

Measurement of the τ electric dipole moment using longitudinal polarization of e^+e^- beams

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Certain CP -odd momentum correlations in the production and subsequent decay of τ pairs in e^+e^- collisions are enhanced significantly when the e^+ and e^- beams are longitudinally polarized. These may be used to probe the real and imaginary parts of d_τ^Z , the electric dipole moment of the τ . Closed-form expressions for these “vector correlations” and the standard deviation of the operators defining them due to standard model interactions are presented for the two-body final states of τ decays. If 42% average polarization of each beam is achieved, as proposed for the τ -charm factories, with equal integrated luminosities for each sign of polarization and a total yield of 2×10^7 $\tau^+\tau^-$ pairs, it is possible to attain sensitivities for $|\delta \text{Re} d_\tau^Z|$ of 8×10^{-19} , 1×10^{-19} , 1×10^{-19} e cm, respectively, and for $|\delta \text{Im} d_\tau^Z|$ of 4×10^{-14} , 6×10^{-15} , 5×10^{-16} e cm, respectively, at the three operating center-of-mass energies of 3.67, 4.25, and 10.58 GeV. These bounds emerge when the effects of a possible weak dipole form factor d_τ^Z are negligible as is the case when it is of the same order of magnitude as d_τ^Z . Furthermore, in such a polarization experiment where different polarizations are possible, a model-independent disentangling of their individual effects is possible, and a technique to achieve this is described. A strong longitudinal polarization physics program at the τ -charm factory appears warranted.

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I. INTRODUCTION

Leptonic CP violation would signal interactions not described in the framework of the standard model since it arises there only at the multiloop level and is way below any measurable level. For example, CP -violating electric dipole moments (EDM's) of leptons vanish at least to three-loop order in the standard model [1,2], and are estimated to be $(m_l/\text{MeV}) \times 1.6 \times 10^{-40}$ e cm, where m_l is the mass of the lepton [2]. Whereas the EDM of the electron is constrained to be $\lesssim 10^{-26}$ e cm and that of the muon is $\lesssim 10^{-19}$ e cm [3], the constraint on the EDM of the τ leptons [4] is less stringent, viz. $\lesssim 5 \times 10^{-17}$ e cm [3]. Thus an important experimental challenge is to measure the τ electric dipole moment far more accurately than at present. The analogous coupling of the τ to the Z boson, the weak dipole form factor (WDF) [4], is better constrained from the CERN e^+e^- collider LEP data to have its real part $\lesssim 7.8 \times 10^{-18}$ e cm and imaginary part $\lesssim 4.5 \times 10^{-17}$ e cm at the Z resonance [5]. The purpose of this paper is to demonstrate that the availability of large polarization will go a long way in improving the measurement of the τ edm. Further, the discussion presented here may also be easily extended to other physical situations which include the measurement of CP -violating form factors in W^+W^- or $t\bar{t}$ production.

An approach has been proposed which consists of measuring CP -odd correlations [6,7] amongst the momenta of the final-state particles in the reaction $e^+e^- \rightarrow \tau^+\tau^- \rightarrow X^+\bar{\nu}_\tau X^-\nu_\tau$, without having to reconstruct the τ momen-

tum or spin directions. In particular, one may construct scalar, vector, and tensor correlations [8] from the momenta \mathbf{q}_+ and \mathbf{q}_- of the decay products of the τ^+ and τ^- . Of these, tensor correlations were suggested to be the most sensitive in experiments at LEP with unpolarized beams. One such tensor correlation was looked for in LEP experiments [9], and a limit on the real part of the WDF was obtained. In [10] it has been shown that the presence of large longitudinal polarization renders certain simple vector correlations sensitive to the real as well as to the imaginary parts of the WDF at the Z factory, the SLAC Linear Collider (SLC).

Here we investigate the sensitivity of these correlations to the real and imaginary parts of the EDM when the production of $\tau^+\tau^-$ is no longer dominated by Z exchange and instead by photon exchange as is typically the case when $\sqrt{s} \ll m_Z$. In particular, we will present much of our numerical results for the proposed τ -charm factories (τ CF) [11] where there exists ample opportunity to have substantial polarization of the e^+ and e^- beams [12]. The prospects for the measurement of the EDM at the τ -charm factory with unpolarized beams has already been considered [13] by measuring tensor correlations among the momenta of final-state particles in the τ decays. Algebraically our approach proves simpler since the vector correlations (more correctly their scalar product with the e^+ beam direction) we consider can be expressed in closed form and the standard deviation of the operators defining the correlations due to the standard model interactions can also be so expressed for the two-body final states of the τ decays. In practice the expressions of Ref. [10] valid

at the Z peak are now generalized to include the pure γ^* as well as the γ^* - Z^* interference terms, using in addition to the standard model (SM), the CP -violating terms in the effective Hamiltonian for the reaction

$$e^+e^- \rightarrow \gamma^*, \quad Z^* \rightarrow \tau^+\tau^- \rightarrow \bar{B}A\bar{\nu}_\tau\nu_\tau \quad (1)$$

given in Ref. [13]. (Note that the expressions obtained here are also valid at much larger center of mass energies where contributions from γ^* and Z^* are significant, and can also be easily modified for W^+W^- and $t\bar{t}$ production [14] where it may be possible to probe CP violation.) For comparable magnitudes of the EDM and WDF, at the τ CF energies, the CP -odd correlations obtain their most significant contribution from the EDM [15].

The CP -odd momentum correlations we consider here are associated with the c.m. momenta \mathbf{p} of e^+ , \mathbf{q}_B of \bar{B} , and \mathbf{q}_A of A , where the \bar{B} and A arise in the decays $\tau^+ \rightarrow \bar{B} + \bar{\nu}_\tau$ and $\tau^- \rightarrow A + \nu_\tau$, and where A and B run over π, ρ, A_1 , etc. In the case when A and B are different, one has to consider also the decays with A and B interchanged, so as to construct correlations which are explicitly CP odd. The calculations include two-body decay modes of the τ in general and is applied specifically to the case of $\tau \rightarrow \pi + \nu_\tau$ and $\tau \rightarrow \rho + \nu_\tau$ due to the fact that these modes possess a good resolving power of the τ polarization, parametrized in terms of the constant α which takes the value 1 for the π channel (with branching fraction of about 11%) and 0.46 for the ρ channel [13] (with branching fraction of about 22%). It may be noted that with these final states the substantive fraction of the channels that are sensitive to such correlations are accounted for; three-body leptonic final states must also be included; they are characterized by a somewhat smaller $\alpha = -0.33$ (with branching fraction of about 35%). Thus with the channels studied here, one more or less reaches the limits of discovery in such experiments. (It would also be possible to apply this to the decay $\tau \rightarrow A_1 + \nu$; α_{A_1} is however too small to be of any experimental relevance.) Further, we also present closed-form expressions for the variance of the correlations considered due to standard model interactions. These, because of finite statistics, provide a measure of the CP -invariant background to the determination of the CP -odd contributions to the correlations. In case of a negative result, the limit on the CP -violating interactions is obtained using the value of the variance and the size of the data sample.

