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Critical analysis of theoretical estimates for B-to-light-meson form factors and $B \to \psi K(K^*)$ data using factorization

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We point out that, if we assume the factorization hypothesis, current estimates of form factors fail to explain the nonleptonic decays $B \to \psi K(K^*)$ and that the combination of data on the semileptonic decays $D \to K(K^*)\ell\nu$ and on the nonleptonic decays $B \to \psi K(K^*)$ (in particular, recent polarization data) severely constrain the form (normalization and q^2 dependence) of the heavy-to-light meson form factors. From a simultaneous fit to $B \to K^{(*)}\psi$ and $D \to K^{(*)}l\nu$ data we find that strict heavy quark limit scaling laws do not hold when going from D to B and must have large corrections that make softer the dependence on the masses. We find that $A_1(q^2)$ should increase slower with q^2 than A_2, V, f_+. We propose a simple parametrization of these corrections based on a quark model or on an extension of the heavy-to-heavy scaling laws to the heavy-to-light case, complemented with an approximately constant $A_1(q^2)$. We analyze in the light of these data and theoretical input various theoretical approaches (lattice calculations, QCD sum rules, quark models) and point out the origin of the difficulties encountered by most of these schemes. In particular we check the compatibility of several quark models with the heavy quark scaling relations.

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I. INTRODUCTION

Heavy-to-heavy meson form factors such as $B \to D^{(*)}$ obey a very constraining principle, namely, the heavy quark symmetry that relates all form factors to the Isgur-Wise function [1]. However, even in this case, large corrections of order $1/m_c$ can occur and many uncertainties remain [2], and most importantly the scaling function ξ remains unknown. In the case of heavy-to-light meson form factors such as $D \to K^{(*)}$ or $B \to K^{(*)}$ there are also rigorous results in the asymptotic heavy quark limit, but which are much weaker, namely, relations between the form factors $D \to K^{(*)}$ and $B \to K^{(*)}$ at fixed $\vec{q}$ near the zero recoil point $\vec{q} = 0$, i.e., $q^2 = q^2_{\rm max}$. This is a small kinematical region, and furthermore, no relation is obtained between the various form factors of a given hadron decay, unlike the heavy-to-heavy case. From semileptonic $D \to K^{(*)}l\nu$ decays we have data for the form factors, although with large errors, in a completely different kinematic region, namely, at small q^2, and we cannot from these data extract information on the $B \to K^{(*)}$ form factors without knowledge of the q^2 dependence, for which we do not have any rigorous result in the heavy-to-light case. In both cases, one must unavoidably appeal to models, such as the quark model, or other estimates such as QCD sum rules and lattice, or just make phenomenological *Ansätze*, for example, assume a pole or a dipole q^2 dependence.

On the other hand, we have very interesting data for the decays $B \to \psi K^{(*)}$, in particular recent polarization data. If we assume factorization, these data can give us precious information on the $B \to K^{(*)}$ form factors at a different kinematic point ($q^2 = m^2_\psi$) than the data on semileptonic D decays (mainly at $q^2 = 0$) or the heavy quark limit QCD scaling laws ($q^2 = q^2_{\rm max}$). The combination of these D semileptonic and B nonleptonic data plus the rigorous scaling law in the asymptotic heavy quark limit can severely constrain the corrections to asymptotic scaling laws and provide rich information on the gross features of the q^2 dependence of the form factors, as we will see below. This is the main object of this paper.

Present models of nonleptonic B decays have trouble describing the $B \to \psi K^{(*)}$ decays, namely, the ratio of decay rates $\psi K/\psi K^*$ and the ψK^* polarization data simultaneously. Moreover, the scaling law in the heavy quark limit is not always verified by current models of heavy-to-light form factors, and it is important to consider this matter to gauge the theoretical consistency of models and not only their phenomenological description of data.

This paper is organized as follows. In Sec. II we address the simplest question, namely, the comparison of the different models with the data on $B \to \psi K^{(*)}$, to see that there is a serious difficulty. In Sec. III we discuss the theoretical constraints for the heavy-to-light form factors in the light of the data to set a general *Ansatz* for the form factors. We use these results in Sec. IV where we compare and discuss the different theoretical schemes. We discuss also the QCD sum rules and the lattice QCD results. In Sec. IV C 6 we propose a quark model that satisfies the theoretical and some phenomenological requirements stated in Sec. III. Finally, our conclusions are given in Sec. V. A short overview of this work has already been given in Ref. [3].

II. $B \to \psi K^{(*)}$ DATA ARE HARDLY COMPATIBLE WITH CURRENT ESTIMATES

To be definite, let us write the form factors

$$\langle P_f|V_\mu|P_i\rangle = \left(p^f_\mu + p^i_\mu - \frac{m_i^2 - m_f^2}{q^2}q_\mu\right) f_+(q^2) + \frac{m_i^2 - m_f^2}{q^2}q_\mu f_0(q^2),$$

$$\langle V_f|A_\mu|P_i\rangle = (m_f + m_i)\, A_1(q^2) \left(\varepsilon^*_\mu - \frac{\varepsilon^*\cdot q}{q^2}q_\mu\right)$$

$$-A_2(q^2)\frac{\varepsilon^*\cdot q}{m_f + m_i}\left(p^i_\mu + p^f_\mu - \frac{m_i^2 - m_f^2}{q^2}q_\mu\right) + 2m_f\ A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q_\mu,$$

$$\langle V_f|V_\mu|P_i\rangle = i\frac{2\ V(q^2)}{m_f + m_i}\varepsilon_{\mu\nu\rho\sigma}p_i^\nu\ p_f^\rho \varepsilon^{*\sigma}, \tag{1}$$

where the indices i (f) refer to the initial (final) state, where V_μ (A_μ) is the vector (axial vector) current with the appropriate flavor to transform the initial active quark into the final one and where we use the convention $\epsilon^{0123} = 1$.

The point that we want to emphasize in this paper is that the nonleptonic decays $B \to \psi K, \psi K^*$ can help to get hints about two questions concerning these form factors: namely, (i) the sign and size of the $1/m_Q$ corrections to the asymptotic heavy-to-light scaling relations as well as (ii) the gross features of the q^2 dependence of the form factors. Of course, we must assume factorization to relate these decays to the form factors.

In the standard Shifman, Vainshtein, and Zakharov (SVZ) [4] factorization assumption, that we will call standard SVZ factorization, one deduces the nonleptonic amplitudes from form factors and annihilation constants. There are two types of two-body decays corresponding to the two different color topologies, the so-called classes I and II of Bauer, Stech, and Wirbel (BSW) [5, 6], respectively, proportional to the effective color factors

$$a_1 = \frac{1}{2}\left[c_+\left(1 + \frac{1}{N_c}\right) + c_-\left(1 - \frac{1}{N_c}\right)\right],$$

$$a_2 = \frac{1}{2}\left[c_+\left(\frac{1}{N_c} + 1\right) + c_-\left(\frac{1}{N_c} - 1\right)\right], \tag{2}$$

where $c_\pm$ are QCD short-distance factors.

The decays we are interested in here, $B \to \psi K, \psi K^*$, are of class II. This standard SVZ factorization, that applies literally with expression (2) using $N_c = 3$, is known to fail definitely in class II decays. On the other hand there is a distinct *phenomenological factorization* prescription proposed by BSW which derives a_1 and a_2 *by fitting* the observed B_d decays. We call this factorization prescription phenomenological in the sense that a_1 and a_2 are fitted from the data and not obtained through theoretical relations (2). It must be stressed that these fitted coefficients have no intrinsic meaning in the sense that they are depending on the models used to estimate the form factors and annihilation constants. The model used has been traditionally chosen to be the BSW model, later modified by Neubert, Rieckert, Stech, and Xu [6]. These authors found, from a fit to the two-body B decays:

$$|a_1| = 1.11, \quad |a_2| = 0.21. \tag{3}$$

The magnitude of $|a_2|$ is incompatible with the expectation from (2) and the short-distance QCD factors for $N_c = 3$: $a_2 \sim 0.1$. More recently, the sign of a_2/a_1 has been unambiguously found positive by considering class III decays [7] that depend on the interference between a_1 and a_2. This sign is inconsistent with the once proposed prescription [5] of taking the limit $N_c \to \infty$ in Eq. (2) since one has $c_+ < c_-$ for the short-distance QCD factors c_+, c_-.

We obtain, within the factorization assumption, the following amplitudes in the B meson rest frame:

$$A\left(\bar{B}^0_d \to \psi K\right) = -\frac{G}{\sqrt{2}}V_{cb}V^*_{cs}\ 2\ f_\psi\ m_B\ f_+(m^2_\psi)a_2 p, \tag{4}$$

$$A^{pv}(\bar{B}^O_d \to \psi(\lambda=0)K^*(\lambda=0)) = -\frac{G}{\sqrt{2}}V_{cb}V^*_{cs}m_\psi f_\psi\left[(m_B + m_{K^*})\left(\frac{p^2 + E_{K^*}E_\psi}{m_{K^*}m_\psi}\right)A_1(m^2_\psi) - \frac{m_B^2}{m_B + m_{K^*}}\frac{2p^2}{m_{K^*}m_\psi}A_2(m^2_\psi)\right]a_2, \tag{5}$$

$$A^{pv}(\bar{B}^0_d \to \psi(\lambda=\pm1)K^*(\lambda=\pm1)) = -\frac{G}{\sqrt{2}}V_{cb}V^*_{cs}m_\psi f_\psi\,(m_B + m_{K^*})\,A_1(m^2_\psi)a_2, \tag{6}$$

$$A^{pc}(\bar{B}^0_d \to \psi(\lambda=\pm1)K^*(\lambda=\pm1)) = \pm\frac{G}{\sqrt{2}}V_{cb}V^*_{cs}m_\psi f_\psi\frac{m_B}{m_B + m_{K^*}}2V(m^2_\psi)a_2 p. \tag{7}$$

We see that the nonleptonic data plus the factorization hypothesis can give us information on the form factors at a different kinematic point ($q^2 = m_\psi^2$) than the data on semileptonic D decays (small q^2) or the heavy quark limit QCD scaling laws (at $q^2_{\rm max}$).

The data for the total rates [7] are

$$B\left(\bar{B}_d^0 \to \psi K^0\right) = (7.5 \pm 2.4 \pm 0.8) \times 10^{-4},$$

$$B\left(B_d^0 \to \psi K^{*0}\right) = (16.9 \pm 3.1 \pm 1.8) \times 10^{-4},$$

$$B\left(B^- \to \psi K^-\right) = (11.0 \pm 1.5 \pm 0.9) \times 10^{-4},$$

$$B\left(B^- \to \psi K^{*-}\right) = (17.8 \pm 5.1 \pm 2.3) \times 10^{-4},$$

and the recent results of ARGUS [8], CLEO [8], and the Collider Detector at Fermilab (CDF) [9] concerning the K^* polarization in the $\bar{B}_d \to \psi K^{*0}$ decay, are

$$\begin{aligned} &\Gamma_L/\Gamma_{\rm tot} > 0.78 \ (95\% \ \text{C.L.}) \quad \text{ARGUS}, \\ &\Gamma_L/\Gamma_{\rm tot} = 0.80 \pm 0.08 \pm 0.05 \quad \text{CLEO}, \\ &\Gamma_L/\Gamma_{\rm tot} = 0.66 \pm 0.10^{+0.10}_{-0.08} \quad \text{CDF}, \end{aligned} \tag{8}$$

where Γ_L is the partial width for the longitudinal polarization whose amplitude is given by (5).

As we have pointed out these decays are affected by the phenomenological factor a_2 which is not well known from other sources. To avoid this uncertainty we will consider the ratio of the total rates,

$$\begin{aligned} R &\equiv \frac{\Gamma\left(\bar{B}_d^0 \to \psi K^{*0}\right)}{\Gamma\left(\bar{B}_d^0 \to \psi K^0\right)} \\ &= 1.64 \pm 0.34 \quad \text{CLEO II [10]}, \end{aligned} \tag{9}$$

and the polarization ratio for ψK^{*0}:

$$R_L \equiv \frac{\Gamma_L\left(\bar{B}_d^0 \to \psi K^{*0}\right)}{\Gamma_{\rm tot}\left(\bar{B}_d^0 \to \psi K^{*0}\right)} \tag{10}$$

that are independent of a_2.

