BRST quantization of anomalous gauge theories

Nelson R. F. Braga*

Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, 21945 Rio de Janeiro, Brazil

(Received 17 August 1994; revised manuscript received 15 December 1994)

It is shown how BRST quantization can be applied to a gauge invariant sector of theories with anomalously broken symmetries. This result is used to show that shifting the anomalies to a classically trivial sector of fields (Wess-Zumino mechanism) makes it possible to quantize the physical sector using a standard BRST procedure, as for a nonanomalous theory. The trivial sector plays the role of a topological sector if the system is quantized without shifting the anomalies.

PACS number(s): 11.15.Ex, 03.70.+k

I. INTRODUCTION

When considering the quantization of an anomalous gauge theory, one possible approach is to quantize the theory without restoring gauge invariance, as was done by Jackiw and Rajaraman [1] for the case of the chiral Schwinger model. A unitary, though nongauge invariant, theory is obtained for this particular solvable model. A more general approach is to use the Wess-Zumino mechanism, usually interpreted as restoring gauge invariance. Following this approach, one can then use a Becchi-Rouet-Stora-Tyutin (BRST) quantization procedure.[1] The main purpose of this article is to discuss a physical interpretation of the Wess-Zumino (WZ) fields, that are introduced to restore gauge invariance, by making a connection with the quantization of the so-called topological field theories. We will make use of the Batalin-Vilkovisky (BV) [4] Lagrangian BRST quantization scheme (also called field-antifield quantization) because it provides, through the master equation, a systematic way of calculating quantum contributions (anomalies and WZ terms), but our interpretation of the WZ fields is valid for a general BRST quantization.

The quantization of a purely quantum field theory (vanishing classical limit) was used by Labastida, Pernici, and Witten [5] as an interesting approach to generate a topological two-dimensional (2D) quantum gravity. The starting points are just the fields and their associated symmetries. At the classical level the Lagrangian is zero. The quantum action corresponds just to the gauge fixing of the initial symmetry. For the particular case of 2D topological quantum gravity, enlarging the usual symmetry of 2D gravity by including the shift symmetry renders a nontrivial ghost structure involving a second generation of ghosts associated with the nonindependence of the transformations. At this stage, without coupling the theory to other sectors there is no nontrivial BRST invariant observable.

The so-called Wess-Zumino mechanism is a well-known technical way of translating the anomalous breaking of classical gauge invariance to the appearance of new dynamical degrees of freedom, the so-called Wess-Zumino fields, originally proposed by Faddeev and Shatashvili [6]. The new enlarged system is invariant under the original symmetries at the quantum level, and one usually says that gauge symmetry is restored. The Batalin-Vilkovisky (BV) formalism [4], also called field-antifield formalism provides a powerful framework for the BRST quantization of gauge theories. A first discussion about the application of the BV formalism to anomalous gauge theories was carried out by Troost, van Nieuwenhuizen, and Van Proeyen [7]. In this reference it was shown how one can regularize a theory in order to make sense of the terms of order higher than zero in $\hbar$ in the master equation. These terms will represent the purely quantum part of the theory, that means, they will take account of the behavior of the path integral measure, and that is why they only make sense when the theory is regularized. When anomalies are present, there is no local solution to the master equation in the standard space of fields and antifields.

In Ref. [8] the BV quantization was applied to the chiral Schwinger model. A nonlocal WZ term was obtained, and it was made local by the introduction of an auxiliary (WZ) field. This procedure is particular since this nonlocal WZ term does not generally exist. Recent investigations show that the application of BV to anomalous gauge theories leads to the appearance of the Wess-Zumino terms if the field-antifield space is extended by the inclusion of field-antifield pairs associated to the broken gauge symmetries. This realization of the mechanism proposed in [6] in the BV framework was first shown for the case of chiral 2D QCD (QCD_2) in [9] and then for general theories with a closed irreducible gauge algebra in [10]. In these two articles it is assumed that the additional fields transform as elements of the original gauge group and that there is no additional symmetry.

