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Sea quark contribution to the dynamical mass and light quark content of a nucleon

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We calculate the flavor mixing in the wave function of a light valence quark. For this, we use the idea of dynamical symmetry breaking. A sea quark of a different flavor may appear through the vacuum polarization of a gluon propagator which appears in the gap equation for the dynamical mass. We have also used the fact that any one of these quark lines may undergo condensation. The dependence of the dynamical mass, generated in this way, on the sea quark mass up to quadratic terms has been retained. The momentum dependence is like $1/p^4$, in contrast with the l/p^2 kind of dependence which occurs for the leading term of the dynamical mass in the subasymptotic region. The extension of the result to the "mass shell" yields $\sigma_{\pi N}$ = 53–54 MeV for the pion-nucleon σ term and $m_s\langle p|\bar{s}s|p\rangle$ = 122–264 MeV for the strange quark contribution to the proton mass, for different values of parameters. These are in reasonable agreement with current phenomenological estimates of these quantities.

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I. INTRODUCTION

Recent analysis and observation of flavor mixing in the wave function of the valence quarks in the baryon sector cast doubt on our understanding of the constituent quark model and the Okubo-Zweig-Iizuka (OZI) rule. Thus the determination of the Σ term ($\Sigma_{\pi N}$), from pion-nucleon scattering and its implications for the strangeness in the proton [1–7], the quark-spin content of the proton, i.e., the flavor singlet axial-charge g_A^0 of the nucleon [8], and intrinsic charm quarks in the nucleon suggested by J/ψ, and open-charm production in hadron-nucleus collisions [9], do not go well with the naive interpretation of the constituent quark model.

From deep inelastic scattering experiments, it is known that the nucleon consists of not only valence quarks but essentially an infinite number of quark-antiquark pairs as well. The interpretation of a constituent quark as a valence quark and many $q\bar{q}$ pairs and gluons is as old as QCD itself. Nevertheless, it is a well known fact that for low energy phenomena, such as the magnetic moments of baryons, the spectroscopy of mesons and baryons, the meson baryon couplings, and ratios of total cross sections such as $\sigma(\pi N)/\sigma(NN)$, etc., the constituent quark model works remarkably well. It is not unreasonable to assume that for quasistatic processes such as g_A^0 of the nucleon and more, especially for $\sigma_{\pi N}$, which is determined from essentially low energy pion-nucleon scattering experiments, the constituent quark picture should not be very much off the mark.

The experimental determination of the Σ term comes from the investigation of $\pi^\pm p \to \pi^\pm p$ scattering. Specifically one takes the isospin-even scattering amplitude with the Born term subtracted (called $\bar{D}^+$ in the literature). The pion-nucleon Σ term is defined as

$$\Sigma_{\pi N} = F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2), \quad \nu = \frac{s-u}{4M_N} \qquad (1.1)$$

where F_π is the pion decay constant and the kinematic location $\nu = 0$, $t = 2M_\pi^2$ is called the Cheng-Dashen (CD) point. Since the CD point is outside the physical region, one needs to extrapolate the available experimental information to this point using analyticity properties and dispersion theory. The Karlsruhe group's value is generally the standard [10], with

$$\Sigma_{\pi N} = 64 \pm 8 \text{ MeV}.$$

In order to do the theoretical evaluations of the Σ term, one first defines another related quantity $\sigma_{\pi N}$ as the nucleon matrix element of the double commutator of the symmetry-breaking term in the baryon Hamiltonian with two axial charges, where an average over proton and neutron states is understood:

$$\sigma_{\pi N} = \langle N(p)|\sum_{i=1}^{3}[Q^{5i},[Q^{5i},H_{\rm SB}(0)]]|N(p)\rangle. \qquad (1.2)$$

A formal evaluation of the commutator in QCD yields

$$\begin{aligned}\sigma_{\pi N} &= \langle N(p)|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle\\ \hat{m} &= \tfrac{1}{2}(m_u + m_d).\end{aligned} \qquad (1.3)$$

The chiral symmetry relates the value of the $\Sigma_{\pi N}$ to the nucleon matrix element of the quark mass term [11]:

$$\begin{aligned}\hat{m}\langle N(p')|(\bar{u}u + \bar{d}d)|N(p)\rangle &= \bar{u}(p')u(p)\sigma(t),\\ t &= (p'-p)^2.\end{aligned}$$

The fact that $\sigma_{\pi N} = \sigma(0)$, thus relates $\Sigma_{\pi N}$ to $\sigma_{\pi N}$:

$$\Sigma_{\pi N} = \sigma_{\pi N} + \Delta_\sigma + \Delta_R, \qquad (1.4)$$

where $\Delta_\sigma = \sigma(2M_\pi^2) - \sigma(0)$ and is the quantity of crucial interest; Δ_R is of the order of $M_\pi^4 \ln M_\pi^2$ and is numerically small: Δ_R = 0.35 MeV. Using the quark mass expansion of the energy levels in the baryon octet, one

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can, on the other hand, express $\sigma_{\pi N}$ in terms of physical masses and an additional parameter y, defined by

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|(\bar{u}u + \bar{d}d)|N\rangle}$$

which reflects the strange quark content of the proton. At the one-loop level in chiral perturbation theory, the expression for $\sigma_{\pi N}$ becomes [3]

$$\sigma_{\pi N} = \frac{\hat{\sigma}}{1-y}, \quad \hat{\sigma} = (35 \pm 5) \text{ MeV}, \tag{1.5}$$

where the 5 MeV error is due to the theoretical uncertainty in the terms of the order $O(m_q^2)$.

At the one-loop level in chiral perturbation theory, Gasser, Sainio, and Svarc [4] have found that $\Delta_\sigma = 4.6$ MeV resulting in

$$\Sigma_{\pi N} - \sigma_{\pi N} \cong 5 \text{ MeV}. \tag{1.6}$$

However, a dispersion analysis gives a higher result [5], $\Delta_\sigma = 15.2 \pm 0.4$ MeV, and this yields

$$\sigma_{\pi N} \cong 49 \pm 8 \text{ MeV}. \tag{1.7}$$

In another method based on forward dispersion relations, the authors [6] have extracted $\Sigma_{\pi N}$ from low-energy data. In this procedure $\Sigma_{\pi N}$ is parametrized as a function of the scattering lengths, Gasser, Leutwyler, and Sainio [7] have used this to obtain

$$\sigma_{\pi N} \cong 45 \pm 4 \pm 4 \pm 4 \text{ MeV}, \tag{1.8}$$

where errors refer to statistics, to database modifications, and to the input uncertainties, respectively. They have confirmed the old estimate $\Sigma_{\pi N} \cong 60$ MeV also. The rapid increase in $\sigma(t)$ from $\sigma(0) \cong 45$ MeV to $\sigma(2M_\pi^2) \cong 60$ MeV is attributed to the anomalous threshold associated with $\pi\pi$ intermediate states.

