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Low-energy theorems for photoinduced reactions in the Skyrme-soliton model

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We show that low-energy theorems for pion photoproduction and Compton scattering can be reproduced within the Skyrme-soliton model using current algebra and gauge invariance. The electromagnetic polarizabilities are evaluated through dispersion integrals of the low-energy pion-photoproduction amplitudes obtained in the model. We explicitly give contributions from the Δ state as the resonance and the $\pi\Delta$ channel coupling.

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I. INTRODUCTION

Recently, there has been a revival of interest in low-energy theorems (LET's) in processes such as pion scattering, pion photoproduction, and Compton scattering off a nucleon. This is due to theoretical efforts to construct models of hadrons based on QCD, and recent experimental activities on low-energy π^0 photoproduction [1] and reanalyses of them [2], and also due to measurement of the dynamical structure constants, electric and magnetic polarizabilities [3]. Many theoretical papers have appeared to examine how to reproduce the data by various models of the nucleon, out of which we cite, here, review works by Drechsel and Tiator [4] and by L'vov [5]. How to reproduce LET's is a crucial test both for models of nucleon and for approximations, because low-energy theorems are derived from general principles such as current algebra, partially conserving axial-vector current (PCAC), and gauge invariance.

Since the Skyrme-soliton model was recognized as a realistic soliton model based on QCD [6,7], much effort has been devoted to investigation of the validity of the model and has shown that the SU(2)×SU(2) original Skyrme model can fairly well describe a low-energy pion-nucleon system within about 30% error in spite of only two parameters for pions. There have been also carried out many attempts to reproduce LET's and the polarizabilities within the Skyrme-soliton model [8–16].

We have shown in a recent paper [17] that the soft-pion theorems, the Goldberger-Treiman [18], Adler-Weisberger [19], and Tomozawa-Weinberg [20] relations, can be reproduced within the Skyrme-soliton model. We have also tried in a previous paper [21] to reproduce the LET's for pion photoproduction [22,23] in the model: The LET's have been reproduced at leading order in m_π/M through the electric and magnetic Born terms, but not completely for terms linear in m_π/M, where M (m_π) is the nucleon (pion) mass.

The analyses of pion scattering and pion photoproduction lead naturally to the investigation of Compton scattering, which involves the electromagnetic polarizabilities of the nucleon. We, thus, attempt to derive LET's for Compton scattering which give the polarizabilities. In doing so, we restudy LET's for pion photoproduction in this paper.

One may think that it must be easy to reproduce LET's within the Skyrme-soliton model, since the original Skyrme Lagrangian is chiral invariant except for the pion mass term, and the electromagnetic current is conserved. However, we should recognize that the Skyrme-soliton model is essentially nonrelativistic in the sense that the production and annihilation of soliton-antisoliton pairs are not well defined, though pions as fluctuation fields around the soliton are treated relativistically. The nonrelativistic kinematics give rise to a restriction against proving LET's: Relativistic effects such as Z-type diagrams are of higher order by $1/M$ than nonrelativistic direct and crossed diagrams, but crucial to reproduce LET's [24]. In order to reach LET's within nonrelativistic kinematics we study the amplitudes written in terms of charge density operators after the classic papers by Low [25]: The amplitudes are derived using the gauge invariance imposed on the corresponding amplitudes given in terms of the spatial currents. The amplitudes are regarded as describing reactions induced by a "longitudinal" photon instead of the real transverse photon. We shall show that the LET's are really reproduced within a nonrelativistic Skyrme-soliton model using these amplitudes.

In a previous paper [26] we have shown that the electric polarizability cannot be attributed to the so-called seagull term, because of the requirement of gauge invariance. In this paper we show that the electric polarizability satisfies a sum rule of the total absorption cross section for the longitudinal photon. From this, the magnetic one is shown to be given by the difference between the total absorption cross sections for the real transverse and the longitudinal photon. We explicitly study contributions from the Δ state. Because both the Δ isobar and the nucleon are the rotational levels of the Skyrme-soliton, the Δ should be treated equally as the nucleon in the

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Skyrme-soliton model. This is in contrast with the chiral perturbation theory [27,28], where the Δ state contributes at higher order as a part of two-loop effects. It is shown that the contributions from the channel coupling to the process $\gamma + N \to \pi + \Delta$ enhance both the electric and magnetic polarizabilities at the empirical $\Delta - N$ mass difference, and that those from the Δ pole in the $\gamma + N \to \pi + N$ channel enhance the magnetic polarizability.

The paper is organized as follows. We first give the Lagrangian of the Skyrme-soliton model with the electromagnetic interaction in the next section. The Hamiltonian expanded in powers of the electric charge e up to $O(e^2)$ and the current are given in terms of the total pion fields. LET's for the Compton scattering amplitudes are next shown to be reproduced, and a dispersion relation is given for the forward scattering amplitude of the longitudinal photon in Sec. III. Section IV is devoted to how LET's for the pion-photoproduction amplitude are reproduced. The amplitudes including the Δ are explicitly given. In Sec. V we show the polarizabilities using the sum rule of the longitudinal photoabsorption cross section calculated with the pion-photoproduction amplitude. Conclusions and discussion are given in the last section.

II. SKYRME LAGRANGIAN AND HAMILTONIAN

In order for this paper to be self-contained, we start with the definition of the total canonical pion field Φ_a through SU(2) field $U(x)$ according to Ref. [17]:

$$U(x) = \frac{1}{f_\pi}[\Phi_0(x) + i\tau_a\Phi_a(x)] \tag{2.1}$$

with a constraint

$$\Phi_0^2(x) = f_\pi^2 - \sum_{a=1}^{3}\Phi_a^2, \tag{2.2}$$

where $x = (x^0, \mathbf{x})$, f_π is the pion decay constant, which is 93 MeV empirically. The total field contains full information about the classical soliton configuration and the fluctuation around it in the one-soliton sector. Any gauge-fixing conditions need not be imposed on the total field. It should be noticed, however, that the total field Φ_a must not be separated into the soliton configuration and the fluctuation fields before the matrix elements of Φ_a's are reduced into those sandwiched between two single-baryon, the nucleon or Δ, states. When a function of Φ_a's is sandwiched between the two single-baryon states, the total field can be replaced by the classical soliton configuration within the tree approximation:

$$\langle B(\mathbf{p}')|\,\Phi_a(x)\,|N(\mathbf{p})\rangle = \langle B(\mathbf{p}')|\,\phi_S^a(\mathbf{x} - \mathbf{X}(x^0))\,|N(\mathbf{p})\rangle\,, \tag{2.3}$$

where $\mathbf{X}(x^0)$ is the center of the soliton, and $\phi_S^a(\mathbf{x})$ is the classical soliton configuration,

$$\phi_S^a(\mathbf{x}) = f_\pi \sin F(r) R_{ai}\hat{x}_i, \qquad \phi_S^0(\mathbf{x}) = f_\pi \cos F(r) \tag{2.4}$$

with $r = |\mathbf{x}|$ and $\hat{x}_i = x_i/r$, and $F(r)$ is the profile function, R_{ai} the orthogonal rotation matrix.

The Skyrme Lagrangian is written as

$$L_S = \int d^3x \left\{\frac{1}{2}\dot{\Phi}_a K_{ab}\dot{\Phi}_b - \mathcal{V}[\Phi, \nabla\Phi]\right\} \tag{2.5}$$

with

$$\mathcal{V}[\Phi, \nabla\Phi] = \frac{1}{2}\partial_i\Phi_c G_{ab}\partial_i\Phi_b + m_\pi^2 f_\pi^2 \mathcal{M}[\Phi], \tag{2.6}$$

where

$$K_{ab} = X_{ab} + \frac{1}{\kappa^2 f_\pi^2} \times (X_{ab}\partial_j\Phi_c X_{cd}\partial_j\Phi_d - X_{ac}\partial_j\Phi_c X_{bd}\partial_j\Phi_d)\,, \tag{2.7a}$$

$$G_{ab} = X_{ab} + \frac{1}{2\kappa^2 f_\pi^2} \times (X_{ab}\partial_j\Phi_c X_{cd}\partial_j\Phi_d - X_{ac}\partial_j\Phi_c X_{bd}\partial_j\Phi_d)\,, \tag{2.7b}$$

with $\kappa = e_S f_\pi$, e_S being the Skyrme constant, and

$$X_{ab} = \delta_{ab} + \frac{\Phi_a\Phi_b}{\Phi_0^2}, \tag{2.8}$$

and the pion mass term is given by

$$\mathcal{M}[\Phi] = 1 - \frac{\Phi_0}{f_\pi}. \tag{2.9}$$

The electromagnetic fields $A_\mu(x)$ are introduced through replacing the derivatives by the covariant ones in L_S:

$$\begin{aligned} D_\mu\Phi_a &= \partial_\mu\Phi_a + eA_\mu\varepsilon_{a3b}\Phi_b, \\ D_\mu\Phi_0 &= \partial_\mu\Phi_0. \end{aligned} \tag{2.10}$$

And also the anomalous interaction comes from gauging the Wess-Zumino term [29]. The interaction Lagrangian exact up to $O(e^2)$ is then given as

$$\begin{aligned} L_{\text{int}} = e &\int d^3x A_\mu(x) J^\mu(x) \\ &+ \frac{1}{2}e^2 \int d^3x A_\mu(x) A_\nu(x) Z^{\mu\nu}(x) \\ &+ e^2 \int d^3x \varepsilon_{\mu\nu\rho\sigma}\partial_\mu A_\nu(x) \cdot A_\rho(x) W_\sigma(x) + O(e^3), \end{aligned} \tag{2.11}$$

where

$$J^\mu = V_3^\mu + \frac{1}{2}B^\mu, \tag{2.12a}$$

$$V_3^\mu = \varepsilon_{3ab}\Phi_a \left(K_{bc} - \frac{1}{\kappa^2 f_\pi^2}(X_{bc}X_{de} - X_{bd}X_{ce})\dot{\Phi}_d\dot{\Phi}_e\right) \times \partial^\mu\Phi_c, \tag{2.12b}$$

$$B^\mu = -\frac{1}{12f_\pi^4}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\Phi_a\partial_\nu\Phi_b\partial_\alpha\Phi_c\partial_\beta\Phi_d, \quad (2.12c)$$

and V_3^μ is the three-component of the isovector current and B^μ the isoscalar current identical to the topological baryon current. The explicit forms of $Z^{\mu\nu}$ and W_σ are given in the Appendix.

