

## Top quark inclusive differential distributions

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The inclusive transverse momentum and rapidity distributions for top quark production at the Fermilab Tevatron are presented both in order  $\alpha_s^3$  in QCD and using the resummation of the leading soft gluon corrections in all orders of QCD perturbation theory. The resummed results are uniformly larger than the  $O(\alpha_s^3)$  results for both distributions.

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### I. INTRODUCTION

At present the top quark has not yet been conclusively discovered at the Fermilab Tevatron, even though there are events which look similar to those expected from top quark decays. The experimental situation is presently confusing because both the Collider Detector at Fermilab (CDF) [1] and D0 [2] Collaborations only have limited statistics. As more events are collected one expects that the situation will be clarified.

At the Tevatron, the top quark should be mainly produced through  $t\bar{t}$  pair production from the light mass quarks and gluons in the colliding proton and antiproton. Both the top quark and the top antiquark then decay to  $(W, b)$  pairs, and each  $W$  boson can decay either hadronically or leptonically. The  $b$  quark becomes an on-mass-shell  $B$  hadron which subsequently decays into leptons and (charmed) hadrons. A large effort is being made to reconstruct the top quark mass from the measured particles in the decay, which is complicated by the fact that the neutrinos are never detected. Also there are additional jets so it is not clear which ones to choose to recombine [3,4]. The best channel for this mass reconstruction is where both  $W$  bosons decay leptonically, one to a  $(e, \nu_e)$  pair, the other to a  $(\mu, \nu_\mu)$  pair (a dilepton event) because the backgrounds in this channel are small. When only a single lepton is detected then it is necessary to identify the  $b$  quark in the decay to remove large backgrounds from the production of  $W$  + jets [5]. In all cases the reconstruction of the particles in the final state involves both the details of the production of the top-quark-antiquark pair as well as the knowledge of their fragmentation and decay products. The CDF Collaboration [1] has reported two events with dilepton final states, six events with a single lepton and a  $b$  quark identified by a secondary vertex, and seven single lepton events with the  $b$  quark identified by a semileptonic decay. The CDF Collaboration then constructed a likelihood function for the invariant mass [6] and quoted the value  $m_{\text{top}} = 174 \pm 10_{-12}^{+13}$  GeV/ $c^2$ . The top quark cross section quoted by the CDF Collaboration is  $13.9_{-4.8}^{+6.1}$  pb. The D0 Collaboration [2] have reported on nine events with an expected background of  $3.8 \pm 0.9$ . If the excess is due to  $t\bar{t}$  production and if the top quark mass is 180 GeV/ $c^2$ , then the top quark cross section is  $8.2 \pm 5.1$  pb.

The values for the top quark production cross section as a function of the top quark mass used by the CDF and D0 Collaborations contain both the next to leading order (NLO) QCD corrections [7,8] and an extension to include the resummation of initial state soft partons to all orders in perturbation theory [9]. A recent summary of the theoretical predictions has been presented in [10]. In the paper of Laenen *et al.* [9] the deep inelastic scattering (DIS) factorization scheme was used with the Martin-Roberts-Stirling set  $D'_-$  (MRSD'\_-) parton distributions [11], the two-loop running coupling constant with five active flavors, and  $\Lambda_{\text{QCD}} = 0.152$  GeV.

In the analysis of the decay distributions one needs knowledge of the inclusive differential distributions of the heavy quarks in transverse momentum  $p_t$  and rapidity  $y$ . These distributions are known in NLO [12,13]. What we would like to discuss in this paper is an updating of the resummation effects on the inclusive transverse momentum distribution of the top quark. In the original paper [14] it was not known which mass to choose whereas now we can assume that the mass is 175 GeV/ $c^2$ . Also we discuss here how the resummation effects modify the rapidity distribution of the top quark. Since there have been suggestions of using the mass and angular distributions in top quark production to look for physics beyond the standard model [15] it is very important to know the normal QCD predictions for these quantities. Recently, a paper appeared with a study of the difference between top quark production and the normal QCD background [16]. In particular, the authors considered the asymmetry in the charged-lepton rapidity distribution.

We first summarize what is known on the top quark cross section. If the top quark mass is 175 GeV/ $c^2$  then the dominant production channel is  $q + \bar{q} \rightarrow t + \bar{t}$ . In lowest order QCD perturbation theory it contributes about 90% of the total cross section with the reaction  $g + g \rightarrow t + \bar{t}$  making up the remaining 10%. One notes that the NLO corrections in the  $q\bar{q}$  channel are small, whereas those in the  $gg$  channel are more than 80%. At this large top quark mass the  $qg$  and  $\bar{q}g$  channels give negligible contributions so we do not consider them. Even though the  $gg$  channel contribution is small in Born approximation it can be significant in NLO due to multiple soft parton radiation. These large corrections are predominantly from the threshold region for heavy quark

production. It was shown previously [17] that initial state gluon bremsstrahlung (ISGB) is responsible for the large corrections at NLO near threshold.

In [14] the dominant logarithms from ISGB, which are the cause of the large corrections near threshold, were carefully examined. Such logarithms have been studied previously in Drell-Yan (DY) [18] production at fixed target energies (again near threshold) where they are responsible for correspondingly large corrections. The analogy between DY and heavy quark production cross sections was exploited in [14] and a formula to resum the leading and next-to-leading logarithms in perturbative QCD (PQCD) to all orders was proposed. Since the contributions due to these logarithms are positive (when all scales  $\mu$  are set equal to the heavy quark mass  $m$ ), the effect of summing the higher order corrections increases the top quark production cross section over that predicted in  $O(\alpha_s^3)$ . This sum, which will be identified as  $\sigma_{\text{res}}$ , depends on a nonperturbative parameter  $\mu_0$ . The reason that a new parameter has to be introduced is that the resummation is sensitive to the scale at which PQCD breaks down. As we approach the threshold region other, nonperturbative, physics plays a role (higher twist, bound states, etc.) indicated by a dramatic increase in  $\alpha_s$  and in the resummed cross section. This is commonly called the effect of the infrared renormalon or Landau pole [19]. We chose to simply cut off the resummation at a specific scale  $\mu_0$  where  $\Lambda_{\text{QCD}} \ll \mu_0 \ll m$  since it is not obvious how to incorporate the nonperturbative effects. Note that our resummed corrections diverge for small  $\mu_0$  but this is *not* physical since they should be joined smoothly onto some nonperturbative prescription and the total cross section will be finite. Another way to make it finite would be to avoid the infrared renormalon by a specific continuation around it, i.e., the principal value resummation method [20,21]. However, at the moment our total resummed corrections depend on the parameter  $\mu_0$  for which we can only make a rough estimate. See [14] for more details.

