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PCAC and the possibility of scaling in electropion production

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This paper is devoted to a study of possible scaling laws, and their logarithmic corrections, occurring in deep inelastic electropion production. Both the exclusive and semiexclusive processes are considered. Scaling laws, originally motivated from PCAC and current algebra considerations, are examined, first in the framework of the parton model and QCD perturbation theory and then from the more formal perspective of the operator product expansion and asymptotic freedom (as expressed through the renormalization group). We emphasize that these processes allow scaling to be probed for the full amplitude rather than just its absorbtive part (as is the case in the conventional structure functions). Because of this it is not possible to give a formal derivation of scaling for deep inelastic electropion production processes even if one believes that they are unambiguously sensitive to the light cone behavior of the operator product. The origin of this is shown to be related to its behavior near $x \approx 0$. Investigations, both theoretical and experimental, of these processes is therefore strongly encouraged.

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I. INTRODUCTION

A. Motivation

The well-known logarithmic scaling violations in the structure functions of nucleons predicted by asymptotic freedom played a crucial role in establishing QCD as the accepted theory of the strong interactions. These predictions, based as they are on the renormalization group and the operator product expansion of two electromagnetic currents near the light cone, are on a strong theoretical footing. This is in contrast with the situation in most other hadronic processes such as, for example, the Drell-Yan process or jet production, which require some input of unknown soft infrared contributions [1]. In spite of this ignorance, recipes for handling such processes have been very successful. These involve a careful mixture of several ingredients, some of which, such as asymptotic freedom and perturbation theory, are well understood, while others, such as the ingredient describing nonperturbative hadronization effects are chosen with an eye for the phenomenological acceptability of the finished product rather than for their firm theoretical basis. This somewhat pragmatic approach clearly incorporates most of the correct physics and is now taken to be sufficiently reliable that it is used to estimate QCD backgrounds in experiments searching for unusual or new phenomena: for example, calculations of jet production are routinely used in searches for potential Higgs candidates.

Given this situation it seems worthwhile to examine other processes that are closely related to conventional deep inelastic scattering but where the derivation of scaling and its violation, for example, is not quite so well justified. It is conceivable that, in this way, experimental results can be used to illuminate the approximations and assumptions used in the recipe and, ultimately to gain further theoretical insight into some of the difficulties. It is somewhat in this spirit that we consider here deep inelastic exclusive electropion production from nucleons. Because the pion is on mass shell the process is not unambiguously controlled by the light cone; however, as was shown some time ago, an application of the ideas of current algebra and PCAC (partial conservation of axial vector current) allows one to finesse some of the usual hadronization problems and derive scaling laws [2,3]. Although these derivations are not rigorous they do lead to predictions of scaling which agree with experiment albeit at relatively low energies [4,5]. Prior discussions of this process did not include the constraints of current algebra and PCAC but, rather, were based on the application of regge ideas to the naive parton model (thereby emphasizing more the hadronic nature of the process) [6–8]. Not surprisingly they concluded that pion electroproduction processes should not scale, in apparent contradiction to the data. The significance of these results in comparison with those of the present paper is discussed in the Appendix.

A second motivation for taking a new look at these

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sorts of processes at this time is to stimulate renewed interest in them since they are now amenable to experiment and one can expect new data from the DESY *ep* collider (HERA) and the Continuous Electron Beam Accelerator Facility (CEBAF) in the not-too-distant future. The main content of this paper will therefore be to reanalyze this process from a more modern viewpoint particularly within the context of perturbative QCD. We shall derive the scaling laws and calculate their expected logarithmic deviations. Some of what we say is also applicable to other processes which can be described theoretically by an amplitude which is proportional to the product of two currents. This should therefore include the deep inelastic electroproduction of ρ's, K's, and most interestingly real photons: the latter process corresponding to a direct measurement on the nonforward Compton amplitude, albeit with one virtual and one real photon. We will show that if the predictions of scaling behavior are indeed verified by experiment, important implications follow about the analytic structure of the amplitudes for these processes.

In Sec. I B, we define the process we are interested in and the corresponding kinematic region. In Sec. II A, we outline the derivation of the scaling law as originally done using PCAC and a current algebra inspired by QCD. As already mentioned the resulting predictions are, in fact, in agreement with the rather coarse data taken at about that time. In Sec. II B we present a rederivation of this scaling law using the language of the parton model [9]. In Sec. II C, we calculate leading logarithmic corrections to the amplitude using standard perturbative QCD techniques. The end result of this approach is an integral representation (2.24) for the amplitude M, which is analogous to the Altarelli-Parisi evolution equation. This equation can then be used much as it is in the conventional analysis to predict the moments of the amplitude, which in principle can be measured. The validity of the procedure is subject to some reservations which we will discuss fully. In Sec. III, we approach the problem from the point of view of the operator product expansion. We explain why the prediction for the moments of the amplitude is sensitive to the analytic properties of the amplitude near $x = 0$. This implies that an experimental study of deep inelastic pion production from which these moments can be determined may yield information on the small x behavior of the amplitude. Finally, in Sec. IV, we discuss the connection of our results to experimental data and suggest future experiments and directions.

B. Kinematics and definitions

The amplitude that we are going to study is defined as

$$M_\mu = \langle p'\pi|J_\mu|p\rangle \tag{1.1}$$

$$= (m_\pi^2 - q'^2)\int d^4x\, e^{iq\cdot x}\langle p'|\theta(x_0)[J_\mu(x),\phi_\pi(0)]|p\rangle \tag{1.2}$$

$$= \frac{(m_\pi^2 - q'^2)}{f_\pi m_\pi^2}\int d^4x\, e^{iq\cdot x}\langle p'|\theta(x_0)[J_\mu(x),\partial^\nu A_\nu(0)]|p\rangle. \tag{1.3}$$

Here $J_\mu(x)$ is the electromagnetic current, $A_\mu(x)$ the axial vector current, m_π the pion mass, f_π its decay coupling constant, and $\phi_\pi(x)$ its field. In going from (1.2) to (1.3), the standard PCAC identification has been used:

$$\partial_\mu A^\mu(x) = f_\pi m_\pi^2 \phi_\pi(x)\ . \tag{1.4}$$

The kinematics are illustrated in Fig. 1: p is the four-momentum of the struck target, p' its final momentum, and q that of the virtual photon delivered by the scattered electron; q' will be used for the pion four-momentum.

