Lorentz symmetry breaking in Abelian vector-field models with Wess-Zumino interaction

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We consider Abelian vector-field models in the presence of the Wess-Zumino interaction with pseudoscalar matter. The occurrence of the dynamic breaking of Lorentz symmetry at classical and one-loop levels is described for massless and massive vector fields. This phenomenon appears to be the nonperturbative counterpart of the perturbative renormalizability and/or unitarity breaking in chiral gauge theories.

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Introduction. The Abelian vector-field models we consider are given by the Lagrangian which contains the Wess-Zumino interaction (in Minkowski space-time):

$$\mathcal{L}_{\rm WZ} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu - b\partial_\mu A^\mu \pm\frac{\beta^2}{2}\partial_\mu\theta\partial^\mu\theta + \frac{\kappa}{4M}\theta F_{\mu\nu}\tilde{F}^{\mu\nu} , \qquad (1)$$

where $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, the universal dimensional scale M is introduced, $m \equiv \beta_V M$ is the mass of the vector field, and the coupling parameters β and κ take particular values depending on their origin as we explain below.

The Wess-Zumino interaction [the last term in (1)] can be equivalently represented in the form

$$\int d^4x \frac{\kappa}{4M}\theta F_{\mu\nu}\tilde{F}^{\mu\nu} = -\int d^4x \frac{\kappa}{2M}\partial_\mu\theta A_\nu \tilde{F}^{\mu\nu} \qquad (2)$$

when it is treated in the action. Therefore the pseudoscalar field is involved in the dynamics only through its gradient $\partial_\mu\theta(x)$ because of topological triviality of Abelian vector fields.

These models have different roots.

(1) They may represent the anomalous part of the chiral Abelian theory when the Lagrangian is prepared in the gauge invariant form by means of integration over the gauge group [1] and after Landau gauge fixing:

$$\mathcal{L}_{\rm ch} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2(A_\mu + \partial_\mu\vartheta)(A^\mu + \partial^\mu\vartheta) - b\partial_\mu A^\mu + \bar{\psi}(\not{\partial} + ie(\not{A} + \not{\partial}\vartheta)P_L)\psi , \qquad (3)$$

where $P_L = (1+\gamma_5)/2$. In this case $\kappa \equiv e^3/12\pi^2$, $\theta = M\vartheta$, and $\beta = m/M$. The latter relation provides the cancellation of the ghost pole in the vector-field propagator coupled to the chiral fermion current or, in other words, it leads to the Proca propagator for a vector field. As is known [2] this is just the anomalous vertex

$$\theta\partial_\mu J^\mu_L = i\frac{e^2}{48\pi^2}\theta F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad (4)$$

which yields the violation of perturbative unitarity at high energies. Therefore the nonperturbative properties of the model (1) might be crucial in understanding what happens to the chiral Abelian model due to the breakdown of perturbative unitarity and of power-counting renormalizability.

Indeed as follows from canonical rescaling arguments

$$x \to \lambda x , \quad \partial_\mu \to \partial_\mu/\lambda , \quad A_\mu \to A_\mu/\lambda ,$$
$$\theta \to \theta/\lambda , \quad b \to b/\lambda^2 , \quad \psi \to \psi/\lambda^{3/2} ,$$
$$S_{\rm kin} + S^\perp_{\rm int} \to S_{\rm kin} + S^\perp_{\rm int} , \quad S^\parallel_{\rm int} \to S^\parallel_{\rm int}/\lambda , \qquad (5)$$

for $\lambda \ll 1$ the notion of high energies is roughly related to the limit $M \to 0$ or $m = \beta_V M \to 0, \kappa/M \to \infty$. Thus one expects that the effective coupling rapidly increases with energies and therefore approaches the strong-coupling regime.