It must be noted that correlations which are CP violating in the absence of initial beam polarization are not strictly CP odd for arbitrary e^+ and e^- polarizations, since the initial state is then not necessarily CP even. We argue, however, that this is true to a high degree of accuracy in the case at hand. In addition, for our numerical results, we restrict ourselves to the case where the e^+ and e^- polarizations are equal and opposite, thus making up a CP -even initial state.

We follow a slightly different notation from Bernreuther *et al.* [13] and use the symbols B_i and B_j to denote the intermediate vector bosons, the photon, and the Z . In the mean, as well as in the variances and in the cross sections, the contributions would eventually

have to be summed over i, j . Our main result is that the contribution to certain CP -odd correlations, which are relatively small in the absence of polarization, since they come with a factor $r_{ij} = (V_e^i A_e^j + V_e^j A_e^i)/(V_e^i V_e^j + A_e^i A_e^j)$ and get enhanced in the presence of polarization, now proportional to $(r_{ij} - P)$, with the corresponding contribution to the cross section being multiplied by $(1 - r_{ij}P)$. Here V_e^i, A_e^i are the vector and axial vector couplings of e^- to B_i , and P is the effective polarization defined by

$$P = \frac{P_e - P_{\bar{e}}}{1 - P_e P_{\bar{e}}},$$

where P_e ($P_{\bar{e}}$) is the polarization of the electron (positron) and is positive for right-circular polarization for each particle in our convention.

The correlations that have this property are those that have an odd number of factors of the e^+ c.m. momentum \mathbf{p} , since this would need P and C violation at the electron vertex. Furthermore, we suggest a procedure for obtaining these correlations from the difference in the event distributions for a certain polarization P and the sign-flipped polarization $-P$. With this procedure, the correlations are further enhanced, leading to increased sensitivity. The inclusion of the ρ channel leads to a considerable improvement in the sensitivity that can be reached in the measurement of $\text{Im}d_\tau$ while improving the measurement of $\text{Re}d_\tau$ less spectacularly.

More specifically, we have considered the observables $O_1 \equiv \frac{1}{2}[\hat{\mathbf{p}} \cdot (\mathbf{q}_B \times \mathbf{q}_A) + \hat{\mathbf{p}} \cdot (\mathbf{q}_A \times \mathbf{q}_B)]$ and $O_2 \equiv \frac{1}{2}[\hat{\mathbf{p}} \cdot (\mathbf{q}_A + \mathbf{q}_B) + \hat{\mathbf{p}} \cdot (\mathbf{q}_{\bar{A}} + \mathbf{q}_{\bar{B}})]$ (the caret denoting a unit vector) and obtained analytic expressions for their mean values and standard deviations in the presence of longitudinal polarization. O_2 , being CPT odd, measures $\text{Im}d_\tau^i$, whereas O_1 measures $\text{Re}d_\tau^i$. Inclusion of other exclusive τ decay modes (not studied here) would improve the sensitivity further.

As a result, we find it possible to define 1 standard deviation (s.d.) sensitivities $|\delta \text{Re}d_\tau^i|$ and $|\delta \text{Im}d_\tau^i|$ from the two-body decay modes when we make the reasonable assumption that the EDM and WDF are of comparable magnitudes. To facilitate comparison with Ref. [13] we assume center-of-mass energies of 3.67, 4.25, and 10.58 GeV.

In order to answer what makes our correlations viable, we now discuss what prospects exist for longitudinal polarization at the τ CF [16]. One proposal [12] is that the e^+ and e^- beams be polarized in separate rings to achieve an average degree of polarization of each beam as large as 42% before being injected in the main ring. (It is also important to note that this would not lead to a large loss in luminosity, in contrast to the situation at linear colliders where the large polarization is accompanied by modest luminosities as, for instance, in the case of the SLC.) This proposal also envisages all four possibilities in the combinations of the polarizations. In particular, as an effective polarization P can be as large as 0.71 and of either sign in the e^+e^- collisions at the τ CF. We show that with equal integrated luminosities with either sign, $\int \mathcal{L}(P)dt = \int \mathcal{L}(-P)dt$ and a total yield $N_{\tau^+\tau^-}$ of 2×10^7 $\tau^+\tau^-$ pairs, we can probe the real part of the EDM of the τ to the remarkable 1 s.d. precision of $\sim 10^{-19}$ e cm.

The imaginary part however is not probed to such a spectacular degree. We finally describe a technique whereby the reasonable assumption of the comparability of magnitudes of the EDM and WDFP can be avoided in such a polarization experiment. These considerations enable us to build a very strong case for introducing longitudinal polarization at the τ CF [17,18].

II. NOTATION AND FORMALISM

Although much of this section has already been described in our previous papers [10] we will repeat it for the sake of completeness and to make the generalization to the inclusion of γ and Z (we drop the asterisk in what follows since no confusion is bound to arise) more transparent.

The process we consider is

$$e^-(p_-) + e^+(p_+) \rightarrow \tau^-(k_-) + \tau^+(k_+) , \quad (2)$$

with the subsequent decays

$$\tau^-(k_-) \rightarrow A(q_A) + \nu_\tau , \quad \tau^+(k_+) \rightarrow \bar{B}(q_B) + \bar{\nu} , \quad (3)$$

together with decays corresponding to A and B interchanged in (2).

Under CP , the various three-momenta transform as

$$\mathbf{p}_- \leftrightarrow -\mathbf{p}_+ , \quad \mathbf{k}_- \leftrightarrow -\mathbf{k}_+ , \quad \mathbf{q}_{A,B} \leftrightarrow -\mathbf{q}_{\bar{A},\bar{B}} . \quad (4)$$

We choose for our analysis the two CP -odd observables $O_1 \equiv \frac{1}{2}[\hat{\mathbf{p}} \cdot (\mathbf{q}_B \times \mathbf{q}_A) + \hat{\mathbf{p}} \cdot (\mathbf{q}_{\bar{A}} \times \mathbf{q}_B)]$ and $O_2 \equiv \frac{1}{2}[\hat{\mathbf{p}} \cdot (\mathbf{q}_A + \mathbf{q}_B) + \hat{\mathbf{p}} \cdot (\mathbf{q}_{\bar{A}} + \mathbf{q}_B)]$, which have an odd number of factors of $\hat{\mathbf{p}}$, the unit vector along \mathbf{p}_+ . As mentioned before, they are expected to get enhanced in the presence of polarization.

Though these observables are CP odd, their observation with polarized e^+ and e^- beams is not necessarily an indication of CP violation, unless the e^+ and e^- longitudinal polarizations are equal and opposite, so that the initial state is described by a CP -even density matrix. The case when only the e^- is polarized has already been discussed [10]. Though our expressions for correlations will be valid for arbitrary polarizations, our results will be only for equal and opposite electron and positron polarizations, so that the correlations are strictly CP odd.

