Vector dominance effects in weak radiative decays of B mesons

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The long-distance vector-meson-dominance (VMD) effects on the weak radiative decays $\bar{B} \to \rho\gamma$ and $\bar{B}^0 \to D^{*0}\gamma$ are studied. For $\bar{B} \to \rho\gamma$ decays, the VMD contribution is (10–20)% of the short-distance penguin amplitude. The pole effect is as important as the VMD one in the decay $B^- \to \rho^-\gamma$, but is suppressed in $\bar{B}^0 \to \rho^0\gamma$. The branching ratio of $\bar{B} \to \rho\gamma$, estimated to be of order 10^{-6}, strongly depends on the sign of the Wolfenstein parameter ρ. A measurement of any deviation of the ratio $R = \Gamma(B^- \to \rho^-\gamma)/\Gamma(\bar{B}^0 \to \rho^0\gamma)$ away from the isospin value 2 will provide a probe on the long-range contribution and possibly indicate the sign of ρ: $R > 2$ for $\rho < 0$ and $R \sim 2$ for $\rho > 0$. The decay $\bar{B}^0 \to D^{*0}\gamma$ does not receive short-distance contributions, and its branching ratio, predicted to be 0.9×10^{-6}, is dominated by W exchange accompanied by a photon emission.

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I. INTRODUCTION

Recently the weak radiative decays of B mesons and bottom baryons have been systematically studied in [1]. At the quark level, there are two essential mechanisms responsible for weak radiative decays: the electromagnetic penguin mechanism and W-exchange (or W-annihilation) bremsstrahlung. The two-body decays of the B meson proceeding through the short-distance electromagnetic penguin diagrams are

$$\begin{aligned} &b \to s\gamma \Rightarrow \bar{B} \to \bar{K}^*\gamma, \bar{B}_s \to \phi\gamma, \\ &b \to d\gamma \Rightarrow \bar{B} \to \rho\gamma, \bar{B}^0 \to \omega\gamma, \bar{B}_s \to K^{*0}\gamma \ , \end{aligned} \tag{1}$$

while the decay modes occurring through W exchange of W annihilation accompanied by a photon emission are

$$W \text{ exchange}: \begin{cases} b\bar{d} \to c\bar{u}\gamma \Rightarrow \bar{B}^0 \to D^{*0}\gamma, \\ b\bar{s} \to c\bar{u}\gamma \Rightarrow \bar{B}_s \to D^{*0}\gamma, \quad b\bar{d} \to c\bar{c}\gamma \Rightarrow \bar{B}^0 \to J/\psi\gamma \end{cases} ,$$

$$W \text{ annihilation}: \quad b\bar{u} \to s\bar{c}\gamma \Rightarrow B^- \to D_s^{*-}\gamma, \quad b\bar{u} \to d\bar{c}\gamma \Rightarrow B^- \to D^{*-}\gamma \ . \tag{2}$$

Note that decay modes in (1) also receive contributions from W-exchange or W-annihilation bremsstrahlung, but they are in general quark mixing suppressed.

At the hadronic level, the W-exchange diagrams manifest as long-distance pole diagrams. However, another possible long-distance effect, namely the vector-meson-dominance (VMD) contribution, was advocated some time ago by Golowich and Pakvasa [2]. For example, $B \to K^*\gamma$ can proceed through $B \to K^*J/\psi \to K^*\gamma$ via $J/\psi - \gamma$ conversion. Since the concept of VMD, though useful, has never been derived from the standard model, it is not clear at all if this VMD contribution to $B \to V\gamma$ is a real one. In fact, it has been argued that at the quark level $b \to sJ/\psi \to s\gamma$ is not allowed at the tree level because of gauge invariance [3]. It is also easily seen at the hadronic level that for a given $B \to VV'$ amplitude with V' being a neutral vector meson, it is no longer gauge invariant after a replacement of the polarization vector $\varepsilon_\mu(V')$ of the vector meson V' by the photon one $\varepsilon_\mu(\gamma)$. This is ascribed to the fact that, as elaborated on in [4,5], the helicity amplitude of $B \to VV'$ has a longitudinal component that spoils gauge invariance after V'-γ conversion. Therefore, in order to retain gauge invariance, one must disregard the longitudinal helicity amplitude of $B \to VV'$ for a correct usage of VMD [4,5].

In the present paper we will assume the validity of VMD and estimate its effect on weak radiative decays. To be specific, we will consider two representative decay modes in (1) and (2): $\bar{B} \to \rho\gamma$ and $\bar{B}^0 \to D^{*0}\gamma$. A generalization of the present work to other radiative decays is straightforward.