(2) On the other hand the model (1) with $\beta^2 \to -\beta^2$ can shed light on the breaking of unitarity in the chiral Abelian vector models (CAVM's) [3] with the anomaly compensating ghost field

$$\mathcal{L}^\perp_{\rm ch} = \mathcal{L}_{\rm ch} - \frac{1}{2}m^2\partial_\mu\eta\partial^\mu\eta - \eta\partial_\mu J^\mu_L . \qquad (6)$$

This model is renormalizable by power counting, exhibits extended gauge invariance [3], and is equivalent to the CAVM (2) with the additional constraint $\Box\theta = 0$ that leads to the nonlocal Lagrangian with purely transversal gauge fields. However the presence of the triangle anomaly is still troublesome since it eventually gives rise to the lack of decoupling of the massless ghost pole [4] in the transversal projector, in spite of the extended gauge or Becchi-Rouet-Stora-Tyutin (BRST) invariance.

Thus model (1) with the negative "ghost" sign of the kinetic term for scalar fields might be helpful to unravel the infrared region of transversal CAVM's.

Equations of motion and spectrum of vector fields in the η background. The Euler-Lagrange equations for the model (1) read

$$\partial_\mu F^{\mu\nu} + m^2 A^\nu + \partial^\nu b - \frac{\kappa}{M}\partial_\mu \theta \tilde{F}^{\mu\nu} = 0 \ ,$$
$$\partial_\mu A^\mu = 0 \Rightarrow \Box b = 0 \ ,$$
$$\pm\beta^2 \Box\theta = \frac{\kappa}{4M} F_{\mu\nu}\tilde{F}^{\mu\nu} \ . \quad (7)$$

One can see that the constant solutions

$$\partial_\mu \theta = M\eta_\mu = \text{const} \ , \quad F_{\mu\nu} = 0 \ , \quad (8)$$

are compatible with Eqs. (7). If $\beta = 0$ then the classical action vanishes for these Lorentz-symmetry-breaking (LSB) solutions and therefore the Lorentz symmetric vacuum configuration $\eta_\mu = 0$ is degenerate. If $\beta \neq 0$ then for $\eta_\mu \eta^\mu \neq 0$ the above solution shifts the potential energy by a constant which might lead to an inequivalent quantum theory. The quantum effective potential (in the Coleman-Weinberg spirit [5]) is needed to select out the true vacuum configuration (see below).

Let us analyze the spectrum of vector fields in the presence of a constant η_μ background. It is evident from Eqs. (7) that the longitudinal components decouple ($\partial_\mu A^\mu = 0$) and the auxiliary field $b(x)$ is free. In the momentum representation Eqs. (7) read

$$\{(p^2 - m^2) g^{\mu\nu} + i\kappa \epsilon^{\mu\nu\rho\sigma}\eta_\rho p_\sigma\} A^\perp_\nu \equiv (\mathbf{K}\cdot A^\perp)^\mu = 0 \ . \quad (9)$$

Let us denote

$$\mathcal{E}^{\mu\nu}(p) = \kappa\epsilon^{\mu\nu\rho\sigma}\eta_\rho p_\sigma = -\mathcal{E}^{\mu\nu}(-p) \ , \quad (10)$$

and multiply Eq. (9) by a transposed matrix $\mathbf{K}^t(p) = \mathbf{K}^*(p) = \mathbf{K}(-p)$ which has the same eigenvalue as $\mathbf{K}$ due to its Hermiticity:

$$\mathbf{K}^t\mathbf{K}\cdot A^\perp = [(p^2 - m^2)^2 \mathbb{1} + \hat{\mathcal{E}}^2] A^\perp = 0 \ . \quad (11)$$

The matrix $\hat{\mathcal{E}}^2$ represents in fact the projector on a two-dimensional plane (for η_μ, p_μ not collinear):

$$\begin{aligned}\mathcal{E}_{\mu\nu}\mathcal{E}^{\nu\lambda} &= \kappa^2[\delta^\lambda_\mu(\eta^2 p^2 - (\eta\cdot p)^2) - (\eta^\lambda\eta_\mu p^2 + p^\lambda p_\mu \eta^2) \\ &\quad + (\eta^\lambda p_\mu + p^\lambda \eta_\mu)(\eta\cdot p)] \\ &\equiv \kappa^2[\eta^2 p^2 - (\eta\cdot p)^2][\mathbf{P}_2]^\lambda_\mu, \quad \mathbf{P}_2^2 = \mathbf{P}_2, \quad \mathrm{tr}\mathbf{P}_2 = 2 \ .\end{aligned} \quad (12)$$

