Calculation of $1/m_c^3$ terms in the total semileptonic width of D mesons

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We calculate the $1/m_c^3$ corrections in the inclusive semileptonic widths of D mesons. We show that these are due to the novel penguin-type operators that appear at this level in the transition operator. Taking into account the nonperturbative corrections leads to the predicted value of the semileptonic width significantly lower than the experimental value. The $1/m_c^3$ corrections worsen the situation or, at the very least, within uncertainty, give a small contribution. We indicate possible ways out of this problem. It seems most probable that violations of duality are noticeable in the energy range characteristic to the inclusive decays in the charm family. Theoretically these deviations are related to divergence of the high-order terms in the power expansion in the inverse heavy quark mass.

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I. INTRODUCTION

Recently a QCD-based approach to the calculation of total inclusive decay rates of heavy (B and D) mesons was developed [1–6]. The approach is based on the systematic expansion in the inverse heavy quark mass within the operator product expansion (OPE) [7]. In this paper we will discuss the application of this formalism to the calculation of the total semileptonic inclusive width of D^0 mesons. Unlike the previous model calculations the OPE-based method gives us full control over all relevant parameters in theoretical expressions.

The leading perturbative $[O(\alpha_s)]$ [8, 9] and nonperturbative $[O(1/m_c^2)]$ [4] corrections have been found previously. It turns out that under a reasonable choice of the c-quark mass the predicted value of $\Gamma_{\rm sl}(D)$ is significantly lower than the corresponding experimental number [10]. We calculate the next-to-leading nonperturbative correction of order $1/m_c^3$ and show that it only worsens the situation, or at the very least, gives a small contribution within uncertainty limits. We then indicate possible ways out of this problem.

At the level of $O(1/m_c^3)$ terms there arise penguin diagrams generating new, four-fermion operators of dimension 6. The penguin graphs were introduced 20 years ago [11] in the strange particle decays where it was crucial that they produce right-handed quarks. In the D meson semileptonic decays the origin of penguin diagrams is quite different—they appear at the level of the *transition operator* and give rise to a contribution of the annihilation type. Usually it is believed that the latter is suppressed by chirality arguments. The suppression is lifted, however, due to the fact that penguin diagrams produce the right-handed quarks, much in the same way as in Ref. [11].

Since the $1/m_c^3$ terms do not eliminate the discrepancy between the theoretical prediction for $\Gamma_{\rm sl}(D)$ and experiment a natural question immediately comes to one's mind: what went wrong? In estimating the D-meson matrix elements of the four-fermion operators we use factorization. In the limit of a large number of colors, $N_c \to \infty$, this approximation becomes exact. One may suspect, however, that at $N_c = 3$ deviations from factorization are substantial. Can these deviations be a solution of the problem?

Although logically this possibility is not ruled out *a priori* it is hard to believe that this is the case. Indeed, if the problem is to be solved in this way not only the matrix element of dimension-6 operators must be enhanced by a factor of ~ 3, its sign also has to be reversed as compared to what one obtains within factorization.

The second logical possibility, an enhanced contribution coming from dimension-7 operators, also seems very unlikely.

Thus, we are inclined to conclude that the failure of the standard m_Q^{-1} expansion in the case of $\Gamma_{\rm sl}(D)$ is due to the fact that the charmed quark mass is too light for duality to set in. This assertion will be explained in more detail in Sec. IV. Here we only note that duality is one of the crucial elements of the calculation of the inclusive widths within the heavy quark expansion. Theoretically the onset of duality is related to the behavior of high-order terms in the $1/m_Q$ expansion, the divergence of the $1/m_Q$ series. The fact that the OPE-based power expansions are actually asymptotic is well established [12]. Very little is known, however, about specific details of the divergence.

An indirect, though a very strong, argument that the charmed quark is only at the border, or even below the boundary, of the duality domain comes from the consideration of the lifetime hierarchy in the charmed family (for a recent discussion see [13]). Although $O(m_c^{-2})$ and $O(m_c^{-3})$ terms qualitatively reproduce the observed pattern some of the predicted lifetime ratios (which span an order of magnitude) are off by a factor ~ 2. The predicted $O(m_c^{-2}, m_c^{-3})$ deviations from the asymptotic

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limit are typically smaller than what is observed experimentally. Needless to say, asymptotically all lifetimes are equal.

As mentioned above, the issue of the inclusive semileptonic D decays was addressed in the recent literature more than once. The approach to the problem accepted, e.g., in Ref. [10] is inverted. It is assumed that the theoretical prediction for $\Gamma_{\rm sl}(D)$ truncated at the leading order of perturbation theory and at the leading order of the $1/m_c$ expansion (i.e., keeping only $1/m_c^2$) is accurate enough to use it to fit the values of the quark mass and other theoretical parameters from $\Gamma_{\rm sl}(D)_{\rm expt}$. The value of the charmed quark mass emerging in this way is unrealistic. At the same time the average value of the heavy quark kinetic energy μ_π^2 remains essentially undetermined. We, instead, use the best available scientific estimates of m_c and μ_π^2. We will see that the results are in direct disagreement with the experimental data.

The organization of the paper is as follows. In Sec. II, we discuss the current situation. The naive parton result is augmented by its perturbative, to α_s, and nonperturbative, to $1/m_c^2$, corrections. Section III shows the situation with $1/m_c^3$ nonperturbative corrections. We will see that they do not improve the match with experiment. Finally, we discuss ways out of the dilemma.

