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Heavy-light mesons in a relativistic potential model

C. B. Yang
Institute of Particle Physics, Hua-Zhong Normal University, 430070 Wuhan, China

X. Cai
*Institut für Theoretische Physik, Freie Universität Berlin, 14195 Berlin, Germany
and Institute of Particle Physics, Hua-Zhong Normal University, 430070 Wuhan, China*
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The energy spectrum of heavy-light meson states is studied in the framework of the full Bethe-Salpeter equation. The heavy-flavor mass expansion method is also used to obtain the heavy-light meson energy spectrum from both the first-order correction and the heavy quark limit. The results calculated by our model for the masses of D^0, $D^{\pm}$, B^0, $B^{\pm}$, D^{*0}, $D^{*\pm}$, B^*, D_1^0, and $D_s^{\pm}$ are in good agreement with the experimental data.

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I. INTRODUCTION

More and more attention has been paid to the test of the standard model, the observation of CP violation in meson decays other than K^0 decays, and the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, so that the experimental and theoretical interest for heavy-light mesons has been growing considerably during the last few years. This interest arose from the discovery of large B^0-$\bar{B}^0$ mixing [1], from the hope that CP violation [2] in B meson systems may be observed in the near future, which is complementary to that of the K mesons, and from the possibility that effects of new physics may be disclosed by anomalous enhancements of the rates of some rare decay channels [3] on the larger accelerators at energies as high as those on the CERN Large Hadron Collider (LHC) [4].

In the frame of the widely accepted quark model, the interaction between a quark and antiquark in a meson has a nonperturbative feature; the bound-state problems still cannot be solved by perturbative calculations and effective theories are needed. For mesons consisting of both a heavy quark and a heavy antiquark, the Schrödinger equation has served to calculate the heavy quarkonia's masses and some rather good fits to experimental data have been obtained. Speaking generally, the motions of quarks and antiquarks in mesons are relativistic, even for heavy ones. As pointed out in Ref. [5], even for the $c\bar{c}$ system the kinetic energy is about 13% of the total energy and the ratio of the relativistic correction to the quark mass will not decrease with the increase of quark mass if the interaction is Coulombic type. As a result the bound-state equation for mesons should in general be relativistic.

As far as we know, the only effective relativistic equation of the bound state is the Bethe-Salpeter (BS) equation [6]. Because of its consistency with quantum field theory, the BS equation can be used, in principle, for all quark-antiquark systems, especially for $Q\bar{q}$ or $q\bar{Q}$ (heavy-light mesons) systems which are yet seldom studied in the frame of the relativistic formulation but of great interest.

At present what one is particularly interested in is not only the energy spectrum of hadrons, needless to say, which is an important source to study the dynamics, but also the wave functions which play a key role in the calculations of form factors, structure functions, decay constants, and related experimental parameters, for quarkonia and heavy-light bound states. So far one of the central difficulties in tests of QCD is the lack of knowledge about the hadron wave functions. Traditional methods used in the perturbative QCD are operator product expansion, the factorization theorem, and the evolution equation which separates parts concerning the hadronic wave function. But the separation of bound-state effects is valid only for the large momentum transfer processes.

If the hadron wave functions can be obtained theoretically, many important quantities, such as decay amplitudes, structure functions for exclusive and inclusive processes, the contributions to inclusive phenomena due to higher twist term, can be connected. In this sense, the wave function can be said to be a bridge between the large-distance nonperturbative physics and the short-distance perturbative physics [7].

In this paper we intend to discuss heavy-light meson bound states in the context of the full BS equation. In Sec. II, we focus our attention on the energy spectrum for heavy-light meson states after discussing the properties of the BS equation and its reductions. A new method to solve the BS equation based on the heavy quark mass expansion is proposed in Sec. III. Some further discussion and numerical results are given in Sec. IV.