II. THE $\bar{B} \to \rho\gamma$ DECAY

The radiative decay $\bar{B} \to \rho\gamma$ is of experimental and theoretical interest since we may learn the quark mixing matrix element V_{td} from its measurement [6]. This decay resembles $B \to K^*\gamma$ in many ways. It is well known that the latter is dominated by the short-distance electromagnetic penguin mechanism $b \to s\gamma$. There are two possible long-distance effects: VMD and W exchange bremsstrahlung; the latter manifested as a long-distance pole contribution at the hadronic level. A recent estimate gives [4]

$$\begin{aligned} &\left|\frac{A_{\rm VMD}}{A_{\rm expt}}\right|_{B \to K^*\gamma} \leq 0.1 \ , \\ &\left|\frac{A_{\rm pole}}{A_{\rm expt}}\right|_{B \to K^*\gamma} \simeq 0.01 \ . \end{aligned} \tag{3}$$

The long-distance contribution is thus dominated by the VMD effect, arising mainly from the process $B \to K^* J/\psi \to K^*\gamma$. The pole contribution is suppressed due to the smallness of the weak mixing $V_{ub}V_{us}$. Apart from the mixing angles, the decay $\bar{B} \to \rho\gamma$ proceeds in the same way as $B \to K^*\gamma$. In this section, we will estimate the short- and long-distance contributions to $\bar{B} \to \rho\gamma$ and see if the pattern (3) is still respected. An estimate of the long-distance effect on $\bar{B} \to \rho\gamma$ was recently made in [7]. We will present in this paper a more quantitative study.

The general amplitude of weak radiative decay with one real photon emission is given by

$$A[\bar{B}(p) \to P^*(q)\gamma(k)] = i\epsilon_{\mu\nu\alpha\beta}\varepsilon^{\mu}k^{\nu}\varepsilon^{*\alpha}q^{\beta}f_1(k^2) + \varepsilon^{\mu}[\varepsilon^*_{\mu}(m_B^2 - m_{P*}^2) - (p+q)_{\mu}\varepsilon^*\cdot k]f_2(k^2)\ , \qquad (4)$$

where ε and ε^* are the polarization vectors of the photon and the vector meson P^*, respectively, the first (second) term on the right-hand side (RHS) is parity conserving (violating), and $k^2 = 0$. The decay width implied by the amplitude (4) is

$$\Gamma(\bar{B} \to P^*\gamma) = \frac{1}{32\pi}\frac{(m_B^2 - m_{P^*}^2)^3}{m_B^3}\left[|f_1(0)|^2 + 4|f_2(0)|^2\right]\ . \qquad (5)$$

To begin with, we consider the transition amplitude induced by the short-distance penguin amplitude $b \to d\gamma$:

$$A(b \to d\gamma) = i\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}\left(\sum_i F_2(x_i)V_{ib}V_{id}^*\right)\varepsilon^{\mu}k^{\nu}\bar{d}\sigma_{\mu\nu}[m_b(1+\gamma_5) + m_d(1-\gamma_5)]b\ , \qquad (6)$$

where $x_i = m_i^2/M_W^2$, m_i is the mass of the quark i, and F_2 is a smooth function of x_i [8]. In the static limit of the heavy b quark, we may use the equation of motion $\gamma_0 b = b$ to derive the relation [9]

$$\langle\rho|\bar{d}i\sigma_{0i}(1\pm\gamma_5)b|B\rangle = \langle\rho|\bar{d}\gamma_i(1\mp\gamma_5)b|B\rangle\ . \qquad (7)$$

As a result, the form factors f_1 and f_2 in (4) can be related to the vector and axial-vector form factors V and A_1 appearing in the matrix element on the RHS of Eq. (7) defined by [10]

$$\langle\rho(p_\rho)|\bar{d}\gamma_\mu b|B(p_B)\rangle = \frac{2i}{m_B + m_\rho}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_\rho^{\alpha}p_B^{\beta} \times V^{B\rho}(q^2)\ ,$$

$$\langle\rho(p_\rho)|\bar{d}\gamma_\mu\gamma_5 b|B(p_B)\rangle = (m_B+m_\rho)\varepsilon^*_\mu A_1^{B\rho}(q^2) - \frac{\varepsilon^*\cdot q}{m_B+m_\rho}(p_B+p_\rho)_\mu A_2^{B\rho}(q^2) - \frac{2\varepsilon^*\cdot q}{q^2}q_\mu m_\rho[A_3^{B\rho}(q^2) - A_0^{B\rho}(q^2)]\ , \qquad (8)$$

with $q = p_B - p_\rho$. At $k^2 = 0$, we obtain (see, e.g., [11])

$$f_1^{\rm peng}(B^- \to \rho^-\gamma) = -\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}\left(\sum_i F_2(x_i)V_{ib}V_{id}^*\right)m_b F^{B\rho}(0)\ ,$$

$$f_2^{\rm peng}(B^- \to \rho^-\gamma) = -\tfrac{1}{2}f_1^{\rm peng}(B^- \to \rho^-\gamma)\ , \qquad (9)$$

and

$$f_i^{\rm peng}(\bar{B}^0 \to \rho^0\gamma) = -\frac{1}{\sqrt{2}}f_i^{\rm peng}(B^- \to \rho^-\gamma)\ , \qquad (10)$$

where

$$B^{B\rho}(0) = \frac{m_B - m_\rho}{m_B}V^{B\rho}(0) + \frac{m_B + m_\rho}{m_B}A_1^{B\rho}(0)\ . \qquad (11)$$