II. THE STARTING POINT

The theoretical expression for the inclusive semileptonic width of the D meson ($c \to sl\nu$ transition), including the leading perturbative and nonperturbative corrections, has the form

$$\Gamma(D \to l\nu X_s) = \frac{G_F^2 m_c^5}{192\pi^3}|V_{cs}|^2 \times \left[1 + A^{(1)}\alpha_s - \frac{3\mu_G^2}{2m_c^2} - \frac{\mu_\pi^2}{2m_c^2}\right] . \quad (1)$$

Here m_c is the charmed quark mass, and we neglected the strange quark mass; V_{cs} is the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix element. The coefficient $A^{(1)}$ of the $O(\alpha_s)$ term has been known for many years, see Ref. [14] whose authors merely adapted the QED radiative correction to $\mu \to e\nu\bar{\nu}$ (the original QED calculations are published in Ref. [15]). The explicit expression for $A^{(1)}$ depends on what definition of the quark mass m_c is accepted. The straightforward borrowing from $\mu \to e\nu\bar{\nu}$ implies the use of the so-called pole mass. Although this parameter is not well defined in full QCD (see Ref. [16] and the discussion below) it is admissible for a limited technical purpose of presenting the $O(\alpha_s)$ correction. Then

$$A^{(1)} = -\frac{2}{3\pi}\left(\pi^2 - \frac{25}{4}\right) . \quad (2)$$

The leading nonperturbative correction in the $1/m_c$ expansion for $\Gamma(D \to l\nu X_s)$ was first calculated in Ref. [4]. The term of the first order in $1/m_c$ is absent, the so-called Chay-Georgi-Grinstein-Bigi-Uraltsev-Vainshtein (CGG-BUV) theorem [3, 4]. At the level $O(m_c^{-2})$ the correction is determined by two parameters:

$$\mu_G^2 = \frac{\langle D|(i/2)\sigma_{\mu\nu}G^{\mu\nu}|D\rangle}{2M_D} \approx \frac{3}{4}(M_{D^*}^2 - M_D^2) ,$$

$$\mu_\pi^2 = \frac{\langle D|(i\vec{D})^2|D\rangle}{2M_D} . \quad (3)$$

Before proceeding to numerical estimates it is worth discussing the parameters in Eq. (1) in more detail. Numerically the most important parameter is the quark mass since it enters in the fifth power. As shown in Refs. [3–6] it is the current quark masses, not the constituent one and not the hadron mass, that appear in the systematic $1/m_Q$ expansions. If we limit ourselves to the $O(\alpha_s)$ expression for Γ and do not ask any questions about higher-order terms we are free, of course, to express the result in terms of the pole mass or in terms of the running mass normalized at any point we like—this will merely redefine the coefficient in front of α_s in a certain way. We are aimed, however, at better accuracy; in particular, we want to include the power nonperturbative terms in analysis.

The use of the pole mass is generically inconsistent; only the running mass can appear in any OPE-based expression, see [16]. The question is what normalization point is relevant. The full expression for the decay rate, including all terms in the α_s expansion, is certainly independent of the choice of the normalization point μ, an auxiliary parameter in the operator product expansion. For a truncated series—and we are forced, of course, to truncate the series at the level of the leading, or, at best, the next-to-leading term—the choice of μ is important since under a "natural" choice the coefficients in the neglected part of the series are small while under "unnatural" choices they can be abnormally large. In Ref. [16] it was explicitly shown that the natural choice for m_c is $\mu = \text{const} \times m_c$ (see also [17, 18]). The leading operator in the expansion in the problem at hand is $\bar{c}(i\not{D})^5 c$. By adopting the normalization point $\mu = \text{const} \times m_c$ we avoid any large corrections. Nonperturbative effects enter through the matrix element of this operator; they are also represented by matrix elements of other (subleading) operators, for instance, $\bar{c}(i\not{D})^3 i\sigma G c$.

Using the equations of motion one reduces the leading operator to $m_c^5\bar{c}c$, where both m_c and $\bar{c}c$ are taken at $\mu = m_c$. We then evolve $\bar{c}c$ down to a low normalization point, $\mu \ll m_c$; the net effect of this evolution is reflected in a factor of the type $c(\mu, m_c) = 1 + a_1(m_c/\mu)\alpha_s(m_c) + a_2(m_c/\mu)\alpha_s^2(m_c) + \cdots$ which is, anyway, included in the perturbative calculation, see Eq. (1).[1] Once the operator

[1]In principle, this factor must contain also terms of order $(\mu/m_c)^n$ due to the exclusion of the domain below μ from the perturbative calculation. It is important that the power n starts from $n = 2$, and this term conspires with μ_G^2 and μ_π^2.

$\bar{c}c$ is evolved down to a low normalization point we use[2] the relation [4]

$$\bar{c}c = \bar{c}\gamma_0 c + \frac{1}{4m_c^2}\bar{c}i\sigma Gc - \frac{1}{2m_c^2}\bar{c}\vec{\pi}^2 c + O(1/m_c^4) \\ +\text{total derivatives.} \tag{4}$$

The numerical value of the (one-loop) pole mass of the charmed quark was determined long ago from the charmonium sum rules [19], $m_c^{\rm pole} \approx 1.35$ GeV (see also [20]). A recent advent of heavy quark effetive theory (HQET) [21] allows one to conduct a consistency check of this estimate. Indeed, let us observe that

$$m_b - m_c = \bar{M}_B - \bar{M}_D + \mu_\pi^2\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) \\ + O(1/m_c^3,\ 1/m_b^3)\ , \tag{5}$$

where

$$\bar{M}_B = \frac{M_B + 3M_{B^*}}{4}$$

and the same for D. Next, for the pole mass of the b quark a very precise evaluation

$$m_b = 4.83 \pm 0.03 \text{ GeV} \tag{6}$$

is obtained in the recent analysis of the QCD sum rules for the Υ system [22]. To be on the safe side we multiplied the original error bars by a factor of 4. It is worth noting that it is very difficult, practically impossible, to go outside the indicated limits. The central value of m_b above implies $m_c \approx 1.33$ GeV (provided we accept the estimate of Ref. [23] for μ_π^2, see below). The most generous error bars in m_b and μ_π^2 (see below) are translated in ± 70 MeV uncertainty in m_c. It seems perfectly safe to say that $m_c^{\rm pole}$ lies between 1.25 and 1.40 GeV—one cannot imagine that the one-loop pole c quark mass is less than 1.25 or larger than 1.40. The HQET result $m_c^{\rm pole} \approx 1.33$ GeV matches very well the QCD sum rule number quoted above.