Assuming factorization, any model or *Ansatz* on the heavy-to-light meson form factors will give a prediction for these ratios that can be compared to experiment. These ratios in terms of the form factors are given by Eqs. (16) and (35).

From these formulas one can already conclude qualitatively that (i) to get R_L sufficiently large, one needs V/A_1 and A_2/A_1 to be small enough and (ii) to get R not too large f_+/A_1 must not be too small.

We will consider the predictions for these ratios from the following theoretical schemes: (1) the pole model of Bauer, Stech, and Wirbel (BSWI) [5]; (2) the pole-dipole model of Neubert *et al.* (BSWII) [6]; (3) the quark model of Isgur, Scora, Grinstein, and Wise (ISGW) [11]; (4) QCD sum rules (QCDSR's).

The results are given in Table I. We do not include the lattice results here on the form factors [12] because they are still affected by large errors; we will discuss these results in Sec. IV. The conclusion of the table is that there is a problem for all known theoretical schemes since both ratios R and R_L cannot be described at the same time. *A priori* there are three possible explanations: (i) The theoretical schemes for *form factors* are to be blamed for the failure; (ii) the experimental numbers are not to be trusted too much; (iii) the basic BSW factorization assumption, which allows to relate $B \to K^{(*)}\psi$ to the form factors, is wrong for class II decays.

In Sec. III we will explore the first possibility by trying to formulate form factors satisfying the relevant theoretical principles and being able to describe the experimental situation. In the light of our discussion in the latter section we will return, in Sec. IV, to these models and try an analysis of their theoretical difficulties.

III. PHENOMENOLOGICAL HEAVY-TO-LIGHT SCALING FORM FACTORS CONFRONTED TO B AND D EXPERIMENTS

A. Setting the problem

Our aim will be to perform a combined experimental and theoretical study of form factors, simultaneously for both of $D \to K^{(*)}l\nu$ and $B \to K^{(*)}\psi$ decays, assuming BSW factorization for the latter. We would like to proceed as independently as possible of the detailed theoretical approaches, using *general Ansätze that respect the heavy-to-light asymptotic scaling laws*, some of them being complemented by ideas derived from heavy-to-heavy scaling law formulas. Only guided by rigorous theoretical laws and some commonly admitted theoretical prejudices, we will try to display general trends suggested

TABLE I. Comparison of different models, a QCD sum rules calculation and our preferred *Ansatz* (soft pole as defined in Table III) to experiment. In the fifth line CDF means fit to this data as explained in Table III.

	$\frac{\Gamma(K^*)}{\Gamma(K)}$	$\frac{\Gamma_L}{\Gamma_{\rm tot}}$	$\frac{A_2^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}$	$\frac{V^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}$	$\frac{f_+^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}$
BSWI [5]	4.23	0.57	1.01	1.20	1.23
BSWII [6]	1.61	0.36	1.41	1.77	1.82
ISGW [11]	1.71	0.07	2.00	2.58	2.30
QCDSR [23]	7.60	0.36	1.19	2.66	1.77
Soft pole (CDF)	2.15	0.45	1.08	2.16	1.86
CLEO II [8, 10]	1.64 ± 0.34	0.8 ± 0.1			
CDF [9]	-	0.66 ± 0.1			

by the experiment. Finally we will underline that experiment, as it stays today, is not easy to account for in a theoretically reasonable manner. We will also advocate the use of a quark-model-inspired (QMI) prescription, that we call the "QMI" *Ansatz*, an extension of some heavy-to-heavy scaling relations to the heavy-to-light system. Although not fully successful, this model is able to account roughly for a large set of data.

We will use the world average [13] for the $D \to K(K^*)\ell\nu$ form factors at $q^2 = 0$:

$$\begin{aligned} &f_+^{sc}(0) = 0.77 \pm 0.08,\\ &V^{sc}(0) = 1.16 \pm 0.16,\\ &A_1^{sc}(0) = 0.61 \pm 0.05, \quad A_2^{sc}(0) = 0.45 \pm 0.09; \end{aligned} \tag{11}$$

$$\begin{aligned} &V^{sc}(0)/A_1^{sc}(0) = 1.9 \pm 0.25,\\ &A_2^{sc}(0)/A_1^{sc}(0) = 0.74 \pm 0.15. \end{aligned} \tag{12}$$

As to the q^2 dependence the indications are poor except for the f_+ form factor where good indications seem to support the relevant vector meson pole dominance. We will use these indications for the q^2 dependence only in a second stage. It should be kept in mind that the data at $q^2 = 0$ are extracted from the integrated rates assuming single pole dominance. Since we will eventually conclude that we believe $A_1(q^2)$ to be flatter than a pole-dominated form factor, we should strictly speaking correct accordingly the value of $A_1(0)$ extracted from experiment. We will assume the correction to be small and neglect it.

To organize the discussion which must handle a great number of possibilities, we will often first concentrate on the evolution from D to B of the *ratios* between different form factors (A_2/A_1, V/A_1, etc.) for which we can formulate more general statements, and then consider the values and evolutions of form factors themselves (A_1, f_+, etc.) which involve additional assumptions.

B. Asymptotic scaling laws for the heavy-to-light form factors

What can be learned from the theory? The only exact results take the form of asymptotic theorems [14] valid for the initial quark mass m_Q large with respect to a typical scale Λ, of QCD, to the final meson mass m_f and to the final momentum[1]

$$\begin{aligned} &\frac{\hat{f}_+(\vec{q}^{\,2})}{c_+}, \quad \frac{\hat{V}(\vec{q}^{\,2})}{c_V}, \quad \frac{\hat{A}_2(\vec{q}^{\,2})}{c_2} = m_Q^{\frac{1}{2}}\left[1 + O\left(\frac{\Lambda}{m_Q}\right) + O\left(\frac{|\vec{q}|}{m_Q}\right) + O\left(\frac{m_f}{m_Q}\right)\right],\\ &\frac{\hat{A}_1(\vec{q}^{\,2})}{c_1} = (m_Q)^{-\frac{1}{2}}\left[1 + O\left(\frac{\Lambda}{m_Q}\right) + O\left(\frac{|\vec{q}|}{m_Q}\right) + O\left(\frac{m_f}{m_Q}\right)\right], \end{aligned} \tag{13}$$

where the c_+, c_2, c_V, c_1 are unknown constants and where we have used carets on form factors to indicate that they depend on *three-momentum, the natural variable in the heavy-to-light case,*

$$\hat{f}(\vec{q}^{\,2}) = f(q^2),$$

with

$$\vec{q}^{\,2} = \left(\frac{m_i^2 + m_f^2 - q^2}{2m_i}\right)^2 - m_f^2. \tag{14}$$

The asymptotic scaling law (13) allows us to relate the form factors, say $D \to K$ and $B \to K$, at small recoil $|\vec{q}| \ll m_D$ (i.e., close to $q^2_{\max}$ for each process):

$$\begin{aligned} &\frac{\hat{f}_+^{sb}(\vec{q}^{\,2})}{\hat{f}_+^{sc}(\vec{q}^{\,2})}, \quad \frac{\hat{V}^{sb}(\vec{q}^{\,2})}{\hat{V}^{sc}(\vec{q}^{\,2})}, \quad \frac{\hat{A}_2^{sb}(\vec{q}^{\,2})}{\hat{A}_2^{sc}(\vec{q}^{\,2})} = \left(\frac{m_B}{m_D}\right)^{\frac{1}{2}}\left[1 + O\left(\frac{\Lambda}{m_D}\right) + O\left(\frac{|\vec{q}|}{m_D}\right) + O\left(\frac{m_f}{m_D}\right)\right],\\ &\frac{\hat{A}_1^{sb}(\vec{q}^{\,2})}{\hat{A}_1^{sc}(\vec{q}^{\,2})} = \left(\frac{m_D}{m_B}\right)^{\frac{1}{2}}\left[1 + O\left(\frac{\Lambda}{m_D}\right) + O\left(\frac{|\vec{q}|}{m_D}\right) + O\left(\frac{m_f}{m_D}\right)\right], \end{aligned} \tag{15}$$

which hold for m_B and m_D much larger than Λ, the spectator quark and final meson masses as well as the final meson momentum.

C. Failure of the simple-minded extrapolation from D to B according to the asymptotic scaling law

In this subsection we will stress qualitatively that $B \to K^{(*)}\psi$ data seem to exclude a simple-minded extrapolation [i.e., neglecting the $O(1/m_D)$ corrective terms in (15)] from $D \to K^{(*)}l\nu$ data at $q^2 = 0$ according to the heavy-to-light asymptotic scaling law.

The ratio $\Gamma_L/\Gamma_{\rm tot}$ is given by

$$\frac{\Gamma_L(B \to K^*\psi)}{\Gamma_{\rm tot}(B \to K^*\psi)} = \frac{\left(3.162 - 1.306\frac{A_2^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}\right)^2}{2\left[1 + 0.189\left(\frac{V^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}\right)^2\right] + \left(3.162 - 1.306\frac{A_2^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}\right)^2}. \tag{16}$$

[1] Unless specified otherwise, we use the initial meson rest frame.

From this expression it is apparent that A_2/A_1 must not be too large[2] in view of the large experimental value of R_L (8), all the more if V/A_1 is large. For example, setting $V=0$ we get the very conservative upper bound $A_2/A_1 \leq 1.3$ for $R_L > 0.5$. For a more realistic value of $V/A_1 \simeq 2$, the upper bound becomes $A_2/A_1 \leq 1$. Now, according to strict application of the asymptotic scaling laws (15), A_2/A_1 (V/A_1) would be multiplied at fixed $\vec{q}$ by $m_B/m_D = 2.83$. From the central experimental D value, $A_2^{sc}/A_1^{sc} = 0.74$ ($V^{sc}/A_1^{sc} = 1.9$), one gets $A_2^{sb}/A_1^{sb} = 2.09$ ($V^{sb}/A_1^{sb} = 5.38$) at $q^2 = 16.56$ GeV (corresponding in B decay to the same $\vec{q}^{\,2}$ as $q^2 = 0$ in D decay). This is in drastic contradiction with experiment unless there is an unexpectedly strong q^2 variation down to $q^2 = m_\psi^2$. A naive insertion of these values in Eq. (16) would indeed give $R_L = 0.014$ which is 4–5 σ's away from the most favorable CDF value. Clearly the message is that *a softening of the increase of the above-considered ratios with respect to the asymptotic scaling law is required.*

D. Extending the heavy-to-heavy Isgur-Wise scaling laws into a heavy-to-light class of *Ansätze*

At this point it is useful to notice that there is an overlap between the domains of validity of the heavy-to-heavy and heavy-to-light scaling laws, namely, when $\Lambda \ll m_{Q_f} \ll m_{Q_i}$. In this domain the heavy-to-heavy scaling law provides corrections of order m_{Q_f}/m_{Q_i} to the asymptotic heavy-to-light scaling law that go in the desired direction of a softening. This will be explained in the next subsection.

1. Reminder about asymptotic scaling laws for heavy-to-heavy transitions

It is well known that a much stronger set of relations than the one in Sec. III B comes from the Isgur-Wise scaling laws [1] for transition form factors between two heavy quarks. Using the notation in [2],

$$\begin{aligned}\frac{\sqrt{4m_{P_i}m_{P_f}}}{m_{P_i}+m_{P_f}}f_+(q^2) &= \frac{\sqrt{4m_{P_i}m_{P_f}}}{m_{P_i}+m_{P_f}}\frac{f_0(q^2)}{1-\frac{q^2}{(m_{P_i}+m_{P_f})^2}}\\ &= \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i}+m_{V_f}}V(q^2) = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i}+m_{V_f}}A_0(q^2) = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i}+m_{V_f}}A_2(q^2) = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i}+m_{V_f}}\frac{A_1(q^2)}{1-\frac{q^2}{(m_{P_i}+m_{V_f})^2}}\\ &= \xi(v_i\cdot v_f)\end{aligned} \tag{17}$$

for m_{P_i}, m_{P_f}, and m_{V_f} much larger than the typical scale Λ of QCD. It must be added that in the same limit m_{P_f} and m_{V_f} are in fact equal so that our writing of different masses is only meant for later use in the real subasymptotic regime, where they are very different ($m_K \neq m_{K^*}$).