Now in order to obtain the Hamiltonian exact up to $O(e^2)$, we have to extract the terms involving the time derivatives of the fields from J^μ and others:

$$J^\mu = j^\mu + J_a^\mu\dot{\Phi}_a + \frac{1}{2}\dot{\Phi}_a J_{ab}^\mu\dot{\Phi}_b, \quad (2.13a)$$

$$Z^{\mu\nu} = \zeta^{\mu\nu} + Z_a^{\mu\nu}\dot{\Phi}_a + \frac{1}{2}\dot{\Phi}_a Z_{ab}^{\mu\nu}\dot{\Phi}_b, \quad (2.13b)$$

$$W_\sigma = g_{\sigma i}\xi^i + g_{\sigma 0}\eta_a\dot{\Phi}_a, \quad (2.13c)$$

where j^μ, J_a^μ, $\zeta^{\mu\nu}$, etc., are functions of Φ and $\partial_i\Phi$, but not containing $\dot{\Phi}$; they are given in the Appendix. Thus, the total Lagrangian is rewritten as

$$L_{\rm tot} = \int d^3x\left\{\frac{1}{2}\dot{\Phi}_a\widetilde{K_{ab}}\dot{\Phi}_b + \widetilde{L_a}\dot{\Phi}_a - \widetilde{\mathcal{V}} - \frac{1}{2}\partial_\nu A_\mu\partial^\nu A^\mu + e^2\varepsilon_{0ijk}\dot{A}_i A_k\xi_j\right\}, \quad (2.14)$$

where

$$\widetilde{K_{ab}} = K_{ab} + eA_\mu J_{ab}^\mu + \frac{1}{2}e^2A_\mu A_\nu Z_{ab}^{\mu\nu}, \quad (2.15a)$$

$$\widetilde{L_a} = eA_\mu J_a^\mu + \frac{1}{2}e^2A_\mu A_\nu Z_a^{\mu\nu} + e^2\varepsilon^{ijk0}(\partial_i A_j)A_k\eta_a, \quad (2.15b)$$

$$\widetilde{\mathcal{V}} = \mathcal{V} - eA_\mu j^\mu - \frac{1}{2}e^2A_\mu A_\nu\zeta^{\mu\nu} - e^2\varepsilon^{i\mu\nu j}(\partial_i A_\mu)A_\nu\xi_j. \quad (2.15c)$$

Then, the momenta $\pi^\mu(x)$ and $\Pi^a(x)$ conjugate to $A_\mu(x)$ and $\Phi_a(x)$, respectively, are written as

$$\pi^\mu = -\dot{A}^\mu + e^2\varepsilon^{0\mu jk}A_j\xi_k, \quad (2.16)$$

$$\Pi_a = \widetilde{K_{ab}}\dot{\Phi}_b + \widetilde{L_a}. \quad (2.17)$$

We obtain the Hamiltonian by using the standard prescription without any constraints as

$$H_{\rm tot} = \int d^3x\{\tfrac{1}{2}(\Pi_a - \widetilde{L_a})\widetilde{K^{-1}}_{ab}(\Pi_b - \widetilde{L_b}) + \widetilde{\mathcal{V}}\} + \frac{1}{2}\int d^3x\{-(\pi^\mu - e^2\varepsilon^{0\mu jk}A_j\xi_k)^2 + \partial_i A_\nu\partial^i A^\nu\} \equiv \widetilde{H}_{\rm sky} + \widetilde{H}_{\rm elmg}, \quad (2.18)$$

where we denote the first integral as $\widetilde{H}_{\rm sky}$ and the second as $\widetilde{H}_{\rm elmg}$. Expanding the Hamiltonian in powers of the electric charge e up to $O(e^2)$, we have

$$\widetilde{H}_{\rm sky} = H_{\rm sky} + H_1 + H_2 + O(e^3), \quad (2.19)$$

where

$$H_{\rm sky} = \int d^3x\{\tfrac{1}{2}\Pi_a K_{ab}^{-1}\Pi_b + \mathcal{V}\}, \quad (2.20a)$$

$$H_1 = e\int d^3x\, A_\mu\{-j^\mu - J_a^\mu K_{ab}^{-1}\Pi_b - \tfrac{1}{2}\Pi_a K_{ac}^{-1}J_{cd}^\mu K_{db}^{-1}\Pi_b\}, \quad (2.20b)$$

$$\begin{aligned} H_2 = {} & \tfrac{1}{2}e^2\int d^3x A_\mu A_\nu\{-\zeta^{\mu\nu} + J_a^\mu K_{ab}^{-1}J_b^\nu + 2J_a^\mu K_{ac}^{-1}J_{cd}^\nu K_{db}^{-1}\Pi_b - Z_a^{\mu\nu}K_{ab}^{-1}\Pi_b\} \\ & -e^2\int d^3x\,(\partial_i A_\mu)A_\nu\{\varepsilon^{i\mu\nu 0}\eta_a K_{ab}^{-1}\Pi_b + \varepsilon^{i\mu\nu j}\xi_j\} \\ & +\frac{1}{2}e^2\int d^3x A_\mu A_\nu\{\Pi_a[K_{ac}^{-1}J_{ce}^\mu K_{ef}^{-1}J_{fd}^\nu K_{db}^{-1} - \tfrac{1}{2}K_{ac}^{-1}Z_{cd}^{\mu\nu}K_{db}^{-1}]\Pi_b\}. \end{aligned} \quad (2.20c)$$

The second term in the first set of curly brackets on the right-hand side of Eq. (2.20c), $J_a^\mu K_{ab}^{-1}J_b^\nu$, is the term which cannot be obtained, if the momentum field Π_a conjugate to Φ_a is not given correctly. We should notice here that $H_{\rm int} \neq -L_{\rm int}$ in general, when the latter involves the time derivatives of the fields. In the above we discarded the ordering problem concerning noncommuting operators, the effect of which is of higher order by $O(\hbar^2)$.

The conserving electromagnetic current operator is defined through the equation of motion

$$\mathcal{J}^\mu(x) = i^2[H_{\rm tot},[H_{\rm tot},A^\mu(x)]] + \partial_i\partial^i A^\mu(x) = \mathcal{J}_1^\mu(x) + \mathcal{J}_2^\mu(x), \quad (2.21)$$

where $\mathcal{J}_1^\mu$ is the current of $O(e)$ and $\mathcal{J}_2^\mu$ of $O(e^2)$:

$$\mathcal{J}_1^\mu = -e\{j^\mu + J_a^\mu K_{ab}^{-1}\Pi_b + \tfrac{1}{2}\Pi_a K_{ac}^{-1}J_{cd}^\mu K_{db}^{-1}\Pi_b\}, \quad (2.22a)$$

$$\mathcal{J}_2^\mu = e^2 A_\nu(x)\{-\zeta^{\mu\nu} + J_a^\mu K_{ab}^{-1} J_b^\nu + J_a^\mu K_{ac}^{-1} J_{cd}^\nu K_{db}^{-1}\Pi_b + J_a^\nu K_{ac}^{-1} J_{cd}^\mu K_{db}^{-1}\Pi_b$$
$$-Z_a^{\mu\nu} K_{ab}^{-1}\Pi_b + \Pi_a[K_{ac}^{-1} J_{ce}^\mu K_{ef}^{-1} J_{fd}^\nu K_{db}^{-1} - \tfrac{1}{2} K_{ac}^{-1} Z_{cd}^{\mu\nu} K_{db}^{-1}]\Pi_b\}$$
$$-e^2\{A_\nu \partial_i[\varepsilon^{i\mu\nu 0}\eta_a K_{ab}^{-1}\Pi_b + \varepsilon^{i\mu\nu j}\xi_j] - 2(\partial_i A_\nu)[\varepsilon^{i\mu\nu 0}\eta_a K_{ab}^{-1}\Pi_b + \varepsilon^{i\mu\nu j}\xi_j]$$
$$+\varepsilon^{0\mu\nu j} A_\nu \Pi_a K_{ab}^{-1}\frac{\partial \xi_j}{\partial \Phi_b} + 2\varepsilon^{0\mu\nu j}\pi_\nu \xi_j\}. \quad (2.22b)$$

We note that A_k does not commute with $\mathcal{J}_2^\mu$ at the equal-time, since the current involves π^k. These are the building blocks for our calculations of the Compton and pion-photoproduction amplitudes in the next section.

III. LET's FOR COMPTON SCATTERING

As stated in the Introduction, the Skyrme-soliton model is nonrelativistic in the sense that the soliton-antisoliton pair production and annihilation are not well defined similar to the static Chew-Low model [30]. Since the Z-type diagrams are not taken into account properly, the intermediate states with the baryon number one in the time-ordered product terms are saturated with the states of a positive-energy baryon plus mesons. If we restrict ourselves to the positive energy single-baryon states, the Born terms with the spatial current cannot give the Thomson limit in the Compton scattering, as shown by Low [25]. Here we stress that the Thomson limit is of order N_c^{-1} except for e^2, the electric charge unit. Since the matrix element of the spatial current sandwiched between two single-baryon states is of $O(N_c)$ in the Skyrme-soliton model, we have to reduce the sum of the Born terms by N_c^{-3} to reach the Thomson limit. It cannot be obtained, however, to reduce the order of the amplitudes by the cancellation among the Born terms constructed with the spatial current. The Thomson limit is hidden in the complicated seagull terms in the Skyrme model, the explicit form of which will be given later.

According to Low [25] we consider the Compton scattering amplitudes made of the charge density operators using the gauge invariance. It turns out that the amplitudes have many advantages for the following reasons: they do not have the seagull terms, and the simple cancellation between the direct and crossed Born terms naturally leads to the Thomson limit of $O(N_c^{-1})$, when the classical charge density of $O(1)$ enters into the Born terms.

We adopt the standard Lehmann-Symanzik-Zimmermann (LSZ) reduction formula [31] to write the scattering amplitudes; then, the Compton scattering amplitude is

$$S_{fi} = \delta_{fi} + (2\pi)^4 i\delta(p + k - p' - k') \times \frac{1}{(2\pi)^{3/2}\sqrt{2\omega_{k'}}}\frac{1}{(2\pi)^{3/2}\sqrt{2\omega_k}} 4\pi\epsilon'_\mu T^{\mu\nu}\epsilon_\nu, \quad (3.1)$$

where $\epsilon'_\mu(\epsilon_\nu)$ is the polarization vector of the final (initial) photon. The amplitude is written as

$$T^{\mu\nu} = \frac{i}{4\pi}\int d^4y e^{ik'y}\langle N(\mathbf{p}')|\, T(\mathcal{J}^\mu(y), \mathcal{J}^\nu(0)) + \delta(y^0)\left\{[\dot{A}^\mu(y), \mathcal{J}^\nu(0)] - i\omega'_k[A^0(y), \mathcal{J}^\nu(0)]\right\}|N(\mathbf{p})\rangle\,, \quad (3.2)$$

where the factor $(1/4\pi)$ in T is for making a coupling constant rationalized. The gauge invariance requires that

$$T^{00}\omega_{k'}\omega_k - k'_i T_{ij} k_j = 0,$$

which leads to

$$T^{00} = \boldsymbol{\kappa}'_i T_{ij}\boldsymbol{\kappa}_j \quad (3.3)$$

with $\boldsymbol{\kappa} = \mathbf{k}/\omega_k$.

The seagull term $S^{\mu\nu}$ is just the sum of the equal-time commutators, which are expressed as

$$S^{\mu\nu} = e^2\Bigg\{\zeta^{\mu\nu} - J_a^\mu K_{ab}^{-1} J_b^\nu - J^\mu K_{ac}^{-1} J_{cd}^\nu K_{db}^{-1}\Pi_b - (\mu \leftrightarrow \nu)$$
$$+Z_a^{\mu\nu} K_{ab}^{-1}\Pi_b - \Pi_a\left[K_{ac}^{-1} J_{ce}^\mu K_{ef}^{-1} J_{fd}^\nu K_{db}^{-1} - \frac{1}{2} K_{ac}^{-1} Z_{cd}^{\mu\nu} K_{db}^{-1}\right]\Pi_b$$
$$-\partial_i[\varepsilon^{i\mu\nu 0}\eta_a K_{ab}^{-1}\Pi_b + \varepsilon i\mu\nu j\xi_j] - \varepsilon^{0\mu\nu j}\left[\Pi_a K_{ab}^{-1}\frac{\delta\xi_j}{\delta\Phi_b} + 2ik'^0\xi_j\right]\Bigg\}. \quad (3.4)$$

For the matrix element of the seagull term the pion fields $\Phi_a(0)$'s can be taken to be the classical soliton fields, in a tree approximation, with the center $\mathbf{X}(0)$ and the rotation matrix R_{ai}'s. The momentum fields Π^a's are also rewritten by the classical fields with the rotational and translational zero modes as shown in Ref. [17].

We have already shown in Ref. [26] that $\langle N|\, S^{00}\, |N\rangle$ vanishes in the Skyrme-soliton model by an explicit calculation of S^{00}, and that the vanishing of S^{00} is attributed to the gauge invariance of the Compton scattering amplitude. Here we prove that $\langle N|\, S^{00}\, |N\rangle = 0$ by another method: Since the current obtained in Eq. (2.22b) does not contain the momentum field $\pi^0(x)$ conjugate to A^0 in the Skyrme-soliton model, we have

$$\delta(y^0 - x^0)[A^0(y), J^\nu(x)] = 0.$$

Then, we have

$$\begin{aligned} 0 &= \int d^4y d^4x e^{ik'y - ikx}(ik_\nu)\langle N(\mathbf{p}')|\delta(y^0 - x^0)[A^0(y), \mathcal{J}^\nu(y)]N(\mathbf{p})\rangle \\ &= \int d^4y d^4x e^{ik'y - ikx}\{i\omega_{k'}\langle N(\mathbf{p}')|\delta(y^0 - x^0)[A^0(y), J^0(x)] + \delta(y^0 - x^0)[\dot{A}^0(y), J^0(x)]|N(\mathbf{p})\rangle\} \\ &= (2\pi)^4\delta(p' + k' - p - k)(4\pi)\langle N||S^{00}|N\rangle, \end{aligned} \tag{3.5}$$

where used are the current conservation $\partial_\nu \mathcal{J}^\nu = 0$. The gauge invariance $T^{0\nu}k_\nu = 0$ also leads to $\langle N|\, S^{00}\, |N\rangle = 0$ by a similar calculation. This is the reason why the electric polarizability cannot be derived from the seagull term [26,5].