The relationship to, and generalization from, the amplitude probed by measuring the conventional structure functions is clear. In that case one is probing only the imaginary part of the forward Compton amplitude whereas in electropion production one measures a *full* amplitude which, in general, is *non*forward. Formally, the difference can be expressed as probing the difference between a time-ordered, or retarded, product, as in (1.3), versus a commutator as in the structure function case:

$$W_{\mu\nu} = \int d^4x\, e^{iq\cdot x}\langle|[J_\mu(x),J_\nu(0)]|p\rangle\ . \tag{1.5}$$

The full forward Compton amplitude is given by

$$\mathcal{J}_{\mu\nu} \equiv \int d^4x e^{iq\cdot x}\langle p|T[J_\mu(x),J_\nu(0)]|p\rangle \tag{1.6}$$

so that $W_{\mu\nu} = \mathrm{Im}\mathcal{J}_{\mu\nu}$. These are represented by the diagrams in Fig. 2.

A further crucial difference between the two cases is, of course, that in (1.3), the kinematics of real pion production dictates that, even in the deep inelastic limit when q^2 is large, q'^2 must remain fixed at m_π^2; on the other hand, in (1.5), the magnitude of the virtual mass of *both* currents is always large in the deep inelastic limit. This latter condition ensures that the light cone is unambiguously being probed and so justifies the use of the light-cone operator product expansion. In spite of the fact that this is not clearly the case in pion production we shall argue below that a short-distance operator product expansion may dominate the process when q^2 is large.

There is a subtlety in this procedure which is also present in the standard forward Compton amplitude case. The point is that this formalism leads to an expansion in powers of $1/x$, where $x \equiv -q^2/2p\cdot q$, and, in the physical region accessible to real experiments, $|x| < 1$. Such an expansion therefore clearly does not converge. In the structure function case, this potential problem is finessed because, there, one is interested in only the imag-

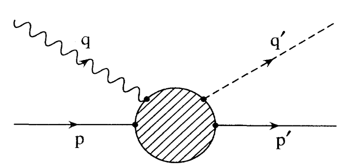

FIG. 1. General electropion production amplitude defined in (1.1) showing the kinematics of the external particles.

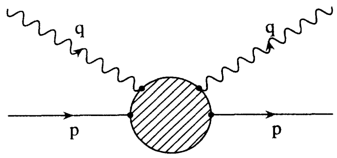

FIG. 2. The forward Compton amplitude defined in (1.6); its imaginary part defines the conventional structure functions (1.5).

inary part, as in (1.5), so an analytic continuation from the unphysical large $|x|$ (where the expansion presumably makes sense) to the physical region can be effected [1]. Indeed this is why the results are expressed in the form of moments of the structure functions rather than the structure functions themselves. We shall discuss this in more detail below. However, it is clear from this that in the pion production case M_μ is sensitive to a potentially interesting part of the formalism not readily accessible to the structure functions. Indeed it may well be that because of this "problem" pion electroproduction can cast interesting light on the general small x behavior as well as the general assumptions that underly the usual derivation. Before reviewing the old scaling arguments, however, let us recall the relationship between the measured cross section and the matrix element M_μ: this is best done in terms of the tensor

$$T_{\mu\nu} \equiv M_\mu M_\nu^* \ . \tag{1.7}$$

The result is given by [10]

$$\frac{d^3\sigma}{dE' d\Omega' d\Omega} = \frac{\alpha}{2\pi^2 q^2} \frac{E'}{E} \frac{(v^2 - q^2)^{1/2}}{1-\epsilon} \frac{d\sigma}{d\Omega} \ , \tag{1.8}$$

where E (E') is the initial (final) energy of the electron in the laboratory (lab) system and ν its energy loss: note that $\nu = E - E' = p \cdot q/M$ where M is the target nucleon mass. The polarization of the virtual photon is given by

$$\epsilon = \left(1 - \frac{2(\nu^2 - q^2)}{q^2} \tan^2 \frac{1}{2}\theta_e\right)^{-1} \ , \tag{1.9}$$

where θ_e is the electron scattering angle in the lab system. The quantity $d\sigma/d\Omega$ represents an equivalent virtual photoproduction cross section in the outgoing hadron center-of-mass (c.m.) system:

$$\frac{d\sigma}{d\Omega} = \frac{M^2|q'|_{\text{c.m.}}}{16\pi^2 W^2 |q|_{\text{c.m.}}} [\tfrac{1}{2}(T_{xx} + T_{yy}) + \tfrac{1}{2}\epsilon(T_{xx} - T_{yy}) - (q^2\nu^2)\epsilon T_{zz} + \{-(2q^2/\nu^2)\epsilon(1+\epsilon)\}^{1/2} T_{xz}] \ . \tag{1.10}$$

Here W is the total c.m. energy so $W^2 \equiv s = (p+q)^2 = (p'+q')^2$. The z axis is defined to be coincident with the direction of q while the electrons define the xy plane. Thus all of the ϕ (azimuthal) dependence is contained in $(T_{xx} - T_{yy}) \sim \cos 2\phi$ and $T_{xz} \sim \cos\phi$. In what follows we shall limit ourselves to the case where the particle spins are unobserved.

Finally, it is worth noting that in the deep inelastic limit ($q^2 \to -\infty$), the square of the momentum transfer

$$t \equiv \Delta^2 \equiv (q' - q)^2 = (p' - p)^2 \tag{1.11}$$

is constrained, in the physical region, to lie between -2ν and

$$t_{\min} \approx -\frac{x^2 M^2}{1-x} \ . \tag{1.12}$$

Typically the limit we will be considering keeps t and x fixed (and finite) with $q^2 \to -\infty$. Thus x must not be too close to unity. Furthermore, the region of interest is predominantly forward scattering in the πN c.m. system. In what follows it is convenient to write

$$q_\mu = E n_\mu + \Delta_\mu \ , \tag{1.13}$$

where n^2 is a null vector, i.e., n^2=0. Thus,

$$q^2 = 2E(n \cdot \Delta) + t \tag{1.14}$$

and

$$x = \frac{-[2E(n\cdot\Delta) + t]}{\{2[E(n\cdot p) + \Delta\cdot p]\}} \ . \tag{1.15}$$

The scaling limit can then be realized by taking $E \to \infty$ with both $x \approx -(n\cdot p)/(n\cdot\Delta)$ and t fixed.