II. BASIC FORMULATION

Considering a system of a fermion-antifermion pair with masses m_1 and m_2, respectively, the BS equation can be written, in the ladder approximation, in the form

$$(\eta_1 \not{P}+ \not{p} - m_1)\chi(p)(\eta_2 \not{P}- \not{p} + m_2) = i \int \frac{d^4q}{(2\pi)^4} V(P;p,q)\chi(q) \ , \quad (1)$$

where $\eta_1 = m_1/(m_1 + m_2)$, $\eta_2 = m_2/(m_1 + m_2)$, $\chi(p)$ is the wave function of the system with total four-momentum P in momentum space, p is the relative four-momentum. The formal product $V(P;p,q)\chi(q)$ of the interaction kernel acting on wave function represents a general interaction which can be written as

$$V(P;p,q)\chi(q) \equiv \sum_{\nu} C_\nu(P;p,q)\Gamma_\nu\chi(q)\Gamma_\nu \ , \tag{2}$$

where Γ_ν are five groups of Dirac matrices, $\nu = S, V, T, A, P$ according to their Lorentz-transformation properties of the interactions, corresponding with scalar, vector, tensor, axial-vector, and pseudoscalar interactions, respectively.

Under the instantaneous approximation, in which the interaction is taken to be dependent only on three-momentum $\mathbf{p}$ and $\mathbf{q}$, after some standard mathematical manipulations one arrives at the full Salpeter equation [8]

$$\Phi(\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3\mathbf{q}\left[\frac{\Lambda_1^+(\mathbf{p})\gamma_0 V(\mathbf{p},\mathbf{q})\Phi(\mathbf{q})\gamma^0\Lambda_2^-(-\mathbf{p})}{M-\omega_1(\mathbf{p})-\omega_2(\mathbf{p})} - \frac{\Lambda_1^-(\mathbf{p})\gamma_0 V(\mathbf{p},\mathbf{q})\Phi(\mathbf{q})\gamma^0\Lambda_2^+(-\mathbf{p})}{M+\omega_1(\mathbf{p})+\omega_2(\mathbf{p})}\right] , \tag{3}$$

where M is the mass of the system, $\omega_\alpha(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_\alpha^2}$, $\alpha = 1, 2$ represents a fermion and antifermion, respectively, and the projection operators

$$\Lambda_\alpha^\pm(\mathbf{p}) = \frac{\omega_\alpha(\mathbf{p}) \pm \gamma_0(\gamma\cdot\mathbf{p} + m_\alpha)}{2\omega_\alpha(\mathbf{p})} = \frac{1}{2}\left[1 \pm \frac{H_\alpha(\mathbf{p})}{\omega_\alpha(\mathbf{p})}\right] , \tag{4}$$

and the wave function

$$\Phi(\mathbf{p}) = \int dp_0 \chi(p_0, \mathbf{p}) \ . \tag{5}$$

The interaction strengths C_ν in Eq. (2) are usually given phenomenologically and should be expected to satisfy two restrictions: (i) asymptotic freedom and (ii) color confinement. One expects that the static potential to be Coulombic at short distances and to grow linearly at large distances. The short-distance behavior of V can be calculated by using the perturbative QCD. Because of the vacuum polarization effect in the QCD, potential incorporates logarithmic modifications of the Coulombic part at short distances. Its Lorentz nature is expected to be a vector, $\Gamma_V = \gamma_\mu$, corresponding to the perturbative one-gluon-exchange interaction. This short-distance property is also supported by nonperturbative calculations of heavy-quark interaction in lattice QCD [9]. It was suggested that vector potential can cause chiral-symmetry breaking as well as confining. But it was also confirmed that pure vector potential cannot explain the energy inversion in J/ψ of $1\,^1P_1$ and $1\,^1P_3$ levels [10]. At large distance, the nature of the coupling can be determined for heavy quarks from lattice calculations, too, by determining the rate of decrease of the spin-dependent part of the potential. The results have shown that the large separation coupling is scalar, as naively expected, $\Gamma_S = 1$.

Though there are only scalar and vector interactions to be considered for quarkonia [11], the real nature of interaction between heavy-light quark-antiquark might contain other kinds of coupling, and tensor potentials between heavy quarks have been evaluated with lattice gauge theory [12]. The change in the nature of the coupling as quark and antiquark move from small to large separation leads to some complications in the choice of coupling for the intermediate region of the potential, but one can assume that it is a mixture of vector and scalar.