The amplitude T^{00} may be regarded as the Compton scattering amplitude for the scattering of a longitudinal photon from Eq. (3.3),

$$T^{00} = \epsilon'_{0i} T^{ij} \epsilon_{0j}, \tag{3.6}$$

where

$$\boldsymbol{\epsilon}_0 = \frac{\mathbf{k}}{\omega_k} = \boldsymbol{\kappa} \tag{3.7}$$

is the polarization vector. The amplitude T^{00} is simply given as

$$T^{00} = \frac{i}{4\pi}\int d^4y e^{ik'y}\, \langle N(\mathbf{p}')|\, T(\mathcal{J}^0(y), \mathcal{J}^0(0))\, |N(\mathbf{p})\rangle\, , \tag{3.8}$$

in which there appears no seagull term $\langle N|\, S^{00}\, |N\rangle$.

The Born terms are obtained by inserting the intermediate positive-energy nucleon states as follows:

$$\begin{aligned} T^N_{00} = \frac{1}{4\pi}\Bigg\{ & \frac{\langle N(\mathbf{p}')|\, \mathcal{J}_0(0)\, |N(\mathbf{p}' + \mathbf{k}')\rangle\, \langle N(\mathbf{p} + \mathbf{k})|\, \mathcal{J}_0(0)\, |N(\mathbf{p})\rangle}{E_N(\mathbf{p} + \mathbf{k}) - E_N(\mathbf{p}) - \omega_k} \\ & + \frac{\langle N(\mathbf{p}')|\, \mathcal{J}_0(0)\, |N(\mathbf{p}' - \mathbf{k})\rangle\, \langle N(\mathbf{p} - \mathbf{k}')|\, \mathcal{J}_0(0)\, |N(\mathbf{p})\rangle}{E_N(\mathbf{p} - \mathbf{k}') - E_N(\mathbf{p}) + \omega'_k}\Bigg\}. \end{aligned} \tag{3.9}$$

The charge densities sandwiched between single-nucleon states are

$$\langle N(\mathbf{p}')|\, \mathcal{J}_0(0)\, |N(\mathbf{p}' + \mathbf{k}')\rangle = e_N + O(\omega_k'^2), \tag{3.10a}$$

$$\langle N(\mathbf{p} + \mathbf{k})|\, \mathcal{J}_0(0)\, |N(\mathbf{p})\rangle = e_N + O(\omega_k^2), \tag{3.10b}$$

where e_N is the charge of the nucleon $|N\rangle$, and we discarded possible spin-flip terms and the effects of the electric form factor. The Δ states do not contribute to the Born terms. For low energy scattering we, thus, have the Thomson limit [25]

$$T^N_{00} = -\frac{e_N^2}{4\pi}\frac{1}{M}\frac{\mathbf{k}'\cdot\mathbf{k}}{\omega_k\omega'_k} + O(\omega_k^2), \tag{3.11}$$

where we used that the coefficient of e_N^2/M in the numerator is given by

$$E_N(\mathbf{p} + \mathbf{k}) - E_N(\mathbf{p}) - \omega_k + E_N(\mathbf{p}' - \mathbf{k}) - E_N(\mathbf{p}') + \omega_k = \mathbf{k}\cdot\mathbf{k}'/M,$$

and that the product of the denominators is $\omega_k\omega_{k'}$ at leading order.

For the forward scattering of the longitudinal photon in the laboratory system, $\mathbf{p} = \mathbf{p}' = 0$ and $\mathbf{k} = \mathbf{k}'$, we see that the relation

$$T_{00}(\omega_k, \mathbf{k}) = T_{00}(\omega_k, -\mathbf{k}) \tag{3.12}$$

holds if we use the parity transformation, and that T_{00} is a function of ω_k. We redefine T_{00} so as to be suitable for the dispersion relation:

$$T_{00}(\omega_k) = \frac{i}{4\pi}\int d^4y e^{iky}\langle N(0)|\,\theta(y^0)[\mathcal{J}_0(y),\mathcal{J}_0(0)]\,|N(0)\rangle\,, \tag{3.13}$$

where $\theta(y^0)$ is the usual step function. This definition is different from Eq. (3.8) in the sign of the imaginary part on negative ω_k [32]:

$$\mathrm{Im}T_{00}(\omega_k) = \pi\sum_n\left\{\delta(E_n(\mathbf{k}) - M - \omega_k)\frac{1}{4\pi}\left|\langle n(\mathbf{k})|\,\mathcal{J}_0(0)\,|N(0)\rangle\right|^2 - \delta[E_n(-\mathbf{k}) - M + \omega_k]\frac{1}{4\pi}\left|\langle n(-\mathbf{k})|\,\mathcal{J}_0(0)\,|N(0)\rangle\right|^2\right\}. \tag{3.14}$$

Since $e^{iky} = \exp[i\omega_k(y^0 - \boldsymbol{\kappa}\mathbf{y})]$ and the commutator $[\mathcal{J}_0(y),\mathcal{J}_0(0)]\theta(y^0)$ survives only for $y_0^2 > \mathbf{y}^2 \geq (\boldsymbol{\kappa}\mathbf{y})^2$ and $y^0 > 0$, we have $y^0 - \boldsymbol{\kappa}\mathbf{y} > 0$, and then we may make $T_{00}(\omega_k)$ to continue into the upper-half complex ω_k plane. The asymptotic behavior of T_{00} is the same as that of the amplitude for the real transverse photon at most, since the charge density commutator at $y^2 \to 0+$ is not much singular than the spatial current commutator. We may, therefore, have a dispersion integral for $T_{00}(\omega_k)$. Since $\mathrm{Im}T_{00}(-\omega_k) = -\mathrm{Im}T_{00}(\omega_k)$ holds, we have

$$T_{00}(\omega_k) = -\frac{1}{4\pi}\frac{e_N^2}{M} + \frac{\omega_k^2}{\pi}\int_0^\infty 2\omega_k' d\omega_k' \frac{\mathrm{Im}T_{00}(\omega_k')}{\omega_k'^2(\omega_k'^2 - \omega_k^2)}, \tag{3.15}$$

which is once subtracted at $\omega_k = 0$ to make the dispersion integral converge as in the forward scattering amplitude for the real photon [33]. The constant is the Thomson limit, $T_{00}(0) = -e_N^2/4\pi M$. Note that $\mathrm{Im}T_{00}$ is proportional to the total absorption cross section of longitudinal photon, and is of $O(N_c)$ as shown in the next section. Thus, the Compton amplitude is of $O(N_c)$. We shall use Eq. (3.15) in Sec. V to study the electromagnetic polarizabilities.

IV. LET's FOR PION PHOTOPRODUCTION

The pion-photoproduction amplitude for $\gamma(k) + N(p) \to \pi^a(q) + B(p')$ is defined as

$$S_{fi} = \delta_{fi} + (2\pi)^4 i\delta(p + k - p' - q) \times\frac{1}{(2\pi)^{3/2}\sqrt{2\omega_q}}\frac{1}{(2\pi)^{3/2}\sqrt{2\omega_k}}4\pi T_\mu^a\epsilon^\mu \tag{4.1}$$

with

$$T_\mu^a = \frac{i}{4\pi}\int d^4y e^{iqy}\langle B(\mathbf{p}')|\,T(\mathcal{J}^a(y),\mathcal{J}_\mu(0)) +\delta(y^0)\left\{[\dot\Phi_a(y),\mathcal{J}_\mu(0)] - i\omega_q[\Phi_a(y),\mathcal{J}_\mu(0)]\right\} \times|N(\mathbf{p})\rangle\,, \tag{4.2}$$

where $\mathcal{J}^a = (\partial_\mu\partial^\mu + m_\pi^2)\Phi_a$ is the pion source term. For the electromagnetic current $\mathcal{J}^\mu$ it is sufficient to take the term, $\mathcal{J}_1^\mu$, defined in the preceding section, which is exact up to $O(e)$.

Now, we rewrite the pion source term $\mathcal{J}^a$ of Φ_a using the axial-vector current A_μ^a according to Refs. [34,4,21]. We divide the axial-vector current into the pion pole and the direct coupling part as

$$A_\mu^a(x) = \tilde{A}_\mu^a(x) - f_\pi\partial_\mu\Phi_a(x), \tag{4.3}$$

where $\tilde{A}_\mu^a$ is the direct coupling part, which is related to the pion source term as

$$\partial^\mu\tilde{A}_\mu^a = f_\pi\mathcal{J}^a. \tag{4.4}$$

Then, the amplitude is written as

$$T_\mu^a = \frac{e}{f_\pi}[q^\nu\Pi_{\mu\nu}^a - iC_\mu^a + \omega_q D_\mu^a], \tag{4.5}$$

where we define $\Pi_{\mu\nu}^a$, etc., as

$$\Pi_{\mu\nu}^a = \frac{1}{4\pi}\int d^4y e^{iqy}\langle B(\mathbf{p}')|\,T(\tilde{A}_\nu^a(y),\mathcal{J}_\mu(0))\,|N(\mathbf{p})\rangle\,, \tag{4.6a}$$

$$C_\mu^a = \frac{1}{4\pi}\int d^4y e^{iqy}\delta(y^0)\langle B(\mathbf{p}')|\,[A_0^a(y),\mathcal{J}_\mu(0)]\,|N(\mathbf{p})\rangle\,, \tag{4.6b}$$

$$D_\mu^a = \frac{f_\pi}{4\pi}\int d^4y e^{iqy}\delta(y^0)\langle B(\mathbf{p}')|\,[\Phi_a(y),\mathcal{J}_\mu(0)]\,|N(\mathbf{p})\rangle\,. \tag{4.6c}$$

Taking the polarization vector as

$$\epsilon^\mu(k) = (0,\boldsymbol{\epsilon}) \quad\text{with}\quad \boldsymbol{\epsilon}\cdot\mathbf{k} = 0, \tag{4.7}$$

we have the amplitude $T_a = -T_i^a\epsilon_i$, which describes the pion production by the transverse photon, and it was used to reproduce LET's in Ref. [21]. Since the equal-time commutators, C_μ^a and D_μ^a, were much involved, we were restricted to approximate calculations there. After Low [25] we transform the amplitude to that of the charge operator in place of the spatial part of the electromagnetic current, under the gauge invariance, and show that LET's are reproduced for the electric part of interaction. The magnetic part of interaction is treated separately, because it is not included in the transformed amplitude.

A. Electric Born terms

The gauge invariance, which is equivalent to the conservation of the electromagnetic current, requires that

$$T_i^a k_i = T_0^a \omega_k, \quad (4.8)$$

which may be rewritten as

$$T_0^a = \boldsymbol{T}^a \cdot \boldsymbol{\epsilon}_0, \quad (4.9)$$

as if it is the production amplitude induced by the longitudinal photon. The amplitude is written in terms of the operators

$$\mathcal{J}_0 = e\left(V_0^3 + \frac{1}{2}B_0\right), \quad (4.10a)$$

$$V_0^3 = \varepsilon_{3ab}\Phi_a\Pi_b, \quad (4.10b)$$

$$B_0 = -\frac{1}{12\pi^2 f_\pi^4}\varepsilon^{0ijk}\varepsilon_{abcd}\Phi_a\partial_i\Phi_b\partial_j\Phi_c\partial_k\Phi_d, \quad (4.10c)$$

$$A_0^a = -\frac{1}{2}\{\Phi_0, \Pi^a\}. \quad (4.10d)$$

The equal-time commutators are much simpler than the case for the spatial current, owing to the current algebra [17]:

$$[A_0^a(0,\mathbf{y}), \mathcal{J}_0(0)] = i\varepsilon_{a3b}A_0^b(0)\delta(\mathbf{y}) + \text{S.t.}, \quad (4.11a)$$

$$[\Phi_a(0,\mathbf{y}), \mathcal{J}_0(0)] = i\varepsilon_{a3b}\Phi_b(0)\delta(\mathbf{y}), \quad (4.11b)$$

where S.t. stands for the Schwinger term. The equal-time commutators are then given as

$$-iC_0^a + \omega_q D_0^a = \frac{\varepsilon_{a3b}}{4\pi}\langle B(\mathbf{p}')|\, A_0^b(0) + if_\pi\omega_q\Phi_b(0)\,|N(\mathbf{p})\rangle. \quad (4.12)$$

The Schwinger term vanishes exactly at the threshold, and we discard it hereafter.