## II. SOFT GLUON APPROXIMATION TO THE INCLUSIVE DISTRIBUTIONS

To make this paper self-contained we list some relevant formulas. The partonic processes under discussion will be denoted by

$$i(k_1) + j(k_2) \rightarrow Q(p_1) + \bar{Q}(p_2) , \quad (2.1)$$

where  $i, j = g, q, \bar{q}$ . The kinematical variables

$$s = (k_1 + k_2)^2, \quad t_1 = (k_2 - p_2)^2 - m^2 , \quad (2.2)$$

$$u_1 = (k_1 - p_2)^2 - m^2$$

are introduced in the calculation of the corrections to the single particle inclusive differential distributions of the heavy (anti)quark. We do not distinguish in the text between the heavy quark and heavy antiquark since the distributions are essentially identical in our calculations. Here  $s$  is the square of the parton-parton c.m. energy and the heavy quark transverse momentum is given by  $p_t = (t_1 u_1 / s - m^2)^{1/2}$ . The rapidity variable is defined by  $\exp(2y) = u_1 / t_1$ . The Born approximation differential cross sections can be expressed by

$$s^2 \frac{d^2 \sigma_{ij}^{(0)}(s, t_1, u_1)}{dt_1 du_1} = \delta(s + t_1 + u_1) \sigma_{ij}^B(s, t_1, u_1) , \quad (2.3)$$

with

$$\sigma_{q\bar{q}}^B(s, t_1, u_1) = \pi \alpha_s^2(\mu^2) K_{q\bar{q}} N C_F \left( \frac{t_1^2 + u_1^2}{s^2} + \frac{2m^2}{s} \right) \quad (2.4)$$

and

$$\begin{aligned} \sigma_{gg}^B(s, t_1, u_1) &= 2\pi \alpha_s^2(\mu^2) K_{gg} N C_F \left( C_F - C_A \frac{t_1 u_1}{s^2} \right) \\ &\times \left[ \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2 s}{t_1 u_1} \left( 1 - \frac{m^2 s}{t_1 u_1} \right) \right] . \end{aligned} \quad (2.5)$$

Here the color factors are  $C_A = N$  and  $C_F = (N^2 - 1)/(2N)$ . The color average factors are  $K_{q\bar{q}} = N^{-2}$  and  $K_{gg} = (N^2 - 1)^{-2}$ . The parameter  $\mu$  denotes the renormalization scale. In [14] the inclusive cross section was examined near threshold ( $s \approx 4m^2$ ) where the contributions from the radiation of soft and collinear gluons are large. A variable  $s_4 = s + t_1 + u_1$  was defined, where  $t_1 = (k_2 - p_2)^2 - m^2$  and  $u_1 = (k_1 - p_2)^2 - m^2$  are inelastic variables in the channel  $i(k_1) + j(k_2) \rightarrow Q(p_1) + \bar{Q}(p_2) + g(k_3)$ . The variable  $s_4 > 0$  now depends on the four-momentum of the extra parton(s) emitted in the reaction. In the Born approximation there are no additional partons so  $s_4 = 0$ . In [14] the NLO contributions were examined in the soft region (where  $s_4 \rightarrow 0$ ) and it was found that the dominant contribution to the NLO cross section in this region had a similar form to the NLO correction in the Drell-Yan process. As the latter correction is known exactly in NNLO [22] this correspondence was used to write the differential cross section in order  $\alpha_s^k(\mu^2)$  as

$$s^2 \frac{d^2 \sigma_{ij}^{(k)}(s, t_1, u_1)}{dt_1 du_1} = \alpha_s^k(\mu^2) \sum_{l=0}^{2k-1} \left[ \frac{1}{s_4} a_l(\mu^2) \ln^l \left( \frac{s_4}{m^2} \right) \theta(s_4 - \Delta) + \frac{1}{l+1} a_l(\mu^2) \ln^{l+1} \left( \frac{\Delta}{m^2} \right) \delta(s_4) \right] \sigma_{ij}^B(s, t_1, u_1) . \quad (2.6)$$

Here a small parameter  $\Delta$  has been introduced to allow us to distinguish between the soft ( $s_4 < \Delta$ ) and the hard ( $s_4 > \Delta$ ) regions in phase space. The quantities  $a_l(\mu^2)$  contain terms involving the QCD  $\beta$  functions and color factors. The variables  $t_1$  and  $u_1$  were then mapped onto the variables  $s_4$  and  $\cos\theta$ , where  $\theta$  is the parton-parton c.m. scattering

angle. After explicit integration over the angle  $\theta$ , the resulting series was exponentiated by the introduction of the  $s_4$  variable into the argument of the running coupling constant.

As noted in the previous paper [14] in addition to the total cross section we can also derive the resummed heavy (anti)quark inclusive  $p_t$  (and below the  $y$ ) distributions. The transverse momentum  $p_t$  of the heavy quark is related to our previous variables by

$$t_1 = -\frac{1}{2}\{s - s_4 - [(s - s_4)^2 - 4sm_t^2]^{1/2}\}, \quad (2.7)$$

$$u_1 = -\frac{1}{2}\{s - s_4 + [(s - s_4)^2 - 4sm_t^2]^{1/2}\}, \quad (2.8)$$

with  $m_t^2 = m^2 + p_t^2$ . The double differential cross section is, therefore,

$$s^2 \frac{d^2\sigma_{ij}(s, t_1, u_1)}{dt_1 du_1} = s[(s - s_4)^2 - 4sm_t^2]^{1/2} \frac{d^2\sigma_{ij}(s, s_4, p_t^2)}{dp_t^2 ds_4}, \quad (2.9)$$

with the boundaries

$$0 < p_t^2 < \frac{s}{4} - m^2, \quad 0 < s_4 < s - 2m_t\sqrt{s}. \quad (2.10)$$

The  $O(\alpha_s^k)$  contribution to the inclusive transverse momentum distribution  $d\sigma_{ij}/dp_t^2$  is given by

$$\begin{aligned} \frac{d\sigma_{ij}^{(k)}(s, p_t^2)}{dp_t^2} &= \frac{2}{s} \alpha_s^k(\mu^2) \sum_{l=0}^{2k-1} a_l(\mu^2) \int_0^{s-2m_t s^{1/2}} ds_4 \left[ \frac{1}{s_4} \ln^l \left( \frac{s_4}{m^2} \right) \theta(s_4 - \Delta) + \frac{1}{l+1} \ln^{l+1} \left( \frac{\Delta}{m^2} \right) \delta(s_4) \right] \\ &\times \frac{1}{[(s - s_4)^2 - 4sm_t^2]^{1/2}} \sigma_{ij}^B(s, s_4, p_t^2), \end{aligned} \quad (2.11)$$

where we have inserted an extra factor of 2 so that  $\int dp_t^2 d\sigma/dp_t^2 = \sigma_{\text{tot}}$ . After some algebra we can rewrite this result as

$$\begin{aligned} \frac{d\sigma_{ij}^{(k)}(s, p_t^2)}{dp_t^2} &= \alpha_s^k(\mu^2) \sum_{l=0}^{2k-1} a_l(\mu^2) \left[ \int_0^{s-2m_t s^{1/2}} ds_4 \frac{1}{s_4} \ln^l \frac{s_4}{m^2} \left( \frac{d\bar{\sigma}_{ij}^{(0)}(s, s_4, p_t^2)}{dp_t^2} - \frac{d\bar{\sigma}_{ij}^{(0)}(s, 0, p_t^2)}{dp_t^2} \right) \right. \\ &\left. + \frac{1}{l+1} \ln^{l+1} \left( \frac{s - 2m_t s^{1/2}}{m^2} \right) \frac{d\bar{\sigma}_{ij}^{(0)}(s, 0, p_t^2)}{dp_t^2} \right], \end{aligned} \quad (2.12)$$

with the definition

$$\frac{d\bar{\sigma}_{ij}^{(0)}(s, s_4, p_t^2)}{dp_t^2} = \frac{2}{s[(s - s_4)^2 - 4sm_t^2]^{1/2}} \sigma_{ij}^B(s, s_4, p_t^2), \quad (2.13)$$

where  $d\bar{\sigma}_{ij}^{(0)}(s, 0, p_t^2)/dp_t^2 \equiv d\sigma_{ij}^{(0)}(s, p_t^2)/dp_t^2$  again represents the Born differential  $p_t$  distribution. For the  $q\bar{q}$  and  $gg$  subprocesses we have the explicit results

$$\frac{d\bar{\sigma}_{q\bar{q}}^{(0)}(s, s_4, p_t^2)}{dp_t^2} = 2\pi\alpha_s^2(\mu^2) K_{q\bar{q}} N C_F \frac{1}{s} \frac{1}{[(s - s_4)^2 - 4sm_t^2]^{1/2}} \left( \frac{(s - s_4)^2 - 2sp_t^2}{s^2} \right) \quad (2.14)$$

and

$$\begin{aligned} \frac{d\bar{\sigma}_{gg}^{(0)}(s, s_4, p_t^2)}{dp_t^2} &= 4\pi\alpha_s^2(\mu^2) K_{gg} N C_F \frac{1}{s} \frac{1}{[(s - s_4)^2 - 4sm_t^2]^{1/2}} \left( C_F - C_A \frac{m_t^2}{s} \right) \\ &\times \left[ \frac{(s - s_4)^2 - 2sm_t^2}{sm_t^2} + \frac{4m^2}{m_t^2} \left( 1 - \frac{m^2}{m_t^2} \right) \right]. \end{aligned} \quad (2.15)$$

Since the above formulas are symmetric under the interchange  $t_1 \leftrightarrow u_1$  the heavy quark and heavy antiquark inclusive  $p_t$  distributions are identical. Note that (2.12) is basically the integral of a plus distribution together with a surface term.