The difference in numerical values of $\hat{\sigma}$ obtained from mass splittings of octet baryons [Eq. (1.5)], and $\sigma_{\pi N}$ deduced from pion-nucleon scattering data is interpreted as a manifestation of nontrivial matrix element of scalar strange quark operator between one nucleon states. Its contribution to the nucleon mass term is believed to be larger than $\sigma_{\pi N}$ [1,2,7]:

$$m_s\langle N|\bar{s}s|N\rangle \cong 130 \text{ MeV}, \tag{1.9}$$

where only $\Delta_\sigma \cong 15$ MeV has been used, since the one-loop result (1.6) is afflicted by large higher-order corrections [7].

Calculations of the σ term ($\sigma_{\pi N}$) have been attempted by various authors in various ways: the chiral soliton model [12], linear σ model [13], Nambu-Jona-Lasinio model [14], bag model [12], QCD sum rule approach [15], etc. Our point of view in this article will be that the method of reconciling theoretical and experimental estimates of σ terms along with the violations of the OZI rule in the form of Eq. (1.9) requires the construction of a constituent quark picture that takes into account contributions of sea quarks as well. The important question here is the following: To what extent can a picture of a constituent quark as a valence quark dressed with sea quarks and gluons be derived from the basic laws of QCD? Being a low-energy phenomena, it is deeply related to the nonperturbative aspects of QCD. In the present article, we have tried to understand the flavor mixing in the valence quark using the idea of dynamical symmetry breaking and have extended it to estimate the light quark scalar condensate between one nucleon states. A reliable theoretical determination of such low-energy parameters using the basic laws of QCD is important.

It is well known that light quarks are attributed to two kinds of masses. The current quark masses are small, typically 5–10 MeV for u and d quarks, and are important for current algebra calculations and for deep inelastic scattering processes. The dynamical masses are relatively large—these are $\sim$ 300 MeV for u and d quarks—and are important to understanding hadron spectroscopy, nucleon magnetic moments, etc. The constituent masses have to be understood as a sum of these two kinds of masses, evaluated at the constituent mass scale [16]. On the other hand, according to the well known Feynman-Hellman theorem, the matrix elements of scalar quark currents between one nucleon states can be derived as

$$\langle N|m_i\bar{q}_i q_i|N\rangle = m_i\frac{\partial m_N}{\partial m_i}, \tag{1.10}$$

where m_N is the nucleon mass and m_i is the current quark mass of the quark in question. In the constituent quark picture, the hadron mass is supposed to be made up of constituent quark masses. If the phenomenological estimate (1.9) has any truth to it, it means that the sea quarks, and, in particular, the strange quarks, have a role in the constitution of the constituent quark mass or the dynamical mass, for that matter. In the process of forming a constituent quark, the quark is "dressed" by gluonic and even $\bar{s}s$ quark fields. It is no longer the naive object that occurs in the QCD Lagrangian. It is this dressed object which may generate strange quark matrix elements [2]. Here, the strange quark may have a special role due to the proximity of its mass to the QCD scale parameter:

$$m_s \sim \Lambda_{\text{QCD}}.$$

The above point of view [16] is not the only one which connects the constituent quark model (CQM) to QCD. Damgaard, Nielsen, and Sollacher [17] have used a gauge symmetric approach for extracting effective degrees of freedom from QCD and have identified chirally rotated quark fields with constituent quarks. A related and perhaps complementary picture has been suggested by Kaplan [18], in which constituent quarks are viewed as Skyrmions in color space. Wilson *et al.* [19] have argued that CQM and QCD can be reconciled in light front dynamics.

In the usual approaches to dynamical symmetry breaking, it is the interaction of the particular (light) quark with gluons which is responsible for the generation of the dynamical mass of the quark [20]. In such an approach, the dynamical mass (evaluated on "mass shell")

$$m_{\rm dyn} \sim \Lambda_{\rm QCD}$$

apart from some calculable numerical factors [21], since $\Lambda_{\rm QCD}$ is the only mass scale available in the theory. However, if we consider a theory which has other mass parameters, such as current quark masses, there is no reason why these additional mass parameters will not contribute to the generation of the dynamical mass.

To simplify the discussion, let us consider QCD with only three flavors of light quarks. The heavier quarks may be taken as decoupled in an effective way in the manner of the Appelquist-Carazzone theorem [2]. Among the light quarks, we may consider the masses of the u and d quarks ($m_q \ll \Lambda_{\rm QCD}$) to be zero, leaving only m_s as the other nontrivial mass parameter. In such a theory, the dynamical mass and the quantities such as quark and gluon condensate, which were earlier expressible in terms of $\Lambda_{\rm QCD}$ only [22], should now involve m_s as well. For instance, the light quark condensate can now be written as

$$\langle \bar{q}q \rangle = a\Lambda^3 + b\Lambda^2 m_s + c\Lambda m_s^2 + \cdots ,$$

where $\Lambda = \Lambda_{\rm QCD}$ and a, b, c are numerical coefficients, which may include logarithms in mass parameters, and are presumably in decreasing order. For instance, the leading chiral behavior of the quark condensate in terms of m_π^2 (or equivalently m_q, since $m_\pi^2 \propto m_q$) as an expansion parameter has been worked out by Novikov *et al.* [23]; Leutwyler and Smilga [24] have tried an expansion of the quark condensate in terms of the current quark mass. Obviously this dependence should be different depending on whether the current quark mass pertains to the same quark or a quark of different flavor. It is this kind of dependence which leads to isospin [25] and SU(3) splitting of quark condensates which otherwise would be the same for all the light quarks in the chiral limit.

The simplest way in which the dependence of $\Sigma(p^2)_{\rm dyn}$ on the current quark mass of a sea quark may be incorporated is through the vacuum polarization of a gluon line that appears in the Schwinger-Dyson equation for the mass function of the quark. In this connection, recently [26] it has been emphasized that in QCD the vacuum polarization can play an important role in the physics of light quarks. We can improve this simple approach by invoking operator product expansion and supplementing this diagram with the ones which have nonperturbative vacuum condensates, which appear in QCD sum rule methods as phenomenological constants [27]. In a systematic approach, all these contributions should be down compared to the leading contribution to the dynamical mass.

In Sec. II, we derive a dependence of the dynamical mass function on the current quark mass of a light sea quark by incorporating the vacuum polarization in the gluon propagator, which will be used for writing the Schwinger-Dyson equation. In Sec. III, we extend our result to compute the pion-nucleon Σ term and the strange quark content of the nucleon. In Sec. IV, we discuss the results and give our conclusions. Finally, we give some mathematical details in the appendices.