It was later pointed out by De Jonghe, Siebelink, and Troost [11] that when one extends the field-antifield space by adding fields that are not present at the classical level one should also take into account an additional symmetry (shift symmetry) that rules out these fields at classical level. Thus, the gauge-fixing part of the action should include an additional term involving the ghost associated

---

*Electronic address: braga@vms1.nce.ufrj.br

[1]See, for example, [2] and [3].

with this symmetry. This fact corresponds to imposing the condition that the gauge fixed action be the proper solution of the (zero order) master equation. Following this approach, they conclude that including WZ terms one is in fact shifting the anomalies to these new symmetries.

Some important questions now arise. If the anomaly is just shifted, can we say that BRST invariance is restored? Can we BRST quantize a theory in which the anomalies are still present? One could follow the approach that anomalies are really canceled, as did Gomis and Paris [12], and impose the proper condition just on the quantum action. This corresponds to neglecting the new symmetries. We will see, however, that following this approach we lose an important physical interpretation for the origin of the WZ fields.

The aim of this article is to show that, including these extra symmetries, the WZ fields will be interpreted as coming from a trivial sector that could also lead to topological field theories depending on the quantization procedure. We will also show that if a general gauge theory has a broken sector of symmetries, we can use BRST quantization for the other sector. This fact is particularly important if the anomaly is shifted to a nonphysical sector.

II. ANOMALOUS GAUGE THEORIES

The BV quantization procedure is defined in an enlarged space of fields and antifields, collectively denoted by Φ^a and Φ^{*a}, respectively. The quantum action has the general $\hbar$ expansion

$$W(\Phi^a,\Phi^{*a}) = S(\Phi^a,\Phi^{*a}) + \sum_{p=1}^{\infty} \hbar^p M_p(\Phi^a,\Phi^{*a}). \quad (1)$$

It should satisfy the so-called (quantum) master equation

$$\frac{1}{2}(W,W) = i\hbar\Delta W, \quad (2)$$

where the antibrackets are defined as $(X,Y) = \frac{\partial_r X}{\partial\Phi^a}\frac{\partial_l Y}{\partial\Phi^{*a}} - \frac{\partial_r X}{\partial\Phi^{*a}}\frac{\partial_l Y}{\partial\Phi^a}$ and the operator delta as $\Delta \equiv \frac{\partial_r}{\partial\Phi^a}\frac{\partial_l}{\partial\Phi^*_a}$. Equation (2) implies that the vacuum functional, defined by

$$Z_\Psi = \int\prod D\Phi^a \exp\left[\frac{i}{\hbar}W\left(\Phi^a,\Phi^{*a} = \frac{\partial\Psi}{\partial\Phi^a}\right)\right], \quad (3)$$

is independent of the gauge fixing fermion Ψ. More details can be found in [4] or [2].

The zero-order term of the action W, $S(\Phi^a,\Phi^{*a})$, is usually called a gauge fixed action and is subject to the boundary condition

$$S(\Phi^a,\Phi^{*a}=0) = \mathcal{S}(\phi^i), \quad (4)$$

where $\mathcal{S}$ is the classical limit of theory. The set of fields Φ^a includes the classical fields ϕ^i, ghost fields c^α associated with the symmetries of $\mathcal{S}(\phi^i)$, and possibly some additional fields necessary to have a standard representation for the gauge conditions [4]. The set of antifields Φ^{*a} contains the corresponding partners of each of the fields.

We can rewrite the master equation (2) in powers of $\hbar$. The two first powers are

$$(S,S) = 0, \quad (5)$$

$$(M_1,S) = i\Delta S. \quad (6)$$

We will consider theories for which the higher-order contributions $M_p (p \geq 2)$ to W can be taken as zero, so we only need these two first-order terms in the master equation.

As mentioned before, we need to regularize the theory in order to make sense of the terms of order higher than zero in $\hbar$ in the master equation, like ΔS. We will not be concerned with the details of the regularization process in this article. One can find them in the literature.[2] We will just present the general idea.