The denominator that divides $A_1(q^2)$ is a straightforward consequence of the heavy quark symmetry and of the definition of the different form factors. It has not the meaning of a dynamical pole related to some intermediate state.[3] It is still in the mathematical sense a pole of the ratio $A_2(q^2)/A_1(q^2)$, etc., and we shall call it for simplicity the "kinematical pole."

When Eq. (17) may be applied, it is much stronger than the heavy-to-light constraint (13). Using this heavy-to-heavy relation (17) for two different values ($m_{P_i} = m_B, m_D$) we automatically obtain the heavy-to-light one (13) when one makes m_{P_i} much larger than[4] m_f. Indeed, at fixed $\vec{q}$, $v_i \cdot v_f = \sqrt{1+\vec{q}^{\,2}/m_f^2}$ (in the rest frame of the initial meson) is fixed. It is also simple to show that

$$\frac{4m_{P_i}m_f}{(m_{P_i}+m_f)^2}\frac{1}{1-\frac{q^2}{(m_{P_i}+m_f)^2}} = \frac{2}{1+v_i\cdot v_f} \tag{18}$$

so that the preceding equations write finally in terms of masses and the fixed $\vec{q}$, with $m_{P_i} \gg m_f$:

$$\begin{aligned}2\left(\frac{m_{P_f}}{m_{P_i}}\right)^{1/2}\left(1-\frac{m_{P_f}}{m_{P_i}}+\cdots\right)\hat{f}_+(\vec{q}^{\,2}) &= \frac{\sqrt{m_{P_i}m_{P_f}}}{m_{P_f}+E_{P_f}}\left(1+\frac{m_{P_f}}{m_{P_i}}\right)\hat{f}_0(\vec{q}^{\,2})\\ &= 2\left(\frac{m_{V_f}}{m_{P_i}}\right)^{1/2}\left(1-\frac{m_{V_f}}{m_{P_i}}+\cdots\right)\hat{V}(q^2) = 2\left(\frac{m_{V_f}}{m_{P_i}}\right)^{1/2}\left(1-\frac{m_{V_f}}{m_{P_i}}+\cdots\right)\hat{A}_0(\vec{q}^{\,2})\\ &= 2\left(\frac{m_{V_f}}{m_{P_i}}\right)^{1/2}\left(1-\frac{m_{V_f}}{m_{P_i}}+\cdots\right)\hat{A}_2(\vec{q}^{\,2}) = \frac{\sqrt{m_{P_i}m_{V_f}}}{m_{V_f}+E_{V_f}}\left(1+\frac{m_{P_f}}{m_{P_i}}\right)\hat{A}_1(\vec{q}^{\,2})\\ &= \xi(E_f/m_f),\end{aligned} \tag{19}$$

[2]Strictly speaking very large values, $A_2/A_1 \geq 3.9$, could also account for a large R_L, but these are unrealistic.
[3]Although the singularity happens to fall at the branching point of a t-channel cut.
[4]In our notations m_f represents generically m_{P_f} and m_{V_f}.

where $E_f = E_{V_f} \simeq E_{P_f}$ are the final energies in the initial rest frame, $E_f = \sqrt{m_f^2 + \vec{q}^{\,2}}$. The "carets" on form factors have been defined in Eq. (14) and we have used

$$1 - \frac{q^2}{(m_{P_i} + m_f)^2} = 2\frac{m_{P_i}(m_f + E_f)}{(m_{P_i} + m_f)^2}. \tag{20}$$

We see that Eq. (19) includes specific values for the $O(m_f/m_{P_i})$ corrections to the heavy-to-light scaling law (13). Of course these corrections are in principle only valid if m_f is heavy. Any specific model claiming to handle the domain of mass $\Lambda \ll m_f \ll m_{P_i}$ should obviously satisfy the relations (19).

An essential effect displayed by formula (19) is that it softens the asymptotic scaling relation (13); i.e., it leads to a slower increase (decrease) of A_2, V, f_+ (A_1) when the initial mass m_{P_i} increases at fixed $\vec{q}$.

Another very important aspect of the Isgur-Wise relations (17) is that all the ratios of form factors for the same quark masses and at the same transfer q^2 (or equivalently $\vec{q}$) are completely fixed by the theory: *quite strikingly, the form factors must be all asymptotically equal at $q^2 = 0$, and therefore their ratios are completely independent of the masses; this extends to any fixed q^2 provided that q^2 is small with respect to m_B^2*:

$$\begin{aligned} f_+(0) &= f_0(0) \\ &= V(0) = A_0(0) = A_2(0) = A_1(0) \\ &= \xi((v_i \cdot v_f)_{q^2=0}), \end{aligned} \tag{21}$$

where

$$(v_i \cdot v_f)_{q^2=0} = \frac{m_{P_i}^2 + m_f^2}{2 m_{P_i} m_f}.$$

This is to be contrasted with the heavy-to-light scaling (13) which leaves these ratios *at the same* q^2 undetermined, because the dependence in $\vec{q}$ is undetermined. Even when nonasymptotic effects are included, we expect from Eq. (21) the ratios f_+/A_1, V/A_1, and A_2/A_1 to behave very softly as a function of the heavy initial mass at $q^2 = 0$.

2. The basic soft-scaling-pole Ansatz

We now formulate our model based on an extension of the heavy-to-heavy scaling relations (17). Let us first assume that we are in a situation described in the preceding section: $m_i \gg m_f \gg \Lambda$. As we have argued, the form factors obey the heavy-to-light scaling relations (13) with specific form factor ratios and specific $O(m_f/m_i)$ corrections, Eq. (19). To these, one should also add the unknown $O(\Lambda/m_f)$ corrections to the heavy quark symmetry.

Let us now consider the intermediate region where the final quark ceases to be heavy. Our ignorance comes from the fact that the $O(\Lambda/m_f)$ corrections become large and may totally modify the above-mentioned specific relations. Our hypothesis will be that it is not so, i.e., that using *some* of the features of Eq. (17) are indeed a good approximation. This hypothesis, although admittedly arbitrary, may be empirically justified by the fact (see Sec. III C) that data demand a softened heavy-to-light scaling, and that formula (17) or equivalently (19) does present such a behavior. Theoretical arguments in favor of the present *Ansatz* will come below and in Sec. III D 3.

It is obvious that an unrestricted extension of Isgur-Wise formulas (17) cannot describe quantitatively the form factors for a simple reason: the $D \to K^{(*)} l\nu$ form factors at $q^2 = 0$, Eq. (11), are obviously not equal to each other, contrary to what is predicted by Eq. (21); and the formula will also completely fail for $B \to K^{(*)}\psi$, since it would predict from formula (35) a much too large ratio $\Gamma(K^*)/\Gamma(K) \simeq 4$. This is after all expected because we do not believe that the D is heavy, not to speak about the K or K^*. Notwithstanding this failure we will try to apply to the heavy-to-light case the q^2 and mass dependence implied by formulas (17). The problem of D decays can be trivially cured by assuming, as we shall do, a "rescaling" of each form factor to put it in agreement with the D data at $q^2 = 0$. We also assume that these rescaling factors (r_+, r_V, r_1, r_2) are independent of the initial heavy quark mass and of q^2. In other words, we assume the $O(\Lambda/m_f)$ corrections to be properly taken into account by these constant rescaling factors. Let us thus start from Eq. (17), multiply for convenience the left-hand side (LHS) and the RHS by $(1 + v_i \cdot v_f)/2$, and rescale the form factors as mentioned above. We obtain

$$\begin{aligned} \frac{m_{P_i} + m_{P_f}}{\sqrt{4 m_{P_i} m_{P_f}}} \left[1 - \frac{q^2}{(m_{P_i} + m_{P_f})^2}\right] \frac{f_+(q^2)}{r_+} &= \frac{m_{P_i} + m_{V_f}}{\sqrt{4 m_{P_i} m_{V_f}}} \left[1 - \frac{q^2}{(m_{P_i} + m_{V_f})^2}\right] \frac{V(q^2)}{r_V} \\ &= \frac{m_{P_i} + m_{V_f}}{\sqrt{4 m_{P_i} m_{V_f}}} \left[1 - \frac{q^2}{(m_{P_i} + m_{V_f})^2}\right] \frac{A_2(q^2)}{r_2} \\ &= \frac{m_{P_i} + m_{V_f}}{\sqrt{4 m_{P_i} m_{V_f}}} \frac{A_1(q^2)}{r_1} = \eta(\vec{q}, m_f), \end{aligned} \tag{22}$$

where m_f is for m_{P_f} or m_{V_f}. In fact, to conform with the asymptotic Isgur-Wise heavy-to-heavy scaling, the rescaling parameters $r_+, r_V, r_2, r_1 = 1 + O(\Lambda/m_f)$ should depend on the final active quark mass m_{q_f} and tend to one when it goes to infinity, but this does not matter here since the final quark will remain the s quark all over this study. In formula (22) we have introduced a function $\eta(\vec{q}, m_f)$, since $\vec{q}$ is the natural variable in the heavy-to-light scaling case.

It should be repeated that at fixed $\vec{q}^2$ formula (22) corrects the asymptotic scaling (13) by the replacement $m_{P_i}^{1/2} \to (m_{P_i}+m_f)/m_{P_i}^{1/2}$. This correction to the asymptotic scaling induces a softer behavior than the asymptotic scaling, Eq. (13), i.e., a slower increase and/or decrease, when m_{P_i} increases. Consequently Eq. (15) becomes specified into

$$\frac{\hat{f}_+^{sb}(\vec{q}^2)}{\hat{f}_+^{sc}(\vec{q}^2)}, \quad \frac{\hat{V}^{sb}(\vec{q}^2)}{\hat{V}^{sc}(\vec{q}^2)},$$

$$\frac{\hat{A}_2^{sb}(\vec{q}^2)}{\hat{A}_2^{sc}(\vec{q}^2)} = \left(\frac{m_B+m_f}{m_D+m_f}\right)\left(\frac{m_B}{m_D}\right)^{-\frac{1}{2}},$$

$$\frac{\hat{A}_1^{sb}(\vec{q}^2)}{\hat{A}_1^{sc}(\vec{q}^2)} = \left(\frac{m_D+m_f}{m_B+m_f}\right)\left(\frac{m_D}{m_B}\right)^{-\frac{1}{2}}. \tag{23}$$

Another important consequence of formula (22) is that at $q^2 = 0$ the form factor ratios stay constant for fixed final masses when m_{P_i} varies, except for a small variation of f_+/A_1 of the order $O((m_{V_f} - m_{P_f})/m_{P_i})$:

$$\frac{A_2(0)}{A_1(0)} = \frac{r_2}{r_1}, \qquad \frac{V(0)}{A_1(0)} = \frac{r_V}{r_1},$$

$$\frac{f_+(0)}{A_1(0)} = \frac{r_+}{r_1}\left(\frac{m_{P_f}}{m_{V_f}}\right)^{\frac{1}{2}} \frac{m_{P_i}+m_{V_f}}{m_{P_i}+m_{P_f}} \frac{\eta(\vec{q}_1, m_{P_f})}{\eta(\vec{q}_2, m_{V_f})}, \tag{24}$$

where $|\vec{q}_1| = (m_{P_i}^2 - m_{P_f}^2)/2m_{P_i}$ and $|\vec{q}_2| = (m_{P_i}^2 - m_{V_f}^2)/2m_{P_i}$ correspond to $q^2 = 0$ for a pseudoscalar and a vector final meson, respectively.

The formula (22) is of course a purely phenomenological assumption, although suggested by our quark model analysis in the weak binding limit (see below); it still presents a large arbitrariness, corresponding to the freedom of $\eta(\vec{q}, m_f)$.