II. SCALING LAW

A. Current algebra

Scaling laws for $T_{\mu\nu}$ can be derived using a current algebra approach augmented by some heuristic assumption about the light-cone behavior of the commutator. This can be checked in perturbation theory and justified by the operator product expansion as sketched below. We begin by setting $q'^2 = 0$ in which case

$$f_\pi M_\mu = C_{\mu\nu} q'^\nu + E_\nu \ , \tag{2.1}$$

where

$$C_{\mu\nu} \equiv i \int d^4x \, \exp(iq\cdot x) \langle p'|\theta(x_0)[J_\mu(x), A_\nu(0)]|p\rangle \tag{2.2}$$

and

$$E_\mu \equiv \int d^4x \, \exp(iq\cdot x) \langle p'|\delta(x_0)[J_\mu(x), A_0(0)]|p\rangle \ . \tag{2.3}$$

Using the usual SU(2)×SU(2) current algebra,

$$\delta(x_0)[A_0^i(x), J_\mu(0)] = i\epsilon^{i3k} A_\mu^k(0)\delta^4(x) \ , \tag{2.4}$$

we immediately get that E_μ is independent of E (it depends only on Δ and, from its usual parametrization, we get the well-known axial vector and induced pseudoscalar form factors of the nucleon).

The scaling result we want to show is that $C_{\mu\nu} q'^\nu$ is also independent of E. We shall first sketch the derivation of this result based on the spacetime behavior of the current commutators. In what follows it is convenient to introduce standard light-cone coordinates for a four-vector a^μ as

$$a_\pm = \frac{\sqrt{2}}{2}(a_0 \pm a_z), \quad \mathbf{a}_\perp = (a_x, a_y) \ . \tag{2.5}$$

Then the scaling limit is equivalent to $q_- \approx \sqrt{2}\nu \to \infty$ with $q_+ \approx \sqrt{2}x$ fixed.

Causality allows us, at least naively, to replace $\theta(x_0)$ in (2.2) by $\theta(x_+)$ in which case the asymptotic behavior of $C_{\mu\nu}$ is given by

$$C_{\mu\nu} \approx -\frac{1}{q_-}\int d^4x \, \exp(iq\cdot x)\langle p'|\delta(x_+)[J_\mu(x), A^i_\nu(0)]|p\rangle. \tag{2.6}$$

Consequently,

$$\begin{aligned} q'^\nu C_{\mu\nu} &\sim q_- C_{\mu+} \\ &\sim -\int d^4x \, \exp(iq\cdot x)\langle p'|\delta(x_+)[J_\mu(x), A^i_+(0)]|p\rangle \ . \end{aligned} \tag{2.7}$$

The commutator in (2.7) can be expressed in the form

$$[J_\mu(x), A^i_+(0)]\delta(x_+) = \tilde{A}^i_\mu(x_-)\delta^2(x_\perp) \ , \tag{2.8}$$

where $\tilde{A}\mu(x_-)$ is, in general, model dependent.

In QCD, canonical commutation relations lead to

$$\tilde{A}^i_\mu(x_-) = A^i_\mu(0)\delta(x_-) + B^i_\mu(x_-) \ , \tag{2.9}$$

where $B^i_\mu(x_-)$ is an unknown (typically bilinear nonlocal) operator which is nonsingular at $x_- \approx 0$. We conclude then that, in the scaling limit,

$$q'^\nu C_{\mu\nu} \sim \int dx_- \exp(iq_+x_-)\langle p'|B_\mu(x_-)|p\rangle \ . \tag{2.10}$$

This is the desired result since it shows that M_μ is independent of E and is only a function of x (through q_+) and Δ.

This result straightforwardly translates into the following scaling constraints on the components of $T_{\mu\nu}$ occurring in the measured cross section, (1.8):

$$\tfrac{1}{2}(T_{xx} + T_{yy}) \to F_1(x,t)(k_x^2 + k_y^2) \ ,$$

$$\tfrac{1}{2}(T_{xx} - T_{yy}) \to \tfrac{1}{2}F_2(x,t)(k_x^2 - k_y^2) \ , \tag{2.11}$$

$$T_{zz} \to F_1(x,t) - F_2(x,t)\Delta_z^2 \ ,$$

and

$$T_{xz} \to -F_2(x,t)k_x\Delta_z \ ,$$

where Δ_z (as expressed in the lab system) $\approx -(\frac{1}{2}t + M\omega)$ and the F_i are Lorentz scalars.

B. Parton model

We now turn to the treatment of this problem in terms of the quark-parton model. We assume that the two currents J_μ and A_ν interact successively with the same quark while the rest act as spectators. Without QCD corrections the amplitude $C_{\mu\nu}$ is given by the sum of the two diagrams in Fig. 3. This is analogous to the usual parton model treatment of the forward Compton scattering.

To calculate these diagrams we assume that the struck quark carries a fraction η of the momentum p of the hadron, in the infinite momentum frame. Immediately after absorbing the photon the virtual quark has momentum $(\eta p + q)$, and hence it is highly off shell. This is the basic reason for considering that the parton model is applicable here. The final quark has momentum $(\eta p + \Delta)$. Consider now the diagram in Fig. 3(a). Its contribution is (τ_i are isospin matrices and $\tilde{Q}$ is the generator corresponding to the electric charge):

$$\begin{aligned} M_{(a)} &= -\frac{\tau_i}{2}\tilde{Q} \\ &\quad \times \frac{\bar{\psi}_{p'}(\eta p + \Delta)\gamma_5\gamma_\nu(\eta \not{p} + E \not{n} + \not{\Delta})\gamma_\nu\psi_p(\eta p)}{2E[\eta(n\cdot p) + (n\cdot\Delta)]} q'_\mu \\ &= -\frac{\tau_i}{2}\tilde{Q}\frac{\bar{\psi}_{p'}(\eta p + \Delta)\gamma_5 \not{n}(\eta \not{p} + \not{\Delta})\gamma_\mu\psi_p(\eta p)}{2[\eta(n\cdot p) + (n\cdot\Delta)]} \ . \end{aligned} \tag{2.12}$$

To this has to be added the contribution from the crossed graph shown in Fig. 3(b). A sum over the various types of quarks has been suppressed. $\psi_p(k)$ represents the amplitude, or wave function, for finding a quark of a particular type carrying momentum k inside a nucleon moving with momentum p. The complete matrix element requires an integration over η, consistent with the requirement that $(\eta p + q)^2 > 0$, i.e., $\eta > x$. Schematically, the parton contribution is thus given by

$$\mathcal{M}^0_\mu = \int_x^1 d\eta \bar{\psi}_{p'}(\eta p + \Delta)M_\mu(\eta, p, \Delta)\psi_p(\eta p) \ . \tag{2.13}$$

The matrix $\mathcal{M}_\mu$ can be read off from (2.12) with an additional contribution coming from the crossed graph. This shows explicitly that, when E is large, M depends only on t and x.