A regularized theory can be built by introducing Pauli-Villars (PV) fields, and adding an extra term $S_{\rm PV}(\chi^a,\chi^{*a},\Phi^a)$ to W. The PV fields χ^a have the same statistics as the corresponding Φ^a, but their path integral is defined in such a way that the contributions from their loops has a relative minus sign. The regularization is obtained by a judicious choice of $S_{\rm PV}$ such that the contribution to ΔS coming from both sets of fields cancel. The mass terms of the PV fields, necessary in order to eliminate their propagators after the appropriate infinity mass limit is taken, will break the zero-order master equation $(S,S) = 0$. There is, as expected an arbitrariness in this regularization process.

A theory is said to be anomalous when there is no local term M_1 involving only the original fields of the theory that satisfies Eq. (6). It can be seen in Refs. [7] and [12] that anomalies correspond to a violation of the master equation that can be put in the form

$$\frac{1}{2}(W,W) - i\hbar\Delta W = c^\gamma A_\gamma, \quad (7)$$

where γ takes some values inside the domain of α (spatial integrations are, as usual, implicit). The symmetries associated with the ghosts c^γ are said to be broken at the quantum level.

For a general gauge theory, when a particular regularization process is chosen and we get a particular form of Eq. (7), we arrive at a quantum theory with two sectors of symmetries. The broken ones (corresponding to c^γ) and the unbroken ones (corresponding to the other ghosts). We can incorporate into the theory the information about the symmetry breaking by defining a vacuum functional

$$\overline{Z}_{\overline{\Psi}} = \int\prod D\Phi^a \prod\delta(c^\gamma)\exp\left[\frac{i}{\hbar}W\left(\Phi^a,\frac{\partial\overline{\Psi}}{\partial\Phi^a}\right)\right], \quad (8)$$

[2]See, for example, [7,12–14].

where $\overline{\Psi}$ is a fermion independent of c_γ. It is easy to show that $\overline{Z}_{\overline{\Psi}}$ is independent of $\overline{\Psi}$. That means we have a BRST invariant theory. We will see in Sec. III that this procedure will enable us, by shifting the anomalies to a trivial sector of fields, to build up a BRST invariant generating functional where the original symmetries of the classical theory are realized.

III. WESS-ZUMINO MECHANISM

We can always associate to any standard field theory (not only topological observables) an additional sector that corresponds to fields with zero Lagrangian at the classical level, in the same spirit of Ref. [5]. We consider a general gauge theory with extended BV action:

$$S = S_{\text{phys}}(\Phi^a, \Phi^{*a}) + S_T(\vartheta^b, \vartheta^{*b}, c^\alpha) \tag{9}$$

subject to the boundary conditions (classical limit)

$$\begin{aligned} &S_{\text{phys}}(\Phi^a, \Phi^{*a} = 0) = \mathcal{S}(\phi^i), \\ &S_T(\vartheta^b, \vartheta^{*b} = 0, c^\alpha) = 0. \end{aligned} \tag{10}$$

The set ϑ^b includes at least the fields θ^β and the ghosts d^β. The classical theory is invariant under two independent groups of gauge transformations:

$$\begin{aligned} \delta\phi^i &= R^i_\alpha(\phi^i)\lambda^\alpha, \\ \delta\theta^\beta &= \rho^\beta, \end{aligned} \tag{11}$$

where λ_α and ρ^β are arbitrary functions.

We will call the first set of transformations physical symmetries because they are manifest symmetries of the classical action $\mathcal{S}(\phi^i)$ that we want to quantize. We want to build up a quantum version of this theory that is also gauge invariant with respect to these symmetries. They will be fixed by the ghosts c^α.

The invariance of the classical theory with respect to these physical symmetries leads to Ward identities relating the Green functions and thus the renormalization parameters that are of extreme importance in proving the renormalizability of the quantum theory [15]. For the case of anomalous gauge theories the Ward identities have higher-order corrections (in loops) that may spoil the renormalizability. One can see, for example, in [16] and [17] that anomalies constitute an obstacle to the proof of renormalizability for gauge theories and that this proof depends on the ability to cancel them out, by, for example, adding extra fermionic fields. The addition of the Wess-Zumino fields at quantum level will also give extra contributions to these identities since the WZ fields will also transform with the physical symmetries. Anyway, we see that these symmetries have an important role when considering the quantization of the physical action.

