Higgs-Yukawa model in curved spacetime

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The Higgs-Yukawa model in curved spacetime (renormalizable in the usual sense) is considered near the critical point, employing the $1/N$ expansion and renormalization group techniques. By making use of the equivalence of this model with the standard NJL model, the effective potential in the linear curvature approach is calculated and the dynamically generated fermionic mass is found. A numerical study of chiral symmetry breaking by curvature effects is presented.

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The Nambu–Jona-Lasinio (NJL) model [1] and the Gross-Neveu model [2] belong to a very restricted class of quantum theoretical models in which an analytical study of the composite bound states is possible. Such models are usually studied in the framework of the $1/N$ expansion (see [3] for a review).

Recently, there has been some interest in the literature [4, 5] about the dynamical symmetry-breaking pattern of NJL-like models for the electroweak interaction, where the top quark plays the role of an order parameter. A study of the NJL model in curved spacetime has been carried out in Refs. [6–8]. Using a block-spin renormalization group (RG), the existence of an IR stable fixed point in the NJL model was pointed out [6, 4].

In Ref. [9] it was suggested that it would be interesting to consider the Higgs-Yukawa (or simply Yukawa) model, which is multiplicatively renormalizable in the usual sense. In frames of the $1/N$ expansion the NJL model and the Higgs-Yukawa model near the critical point become completely equivalent and they describe the same physics of chiral symmetry breaking (CSB).

In the present work we consider the Higgs-Yukawa model in curved spacetime and make use of its equivalence with the NJL model in curved spacetime. The effective potential is found in the large-N (and linear curvature) approximation at a finite cutoff, and also after removing the cutoff in the coupling constants (in the manner of Coleman and Weinberg [10]). The dynamical fermionic mass in curved spacetime is calculated.

We start now the study of the renormalizable Higgs-Yukawa model in curved spacetime. We will be interested in the dynamics of this theory as a kind of four-fermion model near the critical point.

The (multiplicatively renormalizable) Lagrangian of the theory under discussion is the typical one of a Yukawa-type interaction [11]:

$$L=\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma-\frac{1}{2}m^2\sigma^2+\frac{1}{2}\xi R\sigma^2-\frac{\lambda}{4!}\sigma^4$$
$$+\bar{\psi}[i\gamma^\mu(x)\nabla_\mu-h\sigma]\psi$$
$$+\Lambda+\kappa R+a_1R^2+a_2C^2_{\mu\nu\alpha\beta}+a_3G+a_4\Box R, \qquad (1)$$

where σ is a scalar field, ψ a massless, N-component Dirac spinor, $g^{\mu\nu}$ an arbitrary metric of the classical gravitational field, and, for simplicity, we will set $\Box R=0$. Here only σ and ψ will be quantized.

The restricted version of this theory with

$$L=\bar{\psi}[i\gamma^\mu(x)\nabla_\mu-h\sigma]\psi-\frac{1}{2}m^2\sigma^2, \qquad (2)$$

where σ is to be treated as an auxiliary scalar field, represents (after elimination of σ) the standard four-fermion model of NJL type:

$$L_{\text{four fermion}}=\bar{\psi}i\gamma^\mu(x)\nabla_\mu\psi+\frac{h^2}{m^2}(\bar{\psi}\psi)^2, \qquad (3)$$

where it is convenient to put $h=1$ and $m^2=N/(2\lambda_1)$.

The Yukawa model of the kind (1) near the critical point in flat space has been considered in Ref. [9], where it was pointed out that the physics of that model is again the physics of CSB. Under a chiral transformation, σ transforms into $-\sigma$.

We will now study in detail the Higgs-Yukawa model (1), by making use of the fact that it is multiplicatively renormalizable. The standard one-loop β functions can be found easily (see [22] for flat space and [12] for curved space).