II. FLAVOR MIXING IN DYNAMICAL MASS APPROACH

We start with a QCD with three light flavors of quarks u, d, and s. We take $m_u \sim m_d \ll m_s$ and $m_s \sim \Lambda_{\rm QCD}$. The heavier quarks may be considered as decoupled in an effective way, their contribution being $O(m_h^{-2})$ through a gluon vacuum polarization [2]. The dynamical mass of the light quarks has been calculated mainly using two methods: (i) by solving the gap equation obtained by writing the Schwinger-Dyson equation for the quark propagator [20] and (ii) by using the operator product expansion (OPE) of those nonperturbative vacuum expectation values which are known to contribute to QCD sum-rule phenomenology [16]. In either approach, the asymptotic behavior of the dynamical mass function comes out to be

$$\Sigma_D(p^2) \underset{p^2 \to \infty}{\sim} \frac{1}{p^2}$$

apart from a logarithm, which, in any case, we will neglect in our discussion for the sake of simplicity. In fact, for our purpose, we shall use the following convenient parametrization of the dynamical mass function [28]:

$$\Sigma_D(p^2) = \frac{2M^3}{p^2 + M^2}. \tag{2.1}$$

It has the property that at $p^2 = M^2, \Sigma_D = M$ and the asymptotic behavior for large p^2 is retained. It can be extended down to the low-energy region as well, at least when it appears within integration. We first compute the self-energy of a u or d quark for the diagram corresponding to Fig. 1, where the hatched area denotes the quark propagator with the dynamical mass (2.1) incorporated, but the current quark mass neglected. A sea quark contributes through the vacuum polarization of the gluon propagator. For a given circulating quark of mass m_i, we use [29]

$$\begin{aligned} D_{\mu\nu}^{ab(1)}(k) &= \frac{-i\delta^{ab}}{k^2 + i\varepsilon}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{g^2}{(4\pi)^2}\frac{2}{3}(-1)\left[-\ln\frac{m_i^2}{\mu^2} + \int_0^1 dx \frac{(3x^2 - 2x^3)(1-2x)k^2}{k^2 x(1-x) - m_i^2}\right] \\ &= -\delta^{ab}\frac{i}{k^2 + i\varepsilon}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\pi(k^2)(-1) \end{aligned} \tag{2.2}$$

for the gluon propagator with vacuum polarization. μ is the renormalization point and we have written only the m_i-dependent part of the expression. We assume that the mass parameter m_i appearing in the loop is the current quark mass. The evaluation of diagram (1) in Euclidean variables yields

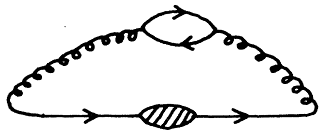

FIG. 1. Sea quark contribution to the dynamical mass in two loops order. Hatched area denotes the dynamical mass of the valence quark.

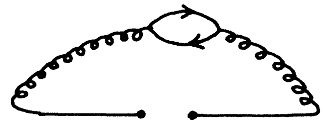

FIG. 2. Sea quark contribution to the dynamical mass with condensing of the valence quark lines.

$$\delta\Sigma_0^{(1)}(p^2) \cong -3C_2 \int \frac{d^4k}{(2\pi)^4} \frac{g^2(k,p)\Sigma_0(k^2)\Pi[-(k-p)^2]}{(k-p)^2[k^2+\Sigma^2(k^2)]}, \tag{2.3}$$

where $\Sigma_0 = \Sigma_D$ given in Eq. (2.1). In order to make the integration analytically doable, we set the arguments of the coupling to be p^2. This is consistent with our earlier approximation wherein we have neglected the logarithm appearing in Σ_D. Obviously this approximation is not going to alter the power behavior of our results below [Eqs. (2.4) and (2.4′)] for $\delta\Sigma_0^{(1)}(p^2)$. In Sec. III, where we have done a numerical evaluation of $m_i\partial\Sigma/\partial m_i$, we have restored the variable nature of the coupling in the integrand. Even now the integration is not so easy. We have given the details of this integration in Appendix A. In the widely used linear approximation [20] where Σ^2 in the denominator is set to zero, the result of the calculation, up to the lowest order in $1/p^2$, is

$$\delta\Sigma_0^{(1)}(p^2) = \frac{\alpha_s^2(p^2)}{3\pi^2}\frac{M^3}{p^2}\left[\frac{3}{2}\frac{M^2}{p^2}\ln\frac{m_i^2}{p^2} - \frac{m_i^2}{p^2}\left(11 + 2\ln\frac{m_i^2}{p^2} + 6\ln\frac{M^2}{p^2}\right)\right]. \tag{2.4}$$

Here we have confined ourselves to only those terms which involve m_i. In a different approximation, where one replaces Σ^2 in the denominator by M^2 [30], we get a somewhat different result:

$$\delta\Sigma_0^{(1)}(p^2) = \frac{\alpha_s^2(p^2)}{3\pi^2}\frac{M^3}{p^2}\left[\frac{3}{2}\frac{M^2}{p^2}\ln\frac{m_i^2}{p^2} - \frac{m_i^2}{p^2}\left(18 + 2\ln\frac{m_i^2}{p^2} + 6\ln\frac{M^2}{p^2}\right)\right]. \tag{2.4′}$$

We shall supplement $\delta\Sigma_0^{(1)}$ with the nonperturbative contributions that appear as a consequence of the OPE of nonperturbative vacuum expectation values. Figures 2 and 3 show the lowest nontrivial contributions where open fermion lines represent the quark condensates. Furthermore, we improve the constant field approximation for the vacuum quarks by taking the value of the parameter (D is the covariant derivative)

$$\lambda_q^2 = \langle\bar{q}D^2q\rangle/\langle\bar{q}q\rangle$$

that specifies the average virtuality of the vacuum quarks to be nonzero. More specifically, in Minkowski space, we have chosen

$$\langle 0| : \bar{q}_i(y)_{j\alpha}q_i(z)_{n\zeta} : |0\rangle = \frac{\delta_{\alpha\zeta}}{12}\langle\bar{q}_i(0)q_i(0)\rangle\left(1 + im_i(\not{y} - \not{z}) - \frac{m_i^2(y-z)^2}{8} + \cdots\right)_{nj}\exp\left[\frac{\lambda_q^2}{2}\frac{(y-z)^2}{4}\right], \tag{2.5}$$

where α, ζ are color indices and j, n are Dirac indices and it is understood that $(y-z)^2 < 0$. The expansion in the mass parameter (taken to be current quark mass) uses equations of motion [16] whereas the exponential function parametrizes the contributions of higher-order operators [31].