The second set in (11) will be called nonphysical symmetries because they just represent the absence of the fields θ^β at the classical level. The ghosts d^β will play the role of gauge fixing these symmetries. When we realize the Wess-Zumino mechanism some of these symmetries will be broken, simply reflecting the fact that, at the quantum level, the theory will no more be independent of the WZ fields. These symmetries are not manifest at classical level and are thus not relevant for considering the quantization of $\mathcal{S}(\phi^i)$. That is why, as we will see at the end of this section, we will build a generating functional that does not involve these nonphysical symmetries.

There is actually not a unique way to express the transformations for the θ^β fields. The important thing is that they eliminate their degrees of freedom at the classical level. If we include in the second group of transformations (11) additional factors associated with usual physical symmetries (such as diffeomorphism) what happens is that we possibly get nonindependent gauge transformations leading to the introduction of higher-order ghosts, as in [5]. The presence of c^α [ghosts associated with the symmetries of $\mathcal{S}(\phi^i)$, that we will call physical symmetries] in S_T is associated with the arbitrariness in the transformation of θ^β with respect to this gauge group, since these fields are not present at the classical level.

We can assume that the trivial sector contains a set of fields that have the same structure (Lorentz plus internal symmetries) of the elements of the physical gauge group. Now, introducing the Pauli-Villars fields to regularize the physical sector, we may choose different mass terms that may break some original physical symmetries, some symmetries of the trivial sector or, in general, a linear combination of them [11]. We prefer to consider a choice of mass terms that do not involve the fields θ^β and thus will break only symmetries of the physical sector. Assuming that the new fields in S_T have an invariant path integral measure, we get

$$\Delta S = \Delta S_{\text{phys}} = c^\gamma A_\gamma. \tag{12}$$

Following the idea of [9] and [10] we can write out a quantum contribution $M_1(\phi^i, \theta^\beta)$ that cancels the contribution of (12) to the master Eq. (2). We know that this M_1 must depend on the extra fields θ^β because we are assuming that the theory has genuine anomalies and, as is well known, they cannot be canceled by just counterterms. Therefore M_1 is not invariant under (11) leading to a violation in the master equation that now has the general form

$$\frac{1}{2}(W, W) - i\hbar\Delta W = d^\gamma \overline{A}_\gamma. \tag{13}$$

We say now that we have implemented the Wess-Zumino mechanism.

The anomalies have not been canceled. They have just been shifted to the symmetries associated with the trivial sector. We can define again, in the same spirit of (8),

$$\begin{aligned} \overline{Z}_{\Psi} = \int \prod D\Phi^a \prod D\vartheta^b \prod \delta(d^\gamma) \\ \times \exp\left[\frac{i}{\hbar} W\left(\Phi^a, \frac{\partial\overline{\Psi}}{\partial\Phi^a}, \vartheta^b, \frac{\partial\overline{\Psi}}{\partial\vartheta^b}\right)\right], \end{aligned} \tag{14}$$

where $\overline{\Psi}$ is a fermion independent of d^γ. Now the functional $\overline{Z}_{\overline{\Psi}}$ involves integrations over the whole set of physical fields (all the ghosts c^α are included). We can couple the fields to sources J and also introduce the sources L writing a generating functional:

$$\overline{Z}[J^a,J^b,L^a,L^b]_\Psi = \int \prod D\Phi^a \prod D\vartheta^b \prod \delta(d^\gamma) \exp\left[\frac{i}{\hbar} W\left(\Phi^a, \frac{\partial\overline{\Psi}}{\partial\Phi^a}+L^a, \vartheta^b, \frac{\partial\overline{\Psi}}{\partial\vartheta^b}+L^b\right) + J^a\Phi^a + J^b\Phi^b\right] \quad (15)$$

defining the classical fields and effective action, respectively, as

$$\phi^A_{\rm cl} = \frac{\hbar}{i}\frac{\delta \ln \overline{Z}[J^A,L^A]_\Psi}{\delta J^A}, \quad (16)$$

$$\Gamma_\Psi[\phi^A,L^A] = \frac{\hbar}{i}\ln \overline{Z}_\Psi - J^A\phi^A_{\rm cl}, \quad (17)$$

with $A=(a,b)$.