Now let us return to the wave function. It is easy to see that $\Phi(\mathbf{p})$ in the Salpeter equation satisfies conditions

$$\Lambda_1^+(\mathbf{p})\Phi(\mathbf{p})\Lambda_2^+(-\mathbf{p}) = \Lambda_1^-(\mathbf{p})\Phi(\mathbf{p})\Lambda_2^-(-\mathbf{p}) = 0 \ , \tag{6}$$

or

$$\frac{H_1(\mathbf{p})}{\omega_1}\Phi(\mathbf{p}) + \Phi(\mathbf{p})\frac{H_2(-\mathbf{p})}{\omega_2} = 0 \ . \tag{7}$$

Because of these conditions the independent component number of Φ should be reduced. For example, the general form of instantaneous wave function in the center of mass (c.m.) frame for 0^- meson states, of which the typifiers are π, η, K, D, and B, can be expressed in momentum space as

$$\Phi = \gamma_5\phi_1 + \gamma_5\gamma_0\phi_2 + \gamma_5\boldsymbol{\gamma}\cdot\mathbf{p}\phi_3 + \gamma_5\gamma_0\boldsymbol{\gamma}\cdot\mathbf{p}\phi_4 \ , \tag{8}$$

where ϕ_1, ϕ_2, ϕ_3, and ϕ_4 are even functions of $\mathbf{p}$. Because of the conditions Eqs. (6) or (7) imposed on the wave function, ϕ_1, ϕ_2, ϕ_3, and ϕ_4 have not been independent, and tied together by the relations

$$\phi_3 = \frac{c_2}{a}\phi_1 \ , \qquad \phi_4 = -\frac{c_1}{a}\phi_2 \ , \tag{9}$$

where for short we have defined $c_1 = 1/\omega_1 + 1/\omega_2$, $c_2 = 1/\omega_1 - 1/\omega_2$, $a = m_1/\omega_1 + m_2/\omega_2$. One can see that the wave functions of 0^- mesons are determined by only two independent functions. This statement is also true for 0^+, 1^-, and 1^+ meson states. From Eqs. (3), (4), (8), and (9) one can calculate the masses and wave functions of mesons with arbitrary flavor contents.

It is evident that if one neglects the second term, i.e., the negative energy term, in Eq. (3) further, the conditions imposed on the wave function turn out to be

$$H_1(\mathbf{p})\Phi(\mathbf{p}) = \omega_1\Phi(\mathbf{p}) \tag{10}$$

and

$$H_2(\mathbf{p})\Phi(\mathbf{p}) = -\omega_2\Phi(\mathbf{p}) \ . \tag{11}$$

These reduce the number of independent components of the wave function further then the coupling between quark and antiquark is neglected. As pointed out by the authors of Ref. [13] the results of not good fit to experimental data are discouraging with respect to the utility of the reduced Salpeter equation. All these show that the reduced Salpeter equation has an essential flaw and cannot be used to describe mesons containing light quark.

Thus it compels us to solve the full Salpeter equation for meson systems with light flavor. By using the properties of projection operators we can rewrite the full Salpeter equation as

$$(M-\omega_1-\omega_2)\Lambda_1^+(\mathbf{p})\Phi(\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3\mathbf{p}\Lambda_1^+(\mathbf{p})\gamma_0 V\Phi\gamma_0\Lambda_2^-(-\mathbf{p}) ,$$

$$(M+\omega_1+\omega_2)\Lambda_1^-(\mathbf{p})\Phi(\mathbf{p}) = -\frac{1}{(2\pi)^3}\int d^3\mathbf{p}\Lambda_1^-(\mathbf{p})\gamma_0 V\Phi\gamma^0\Lambda_2^+(-\mathbf{p}) . \quad (12)$$