In the lowest order of $1/p^2$, the parameter λ_q^2 does not contribute to the diagram corresponding to Fig. 2. The diagram corresponding to Fig. 3 diverges in the infrared region due to the appearance of higher powers of momenta in the denominator in the loop integral. As usually done in QCD sum rule methods [32], we regulate it by using an infrared cutoff Λ_c, which we choose to be $\Lambda_{\rm QCD}$. In the next section, we show numerically that the results, for the lighter quarks, are rather insensitive to the choice of Λ_c. Results of calculations of diagrams corresponding to Figs. 2 and 3 give

$$\delta\Sigma_0^{(2)}(p^2) \cong [\alpha_s(p^2)]^2\frac{10}{9}m_i^2\frac{|\langle\bar{q}q\rangle|}{p^4}, \tag{2.6}$$

$$\delta\Sigma_0^{(3)}(p^2) \cong [\alpha_s(p^2)]^2\frac{2}{3}M^3\frac{m_i|\langle\bar{q}_iq_i\rangle|}{p^4}\left[\exp\left(\frac{-2\Lambda_c^2}{\lambda_q^2}\right)\left(\frac{\lambda_q^2}{2\Lambda_c^2} - 3\right)\frac{1}{\Lambda_c^2} - \frac{\lambda_q^2}{2\Lambda_c^4} + \frac{4}{\Lambda_c^2} - \frac{6}{\lambda_q^2}\mathrm{Ei}\left(-\frac{2\Lambda_c^2}{\lambda_q^2}\right)\right]. \tag{2.7}$$

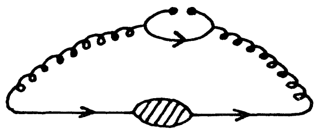

FIG. 3. Sea quark contribution to the dynamical mass with condensing of one of the sea quark lines.

Ei in Eq. (2.7) is the exponential integral function. In (2.6) and (2.7) also, we have retained only m_i-dependent terms and only magnitudes of quark condensates appear as used in Ref. [33] and shown by Barducci *et al.* [20].

The dependence of $\delta\Sigma_0^{(3)}$ on Λ_c could have been weakened overall by one power of Λ_c, had we used the results of Ref. [34] for the two-vector current correlation function, which sums all the powers in m_i^2. Unfortunately, the momentum integration, in that case, becomes analytically undoable and it becomes difficult to extract the asymptotic behavior of $\delta\Sigma_0^{(3)}$. The gluon condensate and other higher-dimensional condensates do not contribute to the lowest-order term in $1/p^2$ considered here.

$\delta\Sigma_0^{(2)}$ is a full solution corresponding to the Feynman diagram shown in Fig. 2, since it involves a trivial loop integration [barring the fermion loop integral given in (2.2)]. However, $\delta\Sigma_0^{(1)}$ and $\delta\Sigma_0^{(3)}$ are not the complete terms corresponding to the respective diagrams, since the complete term would correspond to the solution of an integral equation. We can find a better solution as follows.

The master equation for the dynamical mass function, say in the linear approximation, is

$$\Sigma(p^2) = 3C_2 \int \frac{d^4k}{(2\pi)^4} \frac{g^2(k,p)\Sigma(k^2)}{k^2(k-p)^2} D(-(k-p)^2), \tag{2.8}$$

where D is the scalar function in the full gluon propagator:

$$D_{\mu\nu}^{ab}(k) = \frac{-i\delta^{ab}}{k^2+i\varepsilon}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) D(k^2).$$

Denoting the m_i-dependent part of Σ corresponding to Figs. 1 and 3 as $\delta\Sigma$ and that of D as $-\pi$, we can write

$$\delta\Sigma(p^2) \cong 3C_2 \int \frac{d^4k}{(2\pi)^4} \frac{g^2(k,p)\delta\Sigma(k^2)}{k^2(k-p)^2} - 3C_2 \int \frac{d^4k}{(2\pi)^4} \frac{g^2(k,p)\Sigma_0(k^2)\pi(-k^2)}{k^2(k-p)^2}, \tag{2.9}$$

where the second term on the right-hand side (RHS) is $\delta\Sigma_0 = \delta\Sigma_0^{(1)} + \delta\Sigma_0^{(3)}$. We convert the above nonhomogeneous integral equation in the unknown function $\delta\Sigma$ into a differential equation, as is usually done in dynamical mass calculations [20], and then solve it in the leading log approximation. Writing $\delta\Sigma_0(p^2)$ as

$$\delta\Sigma_0(p^2) = [\alpha_s(p^2)]^2 \frac{1}{p^4}\left[A + B\ln\frac{m_i^2}{p^2} + C\ln\frac{M^2}{p^2}\right] \tag{2.10}$$

we get, for the difference $\delta\Sigma_1 = \delta\Sigma - \delta\Sigma_0$,

$$\delta\Sigma_1(p^2) \underset{\lim p^2\to\infty}{\simeq} \frac{[\alpha_s(p^2)]^3}{\pi p^4}\left[-\frac{B}{2}\ln\frac{m_i^2}{p^2} - \frac{C}{2}\ln\frac{M^2}{p^2} + B\left(\frac{5}{4}+\frac{1}{\beta_0}\right)\frac{\ln(m_i^2/p^2)}{\ln(p^2/\Lambda^2)} + C\left(\frac{5}{4}+\frac{1}{\beta_0}\right)\frac{\ln(M^2/p^2)}{\ln(p^2/\Lambda^2)} - \frac{A}{2} + \frac{3}{4}(B+C)\right], \tag{2.11}$$

where $\beta_0 = 11 - 2n_f/3$ and $\Lambda = \Lambda_{\rm QCD}$. Combining expressions (2.4), (2.6), (2.7), and (2.11), we obtain the total m_i-dependent contribution to the dynamical mass function in the subasymptotic limit in the leading order