The Zinn-Justin equation

$$(\Gamma_{\overline{\Psi}},\Gamma_{\overline{\Psi}}) = 0 \quad (18)$$

(with the antibrackets defined in a space where ϕ^A play the role of the fields and L^A of the antifields) will now express the gauge invariance of the physical sector and possibly some trivial uncoupled symmetries of the trivial sector.

If we do not include $M_1(\phi^i,\theta^\beta)$ in the quantum action the anomaly will show up in the physical sector. The trivial sector will remain uncoupled and may lead to topological theories as in [5]. At least for some simple models, as in [1], one can then possibly quantize the physical sector in a nongauge invariant way and proceed with the BRST quantization for the trivial sector, as in [5].

Gauge invariance is of extreme importance in proving the unitarity [18] of field theories. Implementing the Wess-Zumino mechanism as in the present section we get a nonanomalous version for the potentially anomalous theory.[3] In the BRST language, we get a quantum theory with a nillpotent BRST generator, representing the invariance of the effective action (17). One can then define the physical states in the usual way [2], in terms of the cohomology classes of this generator. If gauge invariance is lost the theory may become nonunitary, and therefore inconsistent.

One could also expect, in principle, that the renormalization properties are improved by restoring gauge invariance. We can see, however, in [21] an example of a four-dimensional gauge theory that after the decoupling of one of the fermion chiralities remains gauge invariant by the generation of a Wess Zumino term, but is nonrenormalizable. So, the issue of renormalizability cannot be analyzed in a general way by just taking gauge invariance into account.

IV. EXAMPLE

Now we will consider an example in order to illustrate our previous development. Let us consider a theory that at the classical level is described by the sum of the actions

$$S_{\rm phys} = \int d^2x\left\{ i\overline{\psi}\,\not{D}\,\frac{(1-\gamma_5)}{2}\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A^*_\mu\partial^\mu c + i\psi^*\psi c - i\overline{\psi}\,\overline{\psi}^*c\right\}, \quad (19)$$

$$S_T = \int d^2x\{\theta^* c + \theta^* d + \overline{c}^*\pi + \overline{d}^*\lambda\}. \quad (20)$$

The action $S_{\rm phys}$ corresponds to the gauge fixed BV action for the chiral Schwinger model [8] and S_T corresponds to the gauge fixed action for a theory of a scalar field that transforms with the gauge group of the Schwinger model (corresponding to the ghost c) and also with an additional symmetry (corresponding to the ghost d). The antighosts $\overline{c}$ and $\overline{d}$ are introduced in order to allow the implementation of the gauge choices in the standard BV way: $\Phi^{*a} = \frac{\partial\Psi}{\partial\Phi^a}$.

The boundary conditions satisfied by $S_{\rm phys}$ and S_T are of the same form as in (10) with $\mathcal{S}(\phi^i)$ being, in this case, the classical action for the chiral Schwinger model.

The BRST transformations for the fields in S_T are

$$\begin{aligned}&\delta c = 0, \quad \delta d = 0,\\ &\delta\theta = c + d, \quad \delta\overline{c} = \pi,\\ &\delta\overline{d} = \lambda, \quad \delta\pi = 0,\\ &\delta\lambda = 0.\end{aligned} \quad (21)$$

We can write S_T as a BRST variation, showing explicitly its topological character

$$S_T = \delta\Omega, \quad (22)$$

with

$$\Omega = -\theta^*\theta + \overline{c}^*\overline{c} + \overline{d}^*\overline{d}. \quad (23)$$

To implement the BV quantization for $S = S_{\rm phys} + S_T$ this theory must be regularized before the calculation of ΔS. Let us first consider S_T. In order to regularize this action we can consider, as already discussed in Sec. II, the Pauli-Villars (PV) regularization. We have to include a Pauli-Villars partner for the field θ, with the same kinetic operator but with a mass term in such a way that after taking the infinity mass limit it would have a vanishing propagator.