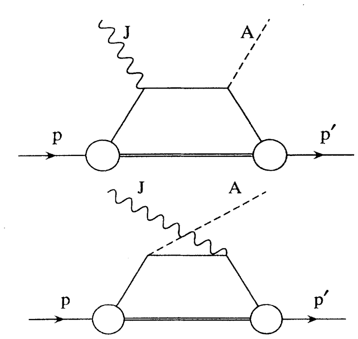

FIG. 3. The leading-order parton model contributions to $C_{\mu\nu}$: (a) the direct and (b) the crossed contributions.

Note, incidentally, that for the parton model description of the conventional structure function, $\Delta = 0$ and only the imaginary part of $\mathcal{M}$, i.e., its δ-function contribution, is required. In that case the integral reduces to $xf(x)$, where $f(\eta) \equiv \bar{\psi}_p(\eta p)\psi_p(\eta p)$, the probability for finding a quark with fraction η of the total momentum. It is worth remarking that in the analogous kinematic configuration here, where $\Delta \approx 0$, the full Born amplitude reduces to

$$M_\mu \approx \left(Q, \frac{\tau^i}{2}\right)(n \cdot p)p_\mu \ . \tag{2.14}$$

C. Leading logarithmic corrections

In this section we sketch a computation of the leading logarithmic corrections to the parton model result. Rather than give a complete detailed description we present here only the salient features for the simpler and more limited kinematic situation where $\Delta \approx 0$. The point is that a subgraph which gives a logarithmic contribution $\sim \ln \Delta^2$ for $\Delta \neq 0$ obviously is singular in the $\Delta \to 0$ limit: hence the same diagrams (i.e., ladders and self-energy insertions to external legs) that give the leading logarithmic corrections in the usual deep inelastic scattering case ($\Delta = 0$) also give the leading logarithmic corrections in the $\Delta \neq 0$ case.

For ease of presentation, we shall from here on use p to denote the momentum of the initial quark rather than of the initial hadron. The initial quark is taken to be off shell by an amount comparable to the inverse of the confinement radius of the nucleon. Since the momentum transfer Δ is typically of the same order of magnitude, we make no distinction between $\ln(-q^2/p^2)$ and $\ln(-q^2/\Delta^2)$. When, for example, we end up with a logarithmic integration of the form $\int dk^2/(k^2 + \Delta^2)$ we shall be free to take the $\Delta \to 0$ limit and write it in the form $\int_{p^2} dk^2/k^2$. Although the final scattered quark is slightly off shell, the $\Delta \to 0$ limit enables us to take the γ matrices between on-shell spinors. It is important, of course, to take the $\Delta \to 0$ limit *after* having secured that the final integration is logarithmic.

It is convenient to work in the light-cone gauge where the propagator for a gluon of momentum k is

$$G_{\mu\nu}(k) = \frac{D_{\mu\nu}(k)}{k^2 + i\epsilon} \tag{2.15}$$

with

$$D_{\mu\nu}(k) = g_{\mu\nu} = -\frac{k_\mu c_\nu + k_\nu c_\mu}{k \cdot c} \ . \tag{2.16}$$

In such a gauge the only diagrams (apart from self-energy parts) giving leading logarithmic corrections are the ladder diagrams of Fig. 4(a). It may be mentioned, in particular, that diagrams of the type of Fig. 4(b) do not give leading logarithmic contributions. Incidentally, had we followed the operator product expansion approach these diagrams would correspond to contributions from gauge noninvariant operators.

Turning now to the calculation of the ladder diagrams we show that a familiar picture emerges: there is strong ordering in the momentum flowing through the ladder and an evolution equation can be derived. We first examine the contribution from the lowest order diagram (Fig. 5). Using a Sudakov parametrization for the quark momentum k,

$$k = \alpha c + \beta p + k_\perp \ , \tag{2.17}$$

$$d^4k = \frac{s}{2} d\alpha d\beta d^2 k_\perp^2 \ , \tag{2.18}$$

(where $s = -q^2/x$). We obtain

$$M_1 = -\frac{\tau i}{2}\tilde{Q}\frac{\alpha_2 C_F}{4\pi^3}\frac{s}{4}\int dk_\perp^2 d\alpha d\beta \frac{\gamma_\sigma(\not{k}+\not{\Delta})\not{n}(\not{k}+\not{\Delta})\gamma_\mu \not{k}\gamma_\rho\gamma_5}{[(k+E_n+\Delta)^2+i\epsilon](k^2+i\epsilon)[(k+\Delta)^2+i\epsilon][(p-k)^2+i\epsilon]}D_{\rho\sigma} \ . \tag{2.19}$$

Initial and final spinors have been suppressed. Note that the amplitude is color singlet in the t channel and that, in terms of Sudakov variables,

$$k^2 = \alpha\beta s - k_\perp^2 \ ,$$
$$(p-k)^2 = -\alpha(1-\beta)s - k_\perp^2 \ , \tag{2.20}$$
$$(k+\Delta)^2 = \alpha\beta s - k_\perp^2 - \alpha s x \ .$$

We first perform the α the integration. In the region $x < \beta < 1$ there is one pole at $\alpha = -k_\perp^2/(1-\beta)s$ due to $(p-k)^2$ lying below the real axis whereas in the region $0 < \beta < x$ there is one pole at $\alpha = k_\perp^2/\beta s$ due to k^2 lying above it. In both cases we close the α contour so as to pick the contributions from those poles. In the regions $\beta < 0$ and $\beta > 1$ all the poles with respect to α lie on one side of the real axis and can be avoided [we do not take $(k+En+\Delta)^2$ into account since it will be combined with the next element of the ladder]. Observe that the leading logarithms come from the wide range of integration $\mu^2 \leq k_\perp^2 \ll s$ where μ is an arbitrary renormalization scale. The parameter β is finite (typically of order x) whereas $\alpha \sim -k_\perp^2/s$ is small. Hence the logarithms come, as expected, from the collinear configuration.

After the α integration the denominator behaves like $(k_\perp^2)^2$, so we have to extract one $k_\perp^2$ from the numerator if the final integration is to be logarithmic. Having done this we can pass to the collinear configuration $k = \beta p$, $\alpha \approx 0$. As already remarked we shall also set $\Delta \approx 0$ and take the γ matrices between on-shell spinors $u(p)$. One might worry whether in the limit $\Delta \to 0$ we lose logarithms multiplied by $q \cdot \Delta/q^2$.