The corresponding formula to (2.12) for the rapidity  $y$  of the heavy quark is obtained by using

$$t_1 = -\frac{s-s_4}{2}(1-\tanh y), \quad (2.16)$$

$$u_1 = -\frac{s-s_4}{2}(1+\tanh y). \quad (2.17)$$

The double differential cross section is therefore

$$s^2 \frac{d^2 \sigma_{ij}(s, t_1, u_1)}{dt_1 du_1} = 2s^2 \frac{\cosh^2 y}{s-s_4} \frac{d^2 \sigma_{ij}(s, s_4, y)}{dy ds_4}, \quad (2.18)$$

with the boundaries

$$-\frac{1}{2} \ln \left( \frac{1+\beta}{1-\beta} \right) < y < \frac{1}{2} \ln \left( \frac{1+\beta}{1-\beta} \right), \quad 0 < s_4 < s - 2\sqrt{s} m \cosh y, \quad (2.19)$$

where  $\beta^2 = 1 - 4m^2/s$ . The  $O(\alpha_s^k)$  contribution to the inclusive rapidity distribution  $d\sigma_{ij}/dy$  is given by

$$\begin{aligned} \frac{d\sigma_{ij}^{(k)}(s, y)}{dy} &= \alpha_s^k(\mu^2) \sum_{l=0}^{2k-1} a_l(\mu^2) \int_0^{s-2ms^{1/2}\cosh y} ds_4 \left[ \frac{1}{s_4} \ln^l \left( \frac{s_4}{m^2} \right) \theta(s_4 - \Delta) + \frac{1}{l+1} \ln^{l+1} \left( \frac{\Delta}{m^2} \right) \delta(s_4) \right] \\ &\times \left( \frac{s-s_4}{2s^2 \cosh^2 y} \right) \sigma_{ij}^B(s, s_4, y). \end{aligned} \quad (2.20)$$

After some algebra we can write this result as

$$\begin{aligned} \frac{d\sigma_{ij}^{(k)}(s, y)}{dy} &= \alpha_s^k(\mu^2) \sum_{l=0}^{2k-1} a_l(\mu^2) \left[ \int_0^{s-2ms^{1/2}\cosh y} ds_4 \frac{1}{s_4} \ln^l \left( \frac{s_4}{m^2} \right) \left( \frac{d\bar{\sigma}_{ij}^{(0)}(s, s_4, y)}{dy} - \frac{d\bar{\sigma}_{ij}^{(0)}(s, 0, y)}{dy} \right) \right. \\ &\left. + \frac{1}{l+1} \ln^{l+1} \left( \frac{s-2ms^{1/2}\cosh y}{m^2} \right) \frac{d\bar{\sigma}_{ij}^{(0)}(s, 0, y)}{dy} \right], \end{aligned} \quad (2.21)$$

with the definition

$$\frac{d\bar{\sigma}_{ij}^{(0)}(s, s_4, y)}{dy} = \frac{s-s_4}{2s^2 \cosh^2 y} \sigma_{ij}^B(s, s_4, y), \quad (2.22)$$

where  $d\bar{\sigma}_{ij}^{(0)}(s, 0, y)/dy \equiv d\sigma_{ij}^{(0)}(s, y)/dy$  again represents the Born differential  $y$  distribution. For the  $q\bar{q}$  and  $gg$  subprocesses we have the explicit formulas

$$\frac{d\bar{\sigma}_{q\bar{q}}^{(0)}(s, s_4, y)}{dy} = \pi \alpha_s^2(\mu^2) K_{q\bar{q}} N C_F \frac{s-s_4}{2s^2 \cosh^2 y} \left( \frac{(s-s_4)^2}{2s^2 \cosh^2 y} (\cosh^2 y + \sinh^2 y) + \frac{2m^2}{s} \right) \quad (2.23)$$

and

$$\begin{aligned} \frac{d\bar{\sigma}_{gg}^{(0)}(s, s_4, y)}{dy} &= 4\pi \alpha_s^2(\mu^2) K_{gg} N C_F \frac{s-s_4}{2s^2 \cosh^2 y} \\ &\times \left( C_F - C_A \frac{(s-s_4)^2}{4s^2 \cosh^2 y} \right) \left[ \cosh^2 y + \sinh^2 y + \frac{8m^2 s \cosh^2 y}{(s-s_4)^2} \left( 1 - \frac{4m^2 s \cosh^2 y}{(s-s_4)^2} \right) \right]. \end{aligned} \quad (2.24)$$

Since the above formulas are symmetric under the interchange  $t_1 \leftrightarrow u_1$  the heavy quark and heavy antiquark inclusive  $y$  distributions are identical. Also (2.21) is again of the form of a plus distribution together with a surface term. Finally, we note that the terms in (2.12) and (2.21) are all finite.

### III. RESUMMATION PROCEDURE IN PARTON-PARTON COLLISIONS

The resummed contribution to the top quark cross section can be written as [14]

$$s^2 \frac{d^2 \sigma_{ij}(s, t_1, u_1)}{dt_1 du_1} = \left[ \frac{df(s_4/m^2, m^2/\mu^2)}{ds_4} \theta(s_4 - \Delta) + f\left(\frac{\Delta}{m^2}, \frac{m^2}{\mu^2}\right) \delta(s_4) \right] \sigma_{ij}^B(s, t_1, u_1), \quad (3.1)$$

where

$$f\left(\frac{s_4}{m^2}, \frac{m^2}{\mu^2}\right) = \exp\left[A \frac{C_{ij}}{\pi} \bar{\alpha}_s\left(\frac{s_4}{m^2}, m^2\right) \ln^2 \frac{s_4}{m^2}\right] \frac{[s_4/m^2]^\eta}{\Gamma(1+\eta)} \exp(-\eta\gamma_E). \quad (3.2)$$

The straightforward expansion of the exponential plus the change of the argument in  $\bar{\alpha}_s$  via the renormalization-group equations generates the corresponding leading logarithmic terms written explicitly in [14]. The scheme-dependent  $A$  and  $\bar{\alpha}_s$  in the above expression are given by

$$A = 2, \quad \bar{\alpha}_s(y, \mu^2) = \alpha_s(y^{2/3}\mu^2) = \frac{4\pi}{\beta_0 \ln(y^{2/3}\mu^2/\Lambda^2)}, \quad (3.3)$$

in the modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme, and

$$A = 1, \quad \bar{\alpha}_s(y, \mu^2) = \alpha_s(y\mu^2) = \frac{4\pi}{\beta_0 \ln(y\mu^2/\Lambda^2)}, \quad (3.4)$$

in the DIS scheme, where  $\beta_0 = 11/3C_A - 2/3n_f$  is the lowest order coefficient of the QCD  $\beta$  function. The color factors  $C_{ij}$  are defined by  $C_{q\bar{q}} = C_F$  and  $C_{gg} = C_A$ , and  $\gamma_E$  is the Euler constant. The quantity  $\eta$  is given by

$$\eta = \frac{8C_{ij}}{\beta_0} \ln\left(1 + \beta_0 \frac{\alpha_s(\mu^2)}{4\pi} \ln \frac{m^2}{\mu^2}\right). \quad (3.5)$$