Two remarks are in order. (i) Equation (9) is subject to $O(1/m_b)$ corrections which are not included here. (ii) Apart from the quark mixing angles, the short-distance $\bar{B} \to \rho\gamma$ amplitude is different from the $B \to K^*\gamma$ one in that the u quark loop contribution is negligible in the latter but not necessarily so in the former. To be precise, the $B \to K^*\gamma$ amplitude is given by

$$f_1^{\rm peng}(B \to K^*\gamma) \cong -\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}F_2(x_t)V_{tb}V_{ts}^* m_b F^{BK^*}(0)\ ,$$

$$f_2^{\rm peng}(B \to K^*\gamma) = -\tfrac{1}{2}f_1^{\rm peng}(B \to K^*\gamma)\ , \qquad (12)$$

where uses of the approximations $F_2(x_t) - F_2(x_c) \cong F_2(x_t)$ and $V_{cb}V_{cs}^* \approx -V_{tb}V_{ts}^*$ due to the smallness of $V_{ub}V_{us}^*$ have been made. Numerically, $F_2(x_t) = 0.65$ for $\Lambda_{\rm QCD} = 200$ MeV and $m_t = 174$ GeV. It follows from (9)–(12) that the short-distance $\bar{B} \to \rho\gamma$ and $B \to K^*\gamma$ amplitudes are related by

$$f_i^{\rm peng}(B^- \to \rho^-\gamma) = \frac{V_{td}^*}{V_{ts}^*}(1+\Delta)\frac{F^{B\rho}(0)}{F^{BK^*}(0)}f_i^{\rm peng}(B \to K^*\gamma)\ , \qquad (13)$$

with

$$\Delta = \frac{F_2(x_u) - F_2(x_c)}{F_2(x_t) - F_2(x_c)}\frac{V_{ub}}{V_{td}^*}\ . \qquad (14)$$

For later purposes of numerical estimate, we will follow [7] to take $[F_2(x_u)-F_2(x_c)]/[F_2(x_t)-F_2(x_c)] \simeq -0.30$.

We next turn to long-distance contributions and first focus on the VMD part. The transitions $\bar{B}\to\rho V$ followed by V-γ conversion are dominated by the virtual vector mesons $V = J/\psi$, ψ', ρ^0, and ω as depicted in Figs. 1 and 2. To illustrate the use of VMD let us consider the hadronic decay $B^-\to\rho^- J/\psi$ as an example. Assuming factorization, its amplitude reads

$$A(B^-\to\rho^- J/\psi) = \frac{G_F}{\sqrt{2}}V_{cb}V_{cd}^* a_2 \varepsilon^\mu(J/\psi)\varepsilon^{*\nu}(\rho) \times\Big(\hat{A}_1(m_{J/\psi}^2)g_{\mu\nu} + \hat{A}_2(m_{J/\psi}^2)p_\mu^B p_\nu^B + i\hat{V}(m_{J/\psi}^2)\epsilon_{\mu\nu\alpha\beta}p_\rho^\alpha p_B^\beta\Big)\,, \tag{15}$$

where

$$\begin{aligned}\hat{A}_1(m_{J/\psi}^2) &= -(m_B+m_\rho)f_{J/\psi}m_{J/\psi}A_1^{B\rho}(m_{J/\psi}^2)\,,\\ \hat{A}_2(m_{J/\psi}^2) &= \frac{2}{m_B+m_\rho}f_{J/\psi}m_{J/\psi}A_2^{B\rho}(m_{J/\psi}^2)\,,\\ \hat{V}(m_{J/\psi}^2) &= \frac{2}{m_B+m_\rho}f_{J/\psi}m_{J/\psi}V^{B\rho}(m_{J/\psi}^2)\,,\end{aligned} \tag{16}$$

and a_2 is a parameter introduced in [12] for the internal W-emission diagram. VMD implies that a possible contribution to $B^-\to\rho^-\gamma$ comes from the decay $B^-\to\rho^- J/\psi$ followed by continuing its amplitude from $p_{J/\psi}^2 = m_{J/\psi}^2$ to $p_{J/\psi}^2 = 0$ and replacing the vector-meson polarization vector $\varepsilon_\mu(J/\psi)$ by the photon one:

$$\varepsilon_\mu(V) \to \frac{e}{\tilde{g}_{\gamma V}}\varepsilon_\mu(\gamma)\,, \tag{17}$$

where $\tilde{g}_{\gamma V}$ is an off-shell extension of the dimensionless quantity $g_{\gamma V}$ defined at $p_V^2 = m_V^2$:

$$\langle 0|J_\mu^{\rm em}|V\rangle = \frac{m_V^2}{g_{\gamma V}}\varepsilon_\mu\,. \tag{18}$$

However we will neglect the difference between $\tilde{g}_{\gamma V}$ and $g_{\gamma V}$ in what follows (for a discussion, see [4]). In order to retain gauge invariance of the $B^-\to\rho^-\gamma$ amplitude, it becomes necessary to demand a vanishing $A(B^-\to\rho^-\gamma)_{\rm VMD}$ when $\varepsilon_\mu(\gamma)\to k_\mu$. This is equivalent to discarding the longitudinal polarization component of the $B^-\to\rho^- J/\psi$ amplitude in the $p_{J/\psi}^2\to 0$ limit [4,5];[1] that is,

$$\hat{A}_1(0) + (p_B\cdot k)\hat{A}_2(0) = \hat{A}_1(0) + \tfrac{1}{2}(m_B^2-m_\rho^2)\hat{A}_2(0) = 0\,. \tag{19}$$