Of O_1 and O_2 , O_1 is even under the combined CPT transformation, and O_2 is CPT odd. A CPT -odd observable can only have a nonzero value in the presence

$$L_{\mu\nu}^{ij}(e) = \{[(1 - P_e P_{\bar{e}})(V_e^i V_e^j + A_e^i A_e^j) - (P_e - P_{\bar{e}})(V_e^i A_e^j + A_e^i V_e^j)]\text{Tr}(\not{p}_- \gamma_\mu \not{p}_+ \gamma_\nu) + [(P_e - P_{\bar{e}})(V_e^i V_e^j + A_e^i A_e^j) - (1 - P_e P_{\bar{e}})(V_e^i V_e^j + A_e^i A_e^j)]\text{Tr}(\gamma_5 \not{p}_- \gamma_\mu \not{p}_+ \gamma_\nu)\} \quad (11)$$

in the limit of vanishing electron mass, where P_e ($P_{\bar{e}}$) is the degree of the e^- (e^+) longitudinal polarization. Equation (11) gives a simple way of incorporating the effect of the longitudinal polarization:

$$(V_e^i V_e^j + A_e^i A_e^j) \rightarrow (V_e^i V_e^j + A_e^i A_e^j) - P(A_e^i V_e^j + A_e^j A_e^i) , \quad (12)$$

of an absorptive part of the amplitude. It is therefore expected that $\langle O_2 \rangle$ will be proportional to the imaginary part of the dipole form factors $\text{Im}d_\tau^i$, since final-state interaction, which could give rise to an absorptive part, is negligible in the weak τ decays. Since $\langle O_1 \rangle$ and mean values of other CPT -even quantities will be proportional to $\text{Re}d_\tau^i$, phase information on d_τ^i can only be obtained if $\langle O_2 \rangle$ (or some other CPT -odd quantity) is also measured.

We assume SM couplings for all particles except τ , for which an additional EDM and WDFP interaction is assumed: viz.,

$$\mathcal{L}_{CPV} = -\frac{i}{2}d_\tau^Z \bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau (\partial_\mu Z_\nu - \partial_\nu Z_\mu) - \frac{i}{2}d_\tau^\gamma \bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau (\partial_\mu A_\nu - \partial_\nu A_\mu) . \quad (5)$$

Using (5), we now proceed to calculate $\langle O_1 \rangle$ and $\langle O_2 \rangle$ in the presence of an effective longitudinal polarization P .

We can anticipate the effect of P in general for the process (1). We can write the matrix element squared for the process in the leading order in perturbation theory, neglecting the electron mass, as

$$|M|^2 = \sum_{i,j} L_{\mu\nu}^{ij}(e) L^{ij\mu\nu*}(\tau) \frac{1}{s - M_i^2} \frac{1}{s - M_j^2} , \quad (6)$$

where the summation is over the gauge bosons (γ, Z, \dots) exchanged in the s channel, and $L_{\mu\nu}^{ij}(e, \tau)$ represent the tensors arising at the e and τ vertices:

$$L_{\mu\nu}^{ij} = V_\mu^i V_\nu^{j*} . \quad (7)$$

For the electron vertex, with only the SM vector and axial-vector couplings,

$$V_\mu^i(e) = \bar{v}(p_+, s_+) \gamma_\mu (V_e^i - \gamma_5 A_e^i) u(p_-, s_-) , \quad (8)$$

We have the definitions

$$V_{e/\tau}^\gamma = -e , \quad A_{e/\tau}^\gamma = 0 ; \quad (9)$$

$$V_{e/\tau}^Z = \left(-\frac{1}{2} + 2 \sin^2 \theta_W \right) \frac{e}{2 \sin \theta_w \cos \theta_w} , \quad A_{e/\tau}^Z = \left(-\frac{1}{2} \right) \frac{e}{2 \sin \theta_w \cos \theta_w} . \quad (10)$$

It is easy to check, by putting in helicity projection operators, that

$$(A_e^i V_e^j + A_e^j V_e^i) \rightarrow (A_e^i V_e^j + A_e^j V_e^i) - P(V_e^i V_e^j + A_e^i A_e^j) ,$$

where P is as defined earlier.

To calculate correlations of O_1 and O_2 , we need the differential cross section for (2) followed by (3) arising from SM γ and Z couplings of e and τ , the dipole couplings of τ arising together from Eq. (5). The calculation

may be conveniently done, following Ref. [13], in steps, by first determining the production matrix χ for $\tau^+\tau^-$ in spin space, and then taking its trace with the decay matrices \mathcal{D}^\pm for τ^\pm decays into single charged particle in addition to the invisible neutrino.

The differential cross section for (1) is given by

$$\frac{d\sigma}{d\Omega_k d\Omega_-^* d\Omega_+^* dE_-^* dE_+^*} = \frac{k}{8\pi s^{3/2}} \frac{1}{(4\pi)^3} \chi^{\beta\beta',\alpha\alpha'} \mathcal{D}_{\alpha'\alpha}^- \mathcal{D}_{\beta'\beta}^+, \quad (13)$$

where $d\Omega_k$ is the solid angle element for \mathbf{k}_+ in the overall c.m. frame, $k = |\mathbf{k}_+|$, and $d\Omega_\pm^*$ are the solid angle elements for $\mathbf{q}_{B,A}^*$, the \vec{B} and A momenta in the τ^\pm rest frame. The \mathcal{D} matrices are given by (for two body final states)

$$\begin{aligned} \mathcal{D}^+ &= \delta(E_B^* - E_{0B}) [1 - \alpha_B \sigma_+ \cdot \hat{\mathbf{q}}_B^*], \\ \mathcal{D}^- &= \delta(E_A^* - E_{0A}) [1 + \alpha_A \sigma_- \cdot \hat{\mathbf{q}}_A^*], \end{aligned} \quad (14)$$

where σ_\pm are the Pauli matrices corresponding to the τ^\pm

spin, E_\pm^* are the charged particle energies in the τ^\pm rest frame, and

$$E_{0A,B} = \frac{1}{2} m_\tau (1 + p_{A,B}), \quad p_{A,B} = m_{A,B}^2 / m_\tau^2. \quad (15)$$

The expressions for χ arising from SM as well as the CP -violating form factor couplings of τ are rather long, and we refer the reader to Ref. [13] for these expressions in the absence of polarization. It is straightforward to incorporate polarization using (12).

III. RESULTS

Using Eqs. (13)–(15) above, as well as the expression for the $\tau^+\tau^-$ production matrix χ from [13], we can obtain expressions for $\langle O_1 \rangle$ and $\langle O_2 \rangle$ by writing O_1 and O_2 in terms of the τ rest frame variables and performing the integrations over them analytically. The expressions for the correlations $\langle O_1 \rangle$ and $\langle O_2 \rangle$ obtained are, neglecting $\sum_{i,j} d_\tau^i d_\tau^j$,

$$\begin{aligned} \langle O_1 \rangle &= -\frac{1}{36x\sigma} \sum_{i,j} K_{ij} s^{3/2} m_\tau^2 (1-x^2) \left(\frac{r_{ij} - P}{1 - r_{ij}P} \right) \left[[(A_\tau^i \text{Red}_\tau^j + A_\tau^j \text{Red}_\tau^i) \alpha_A \alpha_B (1-p_A)(1-p_B)] \right. \\ &\quad \left. - \frac{3}{2} (V_\tau^i \text{Red}_\tau^j + V_\tau^j \text{Red}_\tau^i) [\alpha_A (1-p_A)(1+p_B) + \alpha_B (1-p_B)(1+p_A)] \right], \end{aligned} \quad (16)$$

and

$$\langle O_2 \rangle = \frac{1}{3\sigma} \sum_{i,j} K_{ij} s^{3/2} m_\tau \left(\frac{r_{ij} - P}{1 - r_{ij}P} \right) \frac{1}{4} (A_\tau^i \text{Im}d_\tau^j + A_\tau^j \text{Im}d_\tau^i) (1-x^2) [\alpha_A (1-p_A) + \alpha_B (1-p_B)], \quad (17)$$

where $x = 2m_\tau/\sqrt{s}$ and σ , which is the cross section apart from a normalization factor, is given by

$$\sigma = \sum_{i,j} K_{ij} s \left[V_\tau^i V_\tau^j \left(1 + \frac{x^2}{2} \right) + A_\tau^i A_\tau^j (1-x^2) \right], \quad (18)$$

and

$$K_{ij} = \frac{(V_e^i V_e^j + A_e^i A_e^j)(1 - r_{ij}P)}{(s - M_i^2)(s - M_j^2)} (1-x^2)^{1/2} [1 - P_e P_e]. \quad (19)$$

Here we neglect the width of the Z since we work now at $\sqrt{s} \ll m_Z$. However at the Z peak we neglect γ and treat the system in the narrow-width approximation.