Under the condition of Eq. (7) the last two equations can be formally rewritten to a set of coupled integral equations with two independent functions as

$$M\Psi(\mathbf{p}) = L(\mathbf{p})\Psi(\mathbf{p}) + I(\mathbf{p},\mathbf{q})\Psi(\mathbf{q}) , \quad (13)$$

where

$$\Psi(\mathbf{p}) = \begin{pmatrix}\phi_i(\mathbf{p})\\ \phi_j(\mathbf{p})\end{pmatrix}$$

with components ϕ_i, ϕ_j to be the two independent functions by which to determine the wave functions for 0^-, 0^+, 1^-, and 1^+ mesons. $L(\mathbf{p})$ is a 2×2 matrix acting on Ψ and $I(\mathbf{p},\mathbf{q})$ a 2×2 integro-operator acting on $\Psi(\mathbf{q})$. Both $L(\mathbf{p})$ and $I(\mathbf{p},\mathbf{q})$ can be derived from Eq. (3) and Eq. (7) when J^P of meson is specified. When ϕ_i and ϕ_j is expanded in series of some suitable orthogonal functions, the operator equation can be simplified and the mass M and the coefficients of the expansion can be obtained. One can get some help on the solution of the BS equation by means of the matrix method from Ref. [14]. In our work, Eqs. (12) are solved numerically for D^0, $D^\pm$, B^0, $B^\pm$, D^{*0}, $D^{*\pm}$, B^*, D_1^0, and $D_s^\pm$ mesons. The potentials in our calculation are chosen to be the same form as the $\delta=0$ potentials used in the first two papers of Ref. [13]. Our calculations are listed as "our results 1" in the third column in Table I. Those listed in the second column are experimental data taken from the last updated Particle Data Group compilation [15] for comparison. Our results are in good agreement with the experimental data.

TABLE I. Masses for several heavy-light 0^- mesons in units of MeV. In the calculations, we have used $m_u = 0.30$, $m_d = 0.336$, $m_s = 0.50$, $m_c = 1.636$, and $m_b = 4.962$ in units of GeV. Experimental data are taken from Ref. [15].

	Experimental data	Our results 1
D^0	1864.6± 0.5	1864.8
$D^\pm$	1869.4± 0.4	1865.1
B^0	5279.0± 2.0	5277.2
$B^\pm$	5278.7± 2.0	5271.1
D^{*0}	2006.7± 0.5	1994.3
$D^{*\pm}$	2010.0± 0.5	1999.5
B^*	5324.8± 2.1	5258.6
D_1^0	2422.8± 3.2	2411.9
$D_s^\pm$	1968.5± 0.7	1955.3

III. HEAVY-FLAVOR EXPANSION METHOD

The discussion in the last section is quite general. Though it is possible, in principle, to calculate meson mass and wave function in the BS frame, the relations suggested from the heavy quark effective theory (HQET) [16] between masses, or decay constants, or form factors, of different heavy-light mesons with the same light quark cannot be shown explicitly. To seek such relations we suggest an alternative way to solve the BS equation for heavy-light mesons.

Consider a meson system containing a quark heavy enough. If the mass of heavy quark, m_Q, is much larger than $\Lambda_{\rm QCD}$, one could reasonably regard the heavy quark mass as infinity first, and then take the corrections due to the finite heavy quark mass into account. This is the picture of the HQET. In the heavy quark limit, heavy quark moves steadily and acts as a static color source in the c.m. frame, thus a new spin symmetry emerges. This new symmetry yields relations between the form factors for pseudoscalar and vector B mesons [17]. Another new symmetry emerged in this limit is the additional SU(2) flavor symmetry when masses of two heavy quarks are all regarded as infinity, which relates hadronic matrix elements involving B and D mesons [18]. Though various model-independent predictions can be obtained from the QCD in heavy quark limit, the finite mass corrections are generally model dependent. It is very important to know the magnitude of the $1/m_Q$ corrections. Usually, one may expect that the b quark is heavy enough for a small correction, though the correction may be large in the case of c quark. Lattice calculations [19] have suggested, however, that $1/m_Q$ correction to the B-meson decay constant f_B is very large and negative, meanwhile the corrections to form factors relatively small. This statement has been noted in two-dimensional calculations [20] and sum rule calculations [21].