As mentioned, in the purely theoretical aspect, it is more consistent to use the running mass in the expression for the total semileptonic decay rate. One may choose the so-called Euclidean mass [19, 20] or the running modified minimal subtraction scheme ($\overline{\rm MS}$) mass evaluated at $p^2 = -m_c^2$. Both are close to each other numerically, and are smaller numerically than $m_c^{\rm pole}$ since they are deprived of a part of the (perturbative) gluon cloud compared to $m_c^{\rm pole}$,

$$m_c^{\rm pole} = m_c^{\rm Eucl}\left[1 + \frac{2\ln 2\alpha_s}{\pi} + \cdots\right]$$

and

$$m_c^{\rm pole} = m_c^{\overline{\rm MS}}(m_c)\left[1 + \frac{4\alpha_s}{3\pi} + \cdots\right]. \tag{7}$$

For instance, a fit in the charmonium sum rules yields [19] $m_c^{\rm Eucl} \approx 1.25$ GeV. Since our task is limited—in the numerical evaluation we will not go beyond the first order in α_s—essentially it does not matter which expression for the width is used: the one written in terms of the pole mass or in terms of any other mass from Eq. (7).

Let us first ignore the correction terms in Eq. (1) altogether. Then the naive parton-model expression

$$\Gamma_0(D \to l\nu X_s) = \frac{G_F^2 m_c^5}{192\pi^3}|V_{cs}|^2$$

with $m_c = 1.4$ GeV yields $\Gamma(D \to l\nu X_s) = 1.1 \times 10^{-13}$ GeV, to be compared with

$$\Gamma(D \to l\nu X_s)_{\rm expt} = 1.06 \times 10^{-13}\ \text{GeV}. \tag{8}$$

The value of $m_c = 1.4$ GeV is at the upper boundary of what we believe is allowed for the pole charmed quark mass. We will consistently push the estimates of $\Gamma(D \to l\nu X_s)$ to the high side and, hence, use this value for orientation.

If, instead, $m_c = 1.25$ GeV is substituted then the parton-model formula gives 0.6 of $\Gamma(D \to l\nu X_s)_{\rm expt}$.

The key point, emphasized in the Introduction, is that all known corrections, perturbative and nonperturbative are negative. Consider first the $O(\alpha_s)$ correction in Eq. (1). To evaluate it numerically one needs to know the normalization point of α_s. Usually it is taken to be $\mu \sim m_c$. Again this is the question of higher-order terms. Recently it was argued [9] within the Brodsky-Lepage-Mackenzie hypothesis [24] that the $O(\alpha_s^2)$ terms are negative and large, so that actually the normalization point of α_s constitutes a relatively small fraction of m_c. Since this effect works in the direction of reducing $\Gamma(D \to l\nu X_s)_{\rm theor}$ and we agreed to push the estimate to the high side, we will ignore the $O(\alpha_s^2)$ terms remembering, however, that our estimate will then lie higher than the actual theoretical prediction.

For consistency we use the value $\Lambda_{\rm QCD} \sim 150$ MeV relevant to the one-loop approximation. Then $\alpha_s(m_c) \approx 0.30$, which is consistent with the recent very precise evaluation [22], as well as with the previous analyses of data on deep inelastic scattering [25]. (The corresponding two-loop value $\Lambda_{\rm QCD} \sim 250$ MeV.) Notice that bigger values of $\Lambda_{\rm QCD}$ sometimes advocated in the literature only worsen the disagreement between theory and experiment.

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[2]The $O(m_c^{-2})$ terms in Eq. (4) were derived in Ref. [4]. The fact that the operators of dimension 6 are absent in this expansion can be easily established by using the equation of motion,

$$\frac{1-\gamma_0}{2}c = \frac{1}{2m_c}\hat{\pi}c.$$

Then

$$\bar{c}\frac{1-\gamma_0}{2}c = \bar{c}\frac{1-\gamma_0}{2}\frac{1-\gamma_0}{2}c = \frac{1}{4m_c^2}\bar{c}\hat{\pi}\hat{\pi}c$$

implies that

$$\bar{c}(1-\gamma_0)c = \frac{1}{2m_c^2}\bar{c}\left(\pi^2 + \frac{i}{2}\sigma G\right)c.$$

Moreover, using the equations of motion, again we see that $\bar{c}\pi_0^2 c$ actually reduces to an operator of dimension 7 and we arrive at Eq. (4).

With this value of $\alpha_s(m_c)$ we find that the $O(\alpha_s)$ correction in Eq. (1) is equal to -0.24, i.e., it further reduces $\Gamma(D \to l\nu X_s)_{\rm theor}$ by a quarter.

Let us discuss now the nonperturbative $O(m_c^{-2})$ terms. The value of μ_G^2 is known phenomenologically, see Eq. (3):

$$\mu_G^2 \approx 0.41 \text{ GeV}^2 \tag{9}$$

(small effects due to the anomalous dimension of the chromomagnetic operator [26] are neglected). As for μ_π^2, at present this parameter is not measured, although it is measurable in principle [27]. At least two independent lower bounds are established [28, 27, 29] which turn out to be close numerically:

$$\mu_\pi^2 > 0.4\,\text{GeV}^2 .$$

Moreover, the value of μ_π^2 was evaluated in the QCD sum rules [23], with the result

$$\mu_\pi^2 = 0.6 \pm 0.1\,\text{GeV}^2 . \tag{10}$$

Now we are finally able to estimate the $O(m_c^{-2})$ terms in $\Gamma(D \to l\nu X_s)_{\rm theor}$. Again, trying to increase $\Gamma_{\rm theor}$ we use the lower value of μ_π^2; we then conclude that the chromomagnetic and kinetic terms contribute to the brackets in Eq. (1) -0.31 and -0.13, respectively. So collecting everything, we have [to $O(\alpha_s)$ and $O(m_c^{-2})$]

$$\Gamma(D \to l\nu X_s)_{\rm theor} = \Gamma_0[1 - 0.24 - 0.31 - 0.13].$$

We have less than half of the experimental width. Can the $O(m_c^{-3})$ terms fix the situation? As we will see in the next section the answer to this question is negative.

III. THE $1/m_c^3$ CORRECTIONS TO THE SEMILEPTONIC WIDTH

Since the $1/m_c^2$ terms do not solve the problem of the total semileptonic width, it is natural to consider the corrections due to $1/m_c^3$ terms. They can be calculated in the standard way within the heavy quark expansion [1–6]. Below the basic points of derivation are sketched.