$$\begin{aligned}\delta\Sigma_{\rm tot}(p^2) \underset{\lim p^2\to\infty}{\simeq} \frac{[\alpha_s(p^2)]^2}{p^4}\Bigg(&2M^3\Bigg\{-\frac{11m_i^2}{6\pi^2} + \frac{m_i|\langle\bar{q}_iq_i\rangle|}{3}\left[\exp(-2\Lambda_c^2/\lambda_q^2)\left(\frac{\lambda_q^2}{2\Lambda_c^2}-3\right)\frac{1}{\Lambda_c^2} - \frac{\lambda_q^2}{2\Lambda_c^4} + \frac{4}{\Lambda_c^2}\right.\\ &\left.-\frac{6}{\lambda_q^2}\mathrm{Ei}\left(-\frac{2\Lambda_c^2}{\lambda_q^2}\right)\right]\Bigg\}\left(1-\frac{\alpha_s(p^2)}{2\pi}\right) + \frac{10}{9}m_i^2|\langle\bar{q}q\rangle|\\ &+\frac{2M^3}{3\pi^2}\left(\tfrac{3}{4}M^2 - m_i^2\right)\left\{\ln\frac{m_i^2}{p^2} + \frac{\alpha_s}{\pi}\left[\frac{3}{4} - \frac{1}{2}\ln\frac{m_i^2}{p^2} + \left(\frac{5}{4}+\frac{1}{\beta_0}\right)\frac{\ln(m_i^2/p^2)}{\ln(p^2/\Lambda^2)}\right]\right\}\\ &-\frac{2M^3m_i^2}{\pi^2}\left\{\ln\frac{M^2}{p^2} + \frac{\alpha_s}{\pi}\left[\frac{3}{4} - \frac{1}{2}\ln\frac{M^2}{p^2} + \left(\frac{5}{4}+\frac{1}{\beta_0}\right)\frac{\ln(M^2/p^2)}{\ln(p^2/\Lambda^2)}\right]\right\}\Bigg).\end{aligned} \tag{2.12}$$

Figure 1 has two loops but no operators, whereas Figs. 2 and 3 have only one loop and an operator inserted. This results in increase in powers of p^2 in the denominator, apart from an increase in powers of α_s, compared to the corresponding cases with one loop lesser, in the asymptotic limit. By the same token, we expect that any loop correction and/or operator insertion to these diagrams will result in an increase in powers of $1/p^2$ as well as α_s. Incidentally, this also indicates that the usual derivation of $\Sigma_D \sim 1/p^2$ is stable under loop corrections for asymptotic momenta. All the calculations, here, have been performed in the Landau gauge which has certain advantages for such calculations (Haymaker in [20]) although it may happen that on the mass shell the gauge dependence drops out [16]. We have not tried to improve our results using renormalization group equations.

At this point it should be added that our normalization condition for the mass function $\Sigma_0(p^2 = M^2) = M$ should be augmented to accommodate new terms as well. For this, let us define $\Sigma_D(p^2) = \Sigma_0(p^2) + \delta\Sigma^{(1)}(p^2)$, where $\delta\Sigma^{(1)}(p^2)$ is the contribution to the dynamical mass coming from the complete one-loop corrections and the lowest-dimensional nontrivial operator insertions, which includes (2.12). We should define a new mass scale M' such that $\Sigma_D(p^2 = M'^2) = M'$, then M' is the new generated mass scale which incorporates the effect of the light current quark masses in addition to that of $\Lambda_{\rm QCD}$, whereas M was expressible in terms of $\Lambda_{\rm QCD}$ only.

Using Eq. (2.12), we can find out the contribution of a light sea quark q_i in the constitution of the dynamical mass of a u or d quark:

$$
\begin{aligned}
m_i \frac{\partial \Sigma_D(p^2)}{\partial m_i} \underset{\lim p^2 \to \infty}{\widetilde{\ }} \frac{[\alpha_s(p^2)]^2}{p^4} \Bigg(& 2M^3 \Bigg\{ -\frac{11 m_i^2}{3\pi^2} + \frac{m_i |\langle \bar{q}_i q_i \rangle|}{3} \Bigg[\exp(-2\Lambda_c^2/\lambda_q^2) \frac{1}{\Lambda_c^2} \left(\frac{\lambda_q^2}{2\Lambda_c^2} - 3 \right) - \frac{\lambda_q^2}{2\Lambda_c^4} + \frac{4}{\Lambda_c^2} \\
& - \frac{6}{\lambda_q^2} \mathrm{Ei}\left(-\frac{2\Lambda_c^2}{\lambda_q^2} \right) \Bigg] \Bigg\} \left(1 - \frac{\alpha_s(p^2)}{2\pi} \right) + \frac{20}{9} m_i^2 |\langle \bar{q} q \rangle| - \frac{4M^3 m_i^2}{3\pi^2} \Bigg\{ \ln \frac{m_i^2}{p^2} \\
& + \frac{\alpha_s(p^2)}{\pi} \left[\frac{3}{4} - \frac{1}{2} \ln \frac{m_i^2}{p^2} + \left(\frac{5}{4} + \frac{1}{\beta_0} \right) \frac{\ln(m_i^2/p^2)}{\ln(p^2/\Lambda^2)} \right] \Bigg\} \\
& + \frac{2M^3}{3\pi^2} \left(\frac{3}{4} M^2 - m_i^2 \right) \left\{ 2 + \frac{\alpha_s(p^2)}{\pi} \left[-1 + \left(\frac{5}{4} + \frac{1}{\beta_0} \right) \frac{2}{\ln(p^2/\Lambda^2)} \right] \right\} \\
& - \frac{4M^3 m_i^2}{\pi^2} \left\{ \ln \frac{M^2}{p^2} + \frac{\alpha_s(p^2)}{\pi} \left[\frac{3}{4} - \frac{1}{2} \ln \frac{M^2}{p^2} + \left(\frac{5}{4} + \frac{1}{\beta_0} \right) \frac{\ln(M^2/p^2)}{\ln(p^2/\Lambda^2)} \right] \right\} \Bigg) .
\end{aligned}
\tag{2.13}
$$

$\partial\Sigma_D/\partial m_i$ also describes the mixing of a quark of ith flavor into the wave function of a (constituent) quark of different flavor [14]. Equations (2.12) and (2.13) are the main derivations of this paper. In the next section, we will try to see its phenomenological implications for light baryons.

III. LIGHT QUARK SCALAR CONTENT OF A NUCLEON

In order to find the matrix elements of light quark scalar operators between one nucleon states, we shall assume (i) that the constituent quark mass can be written as [16]

$$
m_{\rm const} = \Sigma_D(p^2 = m_{\rm const}^2) + m_{\rm curr} \tag{3.1}
$$

and (ii) the results obtained from the three diagrams, considered in the previous section, when appropriately extended down to $p^2 = m_{\rm const}^2 \cong M^2$ should be sufficient for the purpose. Finally we shall use Feynman-Hellman theorem (1.10) to calculate the matrix element. The contribution of the expression (2.6) has been extended down to $p^2 = M^2$ as such. However, for the contributions of Figs. 1 and 3, we take the running coupling constant inside the integration. It has been assumed that $g^2(k,p)$ has the widely used form [20]

$$
g^2(k,p) = g^2(k^2)\theta(k^2 - p^2) + g^2(p^2)\theta(p^2 - k^2). \tag{3.2}
$$

Furthermore, for $\alpha_s(Q^2)$, we have used the commonly used hard-freeze form [35]:

$$
\frac{\alpha_s(Q^2)}{\pi} = \begin{cases} (12/27)\{1/\ln(Q^2/\Lambda_{\rm QCD}^2)\} & \text{for } Q^2 \geq Q_0^2, \\ \text{const} \equiv H \quad \text{for } Q^2 \leq Q_0^2, \end{cases} \tag{3.3}
$$

with $H = (12/27)\{1/\ln(Q_0^2/\Lambda_{\rm QCD}^2)\}$.