The problem is that, contrary to Ref. [7], our θ field has no kinetic term at classical level. Thus, θ itself has a vanishing propagator at the classical level. We can overcome this difficulty, for example, by considering an action

$$S_T(\alpha) = S_T + \int d^2x\{\alpha\partial_\mu\theta\partial^\mu\theta\}. \quad (24)$$

The extra kinetic term breaks the gauge invariance, as can be seen from

[3] Helpful discussions about anomalies and their physical implications can be found, for example, in [19] and [20].

$$(S_T(\alpha), S_T(\alpha)) = -2\alpha\,\Box\,\theta(c+d), \qquad (25)$$

but in the limit $\alpha \to 0$ we recover the original theory with its invariances. We can now introduce a PV partner to θ, say $\overline{\theta}$, with the same vanishing classical limit, in the same spirit of [7]:

$$S_{\rm PV}(\alpha) = \int d^2x\{\alpha\partial_\mu\overline{\theta}\partial^\mu\overline{\theta} + M^2\overline{\theta}^2 + \overline{\theta}^*(c+d)\}. \quad (26)$$

The violation of the master equation now takes the form

$$[S_T(\alpha) + S_{\rm PV}(\alpha), S_T(\alpha) + S_{\rm PV}(\alpha)]$$
$$= -2\alpha\,\Box\,(\theta+\overline{\theta})(c+d) + M^2\phi(c+d). \quad (27)$$

Functionally integrating the PV field $\overline{\theta}$, its contribution to the above expression just vanishes because, contrary to the cases considered in [7], the field $\overline{\theta}$ is present only in the kinetic term. So, we recover (25). Taking then the limit of vanishing α we find that the contribution of S_T to ΔS is zero. This was clearly expected from the fact that the generators of the symmetries of the field θ are field independent.

On the other hand, the action $S_{\rm phys}$ is exactly the same action that was considered in [8], where the following result was obtained:

$$\Delta S_{\rm phys} = \frac{i}{4\pi}\int d^2x\; c\,[(1-a)\partial_\mu A^\mu - \epsilon^{\mu\nu}\partial_\mu A_\nu]. \quad (28)$$

Now we must build up a quantum action W of the form of Eq. (1) whose first component is just $S = S_{\rm phys} + S_T$. We can consider two different approaches. The first one is to take all the higher-order contributions M_P to the action (1) as vanishing. Then Eq. (28) implies a violation of the master equation of the same form as (7). In this case, following the lines of Sec. II, we may just take the symmetry associated with the ghost c out of the BRST setting by considering the vacuum functional $\overline{Z}_\Psi$ of (8). The two sectors, associated to $S_{\rm phys}$ and S_T will then remain uncoupled. The quantization of $S_{\rm phys}$ can be performed exactly as in [1], where the chiral Schwinger model was shown to contain a free massive vector boson plus harmonic excitations, while the sector corresponding to S_T will now correspond to the action (removing also the antighosts and auxiliary fields associated with c)

$$\overline{S}_T(\phi^a, \phi^{*a}) = \int d^2x\{\theta^* d + \overline{d}^*\lambda\} \qquad (29)$$

that represents a scalar field with no nontrivial BRST invariant observable, as it happens in [5] for topological 2D gravity. This sector can possibly be coupled to other topological theories in order to generate nontrivial observables. We can gauge fix this action by choosing, for example, the scalar field to be equal to some preferable field θ_0, introducing the fermion

$$\Psi = \overline{d}(\theta - \theta_0) \qquad (30)$$

that leads to the action

$$\overline{S}_T\left(\phi^a, \phi^{*a} = \frac{\partial\Psi}{\partial\phi^a}\right) = \int d^2x\{\overline{d}d + \lambda(\theta-\theta_0)\}. \quad (31)$$