By direct inspection of the β function, one can see in the matter sector that the fixed point $\lambda=g^2=m^2=0$, $\xi=1/6$ is infrared stable. In the IR limit, i.e., at a scale $\mu\ll\Lambda$, one finds, for the dimensionless running couplings $[t=\frac{1}{2}\ln(\mu^2/\Lambda^2)]$,

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$$h^2(t) \sim -\frac{(4\pi)^2}{(4N+6)t},$$
$$\xi(t) \sim \frac{1}{6} + \left(\xi - \frac{1}{6}\right) t^{-\lambda_* - 4N/(4N+6)},$$
$$\lambda(t) \sim -\frac{(4\pi)^2}{t}\lambda_*,$$
$$\lambda_* = \frac{1}{6(2N+3)}\left[-(2N-3) + \sqrt{4N^2+132N+9}\right], \tag{4}$$

and from here one can easily get the corresponding relations for large N. These relations give the logarithmic corrections to the IR fixed-point solution in the matter sector. Moreover, as one can see the behavior of the couplings of the Higgs-Yukawa model in curved spacetime (in the matter sector) near the critical point is the same as for the usual nonrenormalizable NJL model in curved spacetime [6].

Using Eq. (4) we can investigate the behavior of the scalar-graviton coupling constant ξ for the composite bound state. Choosing (for an estimation) $\Lambda \simeq M_{\rm Pl} \simeq 10^{19}$ GeV and $\mu \simeq M_{\rm GUT} \simeq 10^{15}$ GeV, at large N we obtain

$$\xi(t) \sim \frac{1}{6} - \frac{1}{8}\left(\xi - \frac{1}{6}\right); \tag{5}$$

i.e., the IR limit being an asymptotically conformal one, as in [6] [see also [14] and for grand unified theories (GUT's) [13]], our $\xi(t)$ still depends, weakly, on the initial value $\xi = \xi(0)$. For the choice $\xi = 0$, $\xi(t)$ becomes positive and not large, which may be relevant for cosmological applications where, for instance, the model of extended inflation [15] favors very small negative values of ξ [corresponding to $\xi \simeq 4/3$ in (5)], while the inflationary model of Refs. [16] favors very large and negative values of ξ.

Let us now proceed with the study of the effective potential for composite fields in curved spacetime. Rewriting the matter sector of the Lagrangian as

$$L = \frac{1}{2h^2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{N}{2\lambda_1 h^2}\sigma^2 + \frac{N}{2\xi_1 h^2}R\sigma^2 - \frac{N}{4!\lambda_2 h^4}\sigma^4 + \bar\psi[i\gamma^\mu(x)\nabla_\mu - \sigma]\psi, \tag{6}$$

where we have performed the substitutions $m^2 \to N/(2\lambda_1)$, $\xi \to N/(2\xi_1)$, and $\lambda \to N/\lambda_2$, and σ has been rescaled into $h\sigma$, for simplicity, and if we suppose that σ is a constant, we can work out the semiclassical effective action $S_{\rm eff}$. Integrating over the fermion fields, we obtain

$$S_{\rm eff} = \int d^4x\sqrt{-g}\left(-\frac{1}{2\lambda_1 h^2}\sigma^2 + \frac{1}{2\xi_1 h^2}R\sigma^2 - \frac{1}{4!\lambda_2 h^4}\sigma^4\right) - i\ln\det[i\gamma^\mu(x)\nabla_\mu - \sigma], \tag{7}$$

where N has been factored out.

Now, working in the linear curvature approximation and regularizing the divergent integrals by the cutoff method (see, for example, Ref. [17]), we can repeat the calculations of Refs. [7, 8] to obtain, for our model,

$$V(\sigma) = V(0) + \frac{1}{2\lambda_1 h^2}\sigma^2 - \frac{1}{2\xi_1 h^2}R\sigma^2 + \frac{1}{4!\lambda_2 h^4}\sigma^4 - \frac{1}{(4\pi)^2}\left[\sigma^2\Lambda^2 + \Lambda^4\ln\left(1+\frac{\sigma^2}{\Lambda^2}\right) - \sigma^4\ln\left(1+\frac{\Lambda^2}{\sigma^2}\right)\right] + \frac{R}{6(4\pi)^2}\left[-\sigma^2\ln\left(1+\frac{\Lambda^2}{\sigma^2}\right) + \frac{\sigma^2\Lambda^2}{\sigma^2+\Lambda^2}\right]. \tag{8}$$

We have thus obtained the effective potential $V(\sigma)$ in the linear curvature approximation and for a finite cutoff.