Until Sec. III F we shall not need to specify the function $\eta(\vec{q}, m_f)$ as we shall be concerned only with form factor ratios. For completeness we shall now simply state our choice, referring to Sec. III F for justifications:

$$\eta(\vec{q}, m_f) = 1. \tag{25}$$

3. Theoretical justifications

a. Quark model. Our first motivation for this *Ansatz* is that a quark model with relativistic center-of-mass motion described below, the Orsay quark model (OQM) [15–17], fulfilling the Isgur-Wise relations (17) in the heavy-to-heavy limit, will be shown to retain several important features of these relations in the heavy-to-light case. Indeed, it presents the kinematical pole factor differentiating f_+, A_2, V from A_1. It also displays the $O(m_f/m_i)$ corrections predicted by the mass factors in (17) and (19). On the other hand our quark model analysis leads us to expect two types of $O(\Lambda/m_f)$ corrections to (17): (i) corrections already present in the weak-binding limit (which are explicit in OQM); (ii) corrections to the weak-binding limit, not included in OQM. We esteem that both types of corrections can be *very roughly* represented by numerical factors r_i, thus justifying *Ansatz* (22).

b. $B \to K^*\gamma$. An amusing example that exhibits the same trends is provided by the $B \to K^*\gamma$ form factors. Defining the T_i form factors as

$$\langle K^*, k, \epsilon | \bar{s}\sigma^{\mu\nu} q_\nu \frac{1+\gamma_5}{2} b | B, p\rangle = -2\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}p^\lambda k^\sigma T_1(q^2) - i[\epsilon^*{}_\mu(m_B^2 - m_{K^*}^2) - \epsilon^*\cdot q(p+k)_\mu]T_2(q^2)$$
$$-i\epsilon^*\cdot q\left[q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(p+k)_\mu\right]T_3(q^2) \tag{26}$$

it is well known that, for $q^2 = 0$, using the identity $\sigma_{\mu\nu}\gamma_5 = \frac{i}{2}\epsilon_{\mu\nu\lambda\sigma}\sigma^{\lambda\sigma}$, one obtains the exact relation

$$T_1(0) = T_2(0). \tag{27}$$

It has also been shown [14] that

$$T_1(q^2) = d_1\sqrt{m_Q}\left[1 + O\left(\frac{\Lambda}{m_Q}\right) + O\left(\frac{|\vec{q}|}{m_Q}\right)\right],$$

$$T_2(q^2) = d_2\frac{1}{\sqrt{m_Q}}\left[1 + O\left(\frac{\Lambda}{m_Q}\right) + O\left(\frac{|\vec{q}|}{m_Q}\right)\right], \tag{28}$$

where the d_1, d_2 are unknown constants.

In the heavy-to-heavy case one may also show that

$$\frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i}+m_{V_f}}T_1(q^2) = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i}+m_{V_f}}\frac{T_2(q^2)}{1-\frac{q^2}{(m_{P_i}+m_{V_f})^2}}$$
$$= \tfrac{1}{2}\xi(v\cdot v') \tag{29}$$

which is of course fully compatible with the relation (27).

But the new thing here is that *the relation (27) remains exact when the final quark becomes light.* Since the scaling behaviors of the T_1 and T_2 differ in the vicinity of $q^2_{\rm max}$ (28), the equality (27) is a clear indication that the q^2 behavior of both form factors differs sensibly. For example, a pole dominance hypothesis for both form factors is totally excluded by these relations. Furthermore, an extension of relation (29) to the heavy-to-light domain, as we have suggested in Sec. III D 2, would directly comply with both relations (27) and (28). This is a hint that our *Ansatz* may point toward the right direction.

c. Matrix elements. Our *Ansatz* (22), as far as the mass dependence is concerned, amounts to assume that the matrix elements satisfy an uncorrected asymptotic scaling. To illustrate this point let us consider a final vector meson V_f with a polarization ϵ^T orthogonal to the initial and final meson momenta.

From Eqs. (1) and (22) the matrix elements scale as

$$\frac{\langle V_f,\epsilon^T,\vec{q}|A_\mu|P_i\rangle}{\sqrt{4m_B m_{V_f}}} = r_1\,\eta(\vec{q},m_{V_f})\epsilon_\mu,$$

$$\frac{\langle V_f,\epsilon^T,\vec{q}|V_\mu|P_i\rangle}{\sqrt{4m_B m_{V_f}}} = ir_V\,\frac{\eta(\vec{q},m_{V_f})}{1+v_f^0}\left(\vec{v}_f\times\vec{\epsilon}^T\right)_\mu, \tag{30}$$

where $v_f^\mu = p_f^\mu/m_f$ and Eq. (20) has been used .

In this example it is clear that the matrix elements scale exactly like $\sqrt{m_{P_i}m_f}$, which is their asymptotic heavy-to-light scaling behavior. *Our softened scaling Ansatz is equivalent to a precocious asymptotic scaling of the matrix elements.*

d. QCD sum rules, lattice calculations. We will argue in Sec. IV B that QCD sum rules qualitatively favor the q^2 dependence of the form factor ratios as depicted in Eq. (22); i.e., they generally show an increase of the ratios $A_2/A_1, V/A_1, f_+/A_1$ with q^2 not very different from the increase due to the kinematical pole $1/[1-q^2/(m_{P_i}+m_f)^2]$.

Lattice calculations [12] on their side, favor a softened heavy-to-light scaling, as a function of the heavy masses, for the leptonic decay constant F_P and, with large errors, for the form factors except A_2.

E. Confronting the form factors to experimental ratios

We now turn to experiment in order to fix the remaining free parameters and to know whether the data can be really understood in the above phenomenological framework.

Although we would like to stick to our theoretical prejudices, Eqs. (22) and (25), we still feel that the situation is very uncertain in the subasymptotic regime. Therefore we shall also test other competing schemes in order to know by comparison whether experiment is actually giving definite indications on the q^2 and mass dependence of the form factors. A few definite lessons will be drawn but the exercise will not prove as conclusive as we would have wished.

1. Some simple prescriptions for mass and q^2 dependence

We intend to perform several χ^2 fits of the ratios of form factors according to the following method. We assume a given evolution prescription for the dependence of the ratios of form factors as a function of the heavy mass at $q^2_{\max}$, and a given prescription for the behavior of these ratios as a function of q^2 for fixed masses. Next we combine in *one* χ^2 fit the experimental results for $D\to K^{(*)}l\nu$ at $q^2=0$ and the experimental results for the ratios R and R_L for $B\to K^{(*)}\psi$. The evolution prescriptions we have used are now described.

(1) The soft-scaling pole corresponds exactly to assumption (22), i.e., the heavy-to-heavy inspired scaling which implies a softened scaling in heavy quark mass at fixed $\vec{q}$, and a ratio to $A_1(q^2)$ that exhibits the kinematical pole similar to the heavy-to-heavy scaling formulas.

(2) The soft-scaling constant assumes the same softened scaling at $q^2_{\max}$ as before, but a ratio of form factors which stays constant as q^2 varies for fixed masses:

$$\frac{\sqrt{4m_{P_i}m_{P_f}}}{m_{P_i}+m_{P_f}}\frac{f_+(q^2)}{r_+} = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i}+m_{V_f}}\frac{V(q^2)}{r_V} = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i}+m_{V_f}}\frac{A_2(q^2)}{r_2} = \frac{(m_{P_i}+m_{V_f})}{\sqrt{4m_{P_i}m_{V_f}}}\frac{A_1(q^2)}{r_1} = \eta(\vec{q},m_f). \tag{31}$$

(3) The hard-scaling pole assumes hard scaling, i.e., the asymptotic laws without $1/m_Q$ corrections. This is achieved by replacing in the "soft-pole" prescriptions at $q^2_{\max}$,

$$\sqrt{m_{P_i}}/(m_{P_i}+m_f)\to 1/\sqrt{m_{P_i}},$$

but keeping the q^2 dependence at fixed masses as in (22). It corresponds to

$$2\left(\frac{m_{P_f}}{m_{P_i}}\right)^{\frac{1}{2}}\frac{(m_{P_i}+m_{P_f})^2}{4m_{P_i}m_{P_f}}\left[1-\frac{q^2}{(m_{P_i}+m_{P_f})^2}\right]\frac{f_+(q^2)}{r_+} = 2\left(\frac{m_{V_f}}{m_{P_i}}\right)^{\frac{1}{2}}\frac{(m_{P_i}+m_{V_f})^2}{4m_{P_i}m_{V_f}}\left[1-\frac{q^2}{(m_{P_i}+m_{V_f})^2}\right]\frac{V(q^2)}{r_V}$$

$$= 2\left(\frac{m_{V_f}}{m_{P_i}}\right)^{\frac{1}{2}}\frac{(m_{P_i}+m_{V_f})^2}{4m_{P_i}m_{V_f}}\left[1-\frac{q^2}{(m_{P_i}+m_{V_f})^2}\right]\frac{A_2(q^2)}{r_2}$$

$$= \frac{1}{2}\left(\frac{m_{P_i}}{m_{V_f}}\right)^{\frac{1}{2}}\frac{A_1(q^2)}{r_1} = \eta(\vec{q},m_f). \tag{32}$$

(4) The hard-scaling constant assumes the same hard scaling as above and assumes that the ratios do not depend on q^2. It corresponds to

$$2\left(\frac{m_{P_f}}{m_{P_i}}\right)^{\frac{1}{2}}\frac{f_+(q^2)}{r_+}=2\left(\frac{m_{V_f}}{m_{P_i}}\right)^{\frac{1}{2}}\frac{V(q^2)}{r_V}$$
$$=2\left(\frac{m_{V_f}}{m_{P_i}}\right)^{\frac{1}{2}}\frac{A_2(q^2)}{r_2}$$
$$=\frac{1}{2}\left(\frac{m_{P_i}}{m_{V_f}}\right)^{\frac{1}{2}}\frac{A_1(q^2)}{r_1}=\eta(\vec{q},m_f). \quad (33)$$

In prescriptions (2) and (4), the assumption that the form factors have a constant ratio in q^2 is inspired from the popular pole dominance approximation.[5]

2. Lessons from our global χ^2 fits

For the experimental results on $D\to K^{(*)}l\nu$ at $q^2=0$ we take the world average estimated by Witherell [13], and for R_L we have used in two different fits the results from CLEO II [8] and from CDF [9]. For R we have used 1.64 ± 0.34 from CLEO II [10].

Before discussing the outcomes of these fits, let us notice that in this exercise, the result only depends on the double ratios

$$\mathcal{R}_2\equiv(A_2^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2))/(A_2^{sc}(0)/A_1^{sc}(0)),$$
$$\mathcal{R}_+\equiv(f_+^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2))/(f_+^{sc}(0)/A_1^{sc}(0)), \quad (34)$$

since, within our *Ansatz*, the double ratio $\mathcal{R}_V$ satisfies

$$\mathcal{R}_V\equiv(V^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2))/(V^{sc}(0)/A_1^{sc}(0))$$
$$=\mathcal{R}_2.$$

We have first performed a fit restricted to the K^* final state, leaving aside the f_+ form factors and the ratio $\Gamma(B\to K^*\psi)/\Gamma(B\to K\psi)$. The results are displayed in Table II. As a first conclusion from Table II one sees that the best fit is soft-scaling pole. The reason for that is that the large experimental values of $\Gamma_L/\Gamma_{\rm tot}$ and $A_2^{sc}(0)/A_1^{sc}(0)$ impose the double ratio $\mathcal{R}_2$ to be rather small, as argued in Sec. III C, thus suggesting either that the asymptotic scaling law ($A_2/A_1\propto m_Q$) is strongly softened, or that the A_2/A_1 ratio decreases dramatically with decreasing q^2, or some compromise between both effects. Indeed the soft-scaling pole *Ansatz* has both a softened scaling at $q^2_{\rm max}$ and a decrease of the A_2/A_1 ratio with decreasing q^2. Therefore, this *Ansatz* yields the smallest ratio $\mathcal{R}_2$, and thus a larger $\Gamma_L/\Gamma_{\rm tot}$. A roughly acceptable fit is thus obtained for CDF data, with a confidence level of $\sim 20\%$, but not for CLEO. To obtain an acceptable fit with CLEO data, one would need a value of $\mathcal{R}_2$ sensibly smaller than 0.8. Such a low value seems very difficult to obtain in any natural way. Table II also shows that soft scaling is generally favored and hard scaling is in very strong disagreement with data.