Following the procedure in [14] for the resummation of the order  $\alpha_s^k$  contributions to the  $p_t$  distribution we have

$$\begin{aligned} \frac{d\sigma_{ij}(s, p_t^2)}{dp_t^2} &= \sum_{k=0}^{\infty} \frac{d\sigma_{ij}^{(k)}(s, p_t^2)}{dp_t^2} \\ &= \int_{s_0}^{s-2m_t s^{1/2}} ds_4 \frac{df(s_4/m^2, m^2/\mu^2)}{ds_4} \left( \frac{d\bar{\sigma}_{ij}^{(0)}(s, s_4, p_t^2)}{dp_t^2} - \frac{d\bar{\sigma}_{ij}^{(0)}(s, 0, p_t^2)}{dp_t^2} \right) \\ &\quad + f\left(\frac{s-2m_t s^{1/2}}{m^2}, \frac{m^2}{\mu^2}\right) \frac{d\sigma_{ij}^{(0)}(s, p_t^2)}{dp_t^2}. \end{aligned} \quad (3.6)$$

Note that we now have cut off the lower limit of the  $s_4$  integration at  $s_4 = s_0$  because  $\bar{\alpha}_s$  in (3.2) diverges as  $s_4 \rightarrow 0$ . This parameter  $s_0$  must satisfy the conditions  $0 < s_0 < s - 2m_t s^{1/2}$  and  $s_0/m^2 \ll 1$ . It is convenient to rewrite  $s_0$  in terms of the scale  $\mu$  as

$$\frac{s_0}{m^2} = \left(\frac{\mu_0^2}{\mu^2}\right)^{3/2} \quad (\overline{\text{MS}} \text{ scheme}), \quad (3.7a)$$

$$\frac{s_0}{m^2} = \frac{\mu_0^2}{\mu^2} \quad (\text{DIS scheme}). \quad (3.7b)$$

Here  $\mu_0$  is a nonperturbative parameter [14] satisfying  $\Lambda_{\text{QCD}}^2 \ll \mu_0^2 \ll \mu^2$ . The derivative of  $f(s_4/m^2, m^2/\mu^2)$  is obtained from (3.2). It is equal to

$$\begin{aligned} \frac{df(s_4/m^2, m^2/\mu^2)}{ds_4} &= \frac{1}{s_4} \left[ 2A \frac{C_{ij}}{\pi} \bar{\alpha}_s\left(\frac{s_4}{m^2}, m^2\right) \ln \frac{s_4}{m^2} + \eta \right] \exp\left[A \frac{C_{ij}}{\pi} \bar{\alpha}_s\left(\frac{s_4}{m^2}, m^2\right) \ln^2 \frac{s_4}{m^2}\right] \frac{[s_4/m^2]^\eta}{\Gamma(1+\eta)} \\ &\quad \times \exp(-\eta\gamma_E), \end{aligned} \quad (3.8)$$

where we have neglected terms which are higher order in  $\bar{\alpha}_s$ .

The analogous formula for the rapidity distribution is

$$\begin{aligned}
\frac{d\sigma_{ij}(s, y)}{dy} &= \sum_{k=0}^{\infty} \frac{d\sigma_{ij}^{(k)}(s, y)}{dy} \\
&= \int_{s_0}^{s-2ms^{1/2}\cosh y} ds_4 \frac{df(s_4/m^2, m^2/\mu^2)}{ds_4} \left( \frac{d\bar{\sigma}_{ij}^{(0)}(s, s_4, y)}{dy} - \frac{d\bar{\sigma}_{ij}^{(0)}(s, 0, y)}{dy} \right) \\
&\quad + f\left(\frac{s-2ms^{1/2}\cosh y}{m^2}, \frac{m^2}{\mu^2}\right) \frac{d\sigma_{ij}^{(0)}(s, y)}{dy}, \tag{3.9}
\end{aligned}$$

with the conditions  $0 < s_0 < s - 2ms^{1/2}\cosh y$  and  $s_0/m^2 \ll 1$ .

#### IV. NUMERICAL RESULTS

Following the notation in [14] the total hadron-hadron cross section in order  $\alpha_s^k$  is

$$\sigma_H^{(k)}(S, m^2) = \sum_{ij} \int_{4m^2/S}^1 d\tau \Phi_{ij}(\tau, \mu^2) \sigma_{ij}^{(k)}(\tau S, m^2, \mu^2), \tag{4.1}$$

where  $S$  is the square of the hadron-hadron c.m. energy and  $i, j$  run over  $q, \bar{q}$ , and  $g$ . The parton flux  $\Phi_{ij}(\tau, \mu^2)$  is defined via

$$\Phi_{ij}(\tau, \mu^2) = \int_{\tau}^1 \frac{dx}{x} H_{ij}\left(x, \frac{\tau}{x}, \mu^2\right), \tag{4.2}$$

and  $H_{ij}$  is a product of the scale-dependent parton distribution functions  $f_i^h(x, \mu^2)$ , where  $h$  stands for the hadron which is the source of the parton  $i$ :

$$H_{ij}(x_1, x_2, \mu^2) = f_i^{h_1}(x_1, \mu^2) f_j^{h_2}(x_2, \mu^2). \tag{4.3}$$

The mass factorization scale  $\mu$  is chosen to be identical with the renormalization scale in the running coupling constant. Since the  $p_t$  distribution in hadron-hadron collisions is not altered by the Lorentz transformation along the collision axis from the parton-parton c.m. frame, we can write an analogous formula to (4.1) for the heavy-quark inclusive differential distribution in  $p_t^2$ :

$$\begin{aligned}
&\frac{d\sigma_H^{(k)}(S, m^2, p_t^2)}{dp_t^2} \\
&= \sum_{ij} \int_{4m_t^2/S}^1 d\tau \Phi_{ij}(\tau, \mu^2) \frac{d\sigma_{ij}^{(k)}(\tau S, m^2, p_t^2, \mu^2)}{dp_t^2}, \tag{4.4}
\end{aligned}$$

with  $m_t^2 = m^2 + p_t^2$ . In the case of the all-order resummed expressions the lower boundaries in (4.1) and (4.4) have to be modified according to the conditions  $s_0 < s - 2ms^{1/2}$  or  $s_0 < s - 2m_t s^{1/2}$  (see above). Resumming the soft gluon contributions to all orders we obtain

$$\sigma_H^{\text{res}}(S, m^2) = \sum_{ij} \int_{\tau_0}^1 d\tau \Phi_{ij}(\tau, \mu^2) \sigma_{ij}(\tau S, m^2, \mu^2), \tag{4.5}$$

where  $\sigma_{ij}$  is given in (3.24) of [14] and

$$\tau_0 = \frac{[m + (m^2 + s_0)^{1/2}]^2}{S}, \tag{4.6}$$

with  $s_0 = m^2(\mu_0^2/\mu^2)^{3/2}$  ( $\overline{\text{MS}}$  scheme) or  $s_0 = m^2(\mu_0^2/\mu^2)$  (DIS scheme) [see (3.7)]. The all-order resummed differential distribution in  $p_t^2$  is

$$\begin{aligned}
&\frac{d\sigma_H^{\text{res}}(S, m^2, p_t^2)}{dp_t^2} \\
&= \sum_{ij} \int_{\tau_0}^1 d\tau \Phi_{ij}(\tau, \mu^2) \frac{d\sigma_{ij}(\tau S, m^2, p_t^2, \mu^2)}{dp_t^2}, \tag{4.7}
\end{aligned}$$

with  $d\sigma_{ij}/dp_t^2$  given in (3.6) and

$$\tau_0 = \frac{[m_t + (m_t^2 + s_0)^{1/2}]^2}{S}. \tag{4.8}$$

The corresponding formula to (4.4) for the heavy quark inclusive differential distribution in  $y$  is

$$\begin{aligned}
&\frac{d\sigma_H^{(k)}(S, m^2, y)}{dY} \\
&= \sum_{ij} \int_{4m^2 \cosh^2 y/S}^1 d\tau \Phi_{ij}(\tau, \mu^2) \frac{d\sigma_{ij}^{(k)}(\tau S, m^2, y, \mu^2)}{dy}. \tag{4.9}
\end{aligned}$$