We will here adopt a different strategy, by making use of the crucial property that the above model is a multiplicatively renormalizable theory. Owing to this fact, we may repeat the analysis of Coleman and Weinberg [10], and throw away terms which vanish when Λ^2 goes to infinity in (8), to remove the remaining Λ^2-dependent terms via the renormalization of the coupling constants by imposing Coleman-Weinberg-type renormalization conditions (for a general description in the case of a curved spacetime, see [18]). As a result, after some algebra we obtain

$$V(\sigma) = \frac{1}{2\lambda_1 h^2}\sigma^2 - \frac{1}{2\xi_1 h^2}R\sigma^2 + \frac{1}{4!\lambda_2 h^4}\sigma^4 + \frac{\sigma^4}{(4\pi)^2}\left(\ln\frac{\sigma^2}{\mu^2} - \frac{25}{6}\right) + \frac{R\sigma^2}{6(4\pi)^2}\left(\ln\frac{\sigma^2}{\mu^2} - 3\right), \tag{9}$$

where μ^2 is the mass parameter. Notice that using the form (9) of the effective potential it is more difficult to compare the properties of the Higgs-Yukawa model with those of the usual NJL model, because of the fact that some cutoff dependence is hidden in the new parameters ξ_1 and λ_2. (One might also adopt another point of view and hide the new parameters in the cutoff procedure, as was suggested in [9].)

Let us now analyze the phase structure of the effective potential (9). The dynamical mass of the fermion is calculated by using the gap equation

$$\left.\frac{\partial V(\sigma)}{\partial\sigma}\right|_{\sigma=\sigma_0} = \frac{\sigma_0}{\lambda_1 h^2} - \frac{R\sigma_0}{\xi_1 h^2} + \frac{\sigma_0^3}{6\lambda_2 h^4} + \frac{\sigma_0^3}{4\pi^2}\left(\ln\frac{\sigma_0^2}{\mu^2} - \frac{11}{3}\right) + \frac{R\sigma_0}{3(4\pi)^2}\left(\ln\frac{\sigma_0^2}{\mu^2} - 2\right) = 0. \tag{10}$$

The solution σ_0 of Eq. (10) corresponds to the ground state of the composite field $\bar\psi\psi$ (near the critical point) and is equal to the dynamical mass of the fermion. Supposing that such a solution exists, and choosing $\mu^2 = \sigma_0^2$, we get

$$\sigma_0^2 = \left(\frac{R}{\xi_1 h^2} - \frac{1}{\lambda_1 h^2} + \frac{2R}{3(4\pi)^2}\right)\left(\frac{1}{6\lambda_2 h^4} - \frac{11}{12(4\pi)^2}\right)^{-1}. \tag{11}$$

As we can see, in absence of a gravitational field, chiral symmetry breaking takes place at

$$\frac{11}{2(4\pi)^2} > \frac{1}{\lambda_2 h^4}.$$

On the contrary, in the presence of a gravitational field there may arise a completely gravitational effect, which can occur even in the situation when there is no CSB in flat space. Choosing different values for the coupling constants in the effective potential (9), one can numerically investigate the phase structure of the theory, and in particular, the curvature-induced phase transitions [11, 18] between the CS phase and the CSB phase.