Continuing the amplitude (15) from $p_{J/\psi}^2 = m_{J/\psi}^2$ to $p_{J/\psi}^2 = 0$ and substituting (19) into (15) yields

$$A(B^-\to\rho^- J/\psi\to\rho^-\gamma) = \frac{e}{g_{\gamma J/\psi}}\frac{G_F}{\sqrt{2}}V_{cb}V_{cd}^*\Big\{ i\epsilon_{\mu\nu\alpha\beta}\varepsilon^\mu k^\nu\varepsilon^{*\alpha}p_\rho^\beta\hat{V}(0) - \tfrac{1}{2}\varepsilon^\mu\Big[\varepsilon_\mu^*(m_B^2-m_\rho^2) - (p_B+p_\rho)_\mu\varepsilon^*\cdot k\Big]\hat{A}_2(0)\Big\}\,. \tag{20}$$

Comparing (20) with (4), assuming SU(3) flavor symmetry for heavy-light form factors and summing over the intermediate vector meson states gives rise to[2] (see Fig. 1)

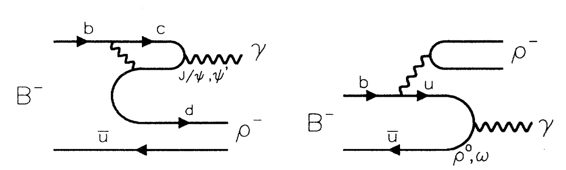

FIG. 1. VMD processes contributing to $B^-\to\rho^-\gamma$ with the vector-meson intermediate states J/ψ, ψ', ρ^0, and ω.

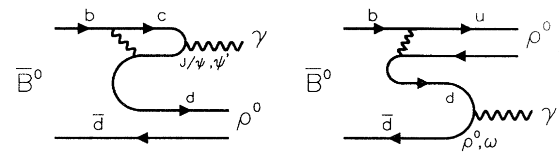

FIG. 2. Same as Fig. 1 except for $\bar{B}^0\to\rho^0\gamma$.

[1]For the process such as $B\to K^* J/\psi\to K^*\gamma$, one may employ the experimental measurement of the transverse polarization component of $B\to K^* J/\psi$ to compute the VMD contribution to $B\to K^*\gamma$. In the absence of experimental information for $\bar{B}\to\rho J/\psi$, etc., we have to appeal to some model calculations for evaluating the VMD $\bar{B}\to\rho\gamma$ amplitude, as we have done here.

[2]The isospin of the final state in the decay $B\to\rho\rho$ is either $I=0$ or 2. Consequently, the p-wave amplitude of $\bar{B}^0\to\rho^0\rho^0$ and $B^-\to\rho^-\rho^0$ vanish when ρ's are on shell. In the factorization calculation, this constraint is satisfied only if Bose symmetry is imposed by hand. Since in our case one of the neutral ρ's is off shell, it is not clear to us how to implement Bose symmetry properly.

$$
\begin{aligned}
f_1^{\mathrm{VMD}}(B^- \to \rho^- \gamma) = eG_F \Bigg\{ & \sqrt{2} V_{cb} V_{cd}^* a_2 \frac{1}{m_B + m_\rho} \left(\frac{f_{J/\psi} m_{J/\psi}}{g_{\gamma J/\psi}} + \frac{f_{\psi'} m_{\psi'}}{g_{\gamma\psi'}} \right) \\
& + V_{ub} V_{ud}^* a_1 f_\rho m_\rho \left(\frac{1}{m_B + m_\rho} \frac{1}{g_{\gamma\rho}} + \frac{1}{m_B + m_\omega} \frac{1}{g_{\gamma\omega}} \right) \Bigg\} V^{B\rho}(0) , \\
f_2^{\mathrm{VMD}}(B^- \to \rho^- \gamma) = & -\frac{1}{2} \frac{A_2^{B\rho}(0)}{V^{B\rho}(0)} f_1^{\mathrm{VMD}}(B^- \to \rho^- \gamma) ,
\end{aligned} \tag{21}
$$

where a_1 is a parameter introduced for the external W-emission diagram [12], and the relative sign between ρ^0- and ω-mediated VMD amplitudes is fixed by the wave functions $\rho^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$ and $\omega = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$. Likewise, for the $\bar{B}^0 \to \rho^0\gamma$ decay (see Fig. 2),

$$
\begin{aligned}
f_1^{\mathrm{VMD}}(\bar{B}^0 \to \rho^0 \gamma) = -\frac{1}{\sqrt{2}} eG_F \Bigg\{ & \sqrt{2} V_{cb} V_{cd}^* a_2 \frac{1}{m_B + m_\rho} \left(\frac{f_{J/\psi} m_{J/\psi}}{g_{\gamma J/\psi}} + \frac{f_{\psi'} m_{\psi'}}{g_{\gamma\psi'}} \right) \\
& + V_{ub} V_{ud}^* a_2 f_\rho m_\rho \left(\frac{1}{m_B + m_\rho} \frac{1}{g_{\gamma\rho}} - \frac{1}{m_B + m_\omega} \frac{1}{g_{\gamma\omega}} \right) \Bigg\} V^{B\rho}(0) , \\
f_2^{\mathrm{VMD}}(\bar{B}^0 \to \rho^0 \gamma) = & -\frac{1}{2} \frac{A_2^{B\rho}(0)}{V^{B\rho}(0)} f_1^{\mathrm{VMD}}(\bar{B}^0 \to \rho^0 \gamma) .
\end{aligned} \tag{22}
$$

Note that the isospin relation for the decay rates $\Gamma(\bar{B}^0 \to \rho^0\gamma) = \frac{1}{2}\Gamma(B^- \to \rho^-\gamma)$, respected by the short-distance penguin interaction [see Eq. (10)], is no longer satisfied by the VMD contributions arising from ρ^0 and ω intermediate states as the decays $\bar{B}^0 \to \rho^0\rho^0$, $\rho^0\omega$ are color suppressed, while $B^- \to \rho^-\rho^0$, $\rho^-\omega$ are not.