Some finite heavy quark mass corrections can be obtained in the frame of BS equation. Consider a meson containing a heavy antiquark and a light quark (for mesons containing a heavy quark and a light antiquark the treatment is similar). Since the mass of the heavy antiquark, m_2, is much larger than that of the light quark, m_1, the effects of the heavy quark mass can be considered perturbatively, and both the binding energy and the wave function can be expanded in reverse power of m_2:

$$\Phi(\mathbf{p}) = \Phi_0(\mathbf{p}) + \frac{\Phi_1(\mathbf{p})}{m_2} + \cdots , \quad (14)$$

$$M - m_2 = E_0 + \frac{\varepsilon_1}{m_2} + \cdots . \quad (15)$$

Correspondingly the projection operators for the heavy quark can be expanded as

$$\Lambda_2^\pm(\mathbf{p}) = \Lambda_0^\pm(\mathbf{p}) + \frac{\Xi^\pm(\mathbf{p})}{m_2} + \cdots , \quad (16)$$

where $\Lambda_0^\pm(\mathbf{p}) = (1\pm\gamma_0)/2$ and therefore, $\Phi_0(\mathbf{p})$, $\phi_1(\mathbf{p})$, and E_0, ε_1 can be determined by the following equations together with the normalization condition:

$$\Phi_0(\mathbf{p}) = \frac{1}{(2\pi)^3} \int d^3\mathbf{q} \frac{\Lambda_1^+(\mathbf{p})\gamma^0 V \Phi_0(\mathbf{q})\gamma_0 \Lambda_0^-(-\mathbf{p})}{E_0 - \omega_1} , \tag{17}$$

$$\begin{aligned}\Phi_1(\mathbf{p}) = &\frac{1}{(2\pi)^3} \int d^3\mathbf{q} \frac{\Lambda_1^+(\mathbf{p})\gamma^0 V \Phi_1(\mathbf{q})\gamma_0 \Lambda_0^-(-\mathbf{p})}{E_0 - \omega_1} - \frac{2\varepsilon_1 - \mathbf{p}^2}{2(E_0 - \omega_1)}\Phi_0(\mathbf{p}) \\ &+ \frac{1}{(2\pi)^3} \int d^3\mathbf{q} \left[\frac{\Lambda_1^+(\mathbf{p})\gamma^0 V \Phi_0(\mathbf{q})\gamma_0 \Xi^-(-\mathbf{p})}{E_0 - \omega_1} - \frac{\Lambda_1^-(\mathbf{p})\gamma^0 V \Phi_0(\mathbf{q})\gamma_0 \Lambda_0^+(-\mathbf{p})}{2} \right] ,\end{aligned} \tag{18}$$

with $\Phi_0(\mathbf{p})$ satisfying $\Lambda_1^-(\mathbf{p})\Phi_0(\mathbf{p}) = \Phi_0(\mathbf{p})\Lambda_0^+(-\mathbf{p}) = 0$. Numerical results will be shown in Sec. IV that this limit will give rather good fits to the masses of heavy-light mesons.

For meson states with given J^P, the wave functions of the zero order, the first order, ..., $(\Phi_0, \Phi_1, \ldots)$ should be demanded to have the same symmetry. As an example, let us consider 0^- mesons, for which the most general forms of Φ_0 and Φ_1 can be expressed in exactly the same form as Eq. (8). The component functions appearing in Φ_0 are denoted by $\phi_1, \phi_2, \phi_3, \phi_4$ and those for Φ_1 denoted by $\phi_1', \phi_2', \phi_3', \phi_4'$.