Numerically, for the u and d content of the proton $\delta\Sigma_0^{(1)}$ is important, whereas for the s content of the proton $\delta\Sigma_0^{(2)}$ gives the dominant contribution. Momentum integration up to Q_0^2 can be done analytically; momentum integration beyond Q_0^2 and the entire parametric integration has been done numerically. We have taken $m_u = 5$ MeV, $m_d = 9$ MeV, $m_s = 180$ MeV and $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle$, $\langle \bar{s}s \rangle = 0.8\langle \bar{u}u \rangle$. We have varied $|\langle \bar{q}q \rangle|$ from $(0.224\ \text{GeV})^3$ to $(0.256\ \text{GeV})^3$, the values normally used in literature [3,16,27]; at the same time M, the parameter that characterizes the dynamical mass of a quark and is close to the constituent quark mass, has been varied from 0.28 to 0.32 GeV, again a range of values that has been normally used in literature [20,33] for the dynamical mass in order to reproduce the correct hadronic phenomenology. $\alpha_s(Q_0^2)/\pi$ has been varied from 0.25 (which has been considered as a critical value for dynamical symmetry breaking to occur by some authors [20]) to 0.26 (which is the freezing value of the coupling obtained in Refs. [35,36]); $\Lambda_{\rm QCD}$ has been varied from 280 to 300 MeV for three flavors [35] (however, see the results below for $\Lambda_{\rm QCD} \cong 200$ MeV and $\alpha_s/\pi = 0.28$, also) and λ_q^2 has been taken to be 0.4 GeV^2 [31]. For u and d quarks in the sea, results for $m_{u,d}(\partial\Sigma_D/\partial m_{u,d})$ lies between (5–6) MeV, giving rise to an almost unique result for the σ term,

$$\sigma_{\pi N} \cong 53 \text{ MeV}, \tag{3.4}$$

where we have included the contributions of current quark masses of the valence quark as implied by (3.1) and have averaged over proton and neutron. In all the cases, contributions from the region where momentum is larger than Q_0 turns out to be numerically insignificant. For the s quark, the value of $m_s(\partial\Sigma_D/\partial m_s)$ has a larger variation (40.5–67.5) MeV which results in

$$m_s\frac{\partial m_N}{\partial m_s} \cong 122\text{–}203 \text{ MeV}. \tag{3.5}$$

In Ref. [37] another form of the "hard freeze" model of coupling has been used to fit the absolute magnitude of the pion-nucleon cross section which needs $\Lambda_{\rm QCD} =$ 0.2 GeV and $H = 0.28$ [see Eq. (3.3)]. The same frozen coupling has been used successfully in subsequent work on deriving nucleon structure functions from the constituent-quark model [38]. With this set of parameters, our result for $\sigma_{\pi N}$ remains almost unchanged with

$$\sigma_{\pi N} \cong 54 \text{ MeV}. \tag{3.6}$$

However, the same set of parameters result in significant increase in the strange contribution to the nucleon mass:

$$m_s(\partial m_N/\partial m_s) \cong 264 \text{ MeV}. \tag{3.7}$$

To see what happens to the integrals if one lets $\alpha_s(Q^2)$ keep on running below Q_0^2 used above, we note that $\alpha_s(Q^2)$ increases indefinitely rendering any perturbative calculation ineffective as $Q^2 \to \Lambda_{\rm QCD}^2$, beyond which it becomes unphysical. Hence in the following, by the running of $\alpha_s(Q^2)$ for small Q^2 we basically mean choosing a different value of H corresponding to the lower values of Q_0^2. The result (3.4) has been obtained using $Q_0/\Lambda_{\rm QCD}$=2.35 and 2.43 for $H = 0.26$ and 0.25, respectively. If we choose $Q_0/\Lambda_{\rm QCD} = 2$ and 1.5 corresponding to $H = 0.32$ and 0.55, then for $\Lambda_{\rm QCD} = 280$ MeV the results for $m_{u,d}(\partial\Sigma_D/\partial m_{u,d})$ increases by approximately 8 and 20 MeV, respectively.

Thus, our calculated results for $\sigma_{\pi N}$,(3.4) and (3.6), obtained using certain model behavior of the theory for the low-energy region, have a rather good overlap with what has been deduced from experimental and phenomenological analyses. Our estimate of scalar strange quark condensate in a nucleon has a larger scatter, but the lower values obtained in (3.5) are consistent with the current determination of this quantity [see Eq. (1.9)].

We also observe that the numerical results, obtained at low Euclidean momentum, are quite stable under the change of various parameters when u and d quarks are appearing in the sea. However, the same is not true when the s quark appears in the sea. This is not unexpected, since we have not included terms of $O(m_i^3)$ in $\delta\Sigma_{\rm tot}$. As a consequence, the extrapolation of the result down to $p^2 = M^2$ may introduce a larger error in this case, since $m_s \sim \Lambda_{\rm QCD}$.

IV. DISCUSSION AND CONCLUSION

Dynamical symmetry breaking is by now a well known idea which has been used in QCD to describe phenomena in subasymptotic region. The existence of vacuum condensates of fundamental fields in QCD has been well established through successful calculations of numerous low- and intermediate-energy hadronic phenomena [39] and has been theoretically established through computer simulations. We have combined these two ideas to calculate the contribution of a light sea quark to the constituent mass of a light quark of different flavor. The sea quarks in a constituent quark have been assumed to arise as a result of vacuum polarization of gluons which dress the valence quark. Thus we have employed only QCD based techniques to understand one of the low-energy hadronic phenomena. We apply our result to two different mass scales ($m_{u,d}$ and m_s) which differ by over an order of magnitude. The fact that in both cases our numerical results come reasonably close to what has been found on experimental and phenomenological grounds gives certain credence to our approach.