We start from the weak Lagrangian describing the semileptonic decays:

$$\mathcal{L}(\mu) = \frac{G_F}{\sqrt{2}} V_{sc} O . \tag{11}$$

Here

$$O = (\bar{s}\Gamma_\mu c)(\bar{\nu}\Gamma^\mu e) , \tag{12}$$

and $\Gamma_\mu = \gamma_\mu(1+\gamma_5)$. Equations (11), and (12) present the Lagrangian relevant to the $c \to s\bar{e}\nu$ transition.

Next, we construct the transition operator $\hat{T}(c \to X \to c)$,

$$\hat{T} = i \int d^4x T\{\mathcal{L}(x)\mathcal{L}(0)\} = \sum_i C_i \mathcal{O}_i \tag{13}$$

describing the diagonal amplitude with the heavy quark c in the initial and final states (with identical momenta). The transition operator $\hat{T}$ is built by means of OPE as an expansion in local operators $\mathcal{O}_i$. The lowest-dimension operator in $T(c \to X \to c)$ is $\bar{c}c$, and the complete perturbative prediction corresponds to the perturbative calculation of the coefficient of this operator. In calculating the coefficient of $\bar{c}c$ we treat the light quarks in X as hard and neglect the soft modes. Say, we ignore the fact that in a part of the phase space the s quark line is soft and cannot be treated perturbatively. Likewise, we ignore interaction with the soft gluons. The presence of the soft quark-gluon "medium" is reflected in higher-dimensional operators.

Once the expansion (13) is built we average $\hat{T}$ over the D meson to obtain the lifetime

$$\Gamma = \frac{1}{M_D} \text{Im}\langle D|\hat{T}|D\rangle .$$

At this stage the nonperturbative large-distance dynamics enter through the matrix elements of the operators of dimension 5 and higher. There are no operators of dimension 4 [3]. The operators of dimension 5 have been already discussed. The only new operators relevant at the level of dimension 6 are the four-fermion operators of the type

$$O_6 = \bar{c}\Gamma q \bar{q}\Gamma c ,$$

where q generically denotes the light quark field and Γ stands for a combination of γ and t_a matrices. What particular combination is relevant will be seen from what follows. There are two distinct sources of $1/m_c^3$ corrections in the total semileptonic width: operators of dimension 6 arising in the expansion for $\hat{T}$, and $1/m_c$ corrections in the D-meson matrix elements of the operator $\bar{c}\sigma Gc$ and $\bar{c}c$. Let us discuss these terms starting from dimension-6 operators in the expansion for $\hat{T}$. We shall see that since Γ is a Lorentz scalar quantity, only Lorentz scalar operators can contribute. Thus, the only operator showing up at this level will be of the four-fermion type, O_6.

A. The four-fermion operators at order α_s^0 and in LLA

A four-fermion operator appears in $\hat{T}$ in the zeroth order in α_s from the diagram of Fig. 1. The corresponding result can be read from Eq. (17c) of Ref. [30], where one must put $C_+ = C_- = 1$ and eliminate the color factor of 3 from the numerator:

$$\text{Im}\,\hat{T}^{(0)} = -\frac{G_F^2 m_c^2}{8\pi}|V_{cs}|^2 \times \{[\bar{c}_i\Gamma_\mu c_k - (2/3)\bar{c}_i\gamma_\mu\gamma_5 c_k](\bar{s}_k\Gamma_\mu s_i)\} . \tag{14}$$

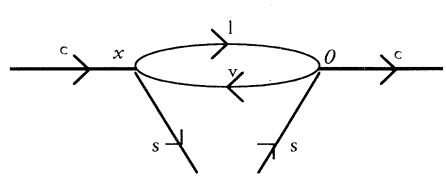

FIG. 1. The four-fermion term in the transition operator as it appears at the level α_s^0.

The expression in the curly brackets includes the left-handed s quarks only. If we use the standard factorization procedure (i.e., saturation by the vacuum state) for estimating the matrix element of this operator over the D-meson state we get zero for two reasons. First, in the factorization approximation $\hat{T}^{(0)}$ corresponds to the annihilation contribution, $c\bar{s} \to l\nu$, which shows up only in D_s. Second, even for D_s the chiral structure is "wrong," and after factorization the matrix element $\langle D|\hat{T}^{(0)}|D\rangle$ vanishes.

Therefore, the four-fermion operator appearing in Eq. (14) by itself is not interesting. Let us recall, however, that the normalization point of this operator in Eq. (14) is $\mu = m_c$, and before estimating its matrix elements we must evolve it down to a low normalization point. (Of course, it is desirable to go to μ of order of the typical off-shellness of quarks inside mesons. Clearly we cannot do this since then the perturbative calculation of the coefficient functions becomes meaningless. We will make a compromise and evolve down to $\mu \sim 0.5$ GeV assuming that, on one hand, the coefficient functions are still calculable and, on the other hand, the factorization procedure can be used for obtaining the D-meson matrix element.) It is straightforward to take this evolution into account in the leading logarithmic approximation (LLA); as a matter of fact, we just parallel the standard penguin analysis [see Fig. 2(a)]. What is crucial is that this evolution brings in new four-fermion operators, of a different flavor and chiral structure, whose matrix elements over D^0 and D^+ do not vanish within factorization. Calculating the diagram of Fig. 2(a) with the logarithmic accuracy we get[3]

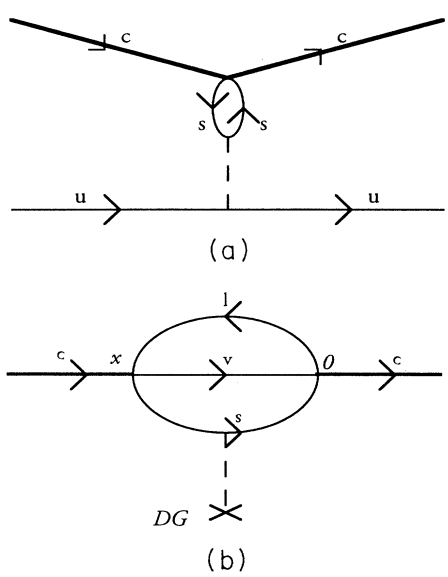

FIG. 2. (a) The penguin graph for the four-fermion operator in $\hat{T}$. (b) The diagram with the DG term in the s-quark line.