Under the condition of Eq. (7), Φ_0 can be expressed only by ϕ_3, and Φ_1 by ϕ_1' and ϕ_2'. Thus one can easily get

$$\begin{aligned}(E_0 - \omega_1)\phi_3(p) &= \frac{1}{(2\pi)^3} \int d^3\mathbf{q}\kappa_0(p,q,\theta)\phi_3(q) , \\ \phi_1'(p) + \phi_2'(p) &= \frac{1}{(2\pi)^3} \int d^3\mathbf{q}\kappa_2(p,q,\theta)\phi_3(q) , \\ \phi_1'(p) - \phi_2'(p) &= -(2\varepsilon_1 - p^2)F(p)\phi_3 \\ &\quad + \frac{1}{(2\pi)^3} \int d^3\mathbf{q}\kappa_1(p,q,\theta)[\phi_1'(q) - \phi_2'(q)] ,\end{aligned} \tag{19}$$

where

$$\begin{aligned}\kappa_0(p,q,\theta) &= \frac{(C_V - C_S)(\omega_1' + m_1)}{\omega_1} + \frac{(C_V + C_S)pq\cos\theta}{\omega_1(\omega_1 + m_1)} , \\ \kappa_1(p,q,\theta) &= \frac{1}{2(E_0 - \omega_1)} \left[\frac{(C_V - C_S)(\omega_1 + m_1)}{\omega_1} + \frac{(C_V + C_S)pq\cos\theta}{w_1(\omega_1' + m_1)} \right] , \\ \kappa_2(p,q,\theta) &= \frac{1}{(E_0 - \omega_1)} \left[\frac{(C_V - C_S)(\omega_1 - m_1)pq\cos\theta}{\omega_1} + \frac{(C_V - C_S)p^2(\omega_1' + m_1)}{\omega_1} \right] ,\end{aligned} \tag{20}$$

and

$$F(p) = \frac{\omega_1 + m_1}{E_0 - \omega_1} , \tag{21}$$

and $\omega_1 = \sqrt{p^2 + m_1^2}$, $\omega_1' = \sqrt{q^2 + m_1^2}$, θ is the angle between $\mathbf{p}$ and $\mathbf{q}$. These equations together with the normalization conditions form a complete set for the determination of masses and wave functions of 0^- meson bound states. Similarly the equations under the heavy-flavor expansion can be derived for other J^P meson bound states. It should be mentioned that C_V and C_S in Eq. (20) are functions of $\mathbf{p}$ and $\mathbf{q}$, and their forms can be gained from the Fourier transformation of the potentials in coordinate space. Equations (20) can be solved by using the matrix method which has been discussed briefly in the last section. Once the masses and the wave functions of heavy-light mesons are obtained to the order of $1/m_2$, some corrections to the relations suggested by the HQET in heavy quark limit can be calculated to the same order.

IV. NUMERICAL RESULTS AND DISCUSSION

Even before the detailed numerical calculations, some qualitative conclusions for the heavy quark limit and first-order corrections can be reached based on some reasonable physical considerations too.

As mentioned before, the spin of the heavy quark has a very small effect on the system, thus the binding energies for mesons containing a heavy quark (or heavy antiquark) and a light antiquark (or light quark) will be quite close to each other for different spin configurations. This is consistent with the small spacings among levels of ground state and excited states for B mesons with the same flavor contents. It can be shown from Eq. (18) that the values of E_0 degenerate for 0^- and 1^- states with the same light flavor contents in the heavy quark limit, meanwhile values of ε_1 are different for those two states. Taking the first order of the heavy quark mass expansion one can approximately get

$$\Delta M = M_{J=1} - M_{J=0} \simeq \frac{\varepsilon_{1J=1} - \varepsilon_{1J=0}}{m_2} . \tag{22}$$

This estimation coincides with

TABLE II. Numerical results from the heavy-flavor expansion. Our results 2 and 3 are those from the first-order under the heavy quark mass expansion and those in heavy quark limit, respectively.