In a second step we have added to our fits the data concerning the K final state. The results are displayed in Table III. Qualitatively there is no big change. Looking in more detail, it appears that the best fits now include on the same level the soft-scaling pole and the soft-scaling-constant cases, with a worst confidence level (around 2%). The reason for that is that all our *Ansätze* correspond to $\mathcal{R}_+\simeq\mathcal{R}_2$, and, while the data on $\Gamma_L/\Gamma_{\rm tot}$ require a small $\mathcal{R}_2$ as just argued, the data on $\Gamma(B\to K^*\psi)/\Gamma(B\to K\psi)$ and $f_+^{sc}(0)/A_1^{sc}(0)$ on the contrary require a not too small double ratio $\mathcal{R}_+$. This can be understood as follows.

The ratio R (9) is given by

$$R=1.081\left(\frac{A_1^{sb}(m_\psi^2)}{f_+^{sb}(m_\psi^2)}\right)^2\left\{2\left[1+0.189\left(\frac{V^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}\right)^2\right]+\left(3.162-1.306\frac{A_2^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}\right)^2\right\}. \quad (35)$$

Multiplying the LHS of Eqs. (16) and (35) we get

$$R(1-R_L)=2.162\left(\frac{A_1^{sb}(m_\psi^2)}{f_+^{sb}(m_\psi^2)}\right)^2\left[1+0.189\left(\frac{V^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}\right)^2\right]. \quad (36)$$

This gives obviously a lower bound on f_+/A_1. For the conservative upper bounds of $R\le2.5$ and $1-R_L\le0.5$ and setting still more conservatively V to zero we get $f_+/A_1\ge1.32$. For a more realistic estimate, $V/A_1\simeq2$, and $R\le2.0$ we get $\mathcal{R}_+\ge1.60$. Contrarly to our discussion in Sec. III C we find here a lower bound which in itself is compatible with the hard-scaling behavior but not with such a soft scaling as required for A_2/A_1. *Clearly the trend for f_+/A_1 is somewhat opposite to the one for A_2/A_1*.

The χ^2 fits tried a compromise between these two opposite trends and this is why the soft-scaling-constant *Ansatz* now fits as well as the soft-scaling-pole one: although the prediction for $\Gamma_L/\Gamma_{\rm tot}$ is worse for the former, it gives a better $\Gamma(B\to K^*\psi)/\Gamma(B\to K\psi)$ ratio, since it corresponds to larger double ratios. Again the hard-scaling cases are rejected. The two best fits are hardly acceptable in the case of the CDF value $R_L=0.66\pm0.14$ and fail with CLEO II's much more restrictive value $R_L=0.80\pm0.095$ (as it would have failed with the Argus bound $R_L>0.78, 95\%$ confidence level). There is a real difficulty, as noted in [18], to account for the data on $D\to K^{(*)}l\nu$ and $B\to K^{(*)}\psi$.

What could be the way out of this dilemma? One may of course question the factorization hypothesis which is the basic hypothesis in all this paper. Although we are

[5]In the nearest pole dominance hypothesis, the form factor ratios slightly differ from a constant when the form factor considered have not the same poles. For simplicity we have used the "constant ratio" hypothesis.

TABLE II. The extrapolation procedures are explained in the text. In each case, the values of the ratios of form factors $V^{sc}(0)/A_1^{sc}(0)$ and $A_2^{sc}(0)/A_1^{sc}(0)$ have been fitted to minimize the χ^2 relative to the experimental numbers in the last line. A two-parameter fit for three constraints leaves one degree of freedom (dof). The experimental value for $R_L = \Gamma_L/\Gamma_{\rm tot}$ is taken from CDF or CLEO according to what is indicated in the first column. "ave" refers to the world average for the form factor ratios in $D \to K^{(*)}l\nu$. Whenever it was needed to combine statistical and systematic errors, we have combined them in quadrature.

Extrapolation	$\frac{A_2^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2)}{A_2^{sc}(0)/A_1^{sc}(0)}$	$\frac{V^{sc}(0)}{A_1^{sc}(0)}$	$\frac{A_2^{sc}(0)}{A_1^{sc}(0)}$	$\frac{\Gamma_L}{\Gamma_{\rm tot}}$	$\chi^2/N_{\rm DF}$
Soft pole: CDF	1.34	1.82	0.656	0.490	1.9
Soft pole: CLEO	1.34	1.67	0.520	0.567	9.0
Soft cons: CDF	1.77	1.75	0.556	0.386	5.7
Soft cons: CLEO	1.77	1.53	0.385	0.520	16.5
Hard pole: CDF	2.14	1.70	0.480	0.322	9.5
Hard pole: CLEO	2.14	1.43	0.292	0.498	22.5
Hard cons: CDF	2.83	1.66	0.375	0.233	16.1
Hard cons: CLEO	2.83	1.29	0.165	0.481	31.9
Expt: ave + CDF	-	$1.9 \pm .25$	$0.74 \pm .15$	$0.66 \pm .14$	-
Expt: CLEO II	-	-	-	$0.80 \pm .10$	-

thoroughly convinced that the factorization hypothesis may very reasonably be doubted in a $1/N_c$ subdominant channel as is $B \to K^{(*)}\psi$, we decided to leave all this discussion outside this paper. We may also hope that more precise experiments will evolve in a direction that will make the problem not so acute. Although the χ^2 may seem horrific when the CLEO II value for R_L is used, it should be kept in mind that a small variation of the experimental value may lead to a dramatic decrease of the χ^2. A comparison with CDF gives a first example of that.

Finally our hypothesis, displayed in Eqs. (22), (31)–(33), leading $V(q^2)/A_1(q^2) \propto A_2(q^2)/A_1(q^2)$ and $f_+(q^2)/A_1(q^2) \propto A_2(q^2)/A_1(q^2)$ may[6] also be criticized. It would of course be meaningless to relax these constraints in the above-described χ^2 fits, since we would have too many free parameters. Qualitatively it is obvious that any prescription with $\mathcal{R}_+$ sensibly larger than $\mathcal{R}_2$ would lessen the χ^2. But we see no sign of such a trend in the models we have considered. Another lessening of the χ^2 would happen if $\mathcal{R}_V = (V^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2))/(V^{sc}(0)/A_1^{sc}(0))$ was sensibly smaller than $\mathcal{R}_2$. This happens to be the case, although in a quantitatively insufficient amount, in the Orsay quark model (see Sec. IV C 6): in Eq. (54) it appears that $V(q^2)$ contains a relatively large corrective factor Y (58), which decreases with the initial mass. This term tends to decrease all the χ^2 in Table III but not enough. It has the effect of providing for the form factor $V(q^2)$ an even larger softening of the increase predicted by the

TABLE III. The only difference with Table II is that we have added to our fits the ratio R (9) and the f_+ form factor. A three parameter fit for five constraints leaves 2 degrees of freedom (dof).

Extrapolation	$\frac{A_2^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2)}{A_2^{sc}(0)/A_1^{sc}(0)}$	$\frac{f_+^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2)}{f_+^{sc}(0)/A_1^{sc}(0)}$	$\frac{f_+^{sc}(0)}{A_1^{sc}(0)}$	$\frac{V^{sc}(0)}{A_1^{sc}(0)}$	$\frac{A_2^{sc}(0)}{A_1^{sc}(0)}$	$\frac{\Gamma(K^*)}{\Gamma(K)}$	$\frac{\Gamma_L}{\Gamma_{\rm tot}}$	$\chi^2/N_{\rm DF}$
Soft pole: CDF	1.34	1.28	1.45	1.62	0.809	2.15	0.449	4.2
Soft pole: CLEO	1.34	1.28	1.47	1.45	0.680	2.21	0.533	8.6
Soft cons: CDF	1.77	1.70	1.32	1.66	0.600	1.81	0.375	3.2
Soft cons: CLEO	1.77	1.70	1.34	1.43	0.443	1.87	0.510	8.9
Hard pole: CDF	2.14	2.14	1.25	1.72	0.472	1.60	0.323	4.7
Hard pole: CLEO	2.14	2.14	1.26	1.43	0.293	1.64	0.498	11.2
Hard cons: CDF	2.83	2.83	1.20	1.79	0.344	1.44	0.234	8.4
Hard cons: CLEO	2.83	2.83	1.20	1.40	0.123	1.44	0.481	16.4
Expt: ave + CDF	-	-	1.26	1.9	0.74	-	0.66	-
	-	-	$\pm .12$	$\pm .25$	$\pm .15$	-	$\pm .14$	-
Expt: CLEO	-	-	-	-	-	1.64	0.80	-
	-	-	-	-	-	$\pm .34$	$\pm .10$	-

[6]The latter equation is only approximately valid in Eqs. (22) and (31) due to the $K - K^*$ mass difference.

asymptotic scaling law (13). It is interesting to notice that such a large correction to the asymptotic scaling law for $V(q^2)$ in the direction of a softening has been found in lattice calculations [12], although the large statistical errors in these calculations do not allow us to draw yet a final conclusion. Finally, our difficulties to get small χ^2 is not too surprising since the χ^2 tends anyhow to become large when experimental errors decrease, unless a very accurate model is available, which is certainly not the case in the present attempts.

To summarize, we have found, using our simple phenomenological *Ansätze* for the dependence of the form factors ratios in q^2 and in masses, that the experimental comparison between $B \to K^{(*)}\psi$ and $D \to K^{(*)}l\nu$ favors a soft heavy-to-light scaling, Eqs. (22) and (31), the best q^2 dependence cannot be selected from this analysis alone, although the separated phenomenological study of the K^* final states (Table II), as well as several theoretical considerations, tend to favor the existence of the "kinematical pole" as in Eq. (22), and there remains a difficulty to reconcile experimental results in $B \to K^{(*)}\psi$ and $D \to K^{(*)}l\nu$ when taking CDF results for R_L ($\chi^2/N_{\rm DF} \simeq 3$) which grows even worse when using CLEO or ARGUS values for R_L. There seems to also be a particular difficulty to fit simultaneously R and R_L. Only fragile indications of possible ways out of these difficulties are known today.

F. q^2 dependence of $A_1(q^2)$, $f_+(q^2)$, etc., from experiment

Up to now we have mainly considered the ratios of form factors, $A_2/A_1, V/A_1$, and f_+/A_1. In this subsection we try to go beyond and consider how the form factors themselves depend on q^2. We shall now gather from different sources information about $A_1(q^2)$, $f_+(q^2)$, and we shall see that these combined informations are rather compatible with what we already know about the ratios.

Of course the above rough agreement of soft-scaling-pole for $\Gamma_L/\Gamma_{\rm tot}$ and for $\Gamma(B \to K^*\psi)/\Gamma(B \to K\psi)$ does not depend on the value of $\eta(\vec{q}, m_f)$ in Eq. (22). This subsection is devoted to argue in favor of our choice in Eq. (25) for $\eta(\vec{q}, m_f)$. The meaning of Eq. (25) is in fact that we choose $A_1(q^2)$ to be a constant:

$$A_1(q^2) = r_1 \frac{\sqrt{4m_{P_i} m_{V_f}}}{m_{P_i} + m_{V_f}}. \tag{37}$$

Of course, only the product $r_1\,\eta(\vec{q}, m_f)$ is relevant, not the separate values of r_1 and $\eta(\vec{q}, m_f)$. Next, let us stress that our QMI *Ansatz*, Eqs. (22) and (25), does not mean that we believe $A_1(q^2)$ to be a constant. We are indeed sure, from its analytic properties (the axial vector current singularities in the t channel), that $A_1(q^2)$ is *not* a constant. Our *Ansatz* simply expresses that we believe $A_1(q^2)$ *to vary slowly with* q^2 in the physically relevant region. Let us now summarize a few arguments in favor of the slow variation of $A_1(q^2)$.

(i) Polelike behavior of $D \to Kl\nu$. In [13] it is argued that $f_+(q^2)$ in $D \to K^{(*)}l\nu$ decay may well be fitted by a vector meson pole, and the fitted pole mass is $M^* =$ $(2.00\pm0.11\pm0.16)$ GeV, in good agreement with the value of 2.1 GeV expected for the mass of the D_s^* meson. This fit does not establish the detailed analytic form of $f_+(q^2)$ since an exponential fit is told to agree as well. But it certainly conveys the message of a $f_+(q^2)$ increasing with q^2 as a pole term rather than, say, a dipole or a constant. Combined with Eq. (22) this q^2 dependence points toward a constant $\eta(q^2, m_K)$.