[3]This result can also be extracted from Eq. (20) of Ref. [30].

$$\mathrm{Im}\,\hat{T}^{(1)} = -\frac{G_F^2 m_c^2}{8\pi}|V_{cs}|^2\left\{-\frac{\alpha_s}{3\pi}\ln\frac{m_c^2}{\mu^2} \times\left(\frac{2}{3}\bar{c}\Gamma_\mu t^a c + \frac{1}{3}\bar{c}\tilde{\Gamma}_\mu t^a c\right)\sum_q \bar{q}\gamma_\mu t^a q\right\}, \tag{15}$$

where

$$\tilde{\Gamma}_\mu = \gamma_\mu(1-\gamma_5)$$

and t^a are generators of SU(3),

$$t^a = \frac{1}{2}\lambda^a .$$

(Here λ^a are the Gell-Mann matrices.) Notice that the light-quark current $\sum \bar{q}\gamma_\mu t^a q$ is actually $\frac{1}{g}D_\nu G_{\mu\nu}$; it includes all light-quark flavors and both left-handed and right-handed fields. This is a typical feature of the penguin diagram contribution [11], leading to a nonvanishing contribution of Eq. (15) to the D-meson matrix elements within factorization. As a matter of fact, we can omit the left-handed part of the light-quark current, since, as was explained above, after factorization the term with the left-handed part of the current will vanish: i.e.,

$$\sum_q \bar{q}\gamma_\mu t^a q = \sum_q \frac{1}{2}\left\{\bar{q}\Gamma_\mu t^a q + \bar{q}\tilde{\Gamma}_\mu t^a q\right\} \to \frac{1}{2}\sum_q \bar{q}\tilde{\Gamma}_\mu t^a q . \tag{16}$$

B. Full $O(\alpha_s)$ calculation

Unfortunately, $\ln m_c^2/\mu^2$ is not a very large numerical parameter and, hence, neglecting nonlogarithmic terms may seem unjustified. Therefore, instead of summing up all logarithms in LLA (which can be readily done, though), it seems reasonable to limit oneself to the $O(\alpha_s)$ calculation including both the logarithmic and nonlogarithmic terms, the more so that we need the result only for the purpose of orientation. We want to convince ourselves that the contribution of dimension-6 operators to $\Gamma(D \to X_s l\nu)$ is negative.

It is most convenient to carry out the full $O(\alpha_s)$ calculation using the background field technique. There are two versions of this technique—the first one was exploited in the context of the inclusive semileptonic decays, e.g., in Ref. [31], the second version, based on the Fock-Schwinger gauge, is reviewed in Ref. [32]. Both versions can be applied in the case at hand. Here, the latter one is more suitable.

The dimension-6 operators come from two sources. To see this, we write explicitly the expression for the transition operator:

$$\hat{T} = -\frac{G_F^2}{2}\int d^4x e^{-iqx}\bar{Q}(x)\Gamma_\mu S_q(x,0)\Gamma_\nu L_{\mu\nu}Q(0) ,$$

where $L_{\mu\nu}$ is the lepton loop, and S_q stands for the light quark Green's function.

As mentioned, in the expression we have the propagator for the s quark. When expanded in the Fock-

Schwinger gauge this propagator is

$$S_q(x,0) = \frac{-im^2}{4\pi^2}\frac{K_1(m\sqrt{-x^2})}{\sqrt{-x^2}} - \frac{m^2\hat{x}}{4\pi^2 x^2}K_2(m\sqrt{-x^2}) + \frac{\tilde{G}_{\rho\lambda}}{8\pi^2}\frac{mK_1(m\sqrt{-x^2})}{\sqrt{-x^2}}(x_\rho\gamma_\lambda\gamma_5) + \frac{G_{\rho\lambda}}{16\pi^2}mK_0(m\sqrt{-x^2})\sigma_{\rho\lambda}$$
$$+\frac{2}{3}g\frac{i}{32\pi^2}\left\{2K_0[m\sqrt{(-x^2m)}]D^\alpha G_{\alpha\beta}\gamma_\beta - (D^\alpha G_{\alpha\beta}\hat{x}x^\beta + x^\gamma x^\alpha D_\gamma G_{\alpha\beta}\gamma^\beta\right.$$
$$\left.-3ix^\gamma x^\alpha D_\gamma\tilde{G}_{\alpha\beta}\gamma_\beta\gamma_5)\frac{mK_1[m\sqrt{(-x^2)}]}{\sqrt{(-x^2)}}\right\} + \cdots , \tag{17}$$

where m is the s quark mass (needed for infrared regularization). We will see that the first of the $1/m_c^3$ corrections will come from inserting the second (free) term of Eq. (17) into $\hat{T}$. The second correction comes from terms of order DG in the propagator. (Note that the term proportional to $\tilde{G}$ and $D\tilde{G}$ in the s-quark propagator yields zero due to their Lorentz structure.)

The third source for corrections, the matrix elements of dimensions 3 and 5, will be discussed later.

We start with the expression for $\hat{T}$ with the free term of the s quark inserted. This term can be read off from the diagram of Fig. 3. Here we use the expression with s-quark mass put to zero from the very beginning since the result is nonlogarithmic (infrared stable). Doing the arithmetic, it is easy to show that, in the Fock-Schwinger gauge,

$$\text{Im}\,\hat{T}_0 = \frac{G_F^2|V_{cs}|^2\bar{Q(0)}p^4\hat{p}Q(0)}{384\pi^3} ,$$

where

$$p_\mu = iD_\mu - gA_\mu$$

and

$$A_\mu = \frac{1}{2\times 0!}x_\rho G_{\rho\mu} + \frac{1}{3\times 1!}x_\rho x_\alpha(D_\alpha G_{\rho\mu}) + \cdots .$$