We now come back to the coupling $g_{\gamma V}$ defined in Eq. (18). In the quark model, $(g_{\gamma V})^{-1}$ is proportional to $\sum_i a_i e_i$ with a_i being the coefficient of the ith quark with charge e_i in the wave function. Consequently, it is expected that

$$g_{\gamma\rho}^{-1} : g_{\gamma\omega}^{-1} : g_{\gamma\phi}^{-1} = 3 : 1 : -\sqrt{2} . \tag{23}$$

Experimentally, $g_{\gamma V}$ can be determined from the measured $V \to l^+l^-$ rate:

$$\Gamma(V \to \ell^+\ell^-) = \frac{4\pi\alpha^2}{3} \frac{m_V}{g_{\gamma V}^2} \left(1 - 4\frac{m_\ell^2}{m_V^2}\right)^{1/2} \left(1 + 2\frac{m_\ell^2}{m_V^2}\right) . \tag{24}$$

From the measured widths [13] we obtain

$$
\begin{aligned}
g_{\gamma\rho} &= 5.05 , \\
g_{\gamma\omega} &= 17.02 , \\
g_{\gamma\phi} &= -12.89 , \\
g_{\gamma J/\psi} &= 11.75 , \\
g_{\gamma\psi'} &= 18.87 ,
\end{aligned} \tag{25}
$$

where we have applied the quark model to fix the sign. Therefore, the relation (23) is satisfied experimentally. The vector-meson decay constant f_V is related to $g_{\gamma V}$ via the relation

$$f_V = m_V \left(g_{\gamma V} \sum_i a_i e_i \right)^{-1} . \tag{26}$$

It follows that the decay constants relevant to our purposes are[3]

$$f_\rho = 216 \text{ MeV}, \quad f_{J/\psi} = 395 \text{ MeV}, \quad f_{\psi'} = 293 \text{ MeV} . \tag{27}$$

Another long-distance contribution to $\bar{B} \to \rho\gamma$ stems from the W-annihilation diagram for $B^- \to \rho^-\gamma$ and the W-exchange diagram for $\bar{B}^0 \to \rho^0\gamma$ (see Fig. 3).[4] Using the formulism developed in Sec. II of [1], the pole contributions are found to be[5]

[3] To determine $f_{J/\psi}$ and $f_{\psi'}$ we have taken into account the momentum dependence of the fine-structure constant.

[4] Contrary to [7], we count the VMD and pole effects as two different long-range contributions since they proceed through different quark diagrams as depicted in Figs. 1–3. As we shall see from Table II, while VMD and pole contributions to $B^- \to \rho^-\gamma$ are comparable, they are different by one order of magnitude in the $\bar{B}^0 \to \rho^0\gamma$ decay amplitude.

[5] Strictly speaking, the formulism developed in Sec. II of [1] is applicable only if both initial and final hadrons can be treated as heavy. Nevertheless, we believe that (28) and (29) here are good for an order-of-magnitude estimate. Basically, our approach is similar to the second method advocated in [7]. As stressed in the Introduction, W-exchange (or W-annihilation) contributions manifest as pole diagrams at the hadronic level. This equivalence has been demonstrated explicitly for $\bar{B}^0 \to D^{*0}\gamma$ in [1].

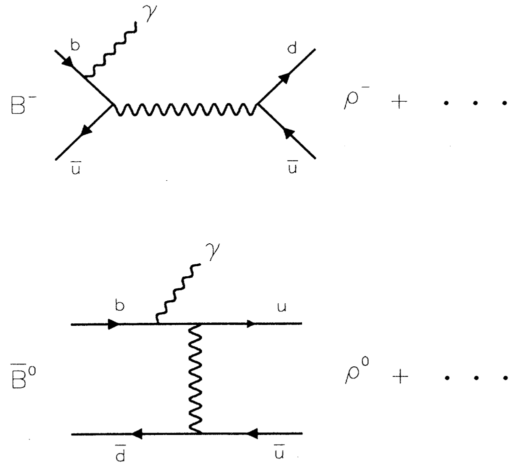

FIG. 3. W-annihilation diagram contributing to $B^- \to \rho^-\gamma$ and W exchange to $\bar{B}^0 \to \rho^0\gamma$. Contributions due to photon emission from other quarks are denoted by ellipses.