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APPENDIX A: INTEGRALS APPEARING IN $\delta\Sigma_0^{(1)}(p^2)$

In this appendix, we shall perform the following integration (momenta are Euclidean):

$$\int \frac{d^4k}{k^2(k-p)^2(M^2+k^2)}\left[-\ln\frac{m^2}{p^2} + \int_0^1 dx\frac{(3x^2-2x^3)(1-2x)(k-p)^2}{m^2+(k-p)^2x(1-x)}\right] \tag{A1}$$

which is required for the evaluation of $\delta\Sigma_0^{(1)}$. First, concentrate on the second term. For the sake of convenience, we shall first perform momentum integration and then x integration. Call $(3x^2 - 2x^3)(1-2x) = A$ and $x(1-x) = B$. The angular integrations can be readily performed:

$$\int \frac{d\Omega_k}{(k-p)^2 + m^2/B} = \frac{\pi^2}{k^2p^2}\left\{\frac{m^2}{B} + k^2 + p^2 - \left[\frac{m^4}{B^2} + \frac{2m^2}{B}(k^2+p^2) + (k^2-p^2)^2\right]^{1/2}\right\}. \tag{A2}$$

Thus, the second part reduces to

$$\frac{\pi^2}{2p^2}\int_0^1 dx\,\frac{A(x)}{B(x)}\int_0^\infty \frac{dk^2}{k^2(M^2+k^2)}\left\{\frac{m^2}{B} + k^2 + p^2 - \left[\frac{m^4}{B^2} + \frac{2m^2}{B}(k^2+p^2) + (k^2-p^2)^2\right]^{1/2}\right\}. \tag{A3}$$

Although the whole integral is convergent, the parts of it may have divergencies. So, we shall keep upper and lower limits of k^2 integration as Λ_u^2 and ε^2. Recalling that $p^2 \gg M^2 > m^2$ and at the end, we have to take the limit $\varepsilon^2 \to 0$, $\Lambda_u^2 \to \infty$, we get for the second part of the momentum integration of (A.3):

$$\int_{\varepsilon^2}^{\Lambda_\mu^2} \frac{dk^2}{k^2(M^2+k^2)}\left[\frac{m^4}{B^2} + \frac{2m^2}{B}(k^2+p^2) + (k^2-p^2)^2\right]^{1/2}$$

$$\cong \frac{p^2 + m^2/B}{M^2}\ln\frac{M^2}{\varepsilon^2} - \frac{Bp^2 - m^2}{Bp^2+m^2} - 1 + \left[\frac{Bp^2-m^2}{Bp^2+m^2} + \frac{2p^2M^2m^2/B}{(p^2+m^2/B)^3}\right]\left[\ln M^2 - 2\,\ln(p^2B+m^2) + 2\,\ln B\right]$$

$$+\left[\frac{B(p^2+M^2/2)-m^2}{p^2B+m^2} - \frac{M^2B(p^2B-m^2)^2}{2(p^2B+m^2)^3}\right]\left[\ln\frac{m^2}{\Lambda_u^2} + \ln(m^2 + B\Lambda_u^2) - 2\,\ln B\right] - \frac{M^2B}{2(p^2B+m^2)}$$

$$-\frac{M^2B[B(p^2+M^2/2)-m^2]}{(p^2B+m^2)^2} - \frac{M^2B(Bp^2-m^2)^2}{2(p^2B+m^2)^3} - \frac{p^2M^2m^2B^2}{(p^2B+m^2)^3} + \ln\frac{B\Lambda_u^2+m^2}{m^2}$$

$$+\left(\frac{3}{2}p^2B + \frac{p^2B}{2}\,\frac{m^2 - B\Lambda_u^2}{m^2+B\Lambda_u^2}\right)\frac{B(p^2+M^2/2)-m^2}{(p^2B+m^2)(B\Lambda_u^2+m^2)} - \frac{m^2}{B\Lambda_u^2}, \tag{A4}$$

where we have used expansion of powers and logarithms, keeping in view of the fact that we will need only leading and next-to-leading order terms in the expansion. Also

$$\int_{\varepsilon^2}^{\Lambda_u^2} dk^2\,\frac{m^2/B + p^2 + k^2}{k^2(M^2+k^2)} \cong \ln\frac{\Lambda_u^2}{M^2} + \frac{m^2/B+p^2}{M^2}\ln\frac{M^2}{\varepsilon^2} - \frac{m^2}{B\Lambda_u^2}. \tag{A5}$$

Subtracting (A4) from (A5), arranging the terms and leaving those terms which drop after x integration on taking the limit of Λ_u^2, we obtain

$$\int_0^{\Lambda_u^2} \frac{dk^2}{k^2(M^2+k^2)}\left\{\frac{m^2}{B} + k^2 + p^2 - \left[\frac{m^4}{B^2} + \frac{2m^2}{B}(k^2+p^2) + (k^2-p^2)^2\right]^{1/2}\right\}$$

$$\cong \left[\frac{Bp^2-m^2}{Bp^2+m^2} + \frac{2p^2m^2M^2B^2}{(Bp^2+m^2)^3}\right]\left[2\,\ln\left(B + \frac{m^2}{p^2}\right) + \ln\frac{p^4}{m^2M^2} - \ln\left(B + \frac{m^2}{\Lambda_u^2}\right)\right]$$

$$+\frac{2Bp^2}{Bp^2+m^2} + \frac{2M^2p^4B^3}{(p^2B+m^2)^3} + \ln\frac{m^2}{M^2} - \ln\left(B + \frac{m^2}{\Lambda_u^2}\right). \tag{A6}$$

The x integration can be conveniently performed, using the factorization of quadratic expressions in x which appear in denominators and logarithms, and by decomposing denominators through partial fractions. We write below our results of prominent integrals only to leading and next-to-leading order in $1/p^2$. Some of the integrals written in isolation diverge at the upper limit. In that case we use $1-\varepsilon'$ as the upper limit:

$$\int_0^{1-\varepsilon'} dx \frac{A}{B} = \tfrac{5}{3} + \ln \varepsilon',$$

$$\int_0^1 dx \frac{A}{p^2 B + m^2} \cong \frac{1}{p^2}\left(\frac{5}{3} + \ln\frac{m^2}{p^2} - \frac{5m^2}{p^2}\right),$$

$$\int_0^{1-\varepsilon'} dx \frac{A}{B}\frac{p^2 B - m^2}{p^2 B + m^2} \cong \tfrac{5}{3} - \ln \varepsilon' + 2\ln\frac{m^2}{p^2} - 12\frac{m^2}{p^2},$$

$$\int_0^1 dx \frac{A}{B}\frac{B\,\ln(B + m^2/\Lambda_u^2)}{(Bp^2 + m^2)^2} \cong \frac{-1}{m^2 p^2}\ln\frac{m^2}{p^2},$$

$$\int_0^{1-\varepsilon'} dx \frac{A}{B}\ln\left(B + \frac{m^2}{\Lambda_u^2}\right) \cong -\frac{28}{9} + \ln\varepsilon' \ln\frac{m^2}{\Lambda_u^2} - \frac{1}{2}\left(\ln\frac{m^2}{\Lambda_u^2}\right)^2,$$

$$\int_0^{1-\varepsilon'} dx \frac{A}{B}\ln\left(B + \frac{m^2}{p^2}\right) \cong -\frac{28}{9} + \ln\varepsilon' \ln\frac{m^2}{p^2} - \frac{1}{2}\left(\ln\frac{m^2}{p^2}\right)^2 + \frac{6m^2}{p^2},$$