Evidently,

$$\hat{\mathcal{E}}\mathbf{P}_2 = \hat{\mathcal{E}} \ , \quad \mathbf{P}_\perp \mathbf{P}_2 = \mathbf{P}_2 \ , \quad \mathbf{P}_2 \cdot p = \mathbf{P}_2\cdot\eta = 0 \ . \quad (13)$$

Therefore in the η_μ direction one finds the free massive field

$$(p^2 - m^2) A^\mu_\eta = 0 \ , \quad A^\mu_\eta \equiv \frac{\eta^\mu}{\eta^2}(\eta \cdot A^\perp) \ , \quad (14)$$

whereas in the two-dimensional plane selected by $\mathbf{P}_2$ the dispersion law is different from a free-particle one,

$$\begin{aligned}[(p^2 - m^2)^2\mathbf{I} + \hat{\mathcal{E}}^2]\mathbf{P}_2 \cdot A &= \{(p^2 - m^2)^2 \\ &\quad + \kappa^2[\eta^2 p^2 - (\eta\cdot p)^2]\}\mathbf{P}_2\cdot A \\ &= 0 \ .\end{aligned} \quad (15)$$

Let us restrict ourselves to spacelike η_μ and choose the coordinate frame where $\eta = (0, \boldsymbol{\eta})$. Then the energy spectrum is defined by the dispersion law

$$p_0^2 = \mathbf{p}^2 + m^2 + \frac{\kappa^2\boldsymbol{\eta}^2}{2} \pm\sqrt{\frac{\kappa^4(\boldsymbol{\eta}^2)}{4} + m^2\kappa^2\boldsymbol{\eta}^2 + \kappa^2(\boldsymbol{\eta}\mathbf{p})^2} \geq 0 \ . \quad (16)$$

Thus in the plane orthogonal to both η_μ and p_μ there appear two types of waves (for two different polarizations). In the soft-momentum region ($\mathbf{p}^2 \ll m^2 + \frac{\kappa^2\eta^2}{4}$) one reveals two massive excitations with masses:

$$m^2_\pm \approx m^2 + \frac{\kappa^2\boldsymbol{\eta}^2}{2} \pm \sqrt{\frac{\kappa^4(\boldsymbol{\eta}^2)^2}{4} + m^2\kappa^2\boldsymbol{\eta}^2} \ . \quad (17)$$

In particular, when the bare vector particle is massless, and after the interaction with the LSB background, the mass splitting arises. In the soft-momentum region we find only one polarization for nearly massless excitations, but a complementary polarization behaves as a massive one.

Effective potential for pseudoscalar field. Let us examine the role of quantum corrections in the formation of the vacuum expectation value (VEV) for $\partial_\mu\theta$ and derive the one-loop effective potential for the gradient of Wess-Zumino (WZ) field induced by the virtual creation of vector particles. We follow the recipe of the background-field method [6,7] to obtain the effective potential and consider the second variation of the action S_{WZ} around constant *spacelike* $\partial_\mu\theta = M\eta_\mu$, $\eta^2 < 0$, and zero vector-field configurations.

For space-like η_μ the energy spectrum of vector fields is real [see Eq. (26)]. Therefore one can employ the causal prescription for vector-field propagators and furthermore perform the Wick rotation in computing the effective action. Then the transversal part of the (Euclidean) vector-field action reads

$$A^\perp_\mu[(-\Box + m^2)\delta_{\mu\nu} - \mathcal{E}_{\mu\nu}(\hat{p})]A^\perp_\nu = (A^\perp \cdot \mathbf{K}A^\perp) \ . \quad (18)$$

