) follows immediately from Eq. (16): namely,

$$\tilde{\Pi}_{\mu\nu}(q) = \hat{\Pi}_{\mu\nu}(q) - 4 \left(\frac{g^2 c_a}{16\pi^2}\right) t_{\mu\nu} \; . \tag{17}$$

Adding the tree-level contribution to $\tilde{T}_0$ of Eq. (16), and using the standard result

$$\Pi_{\mu\nu}^{(\xi=1)} = \frac{g^2 c_a}{16\pi^2} \left\{ \frac{5}{3} \left[C_{\text{UV}} - \ln\left(\frac{-q^2}{\mu^2}\right)\right] + \frac{31}{9} \right\} t_{\mu\nu} \tag{18}$$

together with Eq. (5), we find that $\tilde{T}_0$ is identical to the Fourier transform of the unrenormalized one-loop static potential $V(Q^2)$ for a heavy quark-antiquark system ([15]): namely (we omit the external spinors),

$$\tilde{T}_0 = V(Q^2) = -\frac{g^2}{Q^2} \left[1 + \frac{g^2 c_a}{16\pi^2} \left\{ -\frac{11}{3} C_{\text{UV}} + \frac{11}{3} \ln\left(\frac{Q^2}{\mu^2}\right) - \frac{31}{9} \right\}\right] \; , \tag{19}$$

where the factor $\frac{11}{3} c_a/16\pi^2$ is the coefficient $b_0$ in front of $-g^3$ in the one-loop $\beta$ function. The contribution $\tilde{T}_0$ of Eq. (16) or Eq. (19) to the $S$ matrix is infrared finite. The decomposition of Eq. (9), together with Eq. (16), implies that the PT result for the self-energy gives the dominant contribution to the physical $S$ matrix, when the scattered particles are heavy compared to other mass scales. We have thus arrived at a physical interpretation of this PT subamplitude. In that sense, the mathematical ambiguity in defining a GI propagatorlike subamplitude of the $S$ matrix, which we discussed previously, can be eliminated by imposing a physically motivated boundary condition, i.e., that the effective self-energy should reproduce the $S$ matrix for the scattering of sufficiently heavy external quarks. Since perturbation theory in QCD is reliable only for momentum transfers beyond a few GeV, in practice this subamplitude will provide the dominant contribution to the $S$ matrix only for top and bottom scattering. Nevertheless, it makes sense to *define* this GI subamplitude also for considerably lighter systems, although in such a case it will generally not give the dominant contribution to the $S$ matrix. In addition, Eq. (15) shows that the equivalence between the large-mass limit of the $S$ matrix and the PT result is not trivial, because there is no graph by graph correspondence between the two methods; for example, the pinch contribution (i) of the box diagram (e) vanishes in the Feynman gauge, whereas the large-mass limit of the box in the same gauge gives a momentum-dependent and infrared divergent contribution, which is crucial for the infrared finiteness of the final answer.

It would clearly be of interest to extend this analysis to the vertexlike subamplitudes. Of course, in a theory with massless gauge bosons such subamplitudes are in general infrared divergent; they can therefore not be directly related to a physical process, without including bremsstrahlung. One could nevertheless compare the GI vertexlike amplitudes $\hat{T}_i$ and $\tilde{T}_i$, $i = 1, 2$, of the two schemes, as long as the infrared singularities are regulated in a gauge invariant manner, such as dimensional infrared regularization [21,22]. This goes, however, beyond the scope of the present communication.

The previous considerations can be generalized to the case of multiquark scattering. In particular, from a $2n$-quark amplitude one can define GI gluon $n$-point functions $\tilde{\Gamma}^{(n)}(q_1, \ldots, q_n)$ with all incoming momenta $q_i$, $i = 1, \ldots, n$ off shell. To that end one has to consider the limit of the amplitude as all external fermion masses become large ($m_i \to M, i = 1, \ldots, n$). It would be very interesting to determine if the GI $n$-point functions so obtained are physically equivalent to those obtained with the PT, especially for $n = 3, 4$. It would also be interesting to generalize the previous arguments to the case of theories with spontaneous symmetry breaking, in general, and the electroweak sector of the standard model, in particular.

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[17] This is true for QCD-like theories. In flavor-changing models the effective vertex will generally depend on the masses of the incoming and the outgoing fermion of the vertex, as well as $t$.
[18] The variable $s$ appears only when one computes box diagrams.
[19] It is important to notice that, since the external quarks are "on shell," the incoming momenta $p_i$ are not to be neglected compared to $M$. Therefore our results will in general be different from the case where all incoming momenta are small compared to a large mass inside the loop.
[20] Alternatively, one could use dimensional regularization for both ultraviolet and infrared divergences, as in Ref. [15].
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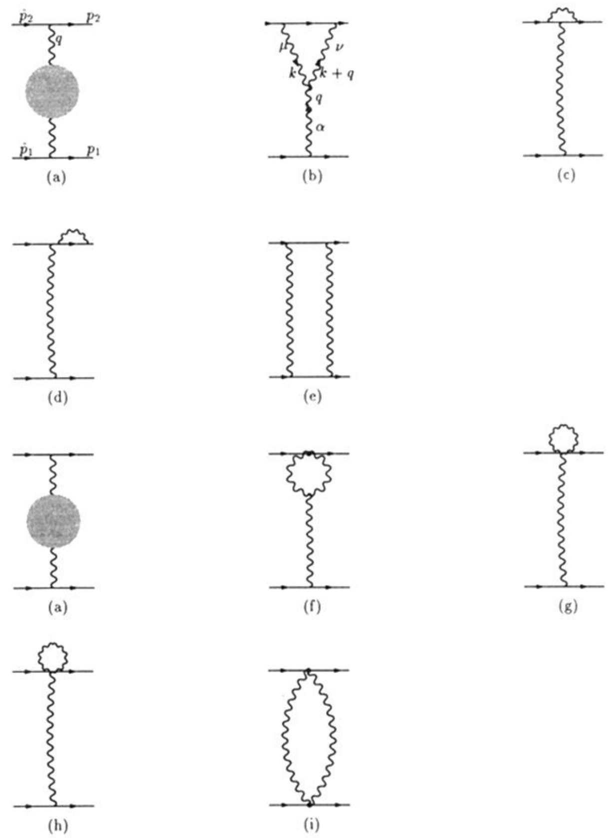


FIG. 1. (a)–(e) are some of the QCD contributions to the $S$ matrix for four-fermion processes. Graphs (f), (g), (h), and (i) are the pinch parts of (b), (c), (d), and (e), respectively. When added to the usual self-energy graphs (a), they give rise to the $\xi$-independent amplitude $\hat{T}_0(t)$. The mirror image graphs corresponding to (b), (c), and (d), as well as the crossed box graph are not shown. Graph (a) contains contributions from virtual fermions, gluons, and Faddeev-Popov ghosts. Solid (wavy) lines represent fermions (gluons).