The axial-vector current and the pion field sandwiched between the single-baryon states can be replaced by the corresponding classical fields:

$$\frac{\varepsilon_{a3b}}{4\pi}\frac{1}{f_\pi}\langle B(\mathbf{p}')|\, A_0^b(0) + if_\pi\omega_q\Phi_b(0)\,|N(\mathbf{p})\rangle = i\varepsilon_{a3b}\mathcal{T}^b\frac{G_{BN\pi}}{8\pi M}\left\{\frac{i\mathbf{S}\cdot(\mathbf{p}'+\mathbf{p})}{2M} + \frac{i\mathbf{S}\cdot(\mathbf{p}'-\mathbf{p})(2\omega_q-\omega_k)}{m_\pi^2+(\mathbf{p}'-\mathbf{p})^2}\right\} + O(N_c^{-1}), \quad (4.13)$$

where we used the relations [21]

$$\langle B(\mathbf{p}')|\,\tilde{A}_0^a\,|N(\mathbf{p})\rangle = f_\pi\frac{G_{BN\pi}}{2M}\mathbf{S}\cdot(\mathbf{p}'+\mathbf{p})\mathcal{T}^a\frac{1}{2M}, \quad (4.14a)$$

$$\langle B(\mathbf{p}')|\,\Phi_a(0)\,|N(\mathbf{p})\rangle = -\frac{G_{BN\pi}}{2M}i\mathbf{S}\cdot(\mathbf{p}'-\mathbf{p})\mathcal{T}^a\frac{1}{m_\pi^2+(\mathbf{p}'-\mathbf{p})^2}, \quad (4.14b)$$

and $\mathcal{T}^b$ (S_i) is the transition isospin (spin) matrix from N to Δ for $B=\Delta$ and τ^b (σ_i) for $B=N$.

We take hereafter the center-of-mass system to write the amplitudes explicitly, where $\mathbf{p}=-\mathbf{k}$ and $\mathbf{p}'=-\mathbf{q}$. Then, Eq. (4.13) reduces to

$$i\varepsilon_{a3b}\mathcal{T}^b\frac{G_{BN\pi}}{8\pi M}\frac{1}{\omega_k}\left\{i\mathbf{S}\cdot(\mathbf{k}+\mathbf{q})\frac{\omega_k}{2M} - i\mathbf{S}\cdot(\mathbf{k}-\mathbf{q})\frac{2\omega_k\omega_q-\omega_k^2}{m_\pi^2+(\mathbf{k}-\mathbf{q})^2}\right\}. \quad (4.15)$$

Next, we proceed to the contribution from the single-baryon intermediate states in the time-ordered product terms:

$$\sum_{B'=N,\Delta}\left\{\frac{i\langle B(\mathbf{p}')|\,\tilde{A}_\mu^a(0)\,|B'(0)\rangle\langle B'(0)|\,\mathcal{J}_0(0)\,|N(\mathbf{p})\rangle}{\omega_k+E_N(\mathbf{p})-E_{B'}(0)} - \frac{i\langle B(\mathbf{p}')|\,\mathcal{J}_0(0)\,|B'(\mathbf{p}'-\mathbf{k})\rangle\langle B'(\mathbf{p}'-\mathbf{k})|\,\tilde{A}_\mu^a(0)\,|N(\mathbf{p})\rangle}{\omega_k+E_{B'}(\mathbf{p}'-\mathbf{k})-E_B(\mathbf{p}')}\right\}. \quad (4.16)$$

Using the relations

$$\langle B'(0)|\,\mathcal{J}_0(0)\,|N(\mathbf{p})\rangle = \delta_{B'N}\left(\frac{1}{2}+\frac{1}{2}\tau^3\right)e + O(k^2), \quad (4.17a)$$

$$\langle B(\mathbf{p}')|\,\mathcal{J}_0(0)\,|B'(\mathbf{p}'-\mathbf{k})\rangle = \delta_{BB'}\left(\frac{1}{2}+\frac{1}{2}\mathcal{T}_{BB}^3\right)e + O(k^2), \quad (4.17b)$$

$$\langle B(\mathbf{p}')|\,\tilde{A}_i^a\,|N(\mathbf{p})\rangle = f_\pi\frac{G_{BN\pi}}{2M}S_i\mathcal{T}^a, \quad (4.17c)$$

we have

$$\frac{1}{f_\pi}q^\mu\Pi^a_{\mu 0} = \frac{eG_{BN\pi}}{8\pi M}\left\{-\frac{i\mathbf{S}\cdot\mathbf{q}\mathcal{T}^a(\frac{1}{2}+\frac{1}{2}\tau^3)}{\omega_k+\frac{k^2}{2M}}+\frac{i\mathbf{S}\cdot\mathbf{q}(\frac{1}{2}+\frac{1}{2}\mathcal{T}^3_{BB})\mathcal{T}^a}{\omega_k+\frac{\mathbf{k}^2+2\mathbf{kq}}{2M}}\right.$$
$$\left.+\frac{i\mathbf{S}\cdot(\mathbf{p}'+\mathbf{p}+\mathbf{k})\mathcal{T}^a(\frac{1}{2}+\frac{1}{2}\tau^3)}{\omega_k+\frac{k^2}{2M}}\frac{\omega_q}{2M}-\frac{i\mathbf{S}\cdot(\mathbf{p}'+\mathbf{p}-\mathbf{k})(\frac{1}{2}+\frac{1}{2}\mathcal{T}^3_{BB})\mathcal{T}^a}{\omega_k+\frac{k^2+2\mathbf{kq}}{2M}}\frac{\omega_q}{2M}\right\} \tag{4.18}$$

as the nucleon and Δ pole terms, where we approximated as $k^2/2M_\Delta = k^2/2M + O(N_c^{-2})$ in the denominators. Here, we ignored the effect of the form factors and spin-flip terms in the matrix element of the charge as in the preceding section. The effect of the form factors will be discussed in the last section related to other definitions of the electric polarizability.

Now we decompose the production amplitude as

$$T^a_0 = i\varepsilon_{a3b}\mathcal{T}^b T^{(-)}_0 + \mathcal{T}^a T^{(0)}_0 + \mathcal{T}^+_{a3} T^{(+)}_0, \tag{4.19}$$

where $\mathcal{T}^+_{a3} \equiv \mathcal{T}^a\frac{1}{2}\tau^3 + \frac{1}{2}\mathcal{T}^3_{BB}\mathcal{T}^a$, which reduces to δ_{a3} for $B = N$. Combining Eqs. (4.15) and (4.18), we obtain the amplitudes for the Born terms together with the equal-time commutator terms in the center-of-mass system as

$$-T^{(-)}_0 = \left(\frac{eG_{BN\pi}}{8\pi M}\right) \times\left\{i\mathbf{S}\cdot\boldsymbol{\kappa}+\frac{i\mathbf{S}\cdot(\mathbf{k}-\mathbf{q})(2\boldsymbol{\kappa}\cdot\mathbf{q}-\boldsymbol{\kappa}\cdot\mathbf{k})}{m_\pi^2+(\mathbf{k}-\mathbf{q})^2}\right\}, \tag{4.20a}$$

$$-T^{(0)}_0 = -T^{(+)}_0 = \left(\frac{eG_{BN\pi}}{8\pi M}\right) i\mathbf{S}\cdot\boldsymbol{\kappa}\left(-\frac{\omega_q}{2M}\right), \tag{4.20b}$$

where we discarded nonleading terms which vanish at the threshold. We call these amplitudes the *longitudinal Born terms* later.

Note that the amplitude T^a_0 is equal to $T^a_i\kappa_i$, but what we want to obtain is $-T^a_i\epsilon_i$, the electric interaction part of which we denote as $T^{(\pm,0)}_E$. Replacing $\boldsymbol{\kappa}$ by $\boldsymbol{\epsilon}$, we have, for $T^{(\pm,0)}_E$,

$$T^{(-)}_E = \left(\frac{eG_{BN\pi}}{8\pi M}\right)\left\{i\mathbf{S}\cdot\boldsymbol{\epsilon}+2\frac{i\mathbf{S}\cdot(\mathbf{k}-\mathbf{q})(\boldsymbol{\epsilon}\cdot\mathbf{q})}{m_\pi^2+(\mathbf{k}-\mathbf{q})^2}\right\}, \tag{4.21a}$$

$$\underset{\mathbf{q}\to 0}{\to}\left(\frac{eG_{BN\pi}}{8\pi M}\right)\left\{i\mathbf{S}\cdot\boldsymbol{\epsilon}+O(\mu^2)\right\}, \tag{4.21b}$$

$$T^{(0)}_E = T^{(+)}_E = \left(\frac{eG_{BN\pi}}{8\pi M}\right) i\mathbf{S}\cdot\boldsymbol{\epsilon}\left(-\frac{\omega_q}{2M}\right) \tag{4.21c}$$

$$\underset{\mathbf{q}\to 0}{\to}\left(\frac{eG_{BN\pi}}{8\pi M}\right)\left\{i\mathbf{S}\cdot\boldsymbol{\epsilon}\,(-\frac{1}{2}\mu)+O(\mu^2)\right\} \tag{4.21d}$$

with being $\mu = m_\pi/M$. We call these terms as the *electric Born terms* later.

We can see that the threshold amplitude in Eq. (4.21d) satisfies LET's except the terms proportional to the magnetic moments of nucleon. Note that the linear term in μ is exactly reproduced in the method. The term is not obtained by means of the transverse polarization condition, because of the nonrelativistic treatment of the time-ordered product terms. Usually, the term is behind the antinucleon propagation. Thus, we have shown that the Skyrme-soliton model can also reproduce the model-independent part of LET's for the pion-photoproduction amplitude. We note that the amplitude $T^{(-)}_{0,E}$ is of $O(N_c^{1/2})$, while $T^{(+,0)}_{0,E}$ are of $O(N_c^{-1/2})$.

B. Magnetic Born terms

It is known that the amplitudes of $N_c^{1/2}$ also come from the magnetic interaction for the spatial current. We now examine these *magnetic Born terms* in $-q^\mu\Pi_{\mu i}\epsilon_i$. These are the terms which vanish in the longitudinal polarization, so that the previous prescription cannot include these terms.

According to Refs. [7,21] the matrix element of the spatial current is given as

$$\langle B(\mathbf{p}')|\,\mathcal{J}_i(0)\epsilon_i\,|N(\mathbf{p})\rangle = i(\boldsymbol{\sigma}\cdot\mathbf{s})\,\delta_{BN}\,\mu^N_S + i(\mathbf{S}\cdot\mathbf{s})\,\mathcal{T}^3\mu^{BN}_V, \tag{4.22}$$

where $\mu^N_{V,S} = \frac{1}{2}(\mu_p \mp \mu_n)/2M_N$, $\mu^{\Delta N}_V = -3/\sqrt{2}\mu^N_V$, and $\mathbf{s} = (\mathbf{p}' - \mathbf{p}) \times \boldsymbol{\epsilon}$. Here, we discarded the terms of the translational zero modes in the isoscalar current. Note that μ^N_V is of $O(N_c)$, while μ^N_S of $O(N_c^{-1})$. The magnetic Born terms are composed of two parts; we denote one of them as $T^{(\alpha)}_M$ with being $\alpha = \pm, 0$, which is proportional to $\mathbf{s} = \mathbf{k}\times\boldsymbol{\epsilon}$, and the other as $T^{(\alpha)}_m$ proportional to $\boldsymbol{\sigma}\cdot\mathbf{s}\boldsymbol{\sigma}\cdot\mathbf{k}$. The former vanishes at threshold, while the latter remains finite. Both amplitudes are transversal, so that they vanish for the longitudinal polarization.