We have also obtained analytic expressions for the variance $\langle O^2 \rangle - \langle O \rangle^2 \approx \langle O^2 \rangle$ in each case, arising from the CP -invariant SM part of the interaction:

$$\begin{aligned} \langle O_1^2 \rangle &= \frac{1}{720x^2\sigma} \sum_{i,j} K_{ij} s m_\tau^4 \left[(1-p_A)^2 (1-p_B)^2 [V_\tau^i V_\tau^j (6 + 8x^2 + x^4) + A_\tau^i A_\tau^j (6 - 2x^2 - 4x^4)] \right. \\ &\quad + (1-x^2) \{ [(1+p_A)^2 (1-p_B)^2 + (1+p_B)^2 (1-p_A)^2] [3V_\tau^i V_\tau^j (3 + 2x^2) + 9A_\tau^i A_\tau^j (1-x^2)] \\ &\quad + 4\alpha_A \alpha_B (1-p_B^2)(1-p_A^2)(1-x^2) [V_\tau^i V_\tau^j - A_\tau^i A_\tau^j] \} \\ &\quad \left. - 6(1-p_A)(1-p_B) (V_\tau^i A_\tau^j + V_\tau^j A_\tau^i) (1-x^2) \left(1 - \frac{x^2}{6} \right) [\alpha_A (1+p_A)(1-p_B) + \alpha_B (1+p_B)(1-p_A)] \right], \end{aligned} \quad (20)$$

$$\begin{aligned}
\langle O_2^2 \rangle = & \frac{1}{360x^2\sigma} \sum_{i,j} K_{ij} sm_\tau^2 \left[\left(3[(1-p_A)^2 + (1-p_B)^2][V_\tau^i V_\tau^j (4+7x^2+4x^4) + A_\tau^i A_\tau^j 2(1-x^2)(2+3x^2)] \right. \right. \\
& - 2\alpha_A \alpha_B (1-p_A)(1-p_B)[V_\tau^i V_\tau^j (4+7x^2+4x^4) + A_\tau^i A_\tau^j 4(1-x^2)^2] \Big) \\
& + 6 \left(6(1-x^2)(p_A-p_B)^2[V_\tau^i V_\tau^j (1+x^2/4) + A_\tau^i A_\tau^j (1-x^2)] \right. \\
& \left. \left. - (V_\tau^i A_\tau^j + V_\tau^j A_\tau^i)(1-x^2)(4+x^2)(p_A-p_B)[\alpha_A(1-p_A) - \alpha_B(1-p_B)] \right) \right]. \quad (21)
\end{aligned}$$

The results for the significant two-body decay channels are presented in the tables. In Tables I–VI we have presented, for three typical values of \sqrt{s} at which the τ CF is expected to run, the values of c_{AB} for O_1 and O_2 , respectively, defined as the correlation for a value of $\text{Re}d_\tau^\gamma$ or $\text{Im}d_\tau^\gamma$ (as the case may be) equal to e/\sqrt{s} , for some values of P chosen to correspond to average beam polarizations of 0, 35%, 42%, and 100%. We have also presented the value of $\sqrt{\langle O_a^2 \rangle}$, ($a = 1, 2$). This 1 s.d. limit is the value of d_τ^γ which gives a mean value of O_a equal to the s.d. $\sqrt{\langle O_a^2 \rangle}/N_{AB}$ in each case:

$$c_{AB}^{1(2)} \delta \text{Re}(\text{Im})d_\tau^\gamma = \frac{e}{\sqrt{s}} \frac{1}{\sqrt{N_{AB}}} \sqrt{\langle O_{1(2)}^2 \rangle}. \quad (22)$$

Here N_{AB} is the number of events in the channel $A\bar{B}$ (or $\bar{A}B$), and is given by

$$N_{AB} = N_{\tau^+\tau^-} B(\tau^- \rightarrow A\nu_\tau) B(\tau^+ \rightarrow \bar{B}\bar{\nu}_\tau), \quad (23)$$

where we take $N_{\tau^+\tau^-}(P) = 10^7$.

These limits can be improved by looking at correlations of the same observables, but in a sample obtained by counting the difference between the number of events for a certain polarization, and for the corresponding sign-flipped polarization. If the partial cross section for the process for a polarization P is given by

TABLE I. (a) c_{AB} , standard deviation and $|\delta \text{Re}d_\tau^\gamma|$ computed for 10^7 $\tau^+\tau^-$ pairs for $\pi\pi$ channel and $\sqrt{s} = 3.67$ GeV for operator O_1 for different P . (b) Same as above for $\pi\rho$ channel. (c) Same as above for $\rho\rho$ channel.

P	c_{AB} (GeV ²)	$\sqrt{\langle O_1^2 \rangle}$ (GeV ²)	$ \delta \text{Re}d_\tau^\gamma $ (e cm)
(a)			
0.00	-3.12×10^{-6}	0.399	1.88×10^{-12}
-0.62	-1.36×10^{-2}	0.399	4.32×10^{-16}
+0.62	1.35×10^{-2}	0.399	4.32×10^{-16}
-0.71	-1.55×10^{-2}	0.399	3.77×10^{-16}
+0.71	1.55×10^{-2}	0.399	3.77×10^{-16}
-1.00	-2.19×10^{-2}	0.399	2.68×10^{-16}
+1.00	2.19×10^{-2}	0.399	2.68×10^{-16}
(b)			
0.00	-7.72×10^{-7}	0.336	4.64×10^{-12}
-0.62	-1.05×10^{-2}	0.336	3.39×10^{-16}
+0.62	1.05×10^{-2}	0.336	3.39×10^{-16}
-0.71	-1.21×10^{-2}	0.336	2.96×10^{-16}
+0.71	1.21×10^{-2}	0.336	2.96×10^{-16}
-1.00	-1.70×10^{-2}	0.336	2.10×10^{-16}
+1.00	1.70×10^{-2}	0.336	2.10×10^{-16}
(c)			
0.00	-1.39×10^{-7}	0.282	1.57×10^{-11}
-0.62	-6.02×10^{-3}	0.282	3.63×10^{-16}
+0.62	6.02×10^{-3}	0.282	3.63×10^{-16}
-0.71	-6.89×10^{-3}	0.282	3.17×10^{-16}
+0.71	6.89×10^{-3}	0.282	3.17×10^{-16}
-1.00	-9.70×10^{-3}	0.282	2.25×10^{-16}
+1.00	9.70×10^{-3}	0.282	2.25×10^{-16}

TABLE II. (a) c_{AB} , standard deviation, and $|\delta \text{Im}d_\tau^*|$ computed for $10^7 \tau^+ \tau^-$ pairs for $\pi\pi$ channel and $\sqrt{s} = 3.67$ GeV for operator O_2 for different P . (b) Same as above for $\pi\rho$ channel. (c) Same as above for $\rho\rho$ channel.