A different approach to quantize the theory described by (19) plus (20) is to implement the Wess-Zumino mechanism, following the lines of Sec. III. In Ref. [8] it was shown that adding the contribution

$$M_1 = -\frac{1}{4\pi}\int d^2x\left\{\frac{(a-1)}{2}\,\partial_\mu\theta\,\partial^\mu\theta + \theta\,[(a-1)\partial_\mu A^\mu + \epsilon^{\mu\nu}\partial_\mu A_\nu]\right\} \quad (32)$$

to the quantum action of the Schwinger model would cancel the contribution from ΔS to the master equation if the term $\theta^* d$ is not present in the classical action S. The inclusion of this extra term, taking into account, as already explained, the additional symmetry associated to the ghost d, leads to

$$(M_1, S) = i\Delta S_{\rm phys} + \int d^2x\overline{A}(\theta, A_\mu)d, \qquad (33)$$

with

$$\overline{A}(\theta, A_\mu) = \frac{1}{4\pi}[(a-1)\,\Box\,\theta - \partial_\mu((a-1)A_\mu + \epsilon^{\mu\nu}A_\nu)]. \quad (34)$$

Now the quantum action $W = S_{\rm phys} + S_T + \hbar M_1$ satisfies the equation

$$\frac{1}{2}(W,W) - i\hbar\Delta W = d^\gamma\overline{A}_\gamma \qquad (35)$$

representing the fact that the introduction of the Wess-Zumino term M_1 has shifted the anomaly from the physical symmetry, associated with c to the nonphysical symmetry associated to the ghost d (only present in S_T). Following Sec. III we then define a vacuum functional as in (14) that takes into account the breaking of the symmetry associated to the ghost d, ruling out this field from the formulation. The theory will then be described by the action

$$\overline{W} = S_{\rm phys} + \int d^2x\{\theta^* c + \overline{c}^*\pi\} + M_1(A_\mu, \theta), \qquad (36)$$

which corresponds to the Schwinger model with its standard Wess-Zumino term. That means, we arrive at a gauge invariant formulation (with respect to the physical symmetry) for the theory.

Thus we see from this simple example that the Wess-Zumino term corresponds to the coupling of the physical sector to a trivial sector that otherwise would play the role of a topological sector.

V. CONCLUSIONS

It is interesting now to make a parallel with the original discussion of Ref. [6]. There, the anomalies are interpreted as not breaking the gauge symmetry, but just inducing a different representation for the group in which the WZ fields are also present. We can say that in order to build up this representation for the gauge group we are borrowing some fields from a sector that was, in principle, trivial. In fact the name trivial is not so appropriate. We have seen that, although trivial at classical level, depending on the quantization process this sector may lead to topological field theories at quantum level if we do not use it to implement the Wess-Zumino mechanism. We have learned from recent studies [22,23] that some interesting results can emerge from these kinds of theories.

Regarding the WZ mechanism as a breaking in the symmetry of what would be a topological sector leads us to some interesting questions for future investigations. If part of the symmetry of this sector is broken in the quantization process by coupling to another sector, we get possibly a mechanism for generating field theories not only with topological observables beginning with topological invariant actions. In other words, we get a mechanism for coupling topological theories to nontopological ones.

It is worth mentioning that the possibility of BRST quantization of anomalous field theories [24] has been considered by Marnelius. This author considers BRST quantization with $Q^2_{\rm BRST} \neq 0$ but with $Q_{\rm BRST}$ conserved. What we were interested in discussing was exactly the mechanism of "restoring gauge invariance," so what we did was to exclude the broken symmetries from the BRST setting. Thus we can define a nillpotent and conserved BRST charge. In the case of (14) this corresponds to the charge that generates the BRST transformations on the physical sector plus some possible remaining (uncoupled) symmetries of the trivial sector. After the master equation is written in the form of (13) we can compute the generator of the BRST transformations excluding the ones associated to the ghosts d^γ [that means the BRST symmetries of (14)] : $\overline{Q}_{\rm BRST}$. The physical states will be defined by

$$\overline{Q}_{\rm BRST}|{\rm phys}\rangle = 0. \qquad (37)$$

All the standard BRST procedures [2,3] can then be applied.