The one-loop effective potential is then obtained from the functional determinant of the above operator $\mathrm{Det}\mathbf{K} = (\mathrm{Det}\mathbf{K}^t\mathbf{K})^{1/2}$ in the conventional way [6,7]:

$$\begin{aligned}V_{\mathrm{eff}} &\equiv V^{(0)} + V^{(1)} \ , \quad V^{(0)} = \pm\frac{\beta^2_{\mathrm{bare}}M^2}{2}\eta^2 \ , \\ V^{(1)} &= \frac{1}{\mathrm{Vol}}\left\{\frac{1}{4}\mathrm{Tr}[\mathbf{P}_2 \ln \mathbf{K}^t(\eta)\mathbf{K}(\eta)]\right\} - \{\eta_\mu = 0\} \\ &= \frac{1}{2}\int_{|p|<\Lambda}\frac{d^4p}{(2\pi)^4}\ln\left(1 + \frac{\kappa^2[\eta^2p^2 - (\eta\cdot p)^2]}{(p^2+m^2)^2}\right) \ ,\end{aligned} \quad (19)$$

where the relations corresponding to Eqs. (9)–(15) are used for the Euclidean space metric ($\eta^2 > 0$ from now on).

In four dimensions there are divergent terms in (19) and in the finite-cutoff regularization they have the cutoff dependence

$$V^{(1)}_1 = \frac{3\kappa^2}{2^7\pi^2}\left(\Lambda^2 - 2m^2\ln\frac{\Lambda^2}{m^2} + m^2\right)\eta^2 + O\left(\frac{m^2}{\Lambda^2}\right) \ ,$$
$$V^{(1)}_2 = -\frac{5\kappa^4}{2^9\pi^2}\ln\frac{\Lambda^2}{m^2}(\eta^2)^2 + O\left(\frac{m^2}{\Lambda^2}\right) \ . \quad (20)$$

Evidently, the renormalization is required with two counterterms:

$$\Delta V(\theta) = \frac{\Delta\beta^2}{2}\partial_\mu\theta\partial^\mu\theta + \frac{\Delta g}{4M^4}(\partial_\mu\theta\partial^\mu\theta)^2 \ . \quad (21)$$

We see that the second divergence cannot be generally cured in the minimal model (1) and implies the inclusion of the dimension-8 vertex into the bare Lagrangian (1). The appearance of higher-dimensional vertices in the effective Lagrangian is not surprising since the model is not perturbatively renormalizable. Thus we should specify the boundary conditions for the effective potential which provide the minimal form of the Lagrangian (1) at a particular scale (by means of the fine-tuning of all higher-dimensional vertices to zero value). Then the form of the effective Lagrangian for other scales will be governed by the effective renormalization-group (RG) flow [8].[1] It also should be pointed out that, in the case of the perturbative power counting renormalizable model described by the Lagrangian (6), one can obtain the Wess-Zumino Lagrangian (1), with a ghostlike kinetic term for the θ-field, after integration over the longitudinal part of the fermionic action. Consequently, because of the stability under renormalization of the Lagrangian (6), it follows that the fine-tuning to zero of the dimension-8 vertex of the effective Wess-Zumino Lagrangian (1) is indeed consistent, as it originates from the perturbative renormalizable (but nonunitary) model (6). The remaining part of $V^{(1)}$ is finite and one can evaluate the effective potential in the form

$$V_{\rm ren} = \pm\frac{\mu_1^2}{2}\eta^2 + \frac{g}{4}(\eta^2)^2 + \frac{1}{32\pi^2}\ln\left(\frac{m^2+z}{\mu_2^2}\right)(5z^2+6m^2z+m^4) , \quad (22)$$

where we set $z \equiv \kappa^2\eta^2/4$. The constants μ_1, μ_2, g are fixed by boundary conditions. For instance, the soft momentum and weak coupling normalization (at $\eta_\mu = 0$) is given by

$$\pm\mu_1^2 = \pm\beta^2M^2 - \frac{\kappa^2m^2}{64\pi^2} , \quad \mu_2^2 = m^2 , \quad g = -\frac{\kappa^4}{256\pi^2} , \quad (23)$$

once we set by fine-tuning the dimension-8 vertex to zero, as already noticed. However, such boundary conditions do not allow the massless limit or the strong-coupling regime that is expected to take place at high energies (see the Introduction). In the latter case we will use the normalization at the fixed scale M which has the meaning of a scale for our measurements.

Dynamic breaking of the Lorentz symmetry. Let us search for the Coleman-Weinberg [5] instabilities in the effective potential of pseudoscalar field that cause the dynamic symmetry breaking of the Lorentz symmetry (for other scenarios of LSB, see [9]). We examine separately the models with massless and massive vector fields.