$$
\begin{aligned}
f_1^{\rm pole}(B^- \to \rho^-\gamma) &= \kappa a_1 \left[\left(\frac{e_d}{m_d}+\frac{e_u}{m_u}\right)\frac{m_\rho}{m_B} + \left(\frac{e_u}{m_u}+\frac{e_b}{m_b}\right)\right]\frac{m_B m_\rho}{m_B^2-m_\rho^2}\,, \\
f_2^{\rm pole}(B^- \to \rho^-\gamma) &= -\frac{1}{2}\kappa a_1 \left[\left(\frac{e_d}{m_d}-\frac{e_u}{m_u}\right)\frac{m_\rho}{m_B} + \left(\frac{e_u}{m_u}-\frac{e_b}{m_b}\right)\right]\frac{m_B m_\rho}{m_B^2-m_\rho^2}\,,
\end{aligned}
\tag{28}
$$

and

$$
\begin{aligned}
f_1^{\rm pole}(\bar{B}^0 \to \rho^0\gamma) &= \frac{1}{\sqrt{2}}\kappa a_2 \left[2\frac{e_u}{m_u}\frac{m_\rho}{m_B} + \left(\frac{e_d}{m_d}+\frac{e_b}{m_b}\right)\right] \times\frac{m_B m_\rho}{m_B^2-m_\rho^2}\,, \\
f_2^{\rm pole}(\bar{B}^0 \to \rho^0\gamma) &= -\frac{1}{2\sqrt{2}}\kappa a_2 \left(\frac{e_d}{m_d}-\frac{e_b}{m_b}\right)\frac{m_B m_\rho}{m_B^2-m_\rho^2}\,,
\end{aligned}
\tag{29}
$$

where $\kappa = eG_F V_{ub}V_{ud}^* f_B f_\rho/\sqrt{2}$, and m_i is the constituent quark mass. Again, we see that isospin symmetry is violated as the W-exchange amplitude is color suppressed whereas W annihilation is color favored.

III. THE $\bar{B}^0 \to D^{*0}\gamma$ DECAY

The radiative decay $\bar{B}^0 \to D^{*0}\gamma$ receives only long-distance contributions, and yet its branching ratio is large enough for a feasible test in the near future. In [1], an effective Lagrangian for the quark-quark bremsstrahlung $b\bar{d} \to c\bar{u}\gamma$ is derived based on the fact that the intermediate quark state in this process is sufficiently off shell and the emitted photon is hard enough, allowing an analysis of the W exchange bremsstrahlung by perturbative QCD. Applying this formulism to $\bar{B}^0 \to D^{*0}\gamma$ yields (see (3.7) of [1])

$$
\begin{aligned}
f_1^{\rm pole}(\bar{B}^0 \to D^{*0}\gamma) &= \kappa' a_2 \left[\left(\frac{e_c}{m_c}+\frac{e_u}{m_u}\right)\frac{m_{D^*}}{m_B} + \left(\frac{e_d}{m_d}+\frac{e_b}{m_b}\right)\right]\frac{m_B m_{D^*}}{m_B^2-m_{D^*}^2}\,, \\
f_2^{\rm pole}(\bar{B}^0 \to D^{*0}\gamma) &= -\frac{1}{2}\kappa' a_2 \left[\left(\frac{e_c}{m_c}-\frac{e_u}{m_u}\right)\frac{m_{D^*}}{m_B} + \left(\frac{e_d}{m_d}-\frac{e_b}{m_b}\right)\right]\frac{m_B m_{D^*}}{m_B^2-m_{D^*}^2}\,,
\end{aligned}
\tag{30}
$$

with $\kappa' = eG_F V_{cb}V_{ud}^* f_B f_{D^*}/\sqrt{2}$. It has been shown explicitly in [1] that the effective Lagrangian and pole model approaches are equivalent, but the former is much simpler and provides information on the form factors.

It is easily seen that the VMD contributions to $\bar{B}^0 \to D^{*0}\gamma$ come from the processes $\bar{B}^0 \to D^{*0}\rho^0(\omega) \to D^{*0}\gamma$. Following Sec. II, we obtain

$$
\begin{aligned}
f_1^{\rm VMD}(\bar{B}^0 \to D^{*0}\gamma) &= -eG_F V_{cb}V_{ud}^* a_2 f_{D^*} m_{D^*} \times\left(\frac{1}{m_B+m_\rho}\frac{1}{g_{\gamma\rho}} - \frac{1}{m_B+m_\omega}\frac{1}{g_{\gamma\omega}}\right)V^{B\rho}(0)\,, \\
f_2^{\rm VMD}(\bar{B}^0 \to D^{*0}\gamma) &= -\frac{1}{2}\frac{A_2^{B\rho}(0)}{V^{B\rho}(0)} f_1^{\rm VMD}(\bar{B}^0 \to D^{*0}\gamma)\,.
\end{aligned}
\tag{31}
$$

IV. NUMERICAL RESULTS

To estimate the short-distance penguin, long-distance VMD and pole contributions to weak radiative decays, we will use the following values for various quantities.

(i) Decay constants for pseudoscalar and vector mesons. In addition to Eq. (27), we also use

$$
f_B = 190~\text{MeV}, \quad f_{D^*} = 200~\text{MeV}\,. \tag{32}
$$

(ii) a_1 and a_2. The parameters a_1 and a_2 appearing in nonleptonic B decays are recently extracted from the CLEO data [14] of $B \to D^{(*)}\pi(\rho)$ and $B \to J/\psi K^{(*)}$ to be [15]

$$
\begin{aligned}
a_1(B \to D^{(*)}\pi(\rho)) &= 1.01 \pm 0.006\,, \\
a_2(B \to D^{(*)}\pi(\rho)) &= 0.23 \pm 0.06\,, \\
\left|a_2(B \to J/\psi K^{(*)})\right| &= 0.227 \pm 0.013\,.
\end{aligned}
\tag{33}
$$

Hence, in the present paper it is natural to employ

$$
a_1 = 1.01, \quad a_2 = 0.23\,. \tag{34}
$$

TABLE I. A summary of the amplitudes from electromagnetic penguin, VMD, and pole mechanisms for various radiative decays of the B meson in units of $10^{-10}e$ for $\rho = 0.18$ (upper entry) and $\rho = -0.18$ (lower entry).