Order by order in perturbation theory the heavy quark rapidity plots in the parton-parton c.m. frame show peaks away from  $y = 0$  (see Fig. 7 in [13]). However, upon folding with the partonic densities the heavy quark rapidities in the hadron-hadron c.m. frame peak near  $Y = 0$ . Therefore we will assume that the plots for the resummed rapidity distribution show a similar feature. The all-order resummed differential distribution in  $Y$  is therefore taken to be

$$\begin{aligned}
&\frac{d\sigma_H^{\text{res}}(S, m^2, Y)}{dY} \\
&= \sum_{ij} \int_{\tau_0}^1 d\tau \Phi_{ij}(\tau, \mu^2) \frac{d\sigma_{ij}(\tau S, m^2, y, \mu^2)}{dy}, \tag{4.10}
\end{aligned}$$

with  $d\sigma_{ij}/dy$  given in (3.9) and

$$\tau_0 = \frac{[m \cosh y + (m^2 \cosh^2 y + s_0)^{1/2}]^2}{S}. \quad (4.11)$$

The hadronic heavy quark rapidity  $Y$  is related to the partonic heavy quark rapidity  $y$  by

$$Y = y + \frac{1}{2} \ln \frac{x_1}{x_2}. \quad (4.12)$$

We now specialize to top quark production at the Fermilab Tevatron where  $\sqrt{S} = 1.8$  TeV and choose the top quark mass to be  $m = 175$  GeV/ $c^2$ . In the presentation of our results for the exact, approximate, and resummed hadronic cross sections we use the MRSD' parametrization for the parton distributions [11]. Note that the hadronic results only involve partonic distribution functions at moderate and large  $x$ , where there is little difference between the various sets of parton densities. We have used the MRSD' set 34 as given in PDFLIB [23] in the DIS scheme with the number of active light flavors  $n_f = 5$  and the QCD scale  $\Lambda_5 = 0.1559$  GeV. We have used the two-loop corrected running coupling constant as given by PDFLIB. Since we know the exact  $O(\alpha_s^3)$  result we can make an even better estimate of the differential distributions by calculating the perturbation theory improved  $p_t$  and  $y$  distributions. We define the improved  $p_t$  distribution by

$$\frac{d\sigma_H^{\text{imp}}}{dp_t} = \frac{d\sigma_H^{\text{res}}}{dp_t} + \left. \frac{d\sigma_H^{(1)}}{dp_t} \right|_{\text{exact}} - \left. \frac{d\sigma_H^{(1)}}{dp_t} \right|_{\text{app}}, \quad (4.13)$$

and the improved  $Y$  distribution by

$$\frac{d\sigma_H^{\text{imp}}}{dY} = \frac{d\sigma_H^{\text{res}}}{dY} + \left. \frac{d\sigma_H^{(1)}}{dY} \right|_{\text{exact}} - \left. \frac{d\sigma_H^{(1)}}{dY} \right|_{\text{app}}, \quad (4.14)$$

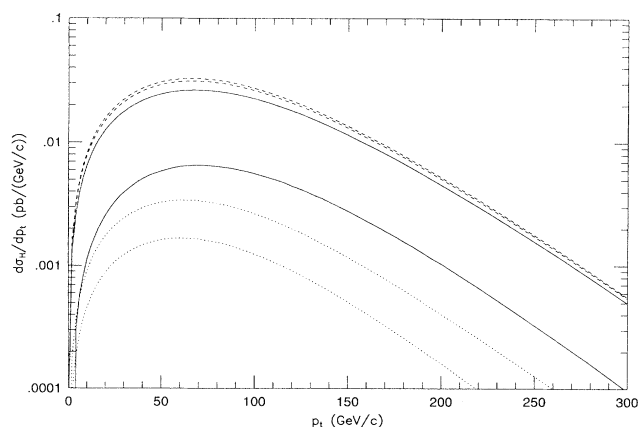


FIG. 1. The top quark  $p_t$  distributions  $d\sigma_H^{(k)}/dp_t$  for the  $q\bar{q}$  channel in the DIS scheme for a top quark mass  $m = 175$  GeV/ $c^2$ . Plotted are  $d\sigma_H^{(0)}/dp_t$  (upper solid line),  $d\sigma_H^{(1)}/dp_t|_{\text{exact}}$  (lower solid line),  $d\sigma_H^{(1)}/dp_t|_{\text{app}}$  (upper dotted line),  $d\sigma_H^{(2)}/dp_t|_{\text{app}}$  (lower dotted line), and  $d\sigma_H^{\text{res}}/dp_t$  ( $\mu_0 = 0.05m$  upper dashed line and  $\mu_0 = 0.1m$  lower dashed line).

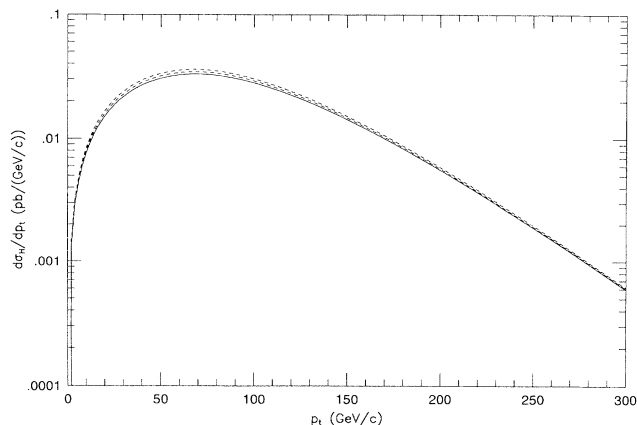


FIG. 2. The top quark  $p_t$  distributions  $d\sigma_H/dp_t$  for the  $q\bar{q}$  channel in the DIS scheme for a top quark mass  $m = 175$  GeV/ $c^2$ . Plotted are  $d\sigma_H^{(0)}/dp_t + d\sigma_H^{(1)}/dp_t|_{\text{exact}}$  (solid line) and  $d\sigma_H^{\text{imp}}/dp_t$  ( $\mu_0 = 0.05m$  upper dashed line and  $\mu_0 = 0.1m$  lower dashed line).

to exploit the fact that  $d\sigma_H^{(1)}/dp_t|_{\text{exact}}$  and  $d\sigma_H^{(1)}/dY|_{\text{exact}}$  are known and  $d\sigma_H^{(1)}/dp_t|_{\text{app}}$  and  $d\sigma_H^{(1)}/dY|_{\text{app}}$  are included in  $d\sigma_H^{\text{res}}/dp_t$  and  $d\sigma_H^{\text{res}}/dY$ , respectively. We note that here  $d\sigma^{(n)}$  denotes the  $O(\alpha_s^{n+2})$  contribution to the differential cross section. Moreover,  $d\sigma^{(n)}|_{\text{exact}}$  denotes the exact calculated differential cross section and  $d\sigma^{(n)}|_{\text{app}}$  the approximate one where only the leading soft gluon corrections are taken into account.