P	c_{AB} (GeV)	$\sqrt{\langle O_2^2 \rangle}$ (GeV)	$ \delta \text{Im}d_\tau^* $ (e cm)
(a)			
0.00	1.36×10^{-5}	0.596	6.44×10^{-13}
-0.62	1.42×10^{-5}	0.596	6.14×10^{-13}
+0.62	1.29×10^{-5}	0.596	6.78×10^{-13}
-0.71	1.43×10^{-5}	0.596	6.10×10^{-13}
+0.71	1.28×10^{-5}	0.596	6.83×10^{-13}
-1.00	1.46×10^{-5}	0.596	5.97×10^{-13}
+1.00	1.25×10^{-5}	0.596	7.01×10^{-13}
(b)			
0.00	9.33×10^{-6}	0.615	7.02×10^{-13}
-0.62	9.79×10^{-6}	0.615	6.69×10^{-13}
+0.62	8.86×10^{-6}	0.615	7.39×10^{-13}
-0.71	9.86×10^{-6}	0.615	6.64×10^{-13}
+0.71	8.88×10^{-6}	0.615	7.44×10^{-13}
-1.00	1.01×10^{-5}	0.615	6.50×10^{-13}
+1.00	8.58×10^{-6}	0.615	7.63×10^{-13}
(c)			
0.00	5.10×10^{-6}	0.575	8.72×10^{-13}
-0.62	5.35×10^{-6}	0.575	8.31×10^{-13}
+0.62	4.85×10^{-6}	0.575	9.18×10^{-13}
-0.71	5.39×10^{-6}	0.575	8.25×10^{-13}
+0.71	4.81×10^{-6}	0.575	9.25×10^{-13}
-1.00	5.51×10^{-6}	0.575	8.08×10^{-13}
+1.00	4.69×10^{-6}	0.575	9.48×10^{-13}

TABLE III. Same as Table I for $\sqrt{s} = 4.25$ GeV.

P	c_{AB} (GeV ²)	$\sqrt{\langle O_1^2 \rangle}$ (GeV ²)	$ \delta \text{Re}d_\tau^* $ (e cm)
(a)			
0.00	-2.67×10^{-5}	0.543	2.58×10^{-13}
-0.62	-8.64×10^{-2}	0.543	7.96×10^{-17}
+0.62	8.63×10^{-2}	0.543	7.97×10^{-17}
-0.71	-9.89×10^{-2}	0.543	6.95×10^{-17}
+0.71	9.88×10^{-2}	0.543	6.96×10^{-17}
-1.00	-1.39×10^{-1}	0.543	4.94×10^{-17}
+1.00	1.39×10^{-1}	0.543	4.94×10^{-17}
(b)			
0.00	-6.59×10^{-6}	0.488	6.80×10^{-13}
-0.62	-6.72×10^{-2}	0.488	6.68×10^{-17}
+0.62	6.72×10^{-2}	0.488	6.68×10^{-17}
-0.71	-7.69×10^{-2}	0.488	5.83×10^{-17}
+0.71	7.69×10^{-2}	0.488	5.83×10^{-17}
-1.00	-1.08×10^{-1}	0.488	4.14×10^{-17}
+1.00	1.08×10^{-1}	0.488	4.14×10^{-17}
(c)			
0.00	-1.19×10^{-6}	0.432	2.43×10^{-12}
-0.62	-3.83×10^{-2}	0.432	7.53×10^{-17}
+0.62	3.83×10^{-2}	0.432	7.53×10^{-17}
-0.71	-4.39×10^{-2}	0.432	6.57×10^{-17}
+0.71	4.39×10^{-2}	0.432	6.57×10^{-17}
-1.00	-6.18×10^{-2}	0.432	4.67×10^{-17}
+1.00	6.18×10^{-2}	0.432	4.67×10^{-17}

TABLE IV. Same as Table II for $\sqrt{s} = 4.25$ GeV.

P	c_{AB} (GeV)	$\sqrt{\langle O_2^2 \rangle}$ (GeV)	$ \delta \text{Im} d_7^* $ (e cm)
(a)			
0.00	1.00×10^{-4}	0.632	8.00×10^{-14}
-0.62	1.05×10^{-4}	0.632	7.62×10^{-14}
+0.62	9.51×10^{-5}	0.632	8.41×10^{-14}
-0.71	1.06×10^{-4}	0.632	7.57×10^{-14}
+0.71	9.44×10^{-5}	0.632	8.48×10^{-14}
-1.00	1.08×10^{-4}	0.632	7.40×10^{-14}
+1.00	9.20×10^{-5}	0.632	8.69×10^{-14}
(b)			
0.00	6.89×10^{-5}	0.655	8.74×10^{-14}
-0.62	7.23×10^{-5}	0.655	8.33×10^{-14}
+0.62	6.54×10^{-5}	0.655	9.20×10^{-14}
-0.71	7.28×10^{-5}	0.655	8.27×10^{-14}
+0.71	6.49×10^{-5}	0.655	9.27×10^{-14}
-1.00	7.44×10^{-5}	0.655	8.09×10^{-14}
+1.00	6.33×10^{-5}	0.655	9.50×10^{-14}
(c)			
0.00	3.76×10^{-5}	0.610	1.08×10^{-13}
-0.62	3.95×10^{-5}	0.610	1.03×10^{-13}
+0.62	3.58×10^{-5}	0.610	1.14×10^{-13}
-0.71	3.98×10^{-5}	0.610	1.02×10^{-13}
+0.71	3.55×10^{-5}	0.610	1.15×10^{-13}
-1.00	4.06×10^{-5}	0.610	1.00×10^{-13}
+1.00	3.46×10^{-5}	0.610	1.18×10^{-13}

TABLE V. Same as Table I for $\sqrt{s} = 10.58$ GeV.