(ii) The phenomenological factorization coefficient a_2. Although there is no theoretical principle to fix a_2 in the phenomenological BSW factorization prescription, it seems reasonable that it cannot be too different from its value, $a_2^{\rm SVZ} \simeq 0.1$, in the standard SVZ factorization, Eq. (2). The results for our favored soft-scaling-pole are displayed in Table IV. It appears that the double pole assumption gives very large values for a_2, while the constant behavior for A_1 is favored as it gives the smallest a_2 (remember that this corresponds to a pole behavior of f_+). This confirms our choice (25). Still our preferred fitted a_2, ranging from 0.22 to 0.28, might be considered as rather large compared to the SVZ value.

(iii) Lattice calculations, QCD sum rules, and Orsay quark model will be discussed in Secs. IV A, IV B, and IV C, respectively.

G. Mass dependence of form factors at $q^2 = 0$

We have already noticed, Eq. (24), that our *Ansatz* (22) for the form factor ratios implies a very simple mass dependence at $q^2 = 0$. In this section we will draw the consequences of our different *Ansätze* on the mass dependence of the form factors at $q^2 = 0$.

Hard-scaling-constant with a pole for A_1. This prescription, using Eq. (33) and

$$A_1(q^2) = \frac{A_1(0)}{1 - \frac{q^2}{m^2_{B_s^{**}}}}, \tag{38}$$

TABLE IV. The fitted values of the phenomenological factorization parameter a_2 from $B \to \psi K$ branching ratios are given in column four, the one fitted from $B \to \psi K^*$ in the last column. The first two lines use BSW models. The starting point for the other lines are the soft-pole form factor ratios in Table III, both with CLEO and CDF values for R_L. Given the ratios, either we take A_1 or f_+ from $D \to K^{(*)}l\nu$ experiment. In the last two columns we report the range of fitted a_2 obtained with four different choices: A_1 or f_+ from experiment, CLEO or CDF for R_L. It appears that these ranges are narrow enough. An additional prescription is used for the A_1 dependence on q^2: pole dominance or constant. The corresponding indication is given in a transparent way in the first column.

Model	$\frac{\Gamma(K^*)}{\Gamma(K)}$	$\frac{\Gamma_L}{\Gamma_{\rm tot}}$	a_2 for K	a_2 for K^*
BSW I [5]	4.23	0.57	0.39	0.24
BSW II [6]	1.61	0.36	0.26	0.26
Soft pole: A_1 pole	4.14	0.45	0.37-0.43	0.30-0.35
Soft pole: A_1 const.	3.21	0.45	0.25-0.28	0.22-0.25

is equivalent to assuming a pole dominance for all form factors, as was done in [5, 19]. At $q^2 = 0$ one obtains

$$\frac{f_+^{sb}(0)}{f_+^{sc}(0)}, \quad \frac{V^{sb}(0)}{V^{sc}(0)},$$

$$\frac{A_2^{sb}(0)}{A_2^{sc}(0)} = \left(\frac{m_D}{m_B}\right)^{\frac{1}{2}} \left[1 + O\left(\frac{\Lambda}{m_D}\right)\right],$$

$$\frac{A_1^{sb}(0)}{A_1^{sc}(0)} = \left(\frac{m_D}{m_B}\right)^{\frac{3}{2}} \left[1 + O\left(\frac{\Lambda}{m_D}\right)\right]. \tag{39}$$

Hard-scaling-pole with a pole for A_1. This prescription starts from (32) also with (38). It yields double poles for the other form factors than A_1. It was used in [6]. At $q^2 = 0$ it gives the same ratio for all form factors:

$$\frac{f_+^{sb}(0)}{f_+^{sc}(0)}, \quad \frac{V^{sb}(0)}{V^{sc}(0)}, \quad \frac{A_1^{sb}(0)}{A_1^{sc}(0)}, \quad \frac{A_2^{sb}(0)}{A_2^{sc}(0)}$$

$$= \left(\frac{m_D}{m_B}\right)^{\frac{3}{2}} \left[1 + O\left(\frac{\Lambda}{m_D}\right)\right]. \tag{40}$$

Hard-scaling-pole with a constant for A_1. Equation (32) with (25). It gives at $q^2 = 0$ also the same ratio for all form factors, with a different power:

$$\frac{f_+^{sb}(0)}{f_+^{sc}(0)}, \quad \frac{V^{sb}(0)}{V^{sc}(0)}, \quad \frac{A_1^{sb}(0)}{A_1^{sc}(0)}, \quad \frac{A_2^{sb}(0)}{A_2^{sc}(0)}$$

$$= \left(\frac{m_D}{m_B}\right)^{\frac{1}{2}} \left[1 + O\left(\frac{\Lambda}{m_D}\right)\right]. \tag{41}$$

The three preceding cases have only an asymptotic validity, for $m_B, m_D \to \infty$. In Eqs. (32) and (33) the corrections have been retained, which explains an $O(1/m_D)$ difference between the latter and Eqs. (39)–(41). On the contrary, the next equation retains the nonasymptotic corrections.

Soft-scaling-pole with a constant for A_1. Equation (22) with (25). As we have already told, this is our preferred *Ansatz*. It gives, at $q^2 = 0$,

$$\frac{f_+^{sb}(0)}{f_+^{sc}(0)} = \left(\frac{m_B}{m_D}\right)^{\frac{1}{2}} \left(\frac{m_D + m_K}{m_B + m_K}\right),$$

$$\frac{V^{sb}(0)}{V^{sc}(0)} = \frac{A_1^{sb}(0)}{A_1^{sc}(0)} = \frac{A_2^{sb}(0)}{A_2^{sc}(0)} = \left(\frac{m_B}{m_D}\right)^{\frac{1}{2}} \left(\frac{m_D + m_{K^*}}{m_B + m_{K^*}}\right), \tag{42}$$

which reduce of course to (41) in the asymptotic regime.

The results (39)–(42) will be useful in Sec. IV to discuss the models.

IV. DISCUSSION OF THEORETICAL APPROACHES

The discussion in Sec. III provides us with some tools to look further into the theoretical schemes considered in the literature. In examining these approaches we will pay attention to two main aspects: (i) To what extent are they satisfying the asymptotic theorems, including the heavy-to-heavy scaling when both masses are heavy? (ii) Why do they fail at explaining the $B \to \psi K(K^*)$ data?

A. Lattice complemented with q^2 *Ansatz*

We have used the lattice Monte Carlo calculations of the form factors performed by the European Lattice Collaboration at $\beta = 6.4$. The details on the lattice parameters can be found in Ref. [12]. What is relevant here is that the lattice spacing is large enough to allow relatively large quark masses, from which to extrapolate up to the B meson. Reversely, the statistics are not too high, leading to large errors. For the light quark we have used the value $\kappa = 0.1495$ which happens to be very close to the "physical" strange quark: $\kappa_s = 0.1495 \pm 0.0001$ [12]. The description of the extrapolation in the heavy quark mass up to the b quark is to be found in [12]. But we have modified the extrapolation procedure in q^2. In [12] a pole dominance approximation was used for all form factors. Since we have strong reasons exposed in this paper to doubt this hypothesis, we have done the following:

The lattice calculations have been performed for the A_1 form factors at five different values of q^2. However, due to the statistical noise we have only used the three closest to $q^2_{\max}$. We then perform a two-parameter fit for A_1:

$$A_1(q^2) = a + \frac{b}{q^2 - M_p^2} \tag{43}$$

or, equivalently,

$$A_1(q^2) = \frac{aq^2 + c}{q^2 - M_p^2}, \quad c = b - aM_p^2, \tag{44}$$

where M_p is the lattice mass of the lightest t-channel axial meson pole. The justification of such a form is twofold: (i) the form factor must indeed present a pole at $q^2 = M_p^2$; (ii) the constant a mimics the subtraction constant of the dispersion relation.

To present the results of our q^2 dependence fit of $A_1(q^2)$ we will define a ratio

$$P_1 = \left(\frac{M_p^2 - q^2_{\max}}{A_1(q^2_{\max})}\right) \left.\frac{\partial A_1(q^2)}{\partial q^2}\right|_{q^2_{\max}} \tag{45}$$

such that $P_1 = 1$ in the pole dominance hypothesis. Using two different analysis methods explained in [12] we find $P_1 = 0.38 \pm 0.50$ (0.92 ± 0.41) for the "analytic" ("ratio") method. One may see an indication in the direction of a flatter behavior of A_1 than predicted by the pole dominance, but the errors prevent any firm statement.

Concerning the other form factors, A_2, f_+, V, the lattice calculations do not give a direct estimate at $q^2_{\max}$ and the same fitting procedure is not possible. We then have chosen to assume that the q^2 dependence of the ratios $A_2(q^2)/A_1(q^2)$, $f_+(q^2)/A_1(q^2)$, and $V(q^2)/A_1(q^2)$ are given by Eq. (22) or by (31), the mass dependence being fitted from the lattice as explained in [20].

Our results are reported in Table V. Clearly the errors are overwhelming for the ratio R, but the results for the ratio R_L delivers a clear message in favor of the "soft-scaling-pole" prescription for the q^2 dependence of form factors ratios. With the latter prescription, the lattice results is within 1σ from CDF, but 3σ from CLEO II.

Preliminary results from APE Lattice Collaboration [20] on $B \to K^*\gamma$ seem to indicate also an increase of $T_2(q^2)$ with q^2 much slower than expected from pole dominance; i.e., the analogous of parameter P_1 defined in Eq. (45) looks much smaller than 1. This is interesting in view of the fact that T_2 is asymptotically equal to A_1. Similarly, T_1 is asymptotically equal to V. Now, the APE Collaboration finds a much faster increase of T_1 with q^2 than T_2, as would be predicted from an extension of Eq. (29) to the heavy-to-light system.

B. QCD sum rules

There have been several studies [21–25] of the q^2 dependence of heavy-to-light form factors using different varieties of QCD sum rules: Laplace sum rules, hybrid sum rules, and light-cone sum rules. Some kind of consensus seems to have emerged, that we could characterize by saying that these authors find an agreement with vector meson dominance for vector current form factors, and a more gentle slope for A_1. Still, when one looks in more detail, the different predictions for A_1 differ somehow. Ali *et al.* [23] find a softly increasing A_1 for all q^2, while Ball [22] finds A_1 decreasing with q^2 for $q^2 \leq 15$ GeV^2. Narison [25] finds a decreasing A_1 in the $q^2 \leq 0$ region, and catches up with Ball's result. Ball finds an increasing A_2 in the same region $q^2 \leq 15$. GeV^2 where A_1 decreases, and interestingly enough, at a first glance the plots show that in this q^2 region, the ratios $A_2(q^2)/A_1(q^2)$ and $V(q^2)/A_1(q^2)$ are not very different from the "kinematical pole" term $1/[1-q^2/(m_{P_i}+m_{V_f})^2]$ [see Eq. (22)]. Unhappily, for the limited $q^2 \geq 15$ GeV^2 region, the trend is reversed and the ratio $A_2(q^2)/A_1(q^2)$ even starts decreasing. Ali *et al.* also find a $V(q^2)/A_1(q^2)$ ratio that has some analogy with the "kinematical pole" in the whole region they plot: $q^2 \leq 17$ GeV^2. Finally, comparing Belyaev *et al.* [24] to Ali *et al.* [23] we see that $f_+(q^2)/A_1(q^2)$ also increases with q^2, although maybe in a milder way than $V(q^2)/A_1(q^2)$. Finally, let us mention that Narison [25] finds asymptotically, when the mass really goes to infinity ($m_Q \gg m_b$), an analytic evidence for a pole behavior of the form factor ratios $A_2/A_1, V/A_1$, and f_+/A_1, but this happens through a polynomial decrease of A_1 and a constancy of the other form factors.

TABLE V. The results from lattice calculations at $\beta = 6.4$. The mass dependence of the form factors and the q^2 dependence of $A_1(q^2)$ has been fitted as explained in the text. The q^2 dependence of the ratios A_2/A_1, f_+/A_1, and V/A_1 have been taken according to a prescription indicated in the first column. The experimental number have been taken from CDF, those from CLEO II being indicated in parentheses.