First we present the differential  $p_t$  distributions at  $\sqrt{S} = 1.8$  TeV for a top quark mass  $m = 175$  GeV/ $c^2$ . For these plots the mass factorization scale is

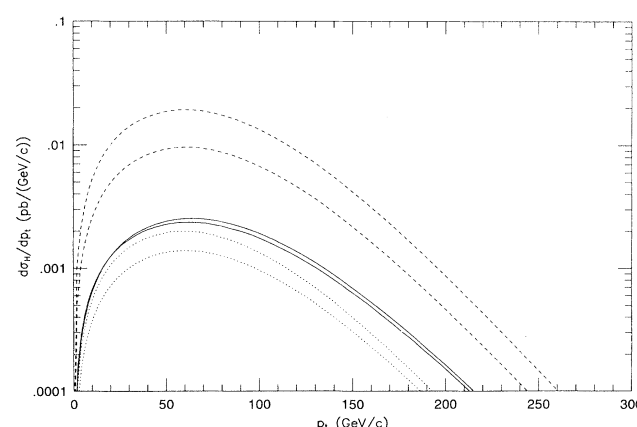


FIG. 3. The top quark  $p_t$  distributions  $d\sigma_H^{(k)}/dp_t$  for the  $gg$  channel in the  $\overline{\text{MS}}$  scheme for a top quark mass  $m = 175$  GeV/ $c^2$ . Plotted are  $d\sigma_H^{(0)}/dp_t$  (upper solid line at large  $p_t$ ),  $d\sigma_H^{(1)}/dp_t|_{\text{exact}}$  (lower solid line at large  $p_t$ ),  $d\sigma_H^{(1)}/dp_t|_{\text{app}}$  (upper dotted line),  $d\sigma_H^{(2)}/dp_t|_{\text{app}}$  (lower dotted line), and  $d\sigma_H^{\text{res}}/dp_t$  ( $\mu_0 = 0.2m$  upper dashed line and  $\mu_0 = 0.25m$  lower dashed line).

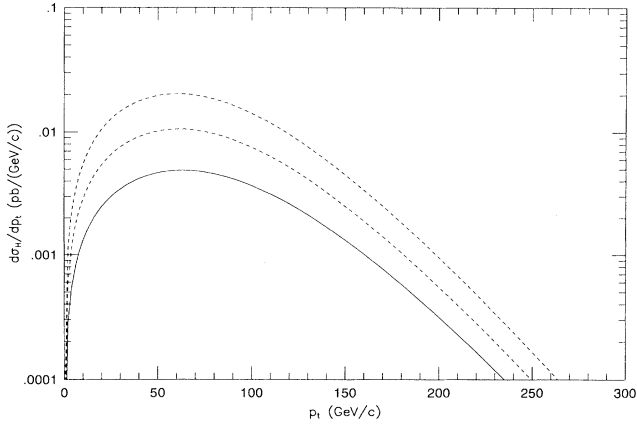


FIG. 4. The top quark  $p_t$  distributions  $d\sigma_H/dp_t$  for the  $gg$  channel in the  $\overline{\text{MS}}$  scheme for a top quark mass  $m = 175$   $\text{GeV}/c^2$ . Plotted are  $d\sigma_H^{(0)}/dp_t + d\sigma_H^{(1)}/dp_t|_{\text{exact}}$  (solid line) and  $d\sigma_H^{\text{imp}}/dp_t$  ( $\mu_0 = 0.2m$  upper dashed line and  $\mu_0 = 0.25m$  lower dashed line).

not everywhere equal to  $m$ . We chose  $\mu = m$  in  $s_0$ ,  $f_k(s_4/m^2, m^2/\mu^2)$ , and  $\bar{\alpha}_s$ , but  $\mu = m_t$  in the MRSD' parton distribution functions and the running coupling constant  $\alpha_s(\mu)$ . It should be noted, however, that it makes little difference if we choose  $\mu = m$  everywhere. The difference in the cross section is only a few percent so that the changes due to scale dependence are insignificant compared with the changes due to higher order resummation. We begin with the results for the  $q\bar{q}$  channel in the DIS scheme. In Fig. 1 we show the Born term  $d\sigma_H^{(0)}/dp_t$ , the first-order exact result  $d\sigma_H^{(1)}/dp_t|_{\text{exact}}$ , the first-order approximation  $d\sigma_H^{(1)}/dp_t|_{\text{app}}$ , the second-order approximation  $d\sigma_H^{(2)}/dp_t|_{\text{app}}$ , and the resummed result  $d\sigma_H^{\text{res}}/dp_t$  for  $\mu_0 = 0.05m$  and for  $\mu_0 = 0.1m$ . These are the same values for  $\mu_0$  that were used in [9]. As we decrease  $\mu_0$  the

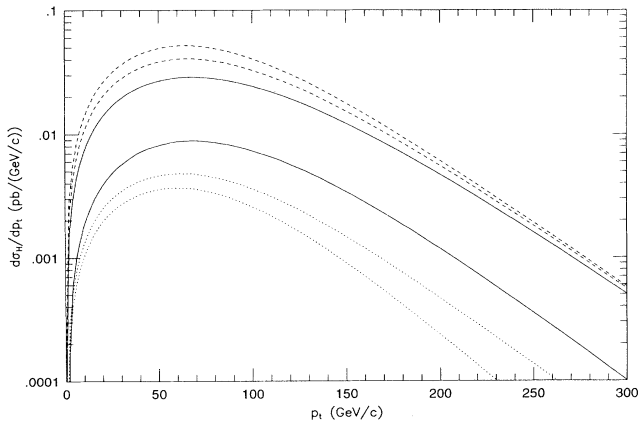


FIG. 5. The top quark  $p_t$  distributions  $d\sigma_H^{(k)}/dp_t$  for the sum of the  $q\bar{q}$  and  $gg$  channels for a top quark mass  $m = 175$   $\text{GeV}/c^2$ . Plotted are  $d\sigma_H^{(0)}/dp_t$  (upper solid line),  $d\sigma_H^{(1)}/dp_t|_{\text{exact}}$  (lower solid line),  $d\sigma_H^{(1)}/dp_t|_{\text{app}}$  (upper dotted line),  $d\sigma_H^{(2)}/dp_t|_{\text{app}}$  (lower dotted line), and  $d\sigma_H^{\text{res}}/dp_t$  (upper and lower dashed lines).

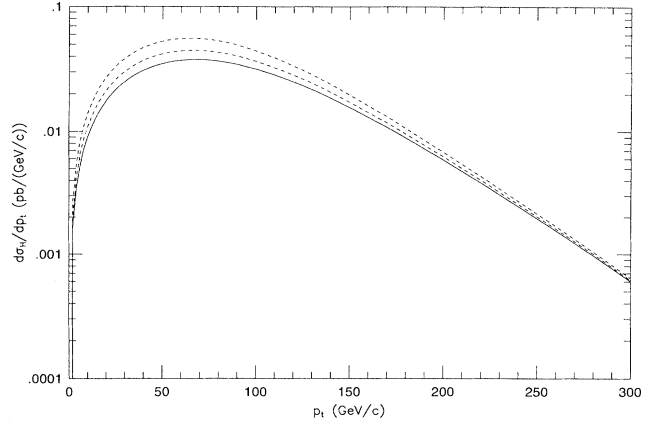


FIG. 6. The top quark  $p_t$  distributions  $d\sigma_H/dp_t$  for the sum of the  $q\bar{q}$  and  $gg$  channels for a top quark mass  $m = 175$   $\text{GeV}/c^2$ . Plotted are  $d\sigma_H^{(0)}/dp_t + d\sigma_H^{(1)}/dp_t|_{\text{exact}}$  (solid line) and  $d\sigma_H^{\text{imp}}/dp_t$  (upper and lower dashed lines).

differential cross sections increase. In Fig. 2 we show the exact  $O(\alpha_s^3)$  result  $d\sigma_H^{(0)}/dp_t + d\sigma_H^{(1)}/dp_t|_{\text{exact}}$ , and, for comparison,  $d\sigma_H^{\text{imp}}/dp_t$  for  $\mu_0 = 0.05m$  and  $\mu_0 = 0.1m$ . The resummed differential cross sections were calculated with the cut  $s_4 > s_0$  while no such cut was imposed on the phase space for the individual terms in the perturbation series. The improved differential distributions are uniformly above the exact  $O(\alpha_s^3)$  result. It is also evident from Fig. 2 that the resummation of the soft gluon contributions to the  $p_t$  distribution modifies the exact  $O(\alpha_s^3)$  result only slightly for the values of  $\mu_0$  that have been chosen.