We give $T^{(\alpha)}_M$ which we call the *magnetic Born terms* for $\gamma + N \to \pi + N$ as

$$T^{(-)}_M = \left(\frac{eG_{NN\pi}}{8\pi M}\right)\mu^N_V\left\{-\frac{(\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})}{\omega_k}-\frac{(\boldsymbol{\sigma}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})}{\omega_k}+\frac{1}{2}\frac{[3\mathbf{s}\cdot\mathbf{q}-(\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})]}{\omega_k-\Delta M}+\frac{1}{2}\frac{[3\mathbf{s}\cdot\mathbf{q}-(\boldsymbol{\sigma}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})]}{\omega_k+\Delta M}\right\}, \tag{4.23a}$$

$$T_M^{(+)} = \left(\frac{eG_{NN\pi}}{8\pi M}\right) \mu_V^N \left\{ -\frac{(\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})}{\omega_k} + \frac{(\boldsymbol{\sigma}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})}{\omega_k} - \frac{[3\mathbf{s}\cdot\mathbf{q} - (\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})]}{\omega_k - \Delta M} + \frac{[3\mathbf{s}\cdot\mathbf{q} - (\boldsymbol{\sigma}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})]}{\omega_k + \Delta M} \right\}, \tag{4.23b}$$

$$T_M^{(0)} = \left(\frac{eG_{NN\pi}}{8\pi M}\right) \mu_S^N \left\{ -\frac{(\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})}{\omega_k} + \frac{(\boldsymbol{\sigma}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})}{\omega_k} \right\}, \tag{4.23c}$$

where we put $(1+\omega_q/2M)/(1+\omega_k/2M) = 1$ and neglect $\mathbf{k}\cdot\mathbf{q}/M$ in the denominator. The amplitudes $T_{\rm M}^{(\alpha)}$ become of P wave owing to neglecting $\mathbf{k}\cdot\mathbf{q}/M$ in the denominator, and vanish at the threshold; we note that $(\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})$ and $3\mathbf{q}\cdot\mathbf{s} - (\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})$ are the P-wave projection operators for $J=1/2$ and $J=3/2$, respectively [30].

It is easy to see that $T_M^{(\pm)}$ reduces to $O(N_c^{1/2})$ by the cancellation among the N- and Δ-pole terms, while $T_M^{(0)}$ is nonleading and of $O(N_c^{-1/2})$.

The amplitudes $T_m^{(\alpha)}$'s which remain finite at threshold are

$$T_m^{(-)} = -\left(\frac{eG_{NN\pi}}{8\pi M}\right) \mu_V^N \left\{ \frac{i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}k^2}{\omega_k}\frac{\omega_q}{M} + \frac{1}{2}\frac{i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}k^2}{\omega_k+\Delta M}\frac{\omega_q}{M} \right\}, \tag{4.24a}$$

$$T_m^{(+)} = \left(\frac{eG_{NN\pi}}{8\pi M}\right) \mu_V^N \left\{ \frac{i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}k^2}{\omega_k}\frac{\omega_q}{M} - \frac{i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}k^2}{\omega_k+\Delta M}\frac{\omega_q}{M} \right\}, \tag{4.24b}$$

$$T_m^{(0)} = \left(\frac{eG_{NN\pi}}{8\pi M}\right) \mu_S^N \left\{ \frac{i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}k^2}{\omega_k}\frac{\omega_q}{M} \right\}, \tag{4.24c}$$

in the same approximation as Eqs.(4.23a) to (4.23c). At the threshold we obtain

$$T_m^{(-)} \underset{\mathbf{q}\to 0}{\to} -i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\left(\frac{eG_{NN\pi}}{8\pi M}\right)\mu_V^N\frac{m_\pi^2}{M}\left(1+\frac{1}{2}\frac{1}{1+\Delta M/m_\pi}\right), \tag{4.25a}$$

$$T_m^{(+)} \underset{\mathbf{q}\to 0}{\to} i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\left(\frac{eG_{NN\pi}}{8\pi M}\right)\mu_V^N\frac{m_\pi^2}{M}\left(1-\frac{1}{1+\Delta M/m_\pi}\right), \tag{4.25b}$$

$$T_m^{(0)} \underset{\mathbf{q}\to 0}{\to} i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\left(\frac{eG_{NN\pi}}{8\pi M}\right)\mu_S^N\frac{m_\pi^2}{M}. \tag{4.25c}$$

This result has already been obtained in a previous paper [21], and is the same as that of the covariant perturbation theory [23]. If we take the leading order terms in the $1/N_c$ expansion by expanding Eqs. (4.25a)–(4.25c) in powers of $\Delta M/m_\pi$ which is of order N_c^{-1}, we have

$$T_m^{(-)} \underset{\mathbf{q}\to 0}{\to} -i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\left(\frac{eG_{NN\pi}}{8\pi M}\right)\mu_V^N\frac{m_\pi^2}{M}\left(\frac{3}{2}\right), \tag{4.26a}$$

$$T_m^{(+)} \underset{\mathbf{q}\to 0}{\to} i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\left(\frac{eG_{NN\pi}}{8\pi M}\right)\mu_V^N\frac{m_\pi^2}{M}\left(\frac{\Delta M}{m_\pi}\right). \tag{4.26b}$$

Note that the net threshold value of $T_m^{(-)}$ remains to be of $O(N_c^{1/2})$, but that of $T_m^{(+)}$ reduces to $O(N_c^{-1/2})$; that is, they are of the same order as of the electric Born terms, Eqs. (4.21b) and (4.21d), while $T_m^{(0)}$ is of $O(N_c^{-3/2})$ higher than the pion pole term by $O(N_c^{-1})$.

Under the same approximation the channel-coupling Born terms for the process $\gamma + N \to \pi + \Delta$ through the magnetic interaction are obtained:

$$(T_M^{(-)})^\Delta = \left(\frac{eG_{\Delta N\pi}}{8\pi M}\right)\mu_V^N\left\{-\frac{(\mathbf{S}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})}{\omega_k} - \frac{4}{5}\frac{(\mathbf{S}_{\Delta\Delta}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})}{\omega_q} + 2\frac{(\mathbf{S}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})}{\omega_q} - \frac{1}{5}\frac{(\mathbf{S}_{\Delta\Delta}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})}{\omega_k}\right\}, \tag{4.27a}$$

$$(T_M^{(+)})^\Delta = \left(\frac{eG_{\Delta N\pi}}{8\pi M}\right)\mu_V^N\left\{-\frac{(\mathbf{S}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})}{\omega_k} - \frac{1}{5}\frac{(\mathbf{S}_{\Delta\Delta}\mathbf{q})(\mathbf{S}\cdot\mathbf{s})}{\omega_q} + \frac{(\mathbf{S}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})}{\omega_q} + \frac{1}{5}\frac{(\mathbf{S}_{\Delta\Delta}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})}{\omega_k}\right\}, \tag{4.27b}$$

$$(T_M^{(0)})^\Delta = \left(\frac{eG_{\Delta N\pi}}{8\pi M}\right)\mu_S^N\left\{-\frac{(\mathbf{S}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{s})}{\omega_k} + \frac{(\mathbf{S}_{\Delta\Delta}\cdot\mathbf{s})(\boldsymbol{\sigma}\cdot\mathbf{q})}{\omega_k}\right\}, \tag{4.27c}$$

where $(\mathbf{S}\cdot\mathbf{q})(\mathbf{S}\cdot\mathbf{s})$ and $(\mathbf{S}_{\Delta\Delta}\cdot\mathbf{q})(\mathbf{S}\cdot\mathbf{s})$ are also the P wave projection operators for $J=1/2$ and $J=3/2$, respectively, in the reactions. We have the counterparts of T_m in the $\gamma + N \to \pi + \Delta$ process, but we do not write them here.

Collecting the above electric and magnetic amplitudes, we write, for the Compton scattering amplitude of nucleon at threshold,

$$T^{(-)} = \left(\frac{eG_{NN\pi}}{8\pi M}\right) i\boldsymbol{\sigma}\cdot\,\boldsymbol{\epsilon}\left\{1+O(\mu^2)\right\}, \tag{4.28}$$

$$T^{(0,+)} = \left(\frac{eG_{NN\pi}}{8\pi M}\right) i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\left\{-\frac{1}{2}\mu+\frac{1}{2}\mu^2\mu^N_{V,S}+O(\mu^3)\right\}, \tag{4.29}$$

where $\mu^N_{S,V}$ are in the units of nuclear magneton. Thus, we have succeeded in reproducing LET's for the pion-photoproduction amplitude through the electric and magnetic Born terms within the Skyrme-soliton model.

V. ELECTROMAGNETIC POLARIZABILITIES

The low-energy Compton scattering amplitude of no spin-flip can be expanded in powers of ω_k as

$$\epsilon'_i T_{ij}\epsilon_j = \epsilon'_i T^N_{ij}\epsilon_j + (\bar\alpha\boldsymbol{\epsilon}'\cdot\boldsymbol{\epsilon}+\bar\beta\mathbf{s}'\cdot\mathbf{s})\,\omega_k^2 + O(\omega_k^3), \tag{5.1}$$

where T^N_{ij} is the nucleon Born amplitude, and $\bar\alpha$ ($\bar\beta$) is the electric (magnetic) polarizability. In the above $\mathbf{s}=\boldsymbol{\kappa}\times\,\boldsymbol{\epsilon}$. If we put the longitudinal polarization $\boldsymbol{\epsilon}_0=\boldsymbol{\kappa}$ in place of the transverse polarization, we have

$$T_{00} = \kappa'_j T_{ij}\kappa_i = \kappa'_j T^N_{ij}\kappa_i + \bar\alpha\omega_k^2\boldsymbol{\epsilon}'_0\cdot\boldsymbol{\epsilon}_0 + O(\omega_k^3), \tag{5.2}$$

and $\kappa'_j T^N_{ij}\kappa_i = T^N_0$. This equation shows that the electric polarizability can be obtained by using the amplitude T_{00}.

Taking the single-pion plus baryon states $|\pi^a(\mathbf{q}),B(\mathbf{k}-\mathbf{q})\rangle$ with $B=\Delta$ or N as $|n(\mathbf{k})\rangle$ in the imaginary part of the amplitude, Eq. (3.14), for the dispersion relation of T_{00}, we rewrite $\langle n(\mathbf{k})|\,\mathcal{J}_0(0)\,|N(0)\rangle$ as

$$\langle n(\mathbf{k})|\,\mathcal{J}_0(0)\,|N(0)\rangle = \frac{1}{(2\pi)^{3/2}\sqrt{2\omega_q}}4\pi T^{aB}_0(\mathbf{q},\mathbf{k}), \tag{5.3}$$

where T^{aB}_0 is the production amplitude of a pion a with a baryon $B=\Delta$ or N in the laboratory frame. The contribution to the imaginary part of T_{00} is then given as

$$4\pi^2\int\frac{d^3q}{(2\pi)^3 2\omega_q}\delta(E_B(\mathbf{k}-\mathbf{q})+\omega_q-M-\omega_k)\left|T^{aB}_0(\mathbf{q},\mathbf{k})\right|^2 = \frac{q}{4\pi}\int d\Omega_{\hat q}\left|T^{aB}_0(\mathbf{q},\mathbf{k})\right|^2. \tag{5.4}$$

Then, we obtain a dispersion integral for $\bar\alpha$:

$$\bar\alpha = \frac{1}{2\pi^2}\int_{\rm th}^{\infty} d\omega_k\frac{\sigma_0(\omega_k)}{\omega_k^2}, \tag{5.5}$$

where σ_0 is the total pion-production cross section by the longitudinal photon defined as

$$\sigma_0(\omega_k) = \frac{q}{\omega_k}\sum_{B,a}\int d\Omega\left|T^{aB}_0(\mathbf{q},\mathbf{k})\right|^2 + \text{multipions}. \tag{5.6}$$

If the total photoabsorption cross section $\sigma_{\rm tot}$ is used in place of σ_0 in the right-hand side of Eq. (5.5), the dispersion integral becomes the Baldin sum rule [35]:

$$\bar\alpha+\bar\beta = \frac{1}{2\pi^2}\int_{\rm th}^{\infty} d\omega_k\frac{\sigma_{\rm tot}(\omega_k)}{\omega_k^2}. \tag{5.7}$$

Note that $T^{aB}_0(\mathbf{q},\mathbf{k})$ is of $O(N_c^{1/2})$, so that $\bar\alpha$ is of $O(N_c)$. Introducing a Lorentz invariant variable $\nu\equiv p\cdot k/M$ which reduces to ω_k in the laboratory system, we write Eq. (5.5) as

$$\bar\alpha = \frac{1}{2\pi^2}\int_{\rm th}^{\infty} d\nu\frac{\sigma_0(\nu)}{\nu^2}, \tag{5.8}$$

which can be used in the center-of-mass system, where

$$\omega_k = \nu = \omega_k^*\frac{E_N(-\mathbf{k}^*)+\omega_k^*}{M} = \omega_k^* + O(N_c^{-2}), \tag{5.9}$$

with ω_k^* and $\mathbf{k}^*$ being the photon energy and momentum in the center-of-mass system. We take an approximation $\omega_k=\omega_k^*$ hereafter. Using the Baldin dispersion integral, we write the magnetic polarizability as

$$\bar\beta = \frac{1}{2\pi^2}\int_{\rm th}^{\infty} d\omega_k\frac{[\sigma_{\rm tot}(\omega_k)-\sigma_0(\omega_k)]}{\omega_k^2}. \tag{5.10}$$

Note that we cannot use experimental data for both expressions of $\bar\alpha$, Eq. (5.5) and $\bar\beta$, Eq. (5.10) in contrast with the finite-energy sum rule developed by L'vov, Petrun'kin, and Startsev [36]. We note here that recently proposed is the backward dispersion relation for $\bar\alpha-\bar\beta$, which is given as the s- and the t-channel contributions in the relativistic pion-nucleon theory [37].