Here we are interested in $1/m_c^3$ corrections and thus look only at the term proportional to DG in A_μ. Our strategy is to pull A_μ to the left since $A(0) = 0$. When doing this, we create commutators which are easily evaluated using the explicit expression for A_μ. Performing this procedure, we obtain the following contribution to the transition operator:

$$\hat{T}|_{\text{free}} = -iG_F^2|V_{cs}|^2\frac{\alpha_s}{48\pi^2}m_c^2\bar{c}\gamma_\beta t^a c\bar{q}\gamma_\beta t^a q , \tag{18}$$

where the subscript "free" means the free term of the s-quark propagator. Note, here we have used the equation of motion

$$D^\alpha G^a_{\alpha\mu} = -g\bar{q}\gamma_\mu t^a q. \tag{19}$$

After factorization (see Sec. III D) this yields

$$\frac{\Delta\Gamma}{\Gamma_0} = \frac{8\alpha_s\pi f_D^2 M_D}{9m_c^3} ,$$

where f_D is the axial vector constant of the D meson. This key constant, f_D, is not measured accurately enough so far, although some experimental results do exist. It seems reasonable to rely on theoretical calculations which were done both on the lattice and in QCD sum rules (see Refs. [32,33], respectively), thus we choose $f_D =$ 170 MeV so as to push our estimate for $\Gamma_{sl}(D)$ to the high side. We also take $\alpha_s = 0.31$, and $m_c = 1.4$ GeV. Plugging in these numericals the above expression gives

$$\frac{\Delta\Gamma}{\Gamma} = 0.016.$$

Next, we consider the diagram of Fig. 2(b). Its contribution to the transition operator is calculated in the Appendix. Figure 2(b) singles out the DG terms in the background field expansion of the quark Green's function, see Eq. (30). The infrared cutoff in the logarithm is achieved by ascribing a mass of μ to the s-quark line. The lepton part of the diagram, which is trivial, must also be inserted, of course. After the Fourier transformation we get

$$\hat{T}|_{DG} = iG_F^2|V_{cs}|^2\left\{\frac{\alpha_s}{72\pi^2}m_c^2\left(\ln\frac{m_c^2}{\mu^2} + \frac{2}{3}\right)\right\}$$
$$\times(2\bar{c}\Gamma_\beta t^a c + \bar{c}\tilde{\Gamma}_\beta t^a c)\sum\bar{q}\gamma_\beta t^a q . \tag{20}$$

Note that the coefficient in front of the logarithm matches the one in Eq. (15), as it should, which was obtained through the logarithmic mixing. Again, after factorization (see Sec. III D), we get

$$\frac{\Delta\Gamma}{\Gamma} = -\frac{16\pi\alpha_s}{9m_c^3}\left(\ln\frac{m_c^2}{\mu^2} + \frac{2}{3}\right)f_D^2 M_D,$$

which is, numerically,

$$\frac{\Delta\Gamma}{\Gamma_0} = -0.08 ,$$

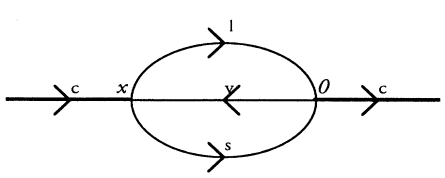

FIG. 3. The diagram with the free s-quark line determining $\hat{T}_{\text{free}}$.

where we have ascribed a value of 0.5 GeV to μ, and used the same values for the other parameters as above.

C. $O(\frac{1}{m_c^3})$ terms from matrix elements of dimension 3 and 5 operators

As mentioned above, the matrix elements of operators of dimension 3 and 5 also give rise to the $1/m_c^3$ corrections in the total semileptonic width. Let us first consider Eq. (4). The matrix element of the first term is exactly unity. The matrix elements of the second and third terms do contain $1/m_c$ corrections, which we discuss here.

The point is that Eq. (3) expressing $\langle D|\bar{c}\frac{i}{2}\sigma Gc|D\rangle$ in terms of the D^*D mass splitting is valid only to the leading (zero) order in $1/m_c$. Let us observe that the spin splitting yielding $M_{D^*}^2 - M_D^2$ is determined by the following terms in the heavy quark Hamiltonian:

$$\Delta\mathcal{H} = \frac{1}{2m_c}\vec{\sigma}\cdot\vec{B} + \frac{1}{4m_c^2}\vec{\sigma}\cdot\vec{E}\times\vec{\pi}\ , \tag{21}$$

where $\vec{B}$ and $\vec{E}$ are the chromoelectric and chromomagnetic fields, respectively, $\vec{B} = g\vec{B}^a t^a$ and $\vec{E} = g\vec{E}^a t^a$. To leading order,

$$\Delta_D \equiv \frac{3}{4}(M_{D^*}^2 - M_D^2) = -\langle\vec{\sigma}\cdot\vec{B}\rangle = 0.405\ \text{GeV}^2\ ,$$

where angular brackets mean $(2M_D)^{-1}\langle D|\bar{c}\cdots c|D\rangle$. At the level of $1/m_c$ the second term in the heavy quark Hamiltonian becomes important in Δ_D, as well as the second-order iteration in $(2m_c)^{-1}\langle\vec{\sigma}\cdot\vec{B}\rangle$. Assuming that both effects are of the same order of magnitude, we can roughly estimate the matrix element $(2m_c^{-1})\ \langle\vec{\sigma}\cdot\vec{E}\times\vec{\pi}\rangle$ as the difference between Δ_D and Δ_B:

$$|\ (2m_c)^{-1}\langle\vec{\sigma}\cdot\vec{E}\times\vec{\pi}\rangle\ | \leq \Delta_D - \Delta_B \approx 0.04\ \text{GeV}^2\ . \tag{22}$$

Next, observe that

$$\frac{i}{2}\bar{c}\sigma_{\mu\nu}G^{\mu\nu}c = -\bar{c}\vec{\sigma}\cdot\vec{B}c - \frac{1}{m_c}\bar{c}\vec{\sigma}\cdot\vec{E}\times\vec{\pi}c - \frac{1}{2m_c}\bar{c}(D_iE_i)c\ . \tag{23}$$

The last term in the above equation reduces to the four-fermion operator which we can take into account explicitly. The second term will be estimated as an uncertainty in the expression relating $\langle\frac{i}{2}\sigma G\rangle$ to Δ_D:

$$\begin{aligned}\mu_G^2 &\equiv \left\langle\frac{i}{2}\sigma G\right\rangle \\ &= \Delta_D \pm 2(\Delta_B - \Delta_D) \\ &\quad -(2m_c)^{-1}4\pi\alpha_s\langle\bar{c}\gamma_\mu t^a c\bar{q}\gamma_\mu t^a q\rangle.\end{aligned}$$

Using factorization for the O_6 term above, and the same values for the parameters as above, we get $+0.01$ for the contribution of O_6 so that

$$\mu_G^2 = \Delta_D \pm 2(\Delta_B - \Delta_D) = 0.42 \pm 0.08\ \text{GeV}^2\ .$$

As for μ_π^2, we will assume that the error bars in Eq. (10) give the estimate of the $1/m_c$ part in μ_π^2. Moreover, it was shown previously that the sign of the $1/m_c$ correction in μ_π^2 is negative (see Eqs. (41) and (42) in Ref. [26]).