Form factors ratios	$\frac{\Gamma(K^*)}{\Gamma(K)}$	$\frac{\Gamma_L}{\Gamma_{tot}}$
Soft pole Eq. (22)	3.5 ± 2.5	0.47 ± 0.11
Soft cons. Eq. (31)	1.9 ± 1.4	0.27 ± 0.16
Expt.	1.64 ± 0.34	$0.64 \pm 0.14\,(0.80 \pm 0.10)$

To summarize, notwithstanding sensibly different predictions for A_1, there is an almost general agreement (except for a small domain near $q^2_{\rm max}$ in [22]) on an increase of the form factor ratios, rather similar to the "kinematical pole" behavior in (22).

Using the results obtained by Ball [22,26] for $B \to \pi, \rho$, and assuming they are also valid for $B \to K^{(*)}$, we have computed the ratios R and R_L that are reported in Table I. R_L comes out rather small due to a too large value: $A_2(m^2_\psi)/A_1(m^2_\psi) \simeq 1.2$: indeed, given the value $V(m^2_\psi)/A_1(m^2_\psi) \simeq 2.8$ in [26], the constraint $R_L \geq 0.5$ translates into $A_2(m^2_\psi)/A_1(m^2_\psi) \leq 0.8$. The ratio R comes out too large due to a too small value $f_+(m^2_\psi)/A_1(m^2_\psi) \simeq 1$, while a reasonable lower bound of ~ 2 may be derived from Eq. (36). We have neglected SU(3) breaking which could change the results, but we doubt this change could be large enough to recover an agreement with $B \to K^{(*)}\psi$ data. Once more we see how difficult a challenge these data are for all known theoretical approaches.

C. Quark models

Under this heading are included very different approaches. This wide range of methods reflects primarily the inability of the simpleminded nonrelativistic model to describe the form factors: this inability consists of two main facts: (i) Away from $q^2_{\rm max}$ the model is highly ambiguous, as one soon reaches relativistic velocities; (ii) the slope ρ^2 of the Isgur-Wise function is definitely too small. Various attempts have been made to cure these defects, either by appealing to ideas connected with vector meson dominance (VMD) or with the relativistic effects [16,17]. We have restricted ourselves to the more extensively used models in literature, leaving aside very interesting ones such as the work by Jaus and Wyler [17].

As a general remark, it must be emphasized that all quark models except the OQM, Sec. IV C 6, do not satisfy the heavy-to-heavy scaling when both m_{P_i} and m_f are large, and in some cases (BSW models) violate also heavy-to-light scaling, as we shall argue in Sec. IV C 1. Also, on the empirical side, all models fail to explain $D \to K^*$ decays. The axial form factors are too large, resulting in a too large $\Gamma(D \to K^* l\nu)/\Gamma(D \to K l\nu)$ ratio. This is easily understandable by the fact that no attempt has been made to incorporate in them the binding effects which are crucial in obtaining a relative reduction of axial vector with respect to vector form factors (remember $D \to K$ is a purely vector transition). A similar situation was discussed in the past about the nucleon axial vector coupling, the G_A/G_V (sometimes noted g_1/f_1) ratio [28].

1. BSW models

The BSW models have been used extensively to analyze nonleptonic D and B decays with the help of the additional BSW factorization assumption. There are two main and very different ingredients in these models, which should not be confused.

(i) A standard quark model, which is used only at $q^2 = 0$. The values at $q^2 = 0$ are found to be approximately the same for all $c \to s$ and for all $b \to s$ form factors:

$$f_+^{sc}(0) \simeq V^{sc}(0) \simeq A_1^{sc}(0) \simeq A_2^{sc}(0) \simeq 0.8, \tag{46}$$

$$f_+^{sb}(0) \simeq V^{sb}(0) \simeq A_1^{sb}(0) \simeq A_2^{sb}(0) \simeq 0.35. \tag{47}$$

One notes that the values of the form factors V and A_2 in (46) are not consistent with what is now known from experimental D semileptonic decays (11) and (12).

From Eq. (35) in [2] one can deduce the common asymptotic behavior of all the form factors at $q^2 = 0$ as function of m_{P_i}:

$$h \propto \frac{1}{m_{P_i}}. \tag{48}$$

(ii) Different possible *Ansätze* about the q^2 dependence away from 0, which lead to two distinct phenomenological models: either a pole for each form factor (BSW I) [5] or a dipole for some and a pole for others (BSW II) [6]. These *Ansätze* are probably motivated by the feeling that naïve application of the quark model would fail, and also by the general idea of pole dominance; the latter reason is why one may speak of "hybrid" models. It must be said that in the case of BSW II, this VMD idea is in addition mixed with still another idea perhaps contradictory to it, the squaring of the pole, which is inspired from Isgur-Wise heavy-to-heavy scaling.

2. First BSW model

The BSW single pole model [5] for all the form factors has been used to analyze the nonleptonic D and B decays. Note that, independently of the precise value of the ratio between the numbers, the ratio between B and D form factors is roughly identical for all form factors. This seems hardly compatible with the relations (39), which result from the combination of hard scaling and the q^2 dependence assumed in the BSW I Model, and would imply asymptotically a different B to D ratio for f_+, V, A_2, and A_1, respectively. This suggests that the model does not satisfy the heavy-to-light scaling properties. One can indeed prove rigorously this fact in the asymptotic limit $m_{P_i} \gg m_f$, by observing that the asymptotic behavior (48) is in contradiction with the asymptotic relations (39) deduced from heavy-to-light scaling.

The BSW I model gives $R_L = 0.59$; this is not too bad, but the ratio $R = 4.23$ is much too large (see Table I). This is due to the fact that the ratio f_+/A_1 is too small as seen in Table I.

3. Second BSW or NRSX model

The pole-dipole model of Neubert, Rieckert, Stech, and Xu (NRSX) [6] uses, to our knowledge, the same values (46) and (47) at $q^2 = 0$. It obviously results that all form factors have the same ratio $0.35/0.8 = 0.44$ for their $q^2 = 0$ value at B versus D. The equality of these ratios is now in agreement with what is expected, in the $m_{P_i} \gg m_f$ limit, from heavy-to-light scaling and the assumed q^2 dependence: (40). But the value of the ratio, 0.44, is larger than expected from the same relations. Although the latter should hold only asymptotically, this suggests somewhat that the model violates heavy-to-light scaling. This can be proven rigorously in the same manner as above by noting that the asymptotic behavior (48) at $q^2 = 0$ contradicts the relations (40).

The model gives a reasonable value $R = 1.61$, but $R_L = 0.36$ is too low, which seems to have escaped notice, with the recent exception of [18]. Overall one could estimate that this model is not faring too badly. This is however obtained by form factors (Table I) rather different from the form factors we advocate by appealing to asymptotic principles (soft-scaling-pole solution in Table I): A_2/A_1 is sensibly higher, and V/A_1 is sensibly lower.

4. Altomari Wolfenstein model

We quote this model [19] as an interesting proposal although it has not been applied to the $D \to K^{(*)}l\nu$ and $B \to K^{(*)}\psi$ phenomenology. In this model one assumes that the nonrelativistic quark model is valid at $q^2_{\rm max}$, completed by a vector dominance assumption for the q^2 dependence. With such asumptions, it is easy to see that heavy-to-light scaling is satisfied asymptotically. On the other hand, it is obvious that the model has not the q^2 dependence required to satisfy heavy-to-heavy scaling away from $q^2_{\rm max}$ when both hadrons are made heavy.

5. ISGW quark model

Although one is tempted to classify it among the nonrelativistic models, it results from a modification of the NR model form factors which is no more the one predicted by the wave functions; this has then some common "hybrid" spirit with the previous A-W model. The justification given is however different: to cure the failure of the NR approximation, an *ad hoc* adjustment of the slope is made to take into account relativistic effects which are indeed expected to enlarge the slope.

More precisely this adjustment consists in making in the NR formulas the replacement

$$\vec{q}^{\,2} \to \frac{1}{\kappa^2}\frac{m_f}{m_i}(q^2_{\rm max} - q^2), \tag{49}$$

where κ^2 is a phenomenological factor $\simeq 0.5$. However we do not think that this prescription suffices to account for the variety of the expected relativistic effects that will be discussed in Sec. IV C 6.

One feature of the model is that in the transition to 0^- and 1^- all the form factors are equal to their $q^2_{\rm max}$ value times a *common* exponential function of q^2; the form factors are then easily found to respect exactly the asymptotic heavy-to-light scaling; on the contrary they obviously violate the heavy-to-heavy scaling except at

$q^2_{\rm max}$, due to an inappropriate q^2 dependence. This is bothersome since the model should apply without any change to the heavy-to-heavy case. This failure is easily understandable since the relativistic boost of spin, which is necessary to obtain the heavy-to-heavy scaling away from $q^2_{\rm max}$, is missing in the model.

The failure of the model for $\Gamma_L/\Gamma_{\rm tot}$ corresponds to the fact that A_2/A_1 is much too large (Table I). This in turn is related to the fact that the form factor ratios A_2/A_1, already too large in $D \to K^{(*)}l\nu$, is still increased up to $B \to K^{(*)}\psi$. Indeed, although this increase is softened at $q^2_{\rm max}$ by the light final masses, being independent of q^2, this ratio is not further depressed by a faster decrease of A_2 with respect to A_1 as would be obtained by our (22) *Ansatz.*

Finally, it should be noticed that in some decays where the final state has a large velocity ($B \to \pi l\nu$) there is a dramatic suppression of the form factor because of the exponential fall-off.

6. Orsay quark model for form factors

The Orsay quark model (OQM) for form factors is a semirelativistic weak-binding model [15,16] that will be described in detail in [29]. In this section, M, P, E refer to hadron masses, energies momenta, while $m, \vec{p}, \epsilon$ refer to quark masses, momenta, energies.

The model incorporates two main relativistic effects of *the center-of-mass motion*: the Lorentz contraction of the wave function and the Lorentz boost of the spinors. On the other hand, we adopt a weak-binding treatment. *One makes everywhere the approximation of retaining only linear terms in internal momenta* and one sets M_i and M_f equal to the sum of corresponding quark constituent masses. Therefore, as we explain in detail below, we do not consider it as a truly phenomenological model; it is rather an analytical instrument to discuss the specific effects of center-of-mass motion. We give now only the general principles behind it and write down the explicit form of the form factors. The total wave function writes

$$\psi^{\rm tot}_{\vec{P}}(\{\vec{p_i}\}) = \delta\left(\sum_i \vec{p_i} - \vec{P}\right)\psi_{\vec{P}}(\{\vec{p_i}\}), \tag{50}$$

where the internal wave function is given by

$$\psi_{\vec{P}}(\{\vec{p_i}\}) = N\left[\prod_i S_i(\vec{P})\right]\psi_{\mathbf{P}=0}(\{\tilde{\mathbf{p}}_i\}) \tag{51}$$

with

$$\vec{\tilde{p}}_{iT} \equiv \vec{p}_{iT},$$

$$\tilde{p}_{iz} \equiv \frac{E}{M}p_{iz} - \frac{P}{M}\varepsilon_i,$$

$$\tilde{\varepsilon}_i \equiv \frac{E}{M}\varepsilon_i - \frac{P}{M}p_{iz} \simeq m_i,$$

and where

$$S_i(\vec{P}) = \sqrt{\frac{E+M}{2M}}\left(1 + \frac{\vec{\alpha}_i\cdot\vec{P}}{E+M}\right)$$

is the Lorentz boost acting on the spinors

$$u_{\vec{P}=0} \simeq \begin{pmatrix}\chi \\ \frac{\vec{\sigma}\cdot\tilde{\mathbf{p}}}{2m}\chi\end{pmatrix}$$

the normalization being global for the internal wave function:

$$\int \psi^+_{\vec{P}}(\{\vec{p_i}\})\psi_{\vec{P}}(\{\vec{p_i}\})\delta\left(\sum_i \vec{p_i} - \vec{P}\right)\prod_i d\vec{p_i} = 2E$$

giving

$$N = \sqrt{2M}\sqrt{1-\beta^2} \quad .$$

The matrix element of an operator acting on the quark 2 will read, in the equal velocity frame (a collinear frame where the velocities are equal in magnitude and opposite in direction), after some algebra:

$$\int \psi^+_{\vec{P}_f}(\{\vec{p}\,'_i\})O(2)\psi_{\vec{P}_i}(\{\vec{p_i}\})\delta\left(\sum_i \vec{p}\,'_i - \vec{P}_f\right)\delta\left(\sum_i \vec{p_i} - \vec{P}_i\right)\prod_i d\vec{p_i}\; d\vec{p}\,'_i\delta\left(\vec{p}_1 - \vec{p}\,'_1\right)\delta\left(\vec{p}_2 - \vec{p}\,'_2 - \vec{q}\right)$$

$$= \frac{N_iN_f}{\sqrt{1-\beta^2}}\delta\left(\vec{P}_i - \vec{P}_f - \vec{q}\right)\int \psi^+_{\vec{P}_f=0}\left(\vec{p}_1 - \frac{m_1}{M_f}\tilde{\mathbf{P}}_f, \vec{p}_2 + \frac{m_1}{M_f}\tilde{\mathbf{P}}_f\right)S^+_2(\vec{P}_f)O(2)S_2(\vec{P}_i)$$
$$\times\psi_{\vec{P}_i=0}\left(\vec{p}_1 - \frac{m_1}{M_i}\tilde{\mathbf{P}}_i, \vec{p}_2 + \frac{m_1}{M_i}\tilde{\mathbf{P}}_i\right)\delta\left(\sum_i \vec{p_i}\right)\prod_i d\vec{p_i}\;. \tag{52}$$

The wave function at rest is assumed to be given by the harmonic oscillator potential.