There is no such danger since we parametrize everything from the start in terms of vectors n, p, and Δ. Then $q\cdot\Delta/q^2 \sim E(n\cdot\Delta)/2E(n\cdot\Delta) \sim \frac{1}{2}$. There will remain a factor $\gamma_5 \not{n}\, \not{k}\gamma_\mu$ from the numerator which will combine with $(k+En+\Delta)^2$ from the denominator to form the Born term $M_B(\beta p, k_\perp^2)$. Note finally that the logarithmic contribution comes from the region $\beta > x$, so that the propagating quark line remains highly off shell.

The numerator in the integrand has the form (apart from the γ_5)

$$\gamma_\sigma(\not{k}+\not{\Delta})\,\not{n}(\not{k}+\not{\Delta})\gamma_\mu\,\not{k}\gamma_\rho\left(g_{\rho\gamma}-\frac{c_\rho(p-k)_\sigma+c_\sigma(p-k)_\rho}{c\cdot(p-k)}\right)\,. \tag{2.21}$$

After some algebra this can be reduced (in the $\Delta\approx 0$ limit) to a familiar form for the leading lowest-order correction:

$$M_1(x,q^2)=2C_F\int_{\mu^2}^{-q^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha_s}{4\pi}\int_x^1 d\beta\left(1-\beta+\frac{2\beta}{1-\beta}\right)M_B(x/\beta,\mu^2)\,. \tag{2.22}$$

Iterating this an arbitrary number of times leads to an evolution equation:

$$M(x,q^2)=M(x,\mu^2)+2C_F\int_{\mu^2}^{-q^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha_s}{4\pi}\int_x^1 d\beta\frac{1+\beta^2}{1-\beta}M(x/\beta,k_\perp^2)\,. \tag{2.23}$$

Up to now we have considered skeleton graphs only. When we dress the ladder with vertex and self-energy corrections further leading logarithmic contributions coming from the ultraviolet region are induced. These can be taken into account simply by replacing the "bare" coupling constant α_s in (2.23) by the running coupling constant $\alpha_s(k_\perp^2)\approx 4\pi/(\beta\ln k_\perp^2)$, where $\beta=(11-2n_f/3)$ (n_f being the number of flavors). In addition, the second term in (2.23) must be multiplied by the quark wave-function renormalization constant Z_F in order to cancel the infrared divergences from the soft gluon region and get a gauge-invariant result. Z_F in the light-cone gauge has been calculated in a number of places. The final result is

$$M(x,q^2)=M(x,\mu^2)+2C_F\int_{\mu^2}^{-q^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha_s(k_\perp^2)}{4\pi}\times\int_x^1 d\beta P(\beta)M(x/\beta,k_\perp^2)\,, \tag{2.24}$$

where

$$P(\beta)=\frac{1+\beta^2}{1-\beta}-\delta(1-\beta)\int_0^1 dx\frac{1+x^2}{1-x}\,. \tag{2.25}$$

Equation (2.24) is somewhat difficult to handle from the phenomenological point of view. If for the moment we disregard the subtleties regarding the low x dependence of M which will be discussed in the following section, then we can disentangle M in (2.24) by taking moments. Defining

$$M_n(q^2)=\int_0^1 dx x^n M(x,q^2)\,, \tag{2.26}$$

we get

$$M_n(q^2)\sim\left(\ln\frac{q^2}{\mu^2}\right)^{dn}\,, \tag{2.27}$$

where

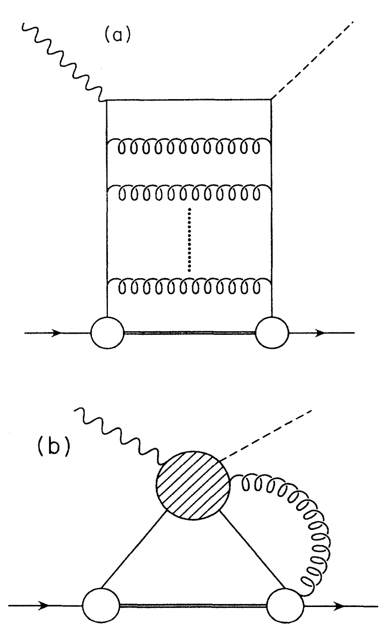

FIG. 4. Ladder graph contributions which lead to the leading logarithmic corrections to the parton model. (b) Typical gluon correction that has no leading logarithmic correction.

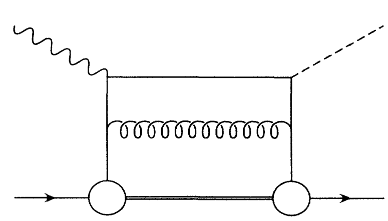

FIG. 5. Lowest-order leading logarithm gluon corrections.

$$d_n = \frac{C_F}{\beta}\left(1 + 4\sum_{j=2}^{n+2}\frac{1}{j}\frac{2}{(n+2)(n+3)}\right) . \qquad (2.28)$$

III. OPERATOR PRODUCT EXPANSION

In this section we discuss an operator product expansion (OPE) analysis of this amplitude. As already intimated, there are some subtleties that prohibit a straightforward prediction for its asymptotic behavior. Before discussing this, however, it is worth reviewing briefly the standard treatment of the conventional structure functions from this viewpoint. In that case the use of perturbation theory to determine the large q^2 behavior can be justified from the application of the OPE to the light-cone expansion of $\mathcal{J}_{\mu\nu}$ (1.6).