P	c_{AB} (GeV ²)	$\sqrt{\langle O_1^2 \rangle}$ (GeV ²)	$ \delta \text{Re} d_7^* $ (e cm)
(a)			
0.00	-1.58×10^{-5}	1.78	5.71×10^{-15}
-0.62	-8.19×10^{-1}	1.78	1.10×10^{-17}
+0.62	8.15×10^{-1}	1.78	1.11×10^{-17}
-0.71	-9.37×10^{-1}	1.78	9.65×10^{-18}
+0.71	9.33×10^{-1}	1.78	9.67×10^{-18}
-1.00	-1.32	1.78	6.86×10^{-18}
+1.00	1.31	1.78	6.86×10^{-18}
(b)			
0.00	-3.91×10^{-4}	1.66	1.57×10^{-14}
-0.62	-6.36×10^{-1}	1.66	9.63×10^{-18}
+0.62	6.35×10^{-1}	1.66	9.64×10^{-18}
-0.71	-7.29×10^{-1}	1.66	8.41×10^{-18}
+0.71	7.27×10^{-1}	1.66	8.42×10^{-18}
-1.00	-1.03	1.66	5.98×10^{-18}
+1.00	1.02	1.66	5.98×10^{-18}
(c)			
0.00	-7.03×10^{-5}	1.51	5.76×10^{-14}
-0.62	-3.63×10^{-1}	1.51	1.12×10^{-17}
+0.62	3.62×10^{-1}	1.51	1.12×10^{-17}
-0.71	-4.15×10^{-1}	1.51	9.77×10^{-18}
+0.71	4.15×10^{-1}	1.51	9.77×10^{-18}
-1.00	-5.85×10^{-1}	1.51	6.93×10^{-18}
+1.00	5.85×10^{-1}	1.51	6.94×10^{-18}

TABLE VI. Same as Table II for $\sqrt{s} = 10.58$ GeV.

P	c_{AB} (GeV)	$\sqrt{\langle O_2^2 \rangle}$ (GeV)	$ \delta \text{Im}d_\tau^\gamma $ (e cm)
		(a)	
0.00	2.83×10^{-3}	1.19	2.53×10^{-15}
-0.62	2.50×10^{-3}	1.19	2.41×10^{-15}
+0.62	2.26×10^{-3}	1.19	2.67×10^{-15}
-0.71	2.52×10^{-3}	1.19	2.40×10^{-15}
+0.71	2.25×10^{-3}	1.19	2.69×10^{-15}
-1.00	2.58×10^{-3}	1.19	2.34×10^{-15}
+1.00	2.19×10^{-3}	1.19	2.75×10^{-15}
(b)			
0.00	1.64×10^{-3}	1.26	2.83×10^{-15}
-0.62	1.72×10^{-3}	1.26	2.69×10^{-15}
+0.62	1.56×10^{-3}	1.26	2.98×10^{-15}
-0.71	1.73×10^{-3}	1.26	2.67×10^{-15}
+0.71	1.55×10^{-3}	1.26	3.00×10^{-15}
-1.00	1.77×10^{-3}	1.26	2.62×10^{-15}
+1.00	1.51×10^{-3}	1.26	3.08×10^{-15}
(c)			
0.00	8.96×10^{-4}	1.15	3.43×10^{-15}
-0.62	9.41×10^{-4}	1.15	3.26×10^{-15}
+0.62	8.51×10^{-4}	1.15	3.61×10^{-15}
-0.71	9.48×10^{-4}	1.15	3.24×10^{-15}
+0.71	8.45×10^{-4}	1.15	3.64×10^{-15}
-1.00	9.69×10^{-4}	1.15	3.17×10^{-15}
+1.00	8.24×10^{-4}	1.15	3.73×10^{-15}

$$d\sigma(P) = \sum_{i,j} \{(X_{ij} + r_{ij}Y_{ij}) - P(r_{ij}X_{ij} + Y_{ij})\}, \quad (24)$$

we can define a polarization asymmetrized distribution:

$$|d\sigma(P) - d\sigma(-P)| = 2 \left| P \sum_{i,j} (r_{ij}X_{ij} + Y_{ij}) \right|. \quad (25)$$

We can then compute the mean and standard deviation for the correlations over this distribution and these are tabulated in Tables VII-IX. The correlations get contributions from the $\pm 2P \sum Y_{ij}$ term in Eq. (25) as compared to the $\sum r_{ij}Y_{ij}$ and is therefore enhanced, since $|r_{ij}| < 1$. However the sensitivities are now computed for smaller event samples whose size is given by $|P \sum_{i,j} r_{ij}N_{ij}|$, where $\sum_{i,j} N_{ij}$ stands for the total num-

TABLE VII. (a) c_{AB} , standard deviation and $|\delta \text{Re}d_\tau^\gamma|$ computed for $\int \mathcal{L}(P)dt = \int \mathcal{L}(-P)dt$ and $\sum_{i,j} N_{ij} = 2 \times 10^7$ $\tau^+\tau^-$ pairs from O_1 for $\pi\pi$, $\pi\rho$, and $\rho\rho$ channels for $\sqrt{s} = 3.67$ GeV from the polarization asymmetrized distribution. (b) c_{AB} , standard deviation, and $|\delta \text{Im}d_\tau^\gamma|$ computed for $\int \mathcal{L}(P)dt = \int \mathcal{L}(-P)dt$ and $\sum_{i,j} N_{ij} = 2 \times 10^7$ $\tau^+\tau^-$ pairs from O_2 for $\pi\pi$, $\pi\rho$, and $\rho\rho$ channels for $\sqrt{s} = 3.67$ GeV from the polarization asymmetrized distribution.

	c_{AB} (GeV ²)	$\sqrt{\langle O_1^2 \rangle}$ (GeV ²)	$ \delta \text{Re}d_\tau^\gamma $ (e cm)
		(a)	
$\pi\pi$	2.39×10^2	0.292	1.32×10^{-18}
$\pi\rho$	1.86×10^2	0.267	1.13×10^{-18}
$\rho\rho$	1.06×10^2	0.240	1.29×10^{-18}
(b)			
	c_{AB} (GeV)	$\sqrt{\langle O_2^2 \rangle}$ (GeV)	$ \delta \text{Im}d_\tau^\gamma $ (e cm)
$\pi\pi$	1.18×10^{-2}	0.596	5.45×10^{-14}
$\pi\rho$	8.15×10^{-3}	0.615	5.94×10^{-14}
$\rho\rho$	4.45×10^{-3}	0.575	7.38×10^{-14}

TABLE VIII. Same as Table VII for $\sqrt{s} = 4.25$ GeV.

	c_{AB} (GeV ²)	$\sqrt{\langle O_1^2 \rangle}$ (GeV ²)	$ \delta \text{Red}_\tau^\gamma $ (e cm)
		(a)	
$\pi\pi$	1.13×10^3	0.529	3.77×10^{-19}
$\pi\rho$	8.82×10^2	0.289	1.90×10^{-19}
$\rho\rho$	5.03×10^2	0.127	1.08×10^{-19}
	c_{AB} (GeV)	$\sqrt{\langle O_2^2 \rangle}$ (GeV)	$ \delta \text{Im}d_\tau^\gamma $ (e cm)
		(b)	
$\pi\pi$	6.52×10^{-2}	0.632	7.83×10^{-15}
$\pi\rho$	4.47×10^{-2}	0.657	8.59×10^{-15}
$\rho\rho$	2.45×10^{-2}	0.610	1.06×10^{-14}

ber of $\tau^+\tau^-$ pairs including both polarizations P and $-P$. The standard deviations are less affected. The net result is an increase in the sensitivity. For the different values of \sqrt{s} we tabulate the associated quantity, $|\sum_{i,j} r_{ij} N_{ij}| / \sum_{i,j} N_{ij}$, the effective polarization asymmetry in Table X. Indeed, the improvement in sensitivity is seen to be by an order of magnitude.

We can combine the sensitivities from the different τ channels in inverse quadrature, to get the improved numbers for $|\delta \text{Red}_\tau^\gamma|$ of 8×10^{-19} , 1×10^{-19} , 1×10^{-19} e cm, respectively, and for $|\delta \text{Im}d_\tau^\gamma|$ of 4×10^{-14} , 6×10^{-15} , 5×10^{-16} e cm, respectively, at the three center-of-mass energies of 3.67, 4.25, and 10.58 GeV.