	$f_1^{\rm peng}$	$f_1^{\rm VMD}$	$f_1^{\rm pole}$	$f_2^{\rm peng}$	$f_2^{\rm VMD}$	$f_2^{\rm pole}$
$B^- \to \rho^-\gamma$	$-13.63 - i6.94$	$-2.60 - i0.72$	$1.64 - i2.74$	$6.82 + i3.47$	$0.75 + i0.21$	$-0.64 + i1.07$
	$-21.96 - i6.94$	$-3.46 - i0.72$	$-1.64 - i2.74$	$10.98 + i3.47$	$1.00 + i0.21$	$0.64 + i1.07$
$\bar{B}^0 \to \rho^0\gamma$	$9.64 + i4.91$	$2.11 + i0.06$	$-0.07 + i0.11$	$-4.82 - i2.46$	$-0.61 - i0.02$	$0.06 - i0.11$
	$15.53 + i4.91$	$2.18 + i0.06$	$0.07 + i0.11$	$-7.77 - i2.46$	$-0.63 - i0.02$	$-0.06 - i0.11$
$\bar{B}^0 \to D^{*0}\gamma$		-3.19	-2.40		0.92	9.75

(iii) Photon-vector meson coupling constants given by Eq. (25).

(iv) Constituent quark masses:

$$\begin{aligned} m_u &= 338 \text{ MeV} , \\ m_d &= 322 \text{ MeV} , \\ m_c &= 1.6 \text{ GeV} , \\ m_b &= 5 \text{ GeV} , \end{aligned} \tag{35}$$

where the light quark masses are taken from p. 1729 of [13].

(v) Form factors $A_{1,2}$ and V at $q^2 = 0$. It has been shown in [15] that the heavy-flavor-symmetry approach for heavy-light form factors in conjunction with a certain type of form factor q^2 dependence provides a satisfactory description of the CLEO data for the ratio $\Gamma(B \to J/\psi K^*)/\Gamma(B \to J/\psi K)$ [14] and the Collider Detector at Fermilab (CDF) measurement of the fraction of longitudinal polarization in $B \to J/\psi K^*$ [16]. Assuming SU(3)-flavor symmetry for heavy-light form factors, we find from Table I of [15] that

$$V^{B\rho}(0) = 0.33, \quad A_1^{B\rho}(0) = 0.29, \quad A_2^{B\rho}(0) = 0.19 . \tag{36}$$

As shown in [15], the CLEO measurement $\mathcal{B}(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ [17] is well explained by the same set of form factors.

(vi) Quark mixing matrix elements. We will take $V_{cb} = 0.040$ [18] and $|V_{ub}/V_{cb}| = 0.08$, which in turn imply the following Wolfenstein parameters ($\lambda = 0.22$) [19]:

$$A = 0.826, \quad \sqrt{\rho^2 + \eta^2} = 0.35 . \tag{37}$$

For the purpose of illustration, we will take $\eta = 0.30$, and hence $\rho = \pm 0.18$. A small and negative ρ is favored by B^0-$\bar{B}^0$ mixing data. In terms of the Wolfenstein parametrization of the quark mixing matrix [19], the quantity appearing in Eq. (13) has the expression

$$\frac{V_{td}^*}{V_{ts}^*}(1 + \Delta) \cong -\lambda(1 - 1.3\rho + i1.3\eta) . \tag{38}$$

With the values given by (32)–(38) for various quantities, we proceed to compute the form factors f_1 and f_2 for $\bar{B} \to \rho\gamma$ and $\bar{B}^0 \to D^{*0}\gamma$ decays. The numerical results are summarized in Table I. It should be stressed that all the relative signs among various amplitudes are fixed in our work. The ratios of the long-distance (VMD and pole) and short-distance (penguin) contributions to $f_{1,2}$ in $\bar{B} \to \rho\gamma$ are shown in Table II. We see from Table II that while VMD and pole amplitudes are comparable in $B^- \to \rho^-\gamma$ decay, estimated to be roughly (10–20)% of the short-distance contribution, the former is the dominant long-distance contribution to $\bar{B}^0 \to \rho^0\gamma$. Taking into account various contributions to $f_{1,2}$,

$$f_{1,2}^{\rm tot} = f_{1,2}^{\rm peng} + f_{1,2}^{\rm VMD} + f_{1,2}^{\rm pole} , \tag{39}$$

we obtain the branching ratios

$$\mathcal{B}(B^- \to \rho^-\gamma) = \begin{cases} 1.8 \times 10^{-6}, \\ 4.6 \times 10^{-6}, \end{cases}$$
$$\mathcal{B}(\bar{B}^0 \to \rho^0\gamma) = \begin{cases} 0.9 \times 10^{-6}, \\ 1.8 \times 10^{-6}, \end{cases} \tag{40}$$