Explicitly, the time-ordered product of the two currents can be Taylor expanded around $x^2 \approx 0$ in terms of a complete set of operators $O_{mn}^{\mu_1\cdots\mu_n}$:

$$T[J(x)J(0)] \approx \sum_{m,n} c_m(x^2) x_{\mu_1\cdots x_{\mu_n}} O_{mn}^{\mu_1\cdots\mu_n}(0) . \qquad (3.1)$$

(For ease of presentation, the currents are here taken to be scalar.) Equivalently, its Fourier transform is given by

$$\int d^4x e^{iq\cdot x} T[J(x)J(0)] \approx \sum_{m,n} C_{mn}(q^2)\frac{q_{\mu_1}\cdots q_{\mu_n}}{(\frac{1}{2}q^2)^n} O_{mn}^{\mu_1\cdots\mu_n} , \qquad (3.2)$$

where

$$C_{mn}(q^2) \equiv \left(-\frac{i\partial}{\partial \ln q^2}\right)^n \int d^4x e^{iq\cdot x} c_m(x^2) . \qquad (3.3)$$

On dimensional grounds the $C_{mn}(q^2)$ behave, up to logarithms, like $(q^2)^{-d_c}$ for large q^2 where $d_c = 2d_J - 4 - (d_0 - n)$ is the dimension of C, d_J that of $J(x)$, and d_0 that of the operator $O_{mn}^{\mu_1\cdots\mu_n}$. This is, of course, the origin of the observation that the asymptotic behavior is controlled by the operator having the lowest twist $\tau_0 \equiv d_0 - n$. Notice that these equations are all properties of the current operators (i.e., in the case of interest here, the pion and the virtual photon) and do not depend on the target state. For forward scattering we require the ground-state target matrix elements

$$\langle p|O_{mn}^{\mu_1\cdots\mu_n}|p\rangle = A_{mn}p_{\mu_1\cdots p_{\mu_n}} + B_{mn}g_{\mu_1\mu_2}p_{\mu_3\cdots p_{\mu_n}} + \cdots . \qquad (3.4)$$

The A_{mn}, B_{mn}, etc., are simply numbers characterizing the target. In the contraction of this with (3.2) it is clear that, in the Bjorken limit, terms involving the A_{mn} dominate: one thereby obtains

$$\mathcal{J}(x,q^2) \equiv \int d^4x e^{iq\cdot x}\langle p|T[J(x)J(0)]|p\rangle \approx \sum_{m,n}\frac{A_{mn}C_{mn}(q^2)}{x^n} . \qquad (3.5)$$

The large q^2 behavior of the $C_{mn}(q^2)$ can be determined from the renormalization group using the asymptotic freedom property of QCD. Typically, for the leading twist operator, the C_{mn} are dimensionless and behave like $(\ln q^2)^{-a_{mn}}$, where a_{mn} is determined by the anomalous dimensions of the O_{mn}. Implications for the structure functions, which are the absorptive part of $\mathcal{J}$, can be obtained using the standard analytic properties of $\mathcal{J}$. This leads to the well-known result relating the moments of W to $C_{mn}(q^2)$:

$$M_n(q^2) \equiv \int_0^1 dx x^{n-2}[\nu W(x,q^2)] \approx \sum_m A_{mn}C_{mn}(q^2) . \qquad (3.6)$$

The sum over m is, of course, finite and typically contains only a rather small number of terms. QCD therefore gives a specific prediction for the q^2 dependence of each moment and it is this that has been successfully checked against experiment [1].

Now, suppose that experiments could be performed that directly measure the large q^2 behavior of the full amplitude $\mathcal{J}(x,q^2)$. What is the QCD prediction for this? One immediately sees the difficulty: the expansion (3.5) presumably only makes sense for $|x| > 1$ and this is outside of the physical region. Indeed the analytic continuation to $|x| < 1$ ultimately leads to the moment equations, (3.6). Ideally, one would like to have a complementary expansion valid for $|x| < 1$; this would require knowledge of the analytic structure near $x \simeq 0$ which, unfortunately is not reliably determined by the renormalization group (RG). Naively, one could proceed with $\mathcal{J}$ just as one proceeded with the M_n; i.e., simply take $q^2 \to \infty$ in (3.6) and pick out the dominant $C_{mn}(q^2)$ as determined by the smallest anomalous dimension. In the singlet case, for example, the conservation of the stress-energy tensor means that it has no anomalous dimension and so $M_2(q^2)$ asymptotically approaches a constant. This, in turn, means that the leading behavior of the $T_{1,2}(q^2,x)$, the two conventional scalar amplitudes occurring in the decomposition of $\mathcal{J}_{\mu\nu}$, is given by

$$T_1(q^2,x) \approx \frac{T_2(q^2,x)}{2x} \approx \frac{\langle Q^2\rangle}{\pi q^2 x}\left(\frac{3n_f}{16+3n_f}\right) . \qquad (3.7)$$

Now let us examine the extension of this to the nonforward case. Equations (3.1)–(3.3) remain valid since they are properties of the currents and expansion (3.1) is supposed to be in terms of a complete set of operators; (3.4) however, clearly needs to be generalized. This can be straightforwardly accomplished by writing

$$\langle p'|O_{mn}^{\mu_1\cdots\mu_n}|p\rangle = \sum_{k=0}^{n}[A_{mnk}(t)p_{\mu_1}\cdots p_{\mu_k}\Delta_{\mu_{k+1}}\cdots\Delta_{\mu_n} + B_{mnk}(t)g_{\mu_1\mu_2}p_{\mu_3}\cdots p_{\mu_k}\Delta_{\mu_{k+1}}\cdots\Delta_{\mu_n} + \cdots] . \qquad (3.8)$$

Clearly $A_{mnn}(0) = A_{mn}$ and $B_{mnn}(0) = B_{mn}$. When contracting this with (3.2) we shall need the quantity

$$\frac{2\Delta \cdot q}{-q^2} = 1 + \frac{t}{q^2} \ . \tag{3.9}$$

It is, therefore, the wider set of coefficients $A_{mnk}(t)$ that dominate the asymptotic behavior: (3.5) is thereby generalized to

$$\mathcal{J}(x, q^2, t) \approx \sum_{n=0}^{\infty} \sum_{m} \sum_{k=0}^{n} \frac{A_{mnk}(t)\tilde{C}_{mn}(q^2)}{x^k} \ . \tag{3.10}$$

The $\tilde{C}_{mn}(q^2)$ are the coefficients appropriate to the axial vector current case of interest here, as expressed in (1.3) and (2.2), and are the analogs of the $C_{mn}(q^2)$ of (3.5). They, too, generally fall with q^2 like powers of $\ln(q^2)$ determined by the appropriate anomalous dimension. PCAC ensures that, in M_μ, there is an operator with vanishing anomalous dimension so that it becomes a function of q^2 and t only. In any case, the corrections to this will, as usual, be powers of $\ln(-q^2)$. As already explained it is not possible, beyond this, to give the precise prediction for the large q^2 behavior without summing the series.

Finally, it should be noted that the result expressed in (3.10) is clearly not valid unless $t \ll q^2$, which means that x must not be too close to 1. On the other hand, probing scaling and its violation should shed some light on the $x \approx 0$ region: if the predictions of this paper are experimentally verified it means that the relevant amplitudes are smooth in the region of small x where it would appear that the operator product expansion breaks down.