A. Contribution from the electric Born terms

For calculating the longitudinal-photon absorption cross section we here take the longitudinal Born term

$T_0^{(-)}$ in Eq. (4.20a) as T_0^{aB} in the static limit, where $\omega_k = \omega_q + \Delta M$ for $B = \Delta$ with being $\Delta M = M_\Delta - M$ and $\omega_k = \omega_q$ for $B = N$. Integrating Eq. (5.6) with neglecting multipion terms, we have, for $B = \Delta$,

$$\sigma_0^\Delta = 4\pi \frac{q}{\omega_k} \frac{8}{9} \left(\frac{eG_{\Delta N\pi}}{8\pi M} \right)^2 F_\Delta(v), \tag{5.11}$$

with

$$F_\Delta(v) = \frac{1}{b^2} \left\{ -(1-v^2) - \frac{1}{4} \frac{1-v^2}{a^2 - b^2 v^2} + \frac{b^2 + 2 - 2v^2}{4bv} \ln\left(\frac{a+bv}{a-bv} \right) \right\}, \tag{5.12}$$

where $v = q/\omega_q$ and

$$b = 1 + \frac{\Delta M}{m_\pi} \sqrt{1-v^2}. \tag{5.13}$$

For $B = N$, we obtain

$$\sigma_0^N = 4\pi \frac{q}{\omega_k} \left(\frac{\sqrt{2} eG_{NN\pi}}{8\pi M} \right)^2 F_N(v), \tag{5.14}$$

with

$$F_N(v) = \left\{ -\frac{5}{4} + v^2 + \frac{3 - 2v^2}{4v} \ln\left(\frac{1+v}{1-v} \right) \right\}. \tag{5.15}$$

This is the same expression as by L'vov [38]. Note that $F_\Delta(v)$ reduces to $F_N(v)$ at $\Delta M = 0$. The integration of σ_0^N with respect to v from 0 to 1 gives

$$\bar{\alpha}^{(N)} = \left(\frac{\sqrt{2} eG_{NN\pi}}{8\pi M} \right)^2 \left\{ \frac{10}{24 m_\pi} \right\}. \tag{5.16}$$

This result is the same as that of the chiral perturbation theory at leading order in powers of m_π [5,38].

In order to calculate the magnetic polarizability we take into account the electric Born term $T_E^{(-)}$ of Eq. (4.21b), through which we obtain

$$\sigma_{\text{tot}}^\Delta = \frac{q}{\omega_k} \frac{8}{9} \left(\frac{eG_{\Delta N\pi}}{8\pi M} \right)^2 G_\Delta(v), \tag{5.17}$$

$$\sigma_{\text{tot}}^N = \frac{q}{\omega_k} \left(\frac{\sqrt{2} eG_{NN\pi}}{8\pi M} \right)^2 G_N(v), \tag{5.18}$$

where

$$G_\Delta(v) = \frac{1}{b^2} \left\{ b^2 + 1 - v^2 - \frac{a(1-v^2)}{2bv} \ln\left(\frac{a+bv}{a-bv} \right) \right\}, \tag{5.19}$$

$$G_N(v) = \left\{ 2 - v^2 - \frac{1-v^2}{2v} \ln\left(\frac{1+v}{1-v} \right) \right\}. \tag{5.20}$$

Since σ_0 diverges logarithmically at $v = 1$, but σ_{tot} tends to a constant, the integrand of $\bar{\beta}$, $\sigma_{\text{tot}} - \sigma_0$, becomes negative for large v. Integration with respect to v gives

$$\bar{\beta}^{(N)} = \left(\frac{\sqrt{2} eG_{NN\pi}}{8\pi M} \right)^2 \left\{ \frac{1}{24 m_\pi} \right\}. \tag{5.21}$$

This result has already been given by L'vov [38] and is equal to the leading $1/m_\pi$ term by the chiral perturbation theory.

The proton-neutron difference between polarizabilities comes from that of the contribution from $T_{0,E}^{(0)}$. Because the amplitudes $T_{0,E}^{(+,0)}$ behave as $O(\omega_k)$ in our nonrelativistic Skyrme-soliton model, it is impossible to integrate terms including $T_E^{(+,0)}$ from the threshold to infinity without unitarization of them. So we do not estimate the proton-neutron difference of the polarizabilities within this paper. At least we can say that the difference is of $O(1)$, that is lower by N_c than the leading order polarizabilities, and that the sign of $T_E^{(0)}$ indicates that the neutron electric polarizability is larger than the proton one.

Here we discuss the effect of the channel coupling with the $\gamma + N \to \pi + \Delta$ process. The relation of coupling constants, $G_{\Delta N\pi} = -3/\sqrt{2} G_{NN\pi}$, holds in the Skyrme-soliton model. Therefore, the size of the coupling constant in σ_0^Δ is *twice* as large as that in σ_0^N. Thus, *if we put* $\Delta M = 0$, $\sigma_0^\Delta + \sigma_0^N = 3\sigma_0^N$. This factor 3 on $\bar{\alpha}$ has been pointed out [39] concerning the difference between the chiral soliton models such as the Skyrme-soliton model, and the chiral perturbation theory; however, the result by the chiral soliton models was obtained from the seagull term at the cost of gauge noninvariance. Our result indicates that the enhancement by the factor 2 in the size of the coupling constant is independent of models, because the factor is due to the spin-isospin contraction in $\sum_a |T^{a\Delta}|^2$ and the ratio of $G_{\Delta N\pi}$ to $G_{NN\pi}$. The same factor is also pointed out in the chiral perturbation theory [28]. If we use Eq. (5.10) for $\bar{\alpha} + \bar{\beta}$, the same factor 2 appears from the single-pion production cross section. Thus, $\bar{\beta}$ is also enhanced by the channel coupling.

It should be recognized, however, that the mass degeneracy between N and Δ is not inevitable to the Skyrme-soliton model. Numerical calculation of the channel coupling effects shows a monotonic decrease of the sizes of the channel coupling and gives $\bar{\alpha}_\Delta = 0.52\bar{\alpha}_N$ and $\bar{\beta}_\Delta = 0.78\bar{\beta}_N$ at the empirical mass difference, the ratio $\Delta M/m_\pi$ being about 2.1. The effect by the channel coupling with the $\gamma + N \to \pi^a + \Delta$ process cannot be negligible both for the electric and magnetic polarizabilities, though the enhancement by the factor 3 is much reduced. The effect of the finite width of Δ to the $\pi\Delta$ channel contributions is not so large; it only reduces about 10% from the above mentioned values.

B. Contribution from the magnetic Born terms

The P-wave magnetic Born terms $T_M^{(\pm)}$ of Eqs. (4.23a) and (4.23b) contribute to the magnetic polarizability at $O(N_c)$.

We rewrite $T_M^{(\pm)}$ as

$$T_M^{(\pm)} = \left(\frac{eG_{NN\pi}}{8\pi M}\right)\mu_V^N \left\{t_1^{(\pm)}P_1(\hat{\mathbf{q}},\hat{\mathbf{s}}) + t_3^{(\pm)}P_3(\hat{\mathbf{q}},\hat{\mathbf{s}})\right\}, \tag{5.22}$$

where μ_V^N is the size of the magnetic moment in units of the nuclear magneton, and

$$\begin{aligned} P_1(\hat{\mathbf{q}},\hat{\mathbf{s}}) &= (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{s}}), \\ P_3(\hat{\mathbf{q}},\hat{\mathbf{s}}) &= 3(\hat{\mathbf{q}}\cdot\hat{\mathbf{s}}) - (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{s}}) \end{aligned} \tag{5.23}$$

are the P-wave projection operators for $J = 1/2$ and $J = 3/2$, respectively [30], and $\hat{\mathbf{q}} = \mathbf{q}/q$ and $\hat{\mathbf{s}} = \mathbf{s}/k$. Using the same variables as in the preceding subsection, $v = q/\omega_q$ and $b = 1 + d\sqrt{1-v^2}$ with $d = \Delta M/m_\pi$, we give the amplitudes $t_{2J}^{(\pm)}$ as

$$t_1^{(-)} = \frac{1}{2M}\left\{-\frac{2}{3}\frac{qk\Delta M}{\omega_k(\omega_k+\Delta M)}\right\} = -\mu\frac{1}{3}v\frac{d}{b}, \tag{5.24a}$$

$$\begin{aligned} t_3^{(-)} = \frac{1}{2M}&\left\{\frac{qk(\Delta_M - i\frac{\Gamma}{4})}{(\omega_k+\Delta M)[\omega_k - \Delta M + i\frac{\Gamma}{2}]}\right. \\ &\left.-\frac{2}{3}\frac{qk\Delta M}{\omega_k(\omega_k+\Delta M}\right\} \\ = \mu\, v &\left\{\frac{1}{2}\frac{d-\frac{1}{2}i\gamma}{b[1-d\sqrt{1-v^2}+i\gamma\sqrt{1-v^2}]} - \frac{1}{3}\frac{d}{b}\right\}, \end{aligned} \tag{5.24b}$$

and similarly

$$t_1^{(+)} = -\mu\frac{2}{3}v\frac{d}{b}, \tag{5.25a}$$

$$t_3^{(+)} = \mu\, v\left\{-\frac{d-\frac{1}{2}i\gamma}{b[1-d\sqrt{1-v^2}+i\gamma\sqrt{1-v^2}]} + \frac{1}{3}v\frac{d}{b}\right\}. \tag{5.25b}$$

In $t_3^{(\pm)}$ we introduced the finite width Γ of Δ state to the direct Δ pole by the simple Breit-Wigner form, and $\gamma = \Gamma/(2m_\pi)$. In order to take account of the P-wave nature of the Δ state near threshold, we may approximate the total width Γ as

$$\Gamma = \Gamma_\Delta\left(\frac{v}{v_\Delta}\right)^3, \tag{5.26}$$

where v_Δ is the velocity at $\omega_q = \Delta M$, and we define Γ_Δ as

$$\Gamma_\Delta = \frac{G^2_{\Delta N\pi}}{12\pi M^2}(q_\Delta^*)^3 \tag{5.27}$$

with q_Δ^* is the c.m. system (c.m.s.) pion momentum at the resonance. Since this definition gives 113 MeV to Γ_Δ at the parameter set by Adkins [7] and the value is close to the empirical one, we fix $\Gamma_\Delta/2m_\pi = 0.4$ in this paper.

