Phase transition in conformally induced gravity with torsion

Jewan Kim and C. J. Park
Department of Physics, Seoul National University, Seoul 151-742, Korea

Yongsung Yoon
Department of Physics, Hanyang University, Seoul 133-791, Korea
(Received 23 May 1994)

We have considered the quantum behavior of a conformally induced gravity in the minimal Riemann-Cartan space. The regularized one-loop effective potential including the quantum fluctuations of the dilaton and the torsion fields in the Coleman-Weinberg sector gives a sensible phase transition for an inflationary phase in de Sitter space. For this effective potential, we have analyzed the semiclassical equation of motion of the dilaton field in the slow-rolling regime.

PACS number(s): 04.62.+v, 11.30.Qc, 98.80.Cq

I. INTRODUCTION

Among the four fundamental interactions in nature, the two feeble interactions are characterized by dimensional coupling constants, Fermi's coupling constant $G_F = (300 \text{ GeV})^{-2}$ and Newton's coupling constant $G_N = (10^{19} \text{ GeV})^{-2}$.

The interactions with dimensional coupling constants of inverse mass dimensions are strongly diverse and nonrenormalizable. However, from the success of the Weinberg-Salam model, the weak interaction at a fundamental level is actually characterized by a dimensionless coupling constant, and the dimensional nature of G_F results from spontaneous symmetry breaking. Indeed $G_F \cong \frac{1}{v_\omega^2}$, where $v_\omega \cong 300$ GeV is the vacuum expectation value of Higgs field. The weakness of the weak interaction comes from the largeness of the vacuum expectation value of the Higgs field [1].

In light of the above remarks, it might be considered that gravity is also characterized by a dimensionless coupling constant ξ, and that the weakness of gravity is associated with symmetry breaking at the high energy scale. Similarly to G_F, G_N could be given by the inverse square of the vacuum expectation value of a scalar field, the dilaton. It was independently proposed by Zee [2], Smolin [3], and Adler [4] that the Einstein-Hilbert action

$$S = -\int d^4x\sqrt{g}\frac{1}{16\pi G_N}R \tag{1}$$

can be replaced by the modified action

$$S = \int d^4x\sqrt{g}\left(-\frac{1}{2}\xi\phi^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right), \tag{2}$$

where the coupling constant ξ is dimensionless. The potential $V(\phi)$ is assumed to attain its minimum value when $\phi = \sigma$; then

$$G_N = \frac{1}{8\pi\xi\sigma^2}. \tag{3}$$

Through spontaneous symmetry breaking, the symmetric phase of the scalar field transits to an asymmetric phase of the scalar field. In analogy with the SU(2) $\times$ U(1) symmetry of weak interactions, we can consider a symmetry which is broken through spontaneous symmetry breaking in gravitational interactions. The most attractive symmetry is conformal symmetry which rejects the Einstein-Hilbert action Eq. (1), but admits the modified action Eq. (2) with the specific coupling $\xi = -\frac{1}{6}$ and quartic potential. We can write down a conformally invariant induced gravity action without introducing the torsion field. However, as discussed in Sec. IV, the spontaneous symmetry-breaking mechanism does not work for scalar field theory with $\xi = -\frac{1}{6}$ in de Sitter space. Introduction of a vector torsion field thus becomes important, because in the regime of interest the background spacetime is well approximated by a de Sitter metric.

In the minimal Riemann-Cartan space, the vector torsion behaves effectively like a conformal gauge field [5]. The introduction of this torsion field makes the dimensionless coupling constant in Eq. (2) free in the conformally invariant induced gravity action. Therefore, it is necessary to introduce the torsion field into conformally induced gravity. Since we expect that conformal symmetry is broken at a very high energy scale, it is natural to consider conformal symmetry with an inflation scenario [6]. We have investigated the quantum behavior of the dilaton and the vector torsion field in conformally induced gravity. In fact quantum fluctuations introduce an anomaly in the gauged scale symmetry. If, nevertheless, we choose some gauge-fixing scheme, one gets a one-loop effective potential with a kind of phase transition which may be responsible for an inflation scenario [7–10].