With these ingredients one can compute all the form factors we are interested in. Calling M_i, M_f the initial and final hadron masses, m_i, m_f the initial and final active quark masses, we find

$$f_+(q^2) = \frac{\sqrt{4M_iM_f}}{M_i+M_f}\frac{1}{1-\frac{q^2}{(M_i+M_f)^2}}I(q^2)\left[1 + \frac{M_i - M_f}{M_i+M_f}X\right], \tag{53}$$

$$V(q^2) = \frac{\sqrt{4M_iM_f}}{M_i + M_f} \frac{1}{1 - \frac{q^2}{(M_i+M_f)^2}} I(q^2)(1+Y), \tag{54}$$

$$A_1(q^2) = \frac{\sqrt{4M_iM_f}}{M_i + M_f} I(q^2) \left[1 + \frac{(M_f - M_i)^2 - q^2}{(M_f + M_i)^2 - q^2} Y \right], \tag{55}$$

$$A_2(q^2) = \frac{\sqrt{4M_iM_f}}{M_i + M_f} \frac{1}{1 - \frac{q^2}{(M_i+M_f)^2}} I(q^2) \left[1 + \frac{\left(M_i^2 - M_f^2\right) - q^2}{(M_i + M_f)^2 - q^2} Y - \frac{2M_f\,(M_i + M_f)}{(M_f + M_i)^2 - q^2} X, \right] \tag{56}$$

where, for the harmonic oscillator potential,

$$I(q^2) = \left(\frac{2R_iR_f}{R_f^2 + R_i^2} \right)^{\frac{3}{2}} \exp\left(- \frac{2m^2R_i^2R_f^2}{R_f^2 + R_i^2} \left[\frac{(M_f - M_i)^2 - q^2}{(M_f + M_i)^2 - q^2} \right] \right) \tag{57}$$

and

$$X = -\frac{m}{R_f^2 + R_i^2} \left(\frac{R_f^2}{m_i} - \frac{R_i^2}{m_f} \right), \quad Y = \frac{m}{R_f^2 + R_i^2} \left(\frac{R_f^2}{m_i} + \frac{R_i^2}{m_f} \right), \tag{58}$$

parametrize corrections to the scaling limit, proportional to the spectator quark mass m, and R_i and R_f are the radii of the initial and final mesons.

Isgur-Wise scaling. It is important to realize that, with such a model, in the limit where both m_i and m_f are made heavy, *one obtains exactly the whole set of scaling relations of Isgur and Wise*, Eq. (17). The scaling function $\xi(v_i \cdot v_f)$ depends of course on the potential except for the relation $\xi(1) = 1$. In the case of the harmonic oscillator potential,

$$\xi(v_i \cdot v_f) = \frac{2}{1 + v_i \cdot v_f} \exp\left[-\frac{m^2R^2}{\sqrt{2}} \left(\frac{v_i \cdot v_f - 1}{v_i \cdot v_f + 1} \right) \right].$$

The corresponding slope at the origin, within the weak binding and linear approximation, is

$$\rho^2 = -\xi'(1) = \frac{1}{2} + \frac{m^2R^2}{2\sqrt{2}} \sim 0.9,$$

where R is the radius of a light-light meson.

q^2 dependence. The expressions for the form factors above show that the q^2 dependence of $A_1(q^2)$ is very weak especially near $q^2 = 0$ and that the q^2 dependence of the form factors $f_+(q^2)$, $V(q^2)$, and $A_2(q^2)$ is dominated by the same kinematic pole that appears in the Isgur-Wise relations in the heavy-heavy case (3). In the model this kinematic pole comes simply from the Lorentz factor

$$1 - \beta^2 = \frac{4M_iM_f}{(M_i + M_f)^2} \frac{1}{1 - \frac{q^2}{(M_i+M_f)^2}}$$

that does not affect $A_1(q^2)$ because this form factor is related to a purely transverse component of the axial vector current. The q^2 dependence of A_1 comes essentially from the exponential and it becomes rather weak when $\vec{q}^{\,2}$ is large, i.e., near $q^2 = 0$; indeed it tends to a constant. This would be true for any potential. It is a simple consequence of the Lorentz contraction.

This model's prediction is therefore similar to the QMI *Ansatz*, Eqs. (22) and (25). The q^2 dependence of form factor ratios is almost the same in both models.

Corrections to scaling. For finite m_i, m_f, the heavy-to-heavy scaling laws are broken in OQM by various effects: (i) the radii in $I(q^2)$ depend on the flavor; (ii) the terms containing X and Y are of order $m/m_{i,f}$. The scaling is broken because the spectator mass is no longer negligible.

On the whole, these scaling violations are rather small except for f_+ and especially for V. That the latter is large is easily understandable even in the most naive nonrelativistic approximation. Indeed, the $1 + Y$ coefficient in (54) varies from 2 for $m_i = m_f = m$ to 1 when $m_i, m_f \gg m$.

Phenomenological shortcomings. Although the model is unique in obeying the full set of asymptotic scaling laws in contrast to previous models, it is not a satisfactory phenomenological model, because it lacks essential effects. First, due to the weak-binding approximation, the axial to vector matrix elements at $q^2_{\rm max}$ are reduced to their static SU(6) value. For instance, one finds in this model for the nucleon axial to vector current coupling ratio: $G_A/G_V = 5/3$ (sometimes written $-g_1/f_1$) [28], which is too large by $\sim \sqrt{2}$. The same thing happens in $D \to K^* l\nu$ and could also explain why $\Gamma(D \to K^* l\nu)/\Gamma(D \to K l\nu)$ is predicted too large by a factor 2 in most quark models. Indeed, the axial form factors A_1, A_2 are found too large with respect to the vector ones. In the OQM,

$$A_1^{sc}(0) = 0.89, \quad A_2^{sc}(0) = 0.87,$$

$$V^{sc}(0) = 1.15, \quad f_+^{sc}(0) = 0.74, \quad \frac{\Gamma_L}{\Gamma_{\rm tot}} = 0.41,$$

$$\frac{\Gamma(B \to K^*)}{\Gamma(B \to K)} = 4.75. \tag{59}$$

This is quite bad. To cure the problem with $G_A/G_V = 5/3$ we have adopted in the past the old recipe of multiplying the axial current by an *ad hoc* factor $g_A = 0.7$ [30]. This gives, multiplying all axial vector form factors by g_A,

$$\frac{\Gamma_L}{\Gamma_{\rm tot}} = 0.34, \quad \frac{\Gamma(B \to K^*)}{\Gamma(B \to K)} = 2.8 \tag{60}$$

Of course this is still unsatisfactory. In addition, such a recipe has no theoretical grounding. One should systematically include the binding corrections, which are known to correct the discrepancy for G_A/G_V [31,32].

D. The phenomenological analysis by Gourdin, Kamal, and Pham

While we were in the process of writing this paper we received a paper by Gourdin, Kamal, and Pham [18] which also studied the relation between $B \to K^{(*)}\psi$ and $D \to K^{(*)}l\nu$ experiments and also confronted some theoretical approaches with $B \to K^{(*)}\psi$ experiments. We agree with these authors on their main conclusion that the current models do not fit the $B \to K^{(*)}\psi$ data. We differ with them in two respects. They have used all over the pole dominance for the q^2 dependence of all form factors, while we conclude to a different q^2 behavior of the different form factors. Second, their Eq. (29) is a different *Ansatz* than ours. They appeal to a direct extrapolation of heavy-to-light scaling rules, which they obtain by assuming that all $1/m_b, 1/m_c$ corrections to Eq. (11) in Ref. [14] are negligible. Note that this equation corresponds to a different choice and to different combinations of form factors than our Eq. (13). It happens that their recipe translated in the language of (13) amounts to a softening, as does our assumption (22) inspired from heavy-to-heavy scaling. Finally we agree with the claim by these authors that $D \to K^{(*)}l\nu$ and $B \to K^{(*)}\psi$ data are difficult to reconcile within the heavy-to-light scaling laws. This difficulty, beyond the $\Gamma_L/\Gamma_{\rm tot}$ problem stressed by the authors, shows up in $\Gamma(K^*)/\Gamma(K) = 2.86$ predicted from their Eq. (33).

V. CONCLUSIONS

All the approaches we have considered in this paper encounter difficulties in accounting for the $B \to K^{(*)}\psi$ data, particularly with the large $\Gamma_L/\Gamma_{\rm tot}$ (CLEO and ARGUS data). At present it seems safer to keep open three possibilities to get out of this problem.

Experiment may not have yet delivered its ultimate word, as the variation between different experiments seem to indicate, and it might evolve toward data easier to account for.

Although we did not discuss the factorization assumption, it should be kept in mind that it rests on no theoretical ground for color-suppressed decay channels, as is the case for $B \to K^{(*)}\psi$.

Finally, models may be wrong. This will now be discussed in more detail.

Of course, the first requirement for any model is to satisfy the heavy-to-light scaling relations. This has been seen not to be the case for the most popular BSW I and BSW II models, notwithstanding their relatively good empirical successes in other areas.

Our analysis has allowed us to extricate from data some general trends, namely, "softened" scaling, a sensibly different q^2 behavior of A_1 versus A_2, V, f_+, and A_1 slowly varying with q^2. *Ansätze* that take these indications as a guide, obtain better values for $\Gamma_L/\Gamma_{\rm tot}$, and a more reasonable a_2, although there remains a general tendency to underestimate $\Gamma_L/\Gamma_{\rm tot}$ with respect to present data. Let us now comment on these general trends.

Data definitely exclude "hard scaling," i.e., the strict application of asymptotic heavy-to-light scaling formulas in the finite mass domain. We have proposed a "softened" *Ansatz* which is based on an extension of heavy-to-heavy scaling relations down to the light final meson case, with some rescaling. In fact this is equivalent to assuming a precocious scaling for the axial vector and vector current *matrix elements*. Consequently, the ratio A_2/A_1 does not increase too fast with the heavy mass.

There are indications from lattice calculations, from quark models, and to some degree from phenomenology, that V should undergo an even softer scaling.

Another consequence of the above *Ansatz*, as well as of the Orsay quark model is that A_2/A_1, V/A_1, and f_+/A_1 should have a polelike behavior in q^2, leading to an increase with q^2. This improves the agreement with $B \to K^{(*)}\psi$ data, and seems to be corroborated by QCD sum rules calculations.

$D \to Kl\nu$ experiments seem to show a polelike behavior for $f_+(q^2)$. Combined with our preceding *Ansatz* for the ratios, this implies an approximately constant $A_1(q^2)$. This particular q^2 behavior is corroborated by the Orsay quark model, while QCD sum rules give q^2 dependence of A_1 that never increases very fast, although different detailed shapes are proposed. Lattice calculations, within large errors, might give the same indication.

Note added in proof. In [33] the authors find a large value $\Gamma_L/T_{\rm tot} \simeq 0.63$ by using $A_2 = 0$ as indicated by the oldest $D \to K^*$ data, which is nowadays excluded by recent and more accurate experiments.

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