In Fig. 1 we show the potential (9) for different values of R and fixed values of λ_1, λ_2, ξ_1, h, and μ. The following combinations of adimensional variables have been chosen (they appear naturally). For the potential itself $f(x) \equiv V(x)/\mu^4$ (this is the y axis), being the variable $x \equiv \sigma^2/\mu^2$, and for the coefficients of the different terms $a_1 \equiv (2\lambda_1 h^2 \mu^2)^{-1}$, $a_2 \equiv (2\xi_1 h^2)^{-1}$, $a_3 \equiv (4!\,\lambda_2 h^4)^{-1}$, and $r \equiv R/\mu^2$, which yields the function

$$f(x) = a_1 x - a_2 r x + a_3 x^2 + \frac{x^2}{(4\pi)^2}(\ln x - 25/6) + \frac{rx}{6(4\pi)^2}(\ln x - 3)\,. \qquad (12)$$

We have simply taken $a_1 = a_2 = a_3 = 1$ and $r = 0, 3, 10, 20$, and the potential is compared in the same range $0 \le x \le 15$. The upper curve corresponds to the zero-curvature case and the curvature increases as we go down.

It is interesting to remark that in the $1/N$ expansion one can consider the gravitational field to be a quantum field as well, since this does not change the picture at all. Working in the $1/N$ expansion (as was proposed some time ago [19]), using the large-N contributions to the β functions, we get in the infrared regime ($t \to -\infty$) Eqs. (4) (dropping the next-to-leading terms) together with the following equations for the gravitational couplings:

$$\Lambda(t) \sim \Lambda - \frac{m^4}{2(4\pi)^2 t}, \qquad \kappa(t) \sim \kappa - \frac{m^2(\xi - 1/6)}{2(4\pi)^2 t},$$

$$\alpha(t) = \frac{\alpha}{1 + \frac{\alpha t}{20(4\pi)^2}}, \qquad \eta^{-1}(t) \simeq -\frac{(\xi - 1/6)^2}{2(4\pi)^2 N t}, \qquad (13)$$

where $\alpha = N/a_2$ and $\eta = N/a_1$. Thus, we see that

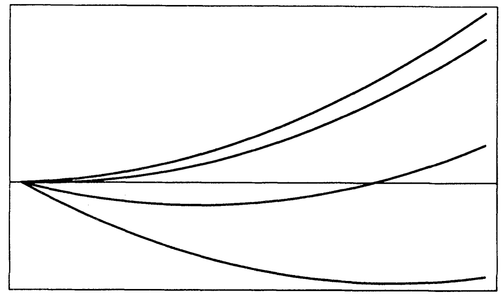

FIG. 1. Plot of the potential $V(\sigma^2/\mu^2)/\mu^4$ in terms of σ^2/μ^2 for different values of R and fixed values of λ_1, λ_2, ξ_1, h, and μ. The upper curve corresponds to $R = 0$ and the curvature increases as we go down.

the cosmological and Newtonian couplings are growing in the infrared regime, as compared with the matter couplings which go to their IR fixed-point values. Notice that proper quantum gravitational (QG) corrections to the β functions are negligible in the $1/N$ expansion. The behavior of $\alpha(t)$ coincides precisely with the corresponding one that was obtained in [19], where a theory of N massless free fermions interacting with a QG was considered. For $\alpha < 0$ we get asymptotic freedom for $\alpha(t)$ (as is the case for η^{-1} in the IR regime). Thus we are able to discuss the running of the effective couplings in the quantum gravity-Higgs-Yukawa system in the infrared region.

We think that the $1/N$ expansion, which is a gauge-invariant procedure, provides very interesting possibilities to deal with QG-matter systems.

In summary, we have studied here the effective potential for the Higgs-Yukawa model in curved spacetime near the critical point where it is equivalent to the standard NJL model. We have also calculated the dynamical fermionic mass and studied the curvature-induced phase transitions of the model. We envisage other possibilities to extend the NJL model while being still able to use the $1/N$ expansion. In particular, let us mention the higher-derivative NJL models [20, 21], where it would be interesting to apply a similar analysis.

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