D. Factorization and estimate of the matrix element

We must estimate the matrix elements of the transition operator over the D-meson state. As mentioned above, to this end we use the factorization procedure. We realize, of course, that it is not exact—deviations from factorization definitely exist; still it seems safe to say that it gives a reasonable estimate of the four-fermion operators, especially as far as the signs are concerned.

The relevant operators are first rearranged, through the Fierz transformations, into the form $\bar{c}\Gamma_\mu q\bar{q}\Gamma^\mu c$, with appropriate coefficients. Then the matrix elements of the latter operators are found by saturating with the vacuum intermediate state:

$$\langle D|\bar{c}\Gamma_\alpha q\bar{q}\Gamma^\alpha c|D\rangle = f_D^2 M_D^2\ , \tag{24}$$

$$\langle D|\bar{c}\Gamma_\alpha t^a q\bar{q}\Gamma^\alpha t^a c|D\rangle = 0\ . \tag{25}$$

Here we used the definition

$$\langle 0|\bar{q}\Gamma_\alpha c|D\rangle = if_D p_\alpha\ .$$

In Eq. (25) we accounted for the fact that the matrix element of the color current between the vacuum and D meson is zero.

The $L-R$ chiral structure in Eq. (20) is crucial. We face here an exact analogy with the usual penguin diagrams. Indeed, if we use factorization while estimating the relevant four-quark matrix elements, we see that the contribution of the left-handed light-quark current is zero, so we can use Eq. (16). The corresponding result is that of the factorized transition operator of Eq. (20).

Let us parenthetically note that the penguin diagrams we obtain here have no relation to the penguin operators of the type

$$(\bar{c}t^a s)_L \sum_f (\bar{q}_f t^a q_f)_{L+R}$$

contributing to charm nonleptonic decays. Although our penguin diagrams look similar, their origin is completely different from the usual ones.

After summing up all new effects, i.e., the effects coming from both the expansion of $\hat{T}$ and the uncertainty of O_G and O_π we get, pushing things to the high side,

$$\frac{\Delta\Gamma}{\Gamma} = -0.06 \pm 0.06 + 0.03; \tag{26}$$

here, the first number is due to the four-quark terms in the transition operator, the second is due to the uncertainty of O_G, and the third is due to O_π.

According to our estimates, the uncertainty in O_G and O_π is enough to possibly make the total contribution of $O(1/m_c^3)$ roughly zero. The sign of the O_G corrections is

undetermined, however, and a minus sign gives a result which worsens the agreement with experiment.

IV. WAYS OUT OF THE PROBLEM

We saw in the previous section that contrary to all the hopes, the $1/m_c^3$ contribution to the inclusive semileptonic width, however exotic it is, does not solve the problem of the deficit of the semileptonic inclusive width. There are several possibilities which might explain why the general heavy quark expansion fails to reproduce the experimental width.

First, the factorization that we used while estimating matrix elements can be suspected. However, the corrections to factorization can be estimated using the method of Ref. [35] and they seem small.

Second, a possibility exists that operators of dimension 7 are important. In principle, this may happen since the expansion parameter is $\sim \sqrt{\mu_\pi^2}/m_c \sim 0.7$ and is of order unity. However, since the correction due to the dimension-6 operators is roughly only 10% it seems unlikely that the dimension-7 contribution will dominate.

At the moment it seems most probable to us that the discrepancy demonstrated above is explained by the fact that the family of charm lies below the duality domain. The violations of the quark-hadron duality can be viewed as a cumulative effect of all high-dimension operators, taken together. Let us elucidate this assertion in more detail.

Constructing the transition operator as an expansion in m_c^{-1} we rely on the Wilson operator product expansion. OPE is well formulated in the Euclidean domain where all field fluctuations can be classified as short distance and long distance. Even in the Euclidean domain the divergence of the nonperturbative series in $1/m_c$ in high orders produces exponential terms of the type $\exp(-m_c^\gamma)$ which are not seen to any finite order in the expansion [12]. To get a rough idea of these terms one has to invoke instantons or similar model considerations. From the QCD sum rules it is known [20] that these terms are essentially unimportant in the Euclidean domain till surprisingly low off-shellness.

Kinematics of the problem at hand is essentially Minkowskian since we have to take the imaginary part of the transition operator at the very end. One justifies an OPE-based procedure by keeping in mind an analytical continuation. In the problem of the semileptonic width this may be a continuation in the momentum of the lepton pair [3]—one considers the transition operator at such momenta that one is actually off the cuts corresponding to production of the hadronic states, in the Euclidean domain. The prediction on the cuts is made by invoking dispersion relations, in full analogy with what is usually done in the problem of the total hadronic cross section in the e^+e^- annihilation. In general, one can analytically continue in some auxiliary momenta which has nothing to do with any of the physical momenta. This becomes the only option, say, in the problem of inclusive nonleptonic widths.

Whatever analytic continuation is done, strictly speaking the prediction for each given term in $1/m_c$ expansion refers to the Euclidean domain and is translated to the Minkowski domain only in the sense of averaging which occurs automatically through the dispersion relations. If the integrand is smooth, however, we can forget about the averaging, because in this case smearing is not needed. This is what happens, in particular, with the total hadronic cross section in the e^+e^- annihilation at high energies—the quark-hadron duality sets in and the OPE-based consideration yields the value of the cross section at a given energy, locally (without smearing). At what energy release is the integrand smooth and can the terms in the $1/m_c$ expansion be predicted locally? The existing theory gives no answer to this question. It may well happen that at a given (Minkowskian) energy E the deviations from duality falloff only as a power of $1/E$. Such a regime takes place, for instance, in a model discussed in Ref. [36]. This model is definitely relevant for the large N_c limit. It seems more likely, however, that in the real QCD, with $N_c = 3$, the violations of duality falloff exponentially, $\exp[-(E/E_0)^\gamma]$, and the rate of this fall off is correlated with the divergence of the high-order terms in the power series [12].