II. CONFORMALLY INDUCED GRAVITY IN MINIMAL RIEMANN-CARTAN SPACE

In this section we construct a conformally invariant induced gravity action with a torsion field. Let us start

0556-2821/95/51(2)/562(6)/$06.00

from the condition of conformal invariance of the tetrad postulation,

$$D_\alpha e^i_\beta \equiv \partial_\alpha e^i_\beta + \omega^i_{j\alpha} e^j_\beta - \Gamma^\gamma_{\beta\alpha} e^i_\gamma = 0, \tag{4}$$

to find how the connection and the torsion behave under the conformal transformation

$$(e^i_\alpha)' = \exp[\Lambda(x)] e^i_\alpha, \quad (\omega^i_{j\alpha})' = \omega^i_{j\alpha}. \tag{5}$$

We have used Latin indices for the tangent space and Greek indices for the curved space. From the above requirement, the asymmetric affine connection and the torsion which is the antisymmetric part of the connection transform as

$$(\Gamma^\gamma_{\beta\alpha})' = \Gamma^\gamma_{\beta\alpha} + \delta^\gamma_\beta \partial_\alpha \Lambda, \tag{6}$$

$$(T^\gamma_{\beta\alpha})' = T^\gamma_{\beta\alpha} + \delta^\gamma_\beta \partial_\alpha \Lambda - \delta^\gamma_\alpha \partial_\beta \Lambda, \tag{7}$$

$$(T^\gamma_{\gamma\alpha})' = T^\gamma_{\gamma\alpha} + 3\partial_\alpha \Lambda. \tag{8}$$

Therefore, the contracted torsion $T^\gamma_{\gamma\alpha}$ is effectively playing the role of a conformal gauge field. We can separate the torsion into two components:

$$T^\alpha_{\beta\gamma} = A^\alpha_{\beta\gamma} - \delta^\alpha_\gamma S_\beta + \delta^\alpha_\beta S_\gamma, \tag{9}$$

$$(S_\alpha)' = S_\alpha + \partial_\alpha \Lambda, \quad (A^\alpha_{\beta\gamma})' = A^\alpha_{\beta\gamma}. \tag{10}$$

To avoid unnecessary complexity, we adopt the conformally invariant torsionless condition

$$A^\alpha_{\beta\gamma} \equiv 0. \tag{11}$$

Because this condition is the conformally invariant extension of the torsionless condition in Riemann space $T^\alpha_{\beta\gamma} \equiv 0$, we call this space the minimal Riemann-Cartan space. For this space, the affine connection is solved in terms of $g_{\mu\nu}$ and S_α:

$$\Gamma^\alpha_{\beta\gamma} = \{^{\ \alpha}_{\beta\gamma}\} + S^\alpha g_{\beta\gamma} - S_\beta \delta^\alpha_\gamma. \tag{12}$$

Let us define the conformally invariant connection ${\Omega^\alpha}_{\beta\gamma}$:

$$\Gamma^\alpha{}_{\beta\gamma} = \Omega^\alpha{}_{\beta\gamma} + \delta^\alpha_\beta S_\gamma, \tag{13}$$

$$\Omega^\alpha{}_{\beta\gamma} = \{^{\ \alpha}_{\beta\gamma}\} + S^\alpha g_{\beta\gamma} - S_\beta \delta^\alpha_\gamma - S_\gamma \delta^\alpha_\beta. \tag{14}$$

The curvature tensor of the affine connection $\Gamma^\alpha_{\beta\mu}$,

$$R^\alpha{}_{\beta\mu\nu}(\Gamma) = \partial_\mu \Gamma^\alpha_{\beta\nu} - \partial_\nu \Gamma^\alpha_{\beta\mu} + \Gamma^\alpha_{\sigma\mu}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\sigma\nu}\Gamma^\sigma_{\beta\mu}, \tag{15}$$

can be expressed in terms of $\Omega^\alpha{}_{\beta\gamma}$ and S_α using Eq. (13):

$$R^\alpha{}_{\beta\mu\nu}(\Gamma) = R^\alpha{}_{\beta\mu\nu}(\Omega) + \delta^\alpha_\beta H_{\mu\nu}, \tag{16}$$

$$R_{\alpha\nu}(\Gamma) = R_{\alpha\nu}(\Omega) + H_{\alpha\nu}, \tag{17}$$

where $H_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu$ is the conformal gauge field strength. With the help of Eqs. (12) and (16), we obtain

$$\sqrt{g}R(\Omega) = \sqrt{g}R(\{\}) + 6\sqrt{g}(\nabla_\alpha S^\alpha - S_\alpha S^\alpha), \tag{18}$$

where ∇_α is the ordinary covariant derivative in Riemann space.