Thus far and for the purposes of Tables I–IX, we have made the altogether reasonable assumption that the contribution of the WDFP is negligible which is justified so long as the EDM and WDFP are of comparable magnitude. However, no such assumption is really necessary in polarization experiments such as these where the ability to run the experiment at different polarizations allows one to disentangle their individual contributions to the correlations considered here. Indeed, it has been pointed out in the context of CP violation in the $t\bar{t}$ system that varying the polarization allows a model independent determination of the separate contributions of the EDM and WDFP to the correlations of the type considered here [14]. The principle is that at a given polarization, a certain linear combination of the two form factors alone can be measured. Performing the experiment at two different

polarizations enables us to disentangle the two form factors. Similarly, the 1 s.d. limits also can only be placed on such a linear combination. Indeed, such 1 s.d. limits would be defined by straight lines given by equations such as

$$\delta \text{Red}_\tau^\gamma/a + \delta \text{Red}_\tau^Z/b = \pm 1, \quad (26)$$

for the limits arising from O_1 and by

$$\delta \text{Im}d_\tau^\gamma/c + \delta \text{Im}d_\tau^Z/d = \pm 1, \quad (27)$$

for the limits arising from O_2 where the numbers a , b , c , and d can be explicitly computed for a given P and N . This is also presented for the polarization asymmetrized distribution for which we have set $P = 1$ (with the understanding that this would have to be scaled by \sqrt{P} if P is the polarization realized in a certain experiment). In particular, a (c) is the sensitivity of the real (imaginary) part of the EDM in inverse quadrature when the real (imaginary) part of the WDFP is set to zero and b (d) is the sensitivity of the real (imaginary) part of the WDFP in inverse quadrature when the real (imaginary) part of the EDM is set to zero. We tabulate these quantities for the three different c.m. energies and for different polarizations for the parent distributions in Table XI and for the asymmetrized distributions in Table XII. Note that one can read from the columns for a and c in Table XII

TABLE IX. Same as Table VII for $\sqrt{s} = 10.58$ GeV.

	c_{AB} (GeV ²)	$\sqrt{\langle O_1^2 \rangle}$ (GeV ²)	$ \delta \text{Red}_\tau^\gamma $ (e cm)
		(a)	
$\pi\pi$	1.72×10^3	3.46	2.61×10^{-19}
$\pi\rho$	1.34×10^3	2.38	1.68×10^{-19}
$\rho\rho$	7.62×10^2	1.48	1.33×10^{-19}
	c_{AB} (GeV)	$\sqrt{\langle O_2^2 \rangle}$ (GeV)	$ \delta \text{Im}d_\tau^\gamma $ (e cm)
		(b)	
$\pi\pi$	2.49×10^{-1}	1.19	6.20×10^{-16}
$\pi\rho$	1.71×10^{-1}	1.28	7.03×10^{-16}
$\rho\rho$	9.35×10^{-2}	1.15	8.39×10^{-16}

TABLE X. The effective polarization asymmetry $|\sum_{i,j} N_{ij} r_{ij}| / (\sum_{i,j} N_{ij})$ for $\sqrt{s} = 3.67, 4.25, \text{ and } 10.58$ GeV.

\sqrt{s} (GeV)	$ \sum N_{ij} r_{ij} / \sum N_{ij}$
3.67	9.2×10^{-5}
4.25	1.2×10^{-4}
10.58	7.7×10^{-4}

the bounds cited in the abstract and in the preceding paragraph (scaled by $\sqrt{0.71}$). These implicitly assume that the magnitudes of the EDM and WDFP are comparable and therefore the latter may be ignored for such considerations.

We now discuss how the results of Table XI may be used in order to essentially define regions in the $\text{Re}(\text{Im}) d_{\tau}^{\gamma} - d_{\tau}^Z$ planes due to finite statistics, say at the 1 s.d. level. By performing the experiment at two values of P , say P_1 and P_2 , one obtains two sets of straight lines defined above. The vertices of the intersection of these four lines defines the parallelogram in each of these planes which cannot be ruled out due to the finite statistics. The best results may be obtained by taking the largest value of polarization realizable P_{max} and taking $P_1 = -P_2 = P_{\text{max}}$. In Table XIII we tab-

ulate for the three different values of P_{max} two pairs: (A, B) for the real and (C, D) for the imaginary planes, which give the coordinates of two vertices in the real and imaginary $d_{\tau}^{\gamma} - d_{\tau}^Z$ planes, respectively, with $(-A, -B)$ and $(-C, -D)$ giving the remaining pairs. Thus the availability of polarization and of either sign provides for a model-independent scheme for constraining regions of the parameter space spanned by the CP -violating form factors. It must be noted that the price to be paid for such a model-independent bound on each of the form factors is large. In particular, from Table XIII, the most stringent such bound on the magnitude of $\text{Re}(\text{Im}) d_{\tau}^{\gamma}$ is only the larger of the $|A|(|C|)$.

IV. CONCLUSIONS

We have presented closed-form expressions for the correlations of O_1 and O_2 parametrized by the real and imaginary parts of the EDM and WDFP and for their standard deviations due to standard model interactions. We have tabulated for unit values of these parameters (in units of e/\sqrt{s}) the values of the correlation and standard deviations for a variety of energies at which the τ CF is expected to operate, and we have computed the 1 s.d.

TABLE XI. (a) The quantities $a, b, c,$ and d in e cm defining the lines of sensitivity for different polarization with $N_{\tau^+\tau^-} = 10^7$ for $\sqrt{s} = 3.67$ GeV. (b) As above for $\sqrt{s} = 4.25$ GeV. (c) As above for $\sqrt{s} = 10.58$ GeV.

P	a	b	c	d
(a)				
0.00	1.73×10^{-12}	1.38×10^{-13}	4.17×10^{-13}	2.70×10^{-9}
-0.62	2.15×10^{-16}	1.32×10^{-13}	3.97×10^{-13}	5.52×10^{-10}
+0.62	2.15×10^{-16}	1.45×10^{-13}	4.39×10^{-13}	9.32×10^{-10}
-0.71	1.88×10^{-16}	1.31×10^{-13}	3.95×10^{-13}	4.94×10^{-10}
+0.71	1.88×10^{-16}	1.47×10^{-13}	4.42×10^{-13}	7.80×10^{-10}
-1.00	1.33×10^{-16}	1.28×10^{-13}	3.86×10^{-13}	3.71×10^{-10}
+1.00	1.33×10^{-16}	1.50×10^{-13}	4.53×10^{-13}	5.11×10^{-10}
(b)				
0.00	2.39×10^{-13}	2.03×10^{-14}	5.18×10^{-14}	2.50×10^{-10}
-0.62	4.23×10^{-17}	1.93×10^{-14}	4.93×10^{-14}	5.11×10^{-11}
+0.62	4.23×10^{-17}	2.13×10^{-14}	5.45×10^{-14}	8.63×10^{-11}
-0.71	3.69×10^{-17}	1.92×10^{-14}	4.90×10^{-14}	4.58×10^{-11}
+0.71	3.70×10^{-17}	2.15×10^{-14}	5.49×10^{-14}	7.22×10^{-11}
-1.00	2.62×10^{-17}	1.88×10^{-14}	4.80×10^{-14}	3.43×10^{-11}
+1.00	2.62×10^{-17}	2.21×10^{-14}	5.63×10^{-14}	4.73×10^{-11}
(c)				
0.00	5.34×10^{-15}	4.66×10^{-16}	1.65×10^{-15}	1.27×10^{-12}
-0.62	6.09×10^{-18}	4.44×10^{-16}	1.57×10^{-15}	2.60×10^{-13}
+0.62	6.09×10^{-18}	4.91×10^{-16}	1.74×10^{-15}	4.40×10^{-13}
-0.71	5.32×10^{-18}	4.41×10^{-16}	1.56×10^{-15}	2.33×10^{-13}
+0.71	5.32×10^{-18}	4.94×10^{-16}	1.75×10^{-15}	3.68×10^{-13}
-1.00	3.78×10^{-18}	4.31×10^{-16}	1.53×10^{-15}	1.75×10^{-13}
+1.00	3.78×10^{-18}	5.07×10^{-16}	1.80×10^{-15}	2.41×10^{-13}