We continue with the results for the  $gg$  channel in the  $\overline{\text{MS}}$  scheme. The corresponding plots are given in Figs. 3 and 4. In this case the values of  $\mu_0$  have been chosen to be  $\mu_0 = 0.2m$  and  $\mu_0 = 0.25m$  to correspond to those in [9]. Note that  $\mu_0$  need not be the same in the  $q\bar{q}$  and  $gg$  reactions because the convergence properties of the QCD perturbation series could be different in these channels and moreover depend on the factorization scheme. The first- and second-order corrections in the  $gg$  channel in the  $\overline{\text{MS}}$  scheme are larger than the respective ones in the  $q\bar{q}$  channel in the DIS scheme. In fact, for the range of  $p_t$  values shown the second-order approximate correction is larger than the first-order approximation. Hence, the relative difference in magnitude between the improved  $d\sigma_H^{\text{imp}}/dp_t$  and the exact  $O(\alpha_s^3)$  results is significantly larger than that in the  $q\bar{q}$  channel in the DIS scheme.

We finish our discussion of the differential  $p_t$  distributions with the results of adding the  $q\bar{q}$  and  $gg$  channels. The plots appear in Figs. 5 and 6. It is evident that resummation produces an enhancement of the exact  $O(\alpha_s^3)$  result, with very little change in shape.

Now we turn to a discussion of the differential  $Y$  distributions at  $\sqrt{S} = 1.8$  TeV for a top quark mass  $m = 175$   $\text{GeV}/c^2$ . In this case we set the factorization mass scale equal to  $m$  everywhere. We begin with the results for the  $q\bar{q}$  channel in the DIS scheme. In Fig. 7 we show the Born term  $d\sigma_H^{(0)}/dY$ , the first-



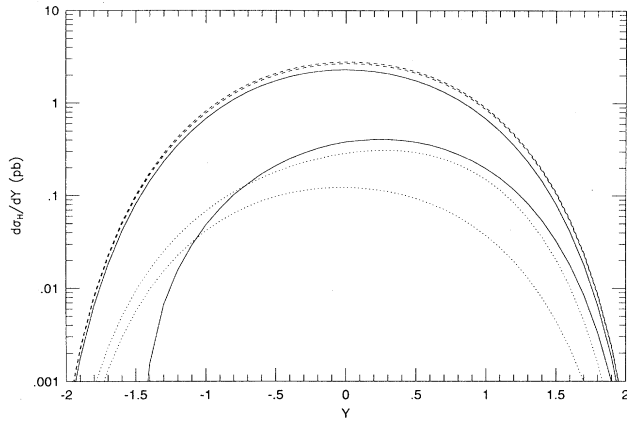


FIG. 7. The top quark  $Y$  distributions  $d\sigma_H^{(k)}/dY$  for the  $q\bar{q}$  channel in the DIS scheme for a top quark mass  $m = 175 \text{ GeV}/c^2$ . Plotted are  $d\sigma_H^{(0)}/dY$  (upper solid line),  $d\sigma_H^{(1)}/dY|_{\text{exact}}$  (lower solid line),  $d\sigma_H^{(1)}/dY|_{\text{app}}$  (upper dotted line),  $d\sigma_H^{(2)}/dY|_{\text{app}}$  (lower dotted line), and  $d\sigma_H^{\text{res}}/dY$  ( $\mu_0 = 0.05m$  upper dashed line and  $\mu_0 = 0.1m$  lower dashed line).

order exact result  $d\sigma_H^{(1)}/dY|_{\text{exact}}$ , the first-order approximation  $d\sigma_H^{(1)}/dY|_{\text{app}}$ , the second-order approximation  $d\sigma_H^{(2)}/dY|_{\text{app}}$ , and the resummed result  $d\sigma_H^{\text{res}}/dY$  for  $\mu_0 = 0.05m$  and  $\mu_0 = 0.1m$ . In Fig. 8 we show the exact  $O(\alpha_s^3)$  result  $d\sigma_H^{(0)}/dY + d\sigma_H^{(1)}/dY|_{\text{exact}}$ , and, for comparison,  $d\sigma_H^{\text{imp}}/dY$  for  $\mu_0 = 0.05m$  and  $\mu_0 = 0.1m$ . Again, the resummed differential cross sections were calculated with the cut  $s_4 > s_0$  while no such cut was imposed on the phase space for the individual terms in the perturbation series. It is also evident from Fig. 8 that the resummation of the soft gluon contributions to the  $Y$  distribution modifies the exact  $O(\alpha_s^3)$  result only slightly for the values of  $\mu_0$  that have been chosen.

We continue with the results for the  $g\bar{g}$  channel in

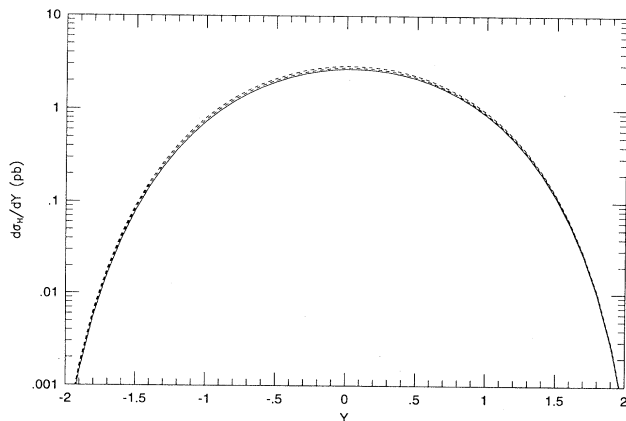


FIG. 8. The top quark  $Y$  distributions  $d\sigma_H/dY$  for the  $q\bar{q}$  channel in the DIS scheme for a top quark mass  $m = 175 \text{ GeV}/c^2$ . Plotted are  $d\sigma_H^{(0)}/dY + d\sigma_H^{(1)}/dY|_{\text{exact}}$  (solid line) and  $d\sigma_H^{\text{imp}}/dY$  ( $\mu_0 = 0.05m$  upper dashed line and  $\mu_0 = 0.1m$  lower dashed line).