	D^0	$D^\pm$	B^0	$B^\pm$	D^{*0}	$D^{*\pm}$	B^*	D_1^0
Our results 2	1865.9	1865.2	5277.2	5271.1	2007.4	2018.8	5257.1	2413.3
Our results 3	1867.9	1873.1	5250.1	5241.8	2004.6	2006.6	5240.1	2412.6

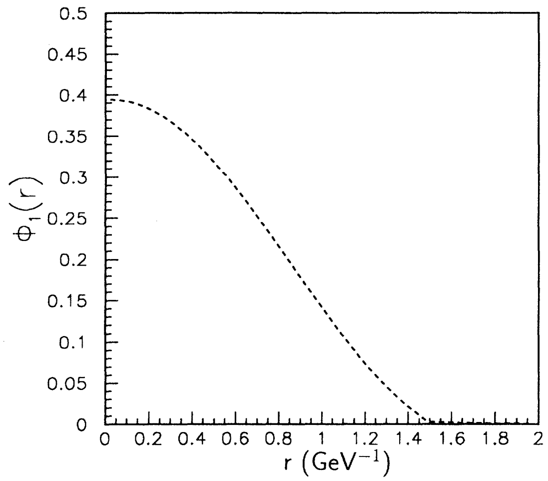

FIG. 1. 0^- meson wave function $\phi_1(r)$ in configuration space.

$$M_{D^*} - M_D = (140.6 \pm 0.15) \text{ MeV } \simeq 3(M_{B^*} - M_B)$$

as a consequence of the fact that the b quark is about three times heavier than the c quark. As a prediction, if mesons T^* and T (analogue to B^* and B with t quark in place of b quark) do exist, one can see $M_{T^*} - M_T \sim 1$ MeV from the recent exciting experimental results of the evidence for the top quark [22]. Both the Collider Detector at Fermilab (CDF) and the D0 Collaborations presented detailed data on the search for top at the Fermilab Tevatron collider. The D0 results are expected for a top quark of mass between 140 and 180 GeV/c^2 [23]. The best mass extracted from the CDF top search is $m_t = 174 \pm 10^{+13}_{-12}$ GeV/c^2 [24].

The $1/M_Q$ correction to the wave function can also bring remarkable improvements on the calculations of decay constant and various kinds of form factors. As is well known, the hadronic wave function provides connection between inclusive and exclusive processes, since the probability amplitude and distribution determining those processes can, in principle, be obtained from hadronic wave function. Some applications of the quarkonium wave function can be found in Ref. [25]. Correction to the wave function may be helpful in the calculations of hadronic matrix elements when nonspectator effects have to be included. When E_0 and the wave function are calculated in the heavy quark limit, the relation $F_M\sqrt{M}$ = const is obvious and deviation from this equality is for heavy mesons mainly due to the $1/M_Q$ correction to the wave function.

We calculate masses for heavy-light mesons D^0, $D^\pm$, B^0, $B^\pm$, D^{*0}, $D^{*\pm}$, B^*, and D_1^0 by using the heavy quark mass expansion method. Numerical results at the first-order approximation and in the heavy quark mass limit are listed in Table II. It is shown that our calculations in the heavy quark limit (our results 3) are quite close to the experimental values (see Table I), and the first-order approximation (our results 2) can give quite good fits. The behavior of the 0^- wave function $\phi_1(r)$ in configuration space which is the Fourier transformation of that in momentum space is shown in Fig. 1.

As a summary, we mention once more that the finite heavy quark mass corrections play an important role in the study of heavy flavor physics and that the heavy flavor expansion method based on the relativistic bound state equation can give a quite conventional description for meson system containing a heavy quark. The heavy flavor expansion method enables one to study meson mass and its wave function both in the heavy quark limit and in the finite heavy quark mass correction, thus one can search for the relations of form factors or decay constants between heavy mesons containing a same light quark not only in the heavy quark limit, but also at higher-order approximations. Based on these studies, a more profound understanding of the meson properties and that of the QCD may be achieved.

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[1] ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **192**, 245 (1987); CLEO Collaboration, A. Jawahery *et al.*, in *Proceedings of the XXIV International Conference on High Energy Physics*, Munich, West Germany, 1988, edited by R. Kotthaus and J. H. Kühn (Springer-Verlag, Berlin, 1988).