for $\eta = 0.30$, $\rho = 0.18$ (upper entry) and $\rho = -0.18$ (lower entry), where we have applied Eq. (5) and the lifetimes $\tau(\bar{B}^0) = 1.50 \times 10^{-12}$ s and $\tau(B^-) = 1.54 \times 10^{-12}$ s [13]. It follows from (40) that

$$R \equiv \frac{\Gamma(B^- \to \rho^-\gamma)}{\Gamma(\bar{B}^0 \to \rho^0\gamma)} \cong \begin{cases} 2.0 \\ 2.5 \end{cases} \quad \text{for } \rho = \begin{cases} 0.18, \\ -0.18, \end{cases} \tag{41}$$

and $\eta = 0.30$. Hence, violation of isospin symmetry for $\bar{B} \to \rho\gamma$ decay rates is at the level of 25% for $\rho = -0.18$, but it is negligible for a positive ρ owing to the destructive interference between VMD and pole amplitudes. Since $R = 2$ due to the electromagnetic penguin contribution, any deviation of R away from 2 gives the indictor of long-distance effects.

As for the $\bar{B}^0 \to D^{*0}\gamma$ decay, we find from Table I that

$$\frac{f_1^{\rm VMD}}{f_1^{\rm pole}} = 1.33, \quad \frac{f_2^{\rm VMD}}{f_2^{\rm pole}} = 0.09 . \tag{42}$$

TABLE II. The ratios of long- and short-distance contributions to the form factors f_1 and f_2 in $\bar{B} \to \rho\gamma$ decays for $\rho = 0.18$ (first entry) and $\rho = -0.18$ (second entry).

	$B^- \to \rho^-\gamma$	$\bar{B}^0 \to \rho^0\gamma$
$\lvert f_1^{\rm VMD}/f_1^{\rm peng}\rvert$	0.18 0.15	0.20 0.13
$\lvert f_1^{\rm pole}/f_1^{\rm peng}\rvert$	0.21 0.14	0.01 0.01
$\lvert f_2^{\rm VMD}/f_2^{\rm peng}\rvert$	0.10 0.09	0.11 0.08
$\lvert f_2^{\rm pole}/f_2^{\rm peng}\rvert$	0.16 0.11	0.02 0.02

We see that the form factor f_2 is dominated by the pole contribution, while the VMD effect plays an essential role in f_1. This is ascribed to the fact that, as can be seen from Eq. (30), there is a large cancellation in $f_1^{\rm pole}$. Since the decay rate is proportional to $|f_1|^2+4|f_2|^2$ and $f_1^{\rm pole}/f_2^{\rm pole}=0.25$, it is easily seen that the branching ratio of $\bar{B}^0 \to D^{*0}\gamma$,

$$\mathcal{B}(\bar{B}^0 \to D^{*0}\gamma) = 0.93 \times 10^{-6} \ , \tag{43}$$

is overwhelmingly dominated by the pole diagrams. In the absence of the VMD contributions, this branching ratio will become 0.74×10^{-6}.[6]

V. DISCUSSION AND CONCLUSION

Assuming the validity of the VMD concept, we have studied in the present paper the effect of VMD on the weak radiative decays $\bar{B} \to \rho\gamma$ and $\bar{B}^0 \to D^{*0}\gamma$. Based on the factorization approach, we found that $\bar{B} \to \rho\gamma$ is dominated by the short-distance penguin diagram and that the VMD contribution is $\sim 10-20\%$ of the penguin amplitude. However, contrary to $B \to K^*\gamma$, the long-range pole effect in $B^- \to \rho^-\gamma$ decay is comparable to the VMD one. The pole contribution in $B \to K^*\gamma$ is suppressed due to the smallness of the weak mixing $V_{ub}V_{us}$ relative to $V_{cb}V_{ud}$ appearing in the VMD process. In the decay $B^- \to \rho^-\gamma$, the mixing matrix elements entering into the pole diagram and the VMD diagram with ρ^0 and ω intermediate states are the same. However, the pole effect in $\bar{B}^0 \to \rho^0\gamma$ is suppressed again (see Table II) owing to the fact that the W exchange diagram is color suppressed. Therefore, as far as the relative magnitudes of the short- and long-distance contributions are concerned, $\bar{B}^0 \to \rho^0\gamma$ resembles most to $B \to K^*\gamma$.

The branching ratio of $\bar{B} \to \rho\gamma$, estimated to be of order 10^{-6}, depends strongly on the sign of the Wolfenstein parameter ρ. A measurement of the ratio $R \equiv \Gamma(B^- \to \rho^-\gamma)/\Gamma(\bar{B}^0 \to \rho^0\gamma)$ is useful for this purpose. Since the short-distance penguin effect alone yields $R=2$, any deviation of R from 2 will provide information on the long-distance contribution and the sign ρ. We found that $R>2$ for $\rho<0$ and $R\sim 2$ for $\rho>0$.

The decay $\bar{B}^0 \to D^{*0}\gamma$ receives only long-distance contributions. It turns out that though the VMD and pole diagrams contribute comparably to the parity-conserving amplitude of $\bar{B}^0 \to D^{*0}\gamma$, the parity-violating part is largely dominated by W exchange bremsstrahlung. Consequently, its branching ratio, predicted to be 0.9×10^{-6}, is overwhelmingly dominated by the pole contributions.

[6]This number is slightly different from the result 0.92×10^{-6} obtained in [1] since a_2 there is identified with $\frac{1}{2}(c_- - c_+)$.

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