$$\int_0^{1-\varepsilon'} dx \frac{A}{B}\frac{p^2 B - m^2}{p^2 B + m^2}\ln\left(B + \frac{m^2}{p^2}\right) \cong -\frac{28}{9} - \frac{\pi^2}{3} + \frac{3}{2}\left(\ln\frac{m^2}{p^2}\right)^2 - \ln\varepsilon' \ln\frac{m^2}{p^2} + 26\frac{m^2}{p^2} - 4\frac{m^2}{p^2}\ln\frac{m^2}{p^2},$$

$$\int_0^{1-\varepsilon'} dx \frac{A}{B}\frac{p^2 B - m^2}{p^2 B + m^2}\ln\left(B + \frac{m^2}{\Lambda_u^2}\right) \cong -\frac{28}{9} - \ln\varepsilon' \ln\frac{m^2}{\Lambda_u^2} + \frac{1}{2}\left(\ln\frac{m^2}{\Lambda_u^2}\right)^2 + \left(\ln\frac{m^2}{p^2}\right)^2 + 16\frac{m^2}{p^2}.$$

Other prominent integrals needed for our purpose may be obtained from the above integrals by one or two differentiations with respect to p^2 and/or m^2. When all these results are substituted in (A3) all the divergences cancel out and the remaining expressions are finite in the limit $\Lambda_u^2 \to \infty$.

APPENDIX B: $\langle \bar{q}q \rangle$ PROJECTION OF ONE-LOOP GLUON PROPAGATOR

Here, we shall compute the vacuum polarization of a gluon propagator with one quark line condensing in a nonlocal manner. The gluon propagator up to second order in the coupling constant is given by

$$D_{\mu\nu}^{ab}(p) = \int d^4x e^{ipx} \langle 0|T\left\{A_\mu^a(x)\exp\left(i\int \mathcal{L}_I(y)d^4y\right)A_\nu^b(0)\right\}|0\rangle$$

$$= D_{\mu\nu}^{0ab}(p) - \frac{g^2}{2}\int d^4x e^{ipx}\int d^4y d^4z \langle 0|T\left\{A_\mu^a(x)\bar{\psi}(y)\frac{\lambda^c}{2}A^c\!\!\!/(y)\psi(y)\bar{\psi}(z)\frac{\lambda^d}{2}A^d\!\!\!/(z)\psi(z)A_\nu^b(0)\right\}|0\rangle. \quad \text{(B1)}$$

Let us call the coefficient of $-g^2/2$ in the second term on the RHS of (B1) as $I_{\mu\nu}^{ab}$. Using Wick's theorem and assuming that one of the quark lines condense, we can write

$$I_{\mu\nu}^{ab} = \int d^4x e^{ipx}\int d^4y d^4z \left[D_{\mu\rho}^{0ac}(x-y)D_{\sigma\nu}^{0bd}(z) + D_{\mu\sigma}^{0ad}(x-z)D_{\rho\nu}^{0bc}(y)\right]$$

$$\left(\frac{\lambda^a}{2}\right)_{\alpha\beta}\left(\frac{\lambda^b}{2}\right)_{\eta\zeta}(\gamma^\rho)_{ij}(\gamma^\sigma)_{ln}\left[iS(y-z)_{jl}^{\beta\eta}\langle 0| : \bar{\psi}(y)_i^\alpha \psi(z)_n^\zeta : |0\rangle + iS(z-y)_{ni}^{\zeta\alpha}\langle 0| : \bar{\psi}(z)_l^\eta \psi(y)_j^\beta : |0\rangle\right], \quad \text{(B2)}$$

where the quark propagator is [40]

$$iS(y-z)_{ni}^{\lambda\alpha} = \frac{1}{4\pi^2}\left[\frac{2i(y\!\!\!/ - z\!\!\!/)}{(y-z)^4} - \frac{m}{(y-z)^2 - i\varepsilon} + O(m^2)\right]_{ni}\delta^{\lambda\alpha}. \quad \text{(B3)}$$

As stated earlier, $(y-z)^2 < 0$. Using Eqs. (2.5) and (B3) we can evaluate (B2). First change the variables as $y - z = Y$, $y + z = Z$, then all the integrations, except those in Y can be performed trivially. The result is

$$I_{\mu\nu}^{ab} = \delta^{ab}\frac{\langle\bar{\psi}\psi\rangle}{96\pi^2}\int d^4y\left[e^{iYp}\left(\frac{-g_{\mu\rho} + p_\mu p_\rho/p^2}{p^2}\right)\left(\frac{-g_{\sigma\nu} + p_\sigma p_\nu/p^2}{p^2}\right) + e^{-iYp}\left(\frac{-g_{\mu\sigma} + p_\mu p_\sigma/p^2}{p^2}\right)\right.$$

$$\left.\times\left(\frac{-g_{\rho\nu} + p_\rho p_\nu/p^2}{p^2}\right)\right]\exp\left(\frac{\lambda_q^2}{8}Y^2\right)8m\left(\frac{Y^\rho Y^\sigma}{Y^4} + \frac{g^{\rho\sigma}}{2Y^2}\right), \quad \text{(B4)}$$

where terms $O(m^2)$ have been neglected. The result of Y integrations are

$$\int d^4Y \exp\left(iYp + \frac{\lambda_q^2}{8}Y^2\right)\frac{1}{Y^2} = -\frac{4i\pi^2}{p^2}\left[\exp\left(\frac{2p^2}{\lambda_q^2}\right) - 1\right], \tag{B5}$$

$$\int d^4Y \exp\left(iYp + \frac{\lambda_q^2}{8}Y^2\right)\frac{Y_\mu Y_\nu}{Y^4} = 4i\pi^2\left[2\left(\frac{g_{\mu\nu}}{p^4} - \frac{4p_\mu p_\nu}{p^6}\right)\frac{\lambda_q^2}{8}\left(1 - \exp(2p^2/\lambda_q^2) + \frac{2p^2}{\lambda_q^2}\right) - \frac{p_\mu p_\nu}{p^4}\left(\exp(2p^2/\lambda_q^2) - 1\right)\right] \tag{B6}$$

This gives, for the total gluon propagator (B1),

$$D^{ab}_{\mu\nu}(p) = \frac{-i\delta^{ab}}{p^2 + i\varepsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\left[1 + \frac{g^2}{12}\langle\bar\psi\psi\rangle\frac{m}{p^4}\left\{\frac{\lambda_q^2}{p^2}\left(1 - \exp(2p^2/\lambda_q^2)\right) - 2\exp(2p^2/\lambda_q^2) + 4\right\}\right]. \tag{B7}$$

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