In the first case the one-loop effective potential cannot be normalized at zero momenta, i.e., at $\eta^2 = 0$. Instead one has to provide the basic Lagrangian at the main scale of the model $\eta^2 = M^2$:

$$V_{\rm ren}\left(\eta, \mu = \frac{\kappa M}{2}\right) = \pm\frac{\beta^2M^2}{2}\eta^2 + \frac{5\kappa^4}{2^9\pi^2}(\eta^2)^2 \ln\frac{\eta^2}{M^2} . \quad (24)$$

[1]We omit for a moment the "dipole-ghost" term with four derivatives $(\partial^2\theta)^2$, which, however, is important in the RG flow for such models.

Here we recall, once again, that we have set to zero the quartic term in η according to the fine-tuning choice discussed above. The minimum is obtained from the conditions

$$\frac{\partial V}{\partial\eta_\mu} = 2\eta_\mu V'(\eta^2) = 0 ,$$

$$V'(\eta^2) \equiv \frac{dV(\eta^2)}{d(\eta^2)} = \pm\frac{\beta^2M^2}{2} + \frac{5\kappa^4}{2^9\pi^2}\eta^2\left(2\ln\frac{\eta^2}{M^2}+1\right) , \quad (25)$$

and

$$\frac{\partial^2 V}{\partial\eta_\mu\partial\eta_\nu} = 2\delta_{\mu\nu}V'(\eta^2) + 4\eta_\mu\eta_\nu V''(\eta^2) \geq 0 , \quad (26)$$

$$V''(\eta^2) \equiv \frac{d^2V(\eta^2)}{[d(\eta^2)]^2} = \frac{5\kappa^4}{2^9\pi^2}\left(2\ln\frac{\eta^2}{M^2}+3\right) .$$

The symmetric solution $\eta_\mu = 0$ leads to the minimum if $V'(0) > 0$ which corresponds to the positive sign in the first term of (24). In the latter case other solutions may arise for a strong coupling κ when $V'(\eta^2) = 0$.

In order to find the critical value of κ let us substitute Eq. (25) into (26) to provide $V''(\eta^2) \geq 0$. Then for $\kappa^4 \geq 128\pi^2e^{3/2}\beta^2/5$ the second minimum appears. However, it lies higher than the symmetric one as compared with the value of the effective potential at $\eta_\mu = 0$. They are degenerate when $\kappa_{\rm cr}^4 = 256\pi^2e\beta^2/5$, and for higher values of κ the LSB vacuum is favorable. By its character the corresponding phase transition is of the first order since at $\kappa_{\rm cr}$ the VEV of η^2 jumps to $\eta_{\rm cr}^2 = M^2/e$. This VEV entails the LSB due to the creation of spacelike constant gradient of pseudoscalar field $\partial_\mu\theta \sim M^2e^{-1/2}(0,\mathbf{n})$, $\mathbf{n}^2 = 1$.

We remark that the different choice of normalization scale μ [or of the bare coupling constant Δg in Eq. (21)] can be absorbed by renormalization of parameters β and M and therefore does not lead to qualitative changes in the LSB phenomenon.

When going back to the Introduction one can conclude that the plausible scenario of what happens in the chiral gauge model (3) with Proca vector fields at high energies is the LSB at strong coupling. This is what might be behind the breaking of perturbative unitarity in such models.

If $\beta = 0$ (the pure WZ interaction without a kinetic term for the θ field) LSB still occurs and the Lorentz symmetric extremum becomes a maximum. For the negative sign of the first term in Eq. (25) the LSB minimum always exists and a normalization scale is not there to prevent the Lorentz symmetric vacuum from decay.