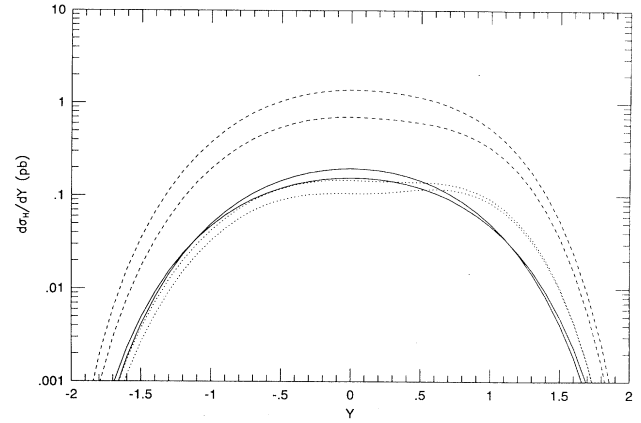


FIG. 9. The top quark  $Y$  distributions  $d\sigma_H^{(k)}/dY$  for the  $g\bar{g}$  channel in the  $\overline{\text{MS}}$  scheme for a top quark mass  $m = 175 \text{ GeV}/c^2$ . Plotted are  $d\sigma_H^{(0)}/dY$  (upper solid line at  $Y = 0$ ),  $d\sigma_H^{(1)}/dY|_{\text{exact}}$  (lower solid line at  $Y = 0$ ),  $d\sigma_H^{(1)}/dY|_{\text{app}}$  (lower dotted line),  $d\sigma_H^{(2)}/dY|_{\text{app}}$  (upper dotted line), and  $d\sigma_H^{\text{res}}/dY$  ( $\mu_0 = 0.2m$  upper dashed line and  $\mu_0 = 0.25m$  lower dashed line).

the  $\overline{\text{MS}}$  scheme. The corresponding plots are given in Figs. 9 and 10. Here, the values of  $\mu_0$  are  $\mu_0 = 0.2m$  and  $\mu_0 = 0.25m$ . As in the case of the  $p_t$  distributions, the first- and second-order corrections in this channel are larger than the respective ones in the  $q\bar{q}$  channel in the DIS scheme. For the range of  $Y$  values shown the second-order approximate correction is larger than the first-order approximation. Again, as in the  $p_t$  distributions, the relative difference in magnitude between the improved  $d\sigma_H^{\text{imp}}/dY$  and the exact  $O(\alpha_s^3)$  results is significantly larger than that in the  $q\bar{q}$  channel in the DIS scheme.

Finally, we conclude our discussion of the differential  $Y$  distributions by showing the results of adding the  $q\bar{q}$  and  $g\bar{g}$  channels. The plots appear in Figs. 11 and 12. Again,

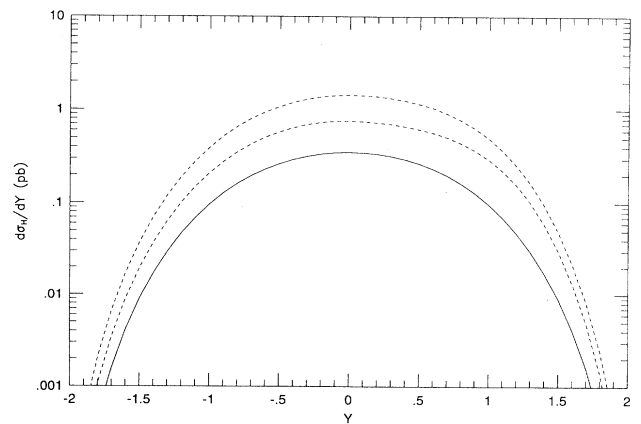


FIG. 10. The top quark  $Y$  distributions  $d\sigma_H/dY$  for the  $g\bar{g}$  channel in the  $\overline{\text{MS}}$  scheme for a top quark mass  $m = 175 \text{ GeV}/c^2$ . Plotted are  $d\sigma_H^{(0)}/dY + d\sigma_H^{(1)}/dY|_{\text{exact}}$  (solid line) and  $d\sigma_H^{\text{imp}}/dY$  ( $\mu_0 = 0.2m$  upper dashed line and  $\mu_0 = 0.25m$  lower dashed line).

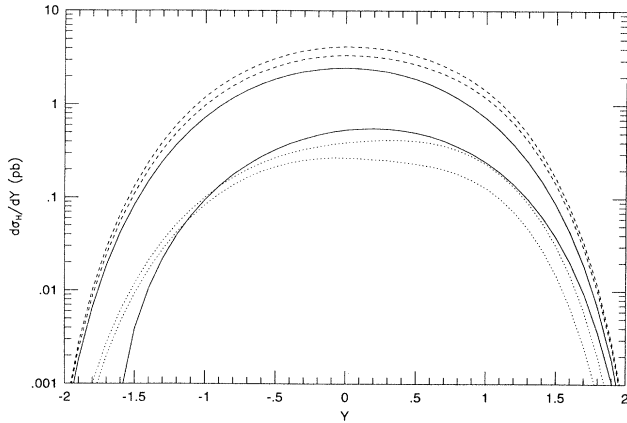


FIG. 11. The top quark  $Y$  distributions  $d\sigma_H^{(k)}/dY$  for the sum of the  $q\bar{q}$  and  $gg$  channels for a top quark mass  $m = 175$  GeV/ $c^2$ . Plotted are  $d\sigma_H^{(0)}/dY$  (upper solid line),  $d\sigma_H^{(1)}/dY|_{\text{exact}}$  (lower solid line),  $d\sigma_H^{(1)}/dY|_{\text{app}}$  (upper dotted line),  $d\sigma_H^{(2)}/dY|_{\text{app}}$  (lower dotted line), and  $d\sigma_H^{\text{res}}/dY$  (upper and lower dashed lines).

it is evident that resummation produces a non-negligible modification of the exact  $O(\alpha_s^3)$  result. However, the shape of the distribution is unchanged.

We have shown that the resummation of soft gluon radiation produces a small difference between the perturbation improved distributions in  $p_t$  and  $Y$  and the exact  $O(\alpha_s^3)$  distributions in  $p_t$  and  $Y$  for the  $q\bar{q}$  reaction in the DIS scheme for the values of  $\mu_0$  chosen. However, for the  $gg$  channel in the  $\overline{\text{MS}}$  scheme the resummation produces a large difference. The difference between the perturbation improved and the exact  $O(\alpha_s^3)$  distributions depends on the mass factorization scheme (DIS or  $\overline{\text{MS}}$ ), the factorization scale  $\mu$ , as well as the specific reaction under consideration ( $q\bar{q}$  or  $gg$ ). For a mass  $m = 175$  GeV/ $c^2$  the  $gg$  channel is not as important numerically as the  $q\bar{q}$  channel. However, since the corrections for the  $gg$  channel are quite large, resummation produces a non-

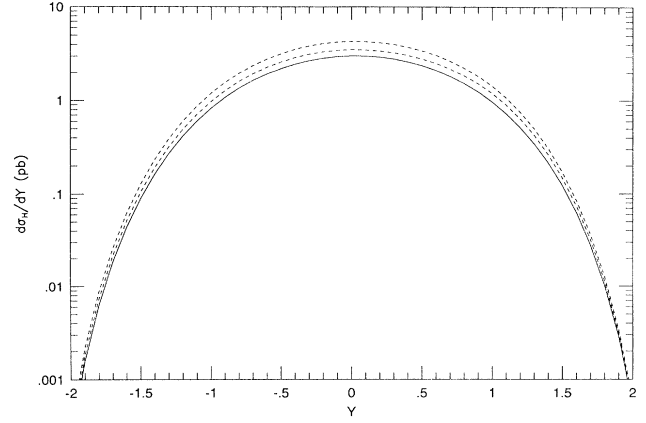


FIG. 12. The top quark  $Y$  distributions  $d\sigma_H/dY$  for the sum of the  $q\bar{q}$  and  $gg$  channels for a top quark mass  $m = 175$  GeV/ $c^2$ . Plotted are  $d\sigma_H^{(0)}/dY + d\sigma_H^{(1)}/dY|_{\text{exact}}$  (solid line) and  $d\sigma_H^{\text{imp}}/dY$  (upper and lower dashed lines).

negligible difference between the perturbation improved and the exact  $O(\alpha_s^3)$  distributions when adding the two channels. However, the shapes of the distributions are essentially unchanged.

*Note added:* The observation of the top quark has recently been announced by both the CDF and D0 collaborations [24,25]. CDF announced a top quark mass of  $176 \pm 8 \pm 10$  GeV/ $c^2$  and a  $t\bar{t}$  production cross section of  $6.8_{-2.4}^{+3.6}$  pb. D0 measured the top quark mass to be  $199_{-21}^{+19} \pm 22$  GeV/ $c^2$  and its production cross section to be  $6.4 \pm 2.2$  pb.

## ACKNOWLEDGMENTS

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