TABLE XII. The quantities a , b , c , and d for the asymmetrized distributions with $P = 1$, $\int \mathcal{L}(P)dt = \int \mathcal{L}(-P)dt$, and $\sum_{ij} N_{ij} = 2 \times 10^7$ for the three different center-of-mass energies.

\sqrt{s} (GeV)	a	b	c	d
3.67	7.14×10^{-19}	9.29×10^{-15}	3.52×10^{-14}	2.91×10^{-12}
4.25	9.11×10^{-20}	8.79×10^{-16}	5.07×10^{-15}	3.13×10^{-13}
10.58	9.68×10^{-20}	1.49×10^{-16}	4.06×10^{-16}	3.99×10^{-15}

sensitivities for a modest sample of 10^7 $\tau^+\tau^-$ pairs. A polarization asymmetry we define is a useful tool to improve this sensitivity. We have described a technique to implement a model independent analysis by varying the polarization which does not require us to neglect the contributions of a possible WDFP that is justified when the EDM and WDFP are of comparable magnitudes. For e^+ and e^- longitudinal beam polarizations of 42% achievable at the τ CF the sensitivities can be as excellent as (few) $\times 10^{-19}$ e cm for the real part and (few) $\times 10^{-16}$ e cm for the imaginary part. We demonstrate that the absence of an axial vector coupling of the electron to the photon is not necessarily a detriment to the use of polarization in probing CP violation. An improvement by at least an

order of magnitude over the sensitivity for the real part of the EDM in the unpolarized case (Table II of Ref. [13]) is noted.

It was noted in [5] recently that optimal variables improve the sensitivity by a significant factor as compared to tensor variables in LEP experiments. This is likely to be the case in experiments at other energies. It may be worthwhile investigating if using appropriate optimal variables would improve the sensitivity in experiments with polarized beams as well. Thus, the sensitivities we quote may be improved upon by choosing optimal variables, rather than the simple vector variables we study. However, our approach has been to obtain analytical expressions for correlations as well as the variances of the

TABLE XIII. (a) A and B , and C and D , defining the parallelograms in the $\text{Re}d_\tau^Z$ - $\text{Re}d_\tau^Z$ and $\text{Im}d_\tau^Z$ - $\text{Im}d_\tau^Z$ planes, respectively, for various values of P_{\max} for a , b , c , and d of Table XI as described in the text at $\sqrt{s} = 3.67$ GeV. (b) As above for $\sqrt{s} = 4.25$ GeV. (c) As above for $\sqrt{s} = 10.58$ GeV.

P_{\max}	A	B	C	D
(a)				
0.62	-4.34×10^{-15}	2.79×10^{-12}	5.17×10^{-13}	-1.66×10^{-10}
	2.16×10^{-16}	-4.55×10^{-16}	-2.02×10^{-12}	3.35×10^{-9}
0.71	-3.31×10^{-15}	2.44×10^{-12}	5.58×10^{-13}	-2.06×10^{-10}
	1.88×10^{-16}	-3.25×10^{-16}	-2.50×10^{-12}	3.62×10^{-9}
1.00	-1.66×10^{-15}	1.73×10^{-12}	8.40×10^{-13}	-4.36×10^{-10}
	1.33×10^{-16}	-1.94×10^{-16}	-5.28×10^{-12}	5.44×10^{-9}
(b)				
0.62	-8.54×10^{-16}	4.10×10^{-13}	6.42×10^{-14}	-1.54×10^{-11}
	4.25×10^{-17}	-6.78×10^{-17}	-2.50×10^{-13}	3.10×10^{-10}
0.71	-6.51×10^{-16}	3.58×10^{-13}	6.94×10^{-14}	-1.91×10^{-11}
	3.70×10^{-17}	-4.83×10^{-17}	-3.10×10^{-13}	3.35×10^{-10}
1.00	-3.28×10^{-16}	2.54×10^{-13}	1.04×10^{-13}	-4.04×10^{-11}
	2.63×10^{-17}	-3.38×10^{-17}	-6.56×10^{-13}	5.04×10^{-10}
(c)				
0.62	-1.24×10^{-16}	9.50×10^{-15}	2.05×10^{-15}	-7.91×10^{-14}
	6.17×10^{-18}	-5.61×10^{-18}	-7.99×10^{-15}	1.58×10^{-12}
0.71	-9.43×10^{-17}	8.26×10^{-15}	2.22×10^{-15}	-9.79×10^{-14}
	5.36×10^{-18}	-3.57×10^{-18}	-9.89×10^{-15}	1.71×10^{-12}
1.00	-4.72×10^{-17}	5.83×10^{-15}	3.34×10^{-15}	-2.07×10^{-13}
	3.78×10^{-18}	-3.86×10^{-19}	-2.09×10^{-14}	2.56×10^{-12}

observables. This is not possible for optimal variables, as they are nonpolynomial in the momenta and spins, and the correlations have necessarily to be computed numerically. In any case, our observation that beam polarization would help in the measurement of the EDM, is likely to be true even if optimal variables are used.

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- [16] It is worthy of note that these correlations and their standard deviations yield an important normalization tool for Monte Carlo simulations that have to be constructed in order to compute three (and larger) body final states and tensor correlations, making them of *more than academic interest* even in the event no polarization experiment is performed.
- [17] An alternative proposal is to polarize the electron beam along [J. Kirkby (private communication)] with the possibility of a degree of polarization of nearly 100%, as for example at SLC where an 80% polarization has now been achieved (B. Schumm, talk given at Third Workshop on Tau Lepton Physics [11]). However, a hard collinear photon bremsstrahlung process would induce a CP -even background to the correlation $\langle O_2 \rangle$ at $O(\alpha)$. Such a background would have to be computed and subtracted. It is possible to do this in the formalism applicable to helicity-flip bremsstrahlung following B. Falk and L. M. Sehgal, *Phys. Lett. B* **325**, 509 (1994). Note that at SLC the cross section of this process is suppressed by a large factor compared to the production cross section (in fact, vanishing in the narrow width approximation) but is not necessarily so at the τCF energies.
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