IV. COMPARISON WITH EXPERIMENT

In this section we begin by reviewing the connection of our results to the existing experimental data. At present, the only such data is for the inclusive reaction (and this was taken over 20 years ago at Cornell [4,5]). It is possible, however, to extend the above arguments to this case provided the mass of the final hadronic "target" state (W') remains relatively small. The main difference is that the scaling function will now depend on W' in addition to x and t so that, instead of

$$s^2 \frac{d\sigma}{dt} \approx F(x, t) \tag{4.1}$$

which is the result implied by (2.11) for the purely exclusive case, one now expects for the inclusive

$$s^2 \frac{d^2\sigma}{dt dW'^2} \approx F(x, t, W'^2) \ . \tag{4.2}$$

In the Cornell experiment, $d^2\sigma/dtdW'^2$ was measured at two different values of $\sqrt{s}$ (2.66 and 3.14 GeV) but at the *same* value of x. The data were averaged over θ and ϕ. The scaling result (4.2) implies that the spectra, when plotted as a function of W',should be identical apart from a normalization factor $(3.14/2.66)^4 \approx 2.41$. The data, as can be readily seen in Fig. 6, are in remarkably good agreement with this prediction.

These data are also presented in terms of the transverse momentum $P_\perp$, the transverse momentum of the pion relative to the direction of the incoming virtual photon, and of a variable x' which depends on the longitudinal momentum of the pion:

$$x' = \frac{P_{11}}{(P_{\max}^2 - P_\perp^2)^{1/2}} \ . \tag{4.3}$$

Here P_{11} is the pion momentum along the direction of the virtual photon and $P_{\max}$ is the maximum pion momentum. In Fig. 7,

$$\frac{E}{\sigma_{\text{tot}}} \frac{d^3\sigma}{dP^3} \tag{4.4}$$

is plotted as a function of $P_\perp^2$ at the two values of W mentioned previously and at two different values of x'. The straight lines are fits of the form $A \exp(-BP_\perp^2)$. The similarity of the spectra suggests that the $P_\perp^2$ distribution (for fixed x) does not depend on q^2, again in striking agreement with the scaling argument.

It is clearly important for new experiments to be carried out at HERA on deep inelastic pion electroproduction at high energies to check whether scaling continues to hold subject to the logarithmic violations which follow from (2.24). Furthermore the arguments of this paper do not only apply to deep inelastic pion electroproduction. A similar argument could be made for deep inelastic electroproduction of any particle which couples to a nucleon

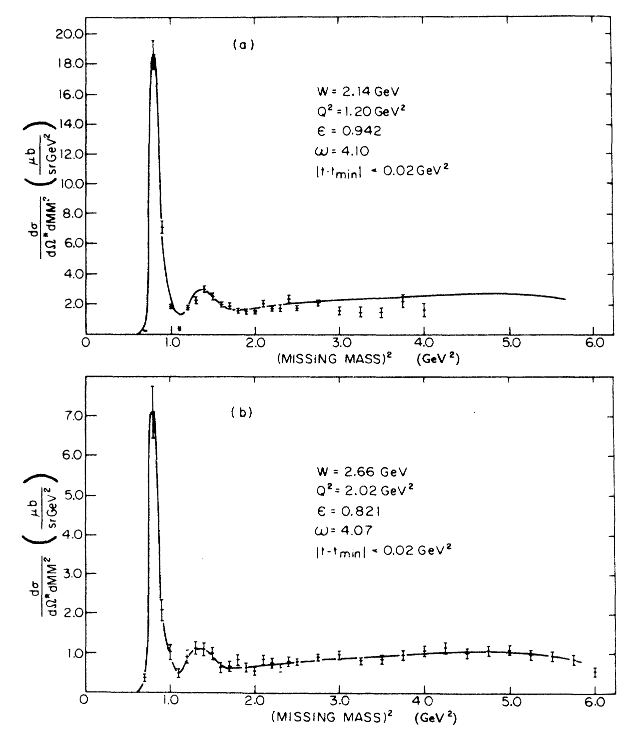

FIG. 6. The virtual photoproduction cross-section at two different values of $W = \sqrt{s}$ but at the same value of $\omega \equiv 1/x$; the data are taken from [4,5]. According to Eqs. (4.1) and (4.2) these should be identical except for a scale factor W^4.

by means of a local current operator. This would include deep inelastic electroproduction of real photons, or of lepton pairs, or of ρ's, or K's, or ψ's, or Υ's, for example. It would be especially interesting to check whether the amplitude for the production of each of these particles scales in the same way, or whether processes which involve heavy quarks are different.

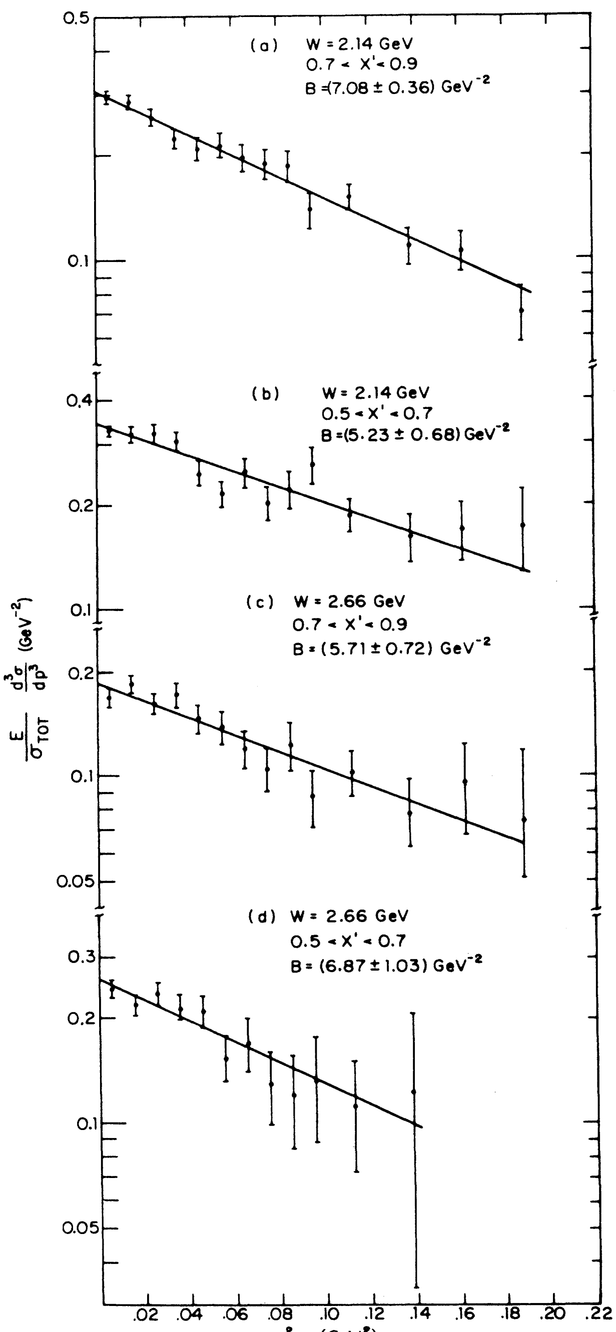

FIG. 7. Transverse-momentum distribution of the produced pions for two regions of longitudinal pion momentum. The similarity of the data at different values of W are in agreement with the scaling argument.