Let us extend our analysis to the massive vector-field models. We pay special attention to the power-counting renormalizable chiral model (6) with transversal vector fields. This model is suitably described in the "ghost" sector by the effective Lagrangian (1), with a "ghost" sign of the kinetic term for the θ field ($\beta^2 \to -\beta^2$, $m = \beta M$), and can be consistently normalized by the choice (23) at the infrared point. The LSB conditions (25), (26) are satisfied both in the strong and the weak coupling regimes, and the LSB minimum is unique since $V''(\eta^2) > 0$ for positive η^2. In particular,

$$\eta^2_{\min} \simeq m^2 \frac{128\pi^2}{5\kappa^4 \ln(32\pi^2/\kappa^2)} \quad \text{for} \quad \kappa \ll 1 \; ;$$
$$\eta^2_{\min} \simeq m^2 (32\pi/\kappa^3\sqrt{7}) \quad \text{for} \quad \kappa \gg 1 \; . \tag{27}$$

Thus the symmetric vacuum in such a model is always unstable.

Once Lorentz symmetry is spontaneously broken one should expect the occurrence of Goldstone modes in the spectrum of fluctuations, $\partial_\mu \tilde{\theta} = \partial_\mu \theta - M\eta_\mu$ around the minimum. In our case it gives rise to the degeneracy in the kinetic term of pseudoscalar fluctuations. As follows from (26), the second variation contains the projector on the η_μ direction. Consequently, after Wick rotating to the Minkowski space-time, the kinetic term $-\frac{1}{2}[(\eta\cdot\partial)\tilde{\theta}(x)]^2$ describes the dynamics of a massless free mode whose support, in the momentum space, lies on the space-like hyperplane $\eta_\mu p^\mu$ (this feature typically arises in the quantization of Yang-Mills fields in algebraic noncovariant gauges [10]). In other directions the dynamics is generated by higher-derivative "ghost-dipole" terms in the effective action, $\sim (\partial^2\theta)^2$.

Conclusions. As a result of the present investigation one can argue that the cancellation of anomalies strongly prevents the chiral theories from the Lorentz symmetry breaking.

On the other hand, in the presence of Wess-Zumino interaction the occurrence of LSB, at least in small domains, seems to be natural. Indeed, the WZ action is invariant under general coordinate transformations, $x_\mu \to n_a(x_\mu)$. This mapping is a local diffeomorphism when

$$\det[\partial^\mu n_a] \equiv e^{\mu\nu\rho\sigma}\partial^\mu n_0 \partial^\nu n_1 \partial^\rho n_2 \partial^\sigma n_3 \neq 0 \; .$$

Under these transformations the fields and derivatives behave as vectors:

$$A^\mu = \overline{A}^a(n)\frac{\partial n_a}{\partial x_\mu} \; .$$

Therefore the pseudoscalar Chern-Pontryagin density in the WZ action turns out to be multiplied by $\det[\partial^\mu n_a]$, just compensating the change in the integration measure. Let us pick out the curvilinear coordinate system with $\theta(x)$ as one of the coordinate vector, i.e., $\theta \equiv \overline{\theta}(n) = M\eta_a n^a$, as it is always possible inside the domains where $\partial_\mu\theta \neq 0$. Then the WZ action (2) in new coordinates takes the form of the Chern-Simons action in the hyperplane orthogonal to the *constant* vector η_a:

$$S_{\rm WZ} = -\frac{\kappa}{2}\int dn_{\parallel}\eta_a \int d^3n^{\perp}\epsilon^{abcd}A_b\partial_c A_d \; , \tag{28}$$

which provides the reduction of dynamics from four to three dimensions. Thus locally, in domains of smooth $\partial_\mu\theta$, one actually deals with the dynamics described in our paper.

We see some similarities of LSB between the (2+1)-dimensional case [11], as it is connected with the dynamics in two dimensions, and the (4+1)-dimensional case in its reduction [12] to our model. These similarities will be discussed elsewhere.

Apart from the benefit for understanding the chiral model, the spectral properties of the light in a pseudoscalar medium ("pseudoscalar optics") could happen to be applicable in two situations: first, to explore the axion matter if it exists in the Universe, and, second, to check the possibility of strong-coupling LSB in the neutral-pion matter under extreme conditions (for other applications, see [13]). If the latter one is conceivably characterized by a vanishing pion decay constant $F_\pi \to 0$ at the phase transition of chiral symmetry restoration, then the WZ interaction of pions and photons may be enhanced considerably to invoke LSB.

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