Very little is known about this aspect of QCD at present; the issue definitely deserves further study which is clearly beyond the scope of the present paper devoted to an applied problem. The genuinely theoretical approach would require determining the rate of the divergence of the high-order terms in the power expansion. Alas, we cannot do that, and the $1/m_c^3$ term found gives no hint on this divergence whatsoever.

In the absence of theoretical considerations we are forced to rely on phenomenological information. The total hadronic width in τ decays is a problem close in essence to that considered here. Moreover, the τ mass is only slightly higher than m_c. A detailed QCD-based analysis of τ decays has been carried out in Ref. [37], and the discussion of the results of Ref. [37] in the context the issue of duality is given in [36]. There are good reasons to believe that the deviation from duality in the τ hadronic width is at the level of 7%. Is it then reasonable to expect that descending from m_τ to m_c we get a deviation at the level of factor of 1.5 or 2?

The fact that the onset of duality is not universal, generally speaking, and depends on the channel considered, is known for a long time [38]. Where specifically the difference lies between $\tau \to \nu X$ and $D \to l\nu X_s$ (apart from the obvious difference, $m_c/m_\tau \sim 0.7$) remains to be found.

Finally, let us emphasize that all attempts to determine parameters of QCD or HQET from the analysis of the heavy quark expansion in the charm family must be viewed with extreme caution and are hardly reliable in view of the uncontrollable theoretical situation discussed above.

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APPENDIX

In this appendix we discuss the calculation of the transition operator associated with Fig. 2(b). We use the Fock-Schwinger gauge $x^\mu A^a_\mu = 0$ and the background field method (see Ref. [32] for details). We shall need the part of the propagator $S(x,0)$, see Eq. (17), for the quark with mass m in external gluon field that contains odd number of γ matrices and is proportional to DG. Note that in the limit $m \to 0$ the propagator becomes singular:

$$\Delta S \to \ln[m\sqrt{(-x^2)}] \ . \tag{A1}$$

The contribution of the diagram of Fig. 2(b) to the transition operator is equal to

$$\hat{T}_{\text{Fig.2(b)}} = \text{Im}|V_{cs}|^2 \frac{G_F^2}{2} \int e^{ipx} \bar{c}\Gamma_\mu \Delta S(x,0)\Gamma_\nu c(0) \times(\text{tr}[S(0,x)\Gamma^\nu S(x,0)\Gamma^\mu]) \ , \tag{A2}$$

where the trace term in the brackets represents the lepton loop, and ΔS stands for the DG part of the s-quark Green's function. Here $S(x,0)$ is the free massless quark propagator:

$$S(x,0) = \hat{x}/(2\pi^2 x^4) \ . \tag{A3}$$

The direct calculation uses the formulas of Ref. [39] for the imaginary part of the Fourier transform of the integrals of the type

$$\int [K_n(m\sqrt{(-x^2)})/(-x^2)^p] d^4x e^{ipx} . \tag{A4}$$

We just write Eq. (A1) explicitly in x space using the propagators from Eqs. (17) and (A3) and then use the formulas of Ref. [39] to convert the resulting expression into the imaginary part of the integral in Eq. (A2). We then see that the diagram of Fig. 2(b) gives the following contribution to Im $\hat{T}$:

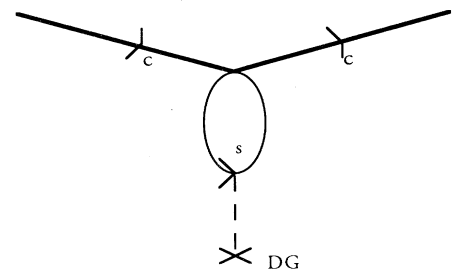

FIG. 4. Subtraction of the infrared part from the coefficient of the operator $\bar{c}DGc$, see Fig. 2(b).

$$\frac{-|V_{cs}|^2 G_F^2}{3^2 2^5 \pi^3} (p^2 g^{\nu\beta} + 2p^\nu p^\beta)\bar{c}(0)\gamma_\nu(1+\gamma_5)D^\alpha G_{\alpha\beta}c(0) \times[\ln(m_c^2/\Lambda^2) - 5/6] \ . \tag{A5}$$

We now go the reference frame connected with the center of mass of the heavy quark where $p^\nu = p^\beta = m_c(1,0,0,0)$. Note also that, for a heavy c quark,

$$\bar{c}\gamma_0 D_\alpha G_{\alpha 0} c = \bar{c}(0)\gamma_\beta D_\alpha G^{\alpha\beta} c \ . \tag{A6}$$

Equation (A6) follows from the fact that $\bar{c}\gamma_i c = O(1/m_c)$. We finally obtain the following "piece" of the transition operator:

$$\text{Im}\hat{T}_{\text{Fig.2(b)}} = G_F^2 |V_{cs}|^2 \frac{\alpha_s}{72\pi^2} m_c^2 [\ln(m_c^2/\Lambda^2) - 5/6] \times 2(\bar{c}\Gamma_\beta t^a c + \bar{c}\gamma_\beta t^a c)\bar{q}\gamma_\beta t^a q \ . \tag{A7}$$

The logarithmic term comes from the singularity, Eq. (A1). This is not all however. Part of the contribution to the transition operator of Eq. (A7) comes from the infrared domain, and its contribution has nothing to do with the contribution we are interested in. The infrared contribution is given by the contracted loop of Fig. 4. It is easily calculated, giving the contribution

$$\text{Im } \hat{T}_{\text{Fig.4}} = G_F^2 |V_{cs}|^2 \frac{\alpha_s}{72\pi^2} m_c^2 [\ln(\mu^2/\Lambda^2) - 3/2] \times(\bar{c}\Gamma_\beta t^a c + \bar{c}\gamma_\beta t^a c)\bar{q}\gamma_\beta t^a q \ . \tag{A8}$$

Subtracting Im $\hat{T}_{\text{Fig.4}}$ from Im $\hat{T}_{\text{Fig.2(b)}}$ we get the transition operator Im$\hat{T}_{DG}$ of Eq. (20).

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