It should be stressed that just due to the cancellation between the N and Δ poles each of the magnetic Born terms for $J = 1/2$ or $3/2$ is asymptotically finite and of $O(N_c^{1/2})$ as like as the electric Born terms. Such a cancellation occurs naturally in the Skyrme-soliton model, where the N and Δ states are treated as the same rotational levels of the Skyrme soliton. On the other hand, $T_M^{(0)}$, which is of $O(N_c^{-1/2})$, is not finite by the lack of the cancellation, so that we need some unitarization to get a finite difference between $\bar{\beta}_p$ and $\bar{\beta}_n$.

We give at first the interference terms as

$$\begin{aligned} 2\int d\Omega_{\hat{\mathbf{q}}}\left(T_E^{(-)\dagger}T_M^{(-)} + T_M^{(-)\dagger}T_E^{(-)}\right)\Bigg|_{\text{spin nonflip}} &= \left(\frac{\sqrt{2}eG_{NN\pi}}{8\pi M}\right)^2\mu_V^N\int d\Omega_{\hat{\mathbf{q}}}\frac{4(\boldsymbol{\epsilon}\cdot\hat{\mathbf{q}})^2qk}{m_\pi^2+(\mathbf{k}-\mathbf{q})^2}[\mathrm{Re}(t_1^{(-)}-t_3^{(-)})] \\ &= 4\pi\left(\frac{\sqrt{2}eG_{NN\pi}}{8\pi M}\right)^2\mu_V^N\left(\frac{m_\pi}{2M}\right)H_{EM}(v), \end{aligned} \tag{5.28}$$

$$H_{EM}(v) = \left\{1 - \frac{1-v^2}{2v}\ln\left(\frac{1+v}{1-v}\right)\right\}\frac{d\,(d\sqrt{1-v^2}-1)+\frac{1}{2}\gamma^2\sqrt{1-v^2}}{(1+d\sqrt{1-v^2})[(1-d\sqrt{1-v^2})^2+\gamma^2(1-v^2)]}. \tag{5.29}$$

The contribution to $\bar{\beta}$ from the interference between the electric and magnetic Born terms is of $O(m_\pi^0)$, that is of the next-to-leading order in powers of m_π, but the leading order term in the $1/N_c$ expansion. There are no $\ln m_\pi$ terms

TABLE I. Numerical results of $\bar{\alpha}$. "Empirical" means that empirical values of constants are used in calculating the Born amplitudes, while "Adkins" that the Skyrme model parameters by Adkins[7] are employed to calculate constants. The second and third columns show the contributions of the N and Δ states to $\bar{\alpha}$, respectively. The sum of both contributions are given in the fourth column.

Parameters	N contribution (fm^3)	Δ contribution (fm^3)	Total (fm^3)
Empirical	13.8×10^{-4}	7.2×10^{-4}	21.0×10^{-4}
Adkins	10.8×10^{-4}	5.6×10^{-4}	16.4×10^{-4}

TABLE II. Numerical results of $\bar{\beta}$. The second column shows the contributions of the electric Born terms in Eq. (4.21b) to the total absorption cross section, the third those of the interference terms in Eq. (5.28), and the fourth those of the magnetic Born terms in Eq. (5.30). Here, the upper cases are the contributions of the π-N channel, and the lower of the π-Δ channel. Total means the full contribution to the $\bar{\beta}$. See the caption of Table I for others.

Parameters	Electric Born (fm^3)	Interference (fm^3)	Magnetic Born (fm^3)	Total (fm^3)
Empirical	1.4×10^{-4}	-1.8×10^{-4}	4.4×10^{-4}	5.8×10^{-4}
	1.1×10^{-4}	-1.1×10^{-4}	1.8×10^{-4}	
Adkins	1.1×10^{-4}	-1.2×10^{-4}	2.0×10^{-4}	2.8×10^{-4}
	0.8×10^{-4}	-0.6×10^{-4}	0.7×10^{-4}	

in the nonrelativistic Skyrme-soliton model. The interference term gives a negative value owing to the resonance behavior of the Δ pole.

The contribution from the square of the magnetic Born terms are given as

$$\int d\Omega_{\hat{\mathbf{q}}}\{2|T_M^{(-)}|^2+|T_M^{(+)}|^2\}=4\pi\left(\frac{\sqrt{2}eG_{NN\pi}}{8\pi M}\right)^2(\mu_V^N)^2\left(\frac{m_\pi}{M}\right)^2 v^2 H_{\mathrm{MM}}(v), \tag{5.30}$$

$$H_{\mathrm{MM}}(v)=\frac{d^2}{b^2}+\frac{d(d^2+\frac{1}{2}\gamma^2)\sqrt{1-v^2}+\frac{\gamma^2}{4}}{b^2[(1-d\sqrt{1-v^2})^2+\gamma(1-v^2)]}. \tag{5.31}$$

This is of $O(m_\pi^2)$, so that it contributes to $\bar{\beta}$ as of order μ.

Since the interference term contributes to $\bar{\beta}$ as of $O(\mu)$ and the square of the magnetic Born term does as of $O(\mu^2)$, one may think that these are effects of higher order. It should be recognized, however, that the order in the $1/N_c$ expansion is the same for all of the terms, that is of $O(N_c)$, though the seeming order in powers of m_π differs from $O(m_\pi^{-1})$ to $O(m_\pi^2)$. We have to take account of all of them from the viewpoint of the Skyrme-soliton model, which is based on the $1/N_c$ expansion.

The contributions from the $\pi\Delta$ channel through the magnetic interaction are given as

$$\frac{4}{3}\int d\Omega[T^{(-)*}(T_M^{(-)})^\Delta+\text{c.c.}]=4\pi\frac{4}{9}\left(\frac{eG_{\Delta N\pi}}{4\pi M}\right)^2\mu_V^N\left(\frac{m_\pi}{2M}\right)\left\{\frac{a}{bv}-\frac{a^2-b^2v^2}{b^2v^2}\ln\left(\frac{a+bv}{a-bv}\right)\right\}\left\{-\frac{7}{15}dv\right\} \tag{5.32}$$

$$\frac{4}{3}\int d\Omega\left(\left|(T_M^{(-)})^\Delta\right|^2+\left|(T_M^{(+)})^\Delta\right|^2\right)=4\pi\frac{4}{9}\left(\frac{eG_{\Delta N\pi}}{4\pi M}\right)^2(\mu_V^N)^2\left(\frac{m_\pi}{2M}\right)^2\left\{\frac{82}{45}d^2v^2\right\}. \tag{5.33}$$

Here, we give numerical results of $\bar{\alpha}$ and $\bar{\beta}$ for the sets of the empirical and the Adkins parameters [7] in Tables I and II, respectively. The net spectrum for $\bar{\beta}$ clearly shows the Δ resonance behavior in contrast with that for $\bar{\alpha}$.

Note that we have only taken into account the Born terms without any unitarization so far, and then we expect that the effects of higher partial waves with resonance behaviors and unitarization of the amplitudes may alter the results. Also we have neglected the effect of the form factors in the vertices, which describes the finite size of the soliton. This effect may alter the contributions to the dispersion integrals, and gives smaller values to the polarizabilities. The realization of the complete calculation is out of the present scope, however.

VI. CONCLUSIONS AND DISCUSSION

We have shown that the low-energy theorems for the photo-induced reactions, pion photoproduction, and Compton scattering can be reproduced within the Skyrme-soliton model. We used in the proof the amplitudes for the longitudinal photon obtained by the gauge invariance imposed on the ones for the real transverse photon; the former has the simplified seagull terms, and the order of the Born terms are of lower in the $1/N_c$ expansion. We explicitly showed that the Compton amplitude is of $O(N_c)$ and the pion-photoproduction amplitudes are of $O(N_c^{1/2})$ owing to the cancellation among the nucleon and Δ pole terms. Such a cancellation occurs naturally in the Skyrme-soliton model, where the N and Δ states are treated as the same rotational levels of the Skyrme-soliton in contrast with the chiral perturbation theory in which the Δ state is treated as of higher-order effects at two-loop levels. The amplitudes for Compton scattering of $O(N_c)$, pion photoproduction of $O(N_c^{1/2})$, and pion-nucleon scattering of $O(1)$ satisfy unitarity in the sense that the powers of $1/N_c$ remain unchanged through the channel coupling, where we consider the electric charge e to be independent of N_c. And the ratio among the pion-coupling constants and the cancellations of the pole terms at the degenerate baryon masses are the same as in the pion scattering amplitudes under the unitarity condition [40,41].

The electromagnetic polarizabilities are calculated using the Born terms in the pion-photoproduction process both with the electric and magnetic interactions in the model: The electric polarizability $\bar{\alpha}$ and the magnetic one $\bar{\beta}$ are given as the dispersion integrals of the Born terms. We have explicitly calculated the Δ state contributions to the polarizabilities: The Δ state contributes to $\bar{\alpha}$ through the channel coupling to the $\gamma + N \to \pi + \Delta$ channel through the electric coupling, while it contributes to $\bar{\beta}$ through the Δ pole both in the direct and crossed magnetic Born terms as well as the channel coupling by the electric and magnetic interactions. Our results are numerically not so far from the experimental data, but the calculation is restricted to the Born contributions within the nonrelativistic Skyrme-soliton model. Studies on contributions from higher partial waves and some unitarization of the amplitudes in the pion-photoproduction process are left for further work.

As a definition of the electromagnetic polarizabilities the following expressions are usually given [42,5]:

$$\bar{\alpha} = 2 \sum_{n \neq N} \frac{|\langle n| D_z |N\rangle|^2}{E_n - M} + \frac{e_N^2 \langle \mathbf{r}^2 \rangle_E}{3M} + O(M^{-3}), \quad (6.1)$$

$$\bar{\beta} = 2 \sum_{n \neq N} \frac{|\langle n| M_z |N\rangle|^2}{E_n - M} + \beta_{\text{seagull}}, \quad (6.2)$$

where $D_z\,(M_z)$ is the z component of the electric (magnetic) dipole-moment operator, and $\langle \mathbf{r}^2 \rangle_E$ is the mean-square radius of the charge distribution of the nucleon. Since the charge radius is of $O(1)$ in the Skyrme-soliton model, the additional term is of not leading order but of $O(N_c^{-1})$. On the other hand, $M_z = \int (\mathbf{x} \times \mathbf{J})_z d^3x$ sandwiched between $|\Delta\rangle$ and $|N\rangle$ behaves as $O(N_c)$ and $E_\Delta - M$ is of $O(N_c^{-1})$, so that the first sum could be of $O(N_c^3)$. However, this definition seems dangerous from the viewpoint of the $1/N_c$ expansion [39,26].

We note at first that the amplitude $\langle n| D_z |N\rangle$ or $\langle n| M_z |N\rangle$ is *off* the energy-shell, that is, $E_n \neq M + \omega_k$ though $\mathbf{p}_n = \mathbf{k}$ in the laboratory system, in contrast with the dispersion integrals in which the amplitudes are *on* the energy shell. The fact that the energy-momentum transfer squared is $(p_n^\mu - p_N^\mu)^2 = -\mathbf{k}^2 + O(N_c^{-2})$ permits the appearance of the form factor of the electromagnetic current, $F(-\mathbf{k}^2) + O(N_c^{-2})$ as the residues of the nucleon pole terms, irrespective of the relativistic or nonrelativistic kinematics. At the same time the off-energy shell amplitude should have extra singularities in addition to the analytic structure of the on-shell amplitude, in order to absorb the singularities coming from the form factor to leave only the simple pole plus unitarity-cut structure of the whole amplitude. We may encounter many complex problems in calculating contributions from continuous states in the off-energy shell amplitudes.

A simple example of such a situation is seen in background scattering of the sine-Gordon theory. The background scattering amplitude is written as

$$T_k(k) = \frac{4mk}{k - im}, \quad (6.3)$$

where m is the meson mass and k is the meson momentum. This is rewritten as a dispersion integral:

$$T_k(k) = 4m - \frac{8m^3}{\omega^2} + \frac{1}{2\pi} \int_{-\infty}^{\infty} dk' \frac{|T_{k'}(k')|^2}{\omega'^2 - \omega^2}, \quad (6.4)$$

where $\omega = \sqrt{k^2 + m^2}$, and we note that the amplitude in the integral is on the energy shell, $T_{k'}(k')$. The same $T_k(k)$ can be written in terms of the off-energy shell amplitude as

$$T_k(k) = 4m - \frac{F^2(\omega)}{\omega^2} + \frac{1}{2\pi} \int_{-\infty}^{\infty} dk' \frac{|T_k(k')|^2}{\omega'^2 - \omega^2}, \quad (6.5)$$

where we note that $T_k(k')$ is the off-shell amplitude. The residue $F(\omega)$ is the form factor: It is $8m^3$ at $\omega = 0$, but has infinite number of double poles which are not present in the original amplitude. Indeed, the off-energy shell amplitude in the integral also has the extra singularities which just absorb the singularities of the form factor [43].