Although the present analysis is based on the BV quantization framework, we can generalize our interpretation about the origin of the Wess-Zumino fields for general BRST quantization.

ACKNOWLEDGMENTS

The author would like to thank R. Amorim, J. Barcelos-Neto, H. Boschi Filho, A. Das, E.C. Marino, J.A. Mignaco, and C. Wotzasek for important discussions and F. De Jonghe for important correspondence. The author is partially supported by CNPq-Brazil.

[1] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985).
[2] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, Princeton, New Jersey, 1992).
[3] R. Marnelius, Nucl. Phys. **B395**, 647 (1993); **B391**, 621 (1993); **B384**, 318 (1992); S. Hwang and R. Marnelius, *ibid.* **B315**, 638 (1989).
[4] I.A. Batalin and G.A. Vilkovisky, Phys. Lett. **102B**, 27 (1981); Phys. Rev. D **28**, 2567 (1983).
[5] J.M.F. Labastida, M. Pernici, and E. Witten, Nucl. Phys. **B310**, 611 (1988).
[6] L.D. Faddeev, Phys. Lett. **145B**, 81 (1984); L.D. Faddeev and S.L. Shatashvili, Phys. Lett. **167B**, 225 (1986).
[7] W. Troost, P. van Nieuwenhuizen, and A. Van Proeyen, Nucl. Phys. **B333**, 727 (1990).
[8] N.R.F. Braga and H. Montani, Phys. Lett. B **264**, 125 (1991).
[9] N.R.F. Braga and H. Montani, Int. J. Mod. Phys. A **8**, 2569 (1993).
[10] J. Gomis and J. Paris, Nucl. Phys. **B395**, 288 (1993).
[11] F. De Jonghe, R. Siebelink, and W. Troost, Phys. Lett. B **306**, 295 (1993).
[12] J. Gomis and J. Paris, Nucl. Phys. **B431**, 378 (1994).
[13] F. De Jonghe, Ph.D. thesis Leuven.
[14] W. Troost and A. Van Proeyen, presented at "Strings 93," Berkeley, California, 1993 (unpublished).
[15] J. Zinn-Justin, in *Trends in Elementary Particle Theory*, edited by H. Rollnik and K. Dietz, Lecture Notes in Physics Vol. 37 (Springer, Berlin, 1975).
[16] D.J. Gross and R. Jackiw, Phys. Rev. D **6**, 477 (1972).
[17] C. Bouchiat, J. Iliopoulos, and Ph. Meyer, Phys. Lett. **38B**, 519 (1972).
[18] G.'t Hooft, Nucl. Phys. **B33**, 173 (1971); **B35**, 167 (1971).
[19] R. Jackiw, in *High Energy Physics 1985*, Proceedings of Theoretical Advanced Study Institute in Elementary Particle Physics, New Haven, Connecticut, edited by M.J. Bowick, and F. Gursey (World Scientific, Singapore, 1985), p. 83.
[20] J. Preskill, Ann. Phys. (N.Y.) **210**, 323 (1991).
[21] E. D'Hoker and E. Farhi, in *Anomalies, Geometry and Topology*, Proceedings of the Symposium, Argonne, Illinois, 1985, edited by W.A. Bardeen and A. White (World Scientific, Singapore, 1985).
[22] E. Witten, Commun. Math. Phys. **117**, 353 (1988).
[23] For a general review on the subject, see D. Birmingham, M. Blau, M. Rakowski, and G. Thompson, Phys. Rep. **209**, 129 (1991).
[24] R. Marnelius, Nucl. Phys. **B315**, 638 (1989).