V. CONCLUSION

In this paper we have shown how scaling laws for deep inelastic electropion production derived on rather general grounds from QCD-inspired current algebra, are manifested in QCD perturbation theory. Leading logarithmic corrections are calculated and an evolution equation for the amplitude derived. These are quite similar in character to the well-known ones occurring in the conventional Compton amplitude but have the interesting twist that the predictions for the full amplitude are, in this case, amenable to experiment. In the Compton case, only the imaginary parts (the conventional deep inelastic structure functions) are, in practice, measurable. However, for the full amplitude we have shown that, contrary to one's naive expectation, the usual deviations from scaling derived from an operator product expansion analysis do not lead to a well-defined prediction in the physical region. Thus, unlike the structure function case, the QCD perturbation theory result cannot be "rigorously" justified from asymptotic freedom. The reason for this can be traced back to the behavior of the amplitude near $x \approx 0$; the OPE leads to an expansion in $1/x$ which cannot converge for a physical process. The conventional moment equations for the structure functions which exploit the known analytic properties of the amplitude are precisely designed to circumvent this difficulty. Thus, observation of the scaling laws and their violation for the *full* amplitude can potentially shed light on the small x behavior and help clarify just how far one can push results based on QCD perturbation theory.

With renewed interest in such problems stimulated by recent HERA results at small x and the potential of detailed data from CEBAF (albeit at relatively low energies) we feel that it is important to examine processes such as these that are natural extensions of the canonical structure functions. It is also worth pointing out that these processes can potentially yield complementary data on the quark-gluon structure of the nucleon which could shed further light on its spin and strangeness content. In future work we intend to explore this aspect of the problem in more detail; meanwhile, the main thrust of this paper is motivated by the desire to rekindle interest in such problems given the real possibility of excellent data in the near future.

ACKNOWLEDGMENTS

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APPENDIX

Exclusive pion electroproduction in the context of the parton model was first examined in the early 1970s by Weiss [6], Roy [7], and Landshoff and Polkinghorne [8] using a Reggeized ladder model. These authors all conclude that the process was governed by a form factor and thus does not scale. Roy predicts that the form factor is $\{F_\pi(q^2)F_N(q^2)\}^{1/2}$ while Landshoff and Polkinghorne predict a simple $F_\pi(q^2)$ dependence. Scaling is derived in our paper starting from the conceptually very different approach of PCAC and current algebra. We shall attempt to elucidate here the connection between this early parton model approach and our work and show why there need not be a form factor suppression.

We should first recall the parton model treatment in the process $\gamma * \pi \to \pi$ where the virtual photon $\gamma *$ carries high q^2 (see, for example, Drell and Yan [11]). The photon interacts with a hard parton which is wide-angled scattered. In this case the hard parton effectively carries all the pion's momentum. Following the interaction with the photon the scattered hard parton, which carries the quark quantum number, interacts with the cloud of soft partons to form a physical pion. It is this hard parton scattering followed by the interaction that gives rise to the form factor. The large quantity q^2 is determined by the wide-angle scattering. This description is model dependent, as is clear from the two different results of [7,8]: the difference between the two results is discussed in [9] in terms of the Reggeon structure assumed. But the common thread of both discussions is that the scattered parton is hard.

Our discussion starts from very different premises. The application of PCAC to pion electroproduction for pions of zero mass was first carried out by Nambu and Schrauner [12] in 1962, who showed that they thereby could generalize the Kroll-Ruderman low-energy theorem of pion photoproduction to pion electroproduction but that the former factor involved was the nucleon axial vector form factor $G_A(q^2)$ and not the pion form factor as often assumed [13]. Read and one of us [14,15] later showed that the pion mass could be included together with PCAC through a dynamical approach to the problem where the effective Lagrangian now involved a pseudovector coupling of the pion to the nucleon. This naturally gives rise to a "seagull" diagram corresponding to the commutator term E_μ in the decomposition of the amplitude (2.1): it is E_μ which involves the axial vector form factor.

The PCAC approach has been and continues to be very productive in the analysis of low-energy pion electroproduction off nucleons and nuclei (for a recent review see Scherer and Koch [16]). The pion pole in this approach is complicated since it appears both in the commutator term E_μ and in the remainder $C_{\mu\nu}$ [14]. Furthermore the pion pole is not gauge invariant by itself and it is easy to choose a gauge where there is no pion pole. There is thus no reason to think that the pion pole dominates the amplitude in the limit in which we are interested.

In the present paper we discuss the PCAC decomposition at high q^2. It is not surprising that the pointlike commutator seagull term gives scaling: indeed in Compton scattering a similar commutator term is responsible for both a low-energy theorem and for scaling at high q^2. That is indeed what we find in our case.

Even if one insists on including a pion pole type of contribution, an alternative possibility exists, namely that, in the crossed channel, an initial hard parton of momentum $-En$ is scattered by a virtual photon of momentum $En + \Delta$ to end up as a parton of momentum Δ, that is to say, as a soft parton. This is a different process in parton terms from the one above and does not necessarily lead to a form factor behavior. A related way of seeing that single-particle exchange may well not be dominant in this process was in fact pointed out by Landshoff and Polkinghorne [8]: in the conventional Regge approach to high-energy scattering at fixed t, the scattering angle $\cos\theta_t$ in the crossed channel increases linearly with s (the invariant center-of-mass energy squared), but in the case of exclusive electroproduction $\cos\theta_t \approx s/q^2$ and does not become large. Hence the importance of single-particle exchange and/or Regge poles for these processes is at best tenuous.

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