The paramagnetic term of $O(N_c^3)$ comes from the fact that the polarizability is defined by making ω_k of $O(1)$ zero, leaving the mass difference ΔM of $O(N_c^{-1})$ finite, as if it is the N_c expansion instead of the $1/N_c$ expansion [26].

Thus, it might be dangerous to calculate the polarizabilities according to the definition Eqs. (6.1) and (6.2).

Since we started with the local and translation invariant field Φ_a's, we expect that the resultant scattering amplitudes are translation-invariant. Indeed, the electric Born terms, $T_{0,E}^{(\alpha)}$, and the P-wave magnetic Born terms, $T_M^{(\alpha)}$, in the laboratory system have the same forms as in the center-of-mass system, where $\alpha = \pm, 0$. The amplitudes $T_m^{(+,0)}$ of the magnetic Born terms seem to depend on the frame of system, however. In the center-of-mass system the crossed Born terms with $\tilde{A}_0^a$ contribute to $T_m^{(\alpha)}$, but the direct terms do to $T_m^{(\alpha)}$ in the laboratory system and the sign of $T_m^{(+,0)}$ of the nucleon pole terms becomes negative at the threshold, though $T_m^{(-)}$ remains the same form. If we include the Δ pole, the resultant $T_m^{(+)}$ is invariant at leading order in the $1/N_c$ expansion, and the resultant one is of $O(N_c^{-1/2})$, which is the same as $T_E^{(-)}$. The frame dependence of higher order terms could not be avoided in the model. This is due to the matrix element of $\langle B| A_0^a |N\rangle$, which is expressed as the classical soliton configuration with the zero-mode wave functions. This is left unsolved.

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APPENDIX A: FUNCTIONS IN LAGRANGIAN AND HAMILTONIAN

We summarize functions appearing in the Lagrangian and Hamiltonian in Sec. II, where we write them in terms of the total pion fields.

(1) J^μ:

$$J^\mu = j^\mu + J^\mu_a \dot{\Phi}_a + \frac{1}{2}\dot{\Phi}_a J_{ab}\dot{\Phi}_b, \tag{A1}$$

where

$$j^i = \varepsilon_{3ab}\Phi_a K_{ab}\partial^i\Phi_c, \tag{A2}$$

$$j^0 = -\frac{1}{24\pi^2 f_\pi^4}\varepsilon^{ijk}\varepsilon_{abcd}[\Phi_a\partial_i\Phi_b\partial_j\Phi_c\partial_k\Phi_d], \tag{A3}$$

$$J^i_a = -\frac{1}{8\pi^2 f_\pi^4}\varepsilon^{ijk}\varepsilon_{bcda}[\Phi_b\partial_j\Phi_c\partial_k\Phi_d], \tag{A4}$$

$$J^0_a = \varepsilon_{3bc}\Phi_b K_{ca}, \tag{A5}$$

$$J^i_{ab} = -\frac{1}{\kappa^2 f_\pi^2}2\varepsilon_{3cd}\Phi_c(X_{ab}X_{de} - X_{ad}X_{be})\partial^i\Phi_e, \tag{A6}$$

$$J^0_{ab} = 0. \tag{A7}$$

(2) $Z^{\mu\nu}$:

$$Z^{\mu\nu} = \zeta^{\mu\nu} + Z^{\mu\nu}_a\dot{\Phi}_a + \frac{1}{2}\dot{\Phi}_a Z^{\mu\nu}_{ab}\dot{\Phi}_b, \tag{A8}$$

where

$$\begin{aligned}\zeta^{\mu\nu} &= g^{\mu\nu}\left\{\Phi_\perp^2 + \frac{1}{\kappa^2 f_\pi^2}[\Phi_\perp^2\partial_i\Phi_a X_{ab}\partial_i\Phi_b - (\vec{\Phi}\times\partial_i\vec{\Phi})_3^2]\right\}\\ &\quad + g^{\mu i}g^{\nu j}\frac{1}{\kappa^2 f_\pi^2}[\Phi_\perp^2\partial_i\Phi_a X_{ab}\partial_j\Phi_b\\ &\quad -(\vec{\Phi}\times\partial_i\vec{\Phi})_3(\vec{\Phi}\times\partial_j\vec{\Phi})_3],\end{aligned} \tag{A9}$$

$$\begin{aligned}Z^{\mu\nu}_a &= (g^{\mu i}g^{\nu 0} + g^{\mu 0}g^{nui})\frac{1}{\kappa^2 f_\pi^2}\\ &\quad\times[\Phi_\perp^2 X_{ab}\partial_i\Phi_b - \varepsilon_{3ca}\Phi_c(\vec{\Phi}\partial_i\vec{\Phi})_3],\end{aligned} \tag{A10}$$

$$Z^{\mu\nu}_{ab} = -(g^{\mu\nu} - g^{\mu 0}g^{\nu 0})\frac{1}{\kappa^2 f_\pi^2}[\Phi_\perp^2 X_{ab} - \varepsilon_{3ca}\varepsilon_{3db}\Phi_c\Phi_d]. \tag{A11}$$

(3) W_σ:

$$W_\sigma = g_{\sigma i}\xi^i + g_{\sigma 0}\eta_a\dot{\Phi}_a, \tag{A12}$$

where

$$\xi^i = -\frac{1}{8\pi^2 f_\pi^2}\Phi_0 X_{3a}\partial^i\Phi_a, \tag{A13}$$

$$\eta_a = -\frac{1}{8\pi^2 f_\pi^2}\Phi_0 X_{3a}. \tag{A14}$$

[1] E. Mazzucato *et al.*, Phys. Rev. Lett. **65**, 3144 (1986); R. Beck *et al.*, *ibid.* **65**, 1841 (1990).
[2] H.W. Naus, Phys. Rev. C **43**, R365 (1991); J.C. Bergstrom, *ibid.* **44**, 1768 (1991); B.R. Holstein, in *Proceedings of the 1991 EPS Nuclear Physics Conference on Hadronic Structure and Electroweak Interactions*, Amsterdam, The Netherlands, edited by J.J. Engelen, J.H. Koch, and P. K. A. DeWitt [Nucl. Phys. **A546**, 213c (1992)].
[3] F.J. Federspiel *et al.*, Phys. Rev. Lett. **67**, 1511 (1991); A. Zieger *et al.*, Phys. Lett. B **278**, 34 (1992); E. Hallin *et al.*, Phys. Rev. C **48**, 1497 (1993); J. Schmiedmeyer *et al.*, Phys. Rev. Lett. **66**, 1015 (1991).
[4] D. Drechsel and L. Tiator, J. Phys. G **18**, 541c (1992), and references therein.
[5] A.I. L'vov, Int. J. Mod. Phys. A **8**, 5267 (1993).
[6] E. Witten, *ibid.* **B160**, 57 (1979); **B223**, 422 (1979); **B223**, 825 (1983); G.S. Adkins, C.R. Nappi, and E. Witten, *ibid.* **B288**, 552 (1983); G.S. Adkins and C.R. Nappi, *ibid.* **B233**, 109 (1984).
[7] G.S. Adkins, *Chiral Solitons,* edited by K-F. Liu (World Scientific, Singapore, 1987), p. 99.
[8] P. Hoodboy, Phys. Lett. B **173**, 111 (1986).
[9] G. Eckart and B. Schwesinger, Nucl. Phys. **A458**, 620 (1986); B. Schwesinger, H. Weigel, G. Holzwarth, and A. Hayashi, Phys. Rep. **173**, 173 (1989).
[10] S. Scherer and D. Drechsel, Nucl. Phys. **A526**, 733 (1991).
[11] T. Ikehashi and K. Ohta, Nucl. Phys. **A556**, 552 (1993).
[12] B. Schwesinger and H. Walliser, Nucl. Phys. **A574**, 836 (1994).
[13] E.M. Nyman, Phys. Lett. **142B**, 388 (1984).
[14] M. Chemtob, Nucl. Phys. **A473**, 613 (1987).
[15] N.N. Scoccola and W. Weise, Nucl. Phys. **A517**, 495 (1990).
[16] S. Scherer and P.J. Mulders, Nucl. Phys. **A549**, 521 (1992).
[17] A. Hayashi, S. Saito, and M. Uehara, Phys. Rev. D **46**, 4856 (1992).
[18] M.L. Goldberger and S.B. Treiman, Phys. Rev. **110**, 1178 (1958).
[19] S.L. Adler, Phys. Rev. **140**, 1471 (1965); W.I. Weisberger, *ibid.* **143**, 707 (1966).
[20] Y. Tomozawa, Nuovo Cimento A **46**, 707 (1966); W.I. Weinberg, Phys. Rev. Lett. **143**, 616 (1966).
[21] S. Saito, F. Takeuti, and M. Uehara, Nucl. Phys. **A556**, 317 (1993).
[22] N.M. Kroll and M.A. Ruderman, Phys. Rev. **93**, 233 (1954).
[23] P. De Baenst, Nucl. Phys. **B24**, 633 (1970).
[24] S. Scherer, G.I. Poulis, and H.W. Fearing, Nucl. Phys. **A570**, 686 (1994).
[25] F. E. Low, Phys. Rev. **96**, 1428 (1954); **97**, 1392 (1955).
[26] S. Saito and M. Uehara, Phys. Lett. B **325**, 20 (1994).

[27] V. Bernard, N. Kaiser, and Ulf-G. Meissner, Nucl. Phys. **B373**, 346 (1992).

[28] N. Kaiser, in *Proceedings of International Workshop on Baryons as Skyrme Solitons*, edited by G. Holzwarth (World Scientific, Singapore, 1994), p. 117.

[29] C.G. Callan and E. Witten, Nucl. Phys. **B239**, 161 (1984); N.K. Pak and P. Rossi, *ibid.* **B250**, 279 (1985).

[30] G.F. Chew and F. Low, Phys. Rev. **101**, 1579 (1956); **101**, 1571 (1956).

[31] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento **1**, 205 (1955).

[32] M.L. Goldberger and M.K. Watson, *Collision Theory* (Wiley, New York, 1964).

[33] M. Gell-Mann, M.L. Goldberger, and W.E. Thirring, Phys. Rev. **95**, 1612 (1954).

[34] S.L. Adler and F. Gilman, Phys. Rev. **152**, 1460 (1966); W.I. Weisberger, in *Elementary Particle Physics and Scattering Theory*, edited by M. Chrétien and S.S. Schweber (Gordon and Breach, New York, 1967), Vol. I, p. 363.

[35] A.M. Baldin, Nucl. Phys. **18**, 310 (1960).

[36] A.I. L'vov, V.A. Petrun'kin, and S.A. Startsev, Sov. J. Nucl. Phys. **29**, 651 (1979); V.A. Petrunkin, Sov. J. Part. Nucl. **12**, 278 (1981).

[37] B.R. Holstein and A.M. Nathan, Phys. Rev. D **49**, 6101 (1994).

[38] A.I. L'vov, Phys. Lett. B **304**, 29 (1993).

[39] W. Broniowski and T.D. Cohen, Phys. Rev. D **47**, 299 (1993).

[40] J.-L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984); Phys. Rev. D **30**, 1795 (1984).

[41] R. Dashen and A.V. Manohar, Phys. Lett. B **315**, 425 (1993); **315**, 438 (1993); R. Dashen, E. Jenkins, and A.V. Manohar, Phys. Rev. D **49**, 4713 (1994).

[42] V.A. Petrun'kin, Sov. J. Nucl. Phys. **34**, 597(1961); S. Ragusa, Phys. Rev. D **11**, 1536 (1975); N.V. Maksimenko and S.G. Shulga, Sov. J. Nucl. Phys. **52**, 355 (1990).

[43] Y.G. Liang, B.A. Li, K.F. Liu, and R.K. Su, Phys. Lett. B **243**, 133 (1990).