[2] For a review, see *CP Violation*, edited by C. Jarlskog (World Scientific, Singapore, 1989); M. Wise, "CP-Violation," Report No. Calt-68-1708, 1991 (unpublished).

[3] S. Bertolini, F. Borzumati, and A. Masiero, Phys. Lett. B **192**, 437 (1987).

[4] P. Chiappetta *et al.*, in *Proceedings of the ECFA Large Hadron Collider Workshop*, Aachen, Germany, 1990, edited by G. Jarlskog and D. Rein (CERN Report No. 90-10, Geneva, Switzerland, 1990), Vol. II.

[5] W. Buchmüller and S.-H.H. Tye, Phys. Rev. D **24**, 132 (1981); M. Kaburagi *et al.*, Phys. Lett. **97B**, 143 (1980); T. Murota, Prog. Theor. Phys. **69**, 181 (1983).

[6] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 1232 (1951).

[7] S. Drell and T. D. Lee, Phys. Rev. D **4**, 1738 (1974); S. J. Brodsky, T. Huang, and G. P. Lepage, Report No. SLAC-PUB-2540, 1980 (unpublished).

[8] E. E. Salpeter, Phys. Rev. **87**, 328 (1952).

[9] P. de Forcrand and J. Stack, Phys. Rev. Lett. **55**, 1254

(1985); C. Michael, *ibid.* **56**, 1219 (1986); M. Campostrini, K. Moriarty, and C. Rebbi, Phys. Rev. D **36**, 3450 (1987).

[10] C. Y. Habe *et al.*, Prog. Theor. Phys. **80**, 1069 (1988).

[11] S. Jacobs, M. G. Olsson, and Casimir J. Suchyta III, Phys. Rev. D **35**, 2448 (1987).

[12] C. Michael and P. E. L. Rakow, Nucl. Phys. **B256**, 640 (1985).

[13] A. Gara, B. Durand, and L. Durand, Phys. Rev. D **40**, 843 (1989); **42**, 1651 (1990); W. Lucha, H. Rupprecht, and F. F. Schöberl, *ibid.* **46**, 1088 (1992).

[14] L. Durand and A. Gara, J. Math. Phys. **31**, 2237 (1990).

[15] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).

[16] H. Georgi, Phys. Lett. B **240**, 447 (1990); B. Grinstein, Nucl. Phys. **B339**, 253 (1990).

[17] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989).

[18] F. Falk *et al.*, Nucl. Phys. **B343**, 1 (1990).

[19] C. R. Allton *et al.*, Nucl. Phys. **B349**, 598 (1991); A. Abada *et al.*, *ibid.* **B376**, 172 (1992); C. Alexandrou *et al.*, Phys. Lett. B **256**, 60 (1991); C. Bernard *et al.*, Phys. Rev. D **43**, 2140 (1991); L. Maiani, Helv. Phys. Acta. **64**, 853 (1991).

[20] B. Grinstein and P. F. Mende, Phys. Rev. Lett. **69**, 1018 (1992); M. Burkardt and E. S. Swanson, Phys. Rev. D **46**, 5083 (1992).

[21] M. Neubert, Phys. Rev. D **46**, 1076 (1992); E. Bagan, P. Ball, V. M. Braun, and H. G. Dosch, Phys. Lett. B **278**, 457 (1993).

[22] M. J. Shochet, in Proceedings of Eighth Meeting of the Division of Particles and Fields of the American Physical Society, Albuquerque, New Mexico, 1994 (unpublished).

[23] D0 Collaboration, S. Abachi *et al.*, Phys. Rev. Lett. **72**, 2138 (1994); D0 Collaboration, P. Grannis, Fermilab Report No. FERMILAB-Pub-94/277, 1994 (unpublished).

[24] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **73**, 224 (1994); **73**, 2668 (1994); Phys. Rev. D **50**, 2966 (1994).

[25] A. B. Henriques, B. H. Kellett, and R. G. Moorhouse, Phys. Lett. **64B**, 85 (1976); P. Ditsas, N. A. Mcdougall, and R. G. Moorhouse, Nucl. Phys. **B146**, 191 (1978).