Under the conformal transformations, the scalar field in four dimensions transforms as

$$\phi'(x) = \exp(-\Lambda)\phi(x). \tag{19}$$

Finally, the conformally invariant Lagrangian function $\sqrt{g}\phi^2 R(\Omega)$ up to total derivatives can be expressed as

$$\sqrt{g}\phi^2 R(\Omega) = \sqrt{g}\phi^2 R(\{\}) - 6\sqrt{g}\phi^2 S_\alpha S^\alpha - 6\sqrt{g} S^\alpha \partial_\alpha \phi^2. \tag{20}$$

Defining the conformally covariant derivative D_α,

$$D_\alpha \phi = \partial_\alpha \phi + S_\alpha \phi, \tag{21}$$

we have the following expression of the conformally invariant induced gravity action in terms of $g_{\alpha\beta}, S_\alpha$, and ϕ:

$$S = \int d^4x \sqrt{g} \left[-\frac{\xi}{2} R(\Omega)\phi^2 + \frac{1}{2} D_\alpha \phi D^\alpha \phi - \frac{1}{4} H_{\alpha\beta} H^{\alpha\beta} - \frac{\lambda}{4!}\phi^4 \right], \tag{22}$$

where we have excluded the curvature squared terms. The parameters ξ and λ are dimensionless constants. Using Eq. (20) we can rewrite this action in terms of the Riemann curvature scalar $R(\{\})$:

$$S = \int d^4x \sqrt{g} \left[-\frac{\xi}{2} R(\{\})\phi^2 + \frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi - \frac{1}{4} H_{\alpha\beta} H^{\alpha\beta} + (1+6\xi) S^\alpha (\partial_\alpha \phi)\phi + \frac{1}{2}(1+6\xi) S_\alpha S^\alpha \phi^2 - \frac{\lambda}{4!}\phi^4 \right]. \tag{23}$$

Here we are interested in the ξ range, $-\frac{1}{6} < \xi < 0$.

III. ONE-LOOP EFFECTIVE POTENTIAL IN de SITTER SPACE

In this section, we have found the one-loop effective potential of the above action in de Sitter space using the background field method and ζ-function regularization in Refs. [11,12].

We consider the quantum fluctuations of the scalar field ϕ and the torsion field S_α, and treat the metric $g_{\mu\nu}$ as a classical background field:

$$g_{\mu\nu} = g^b_{\mu\nu}, \quad S_\mu = \tilde{S}_\mu, \quad \phi = \phi_b + \tilde{\phi}. \tag{24}$$

Let us expand the action Eq. (23) around the background fields; then we have the quadratic action of the quantum fluctuations:

$$I_2 = \int d^4x\sqrt{g}\left[-\frac{1}{2}\xi R(\{\})\tilde{\phi}^2 + \frac{1}{2}\partial_\alpha\tilde{\phi}\partial^\alpha\tilde{\phi} - \frac{\lambda}{4}\phi_b^2\tilde{\phi}^2 - \frac{1}{4}\tilde{H}_{\mu\nu}\tilde{H}^{\mu\nu} - (1+6\xi)\phi_b\nabla_\alpha\tilde{S}^\alpha\tilde{\phi} + \frac{1}{2}(1+6\xi)\tilde{S}_\alpha\tilde{S}^\alpha\phi_b^2\right]. \tag{25}$$

For the sake of convenience, we define a potential $\tilde{V}$ as

$$\tilde{V} \equiv \frac{1}{2}\xi R(\{\})\phi^2 + \frac{\lambda}{4}\phi^4 = \tilde{V}(\phi_b) + \frac{1}{2}\tilde{V}''(\phi_b)\tilde{\phi}^2. \tag{26}$$

By the Hodge decomposition theorem, the one-form S^μ in de Sitter space can be decomposed into two parts, a co-closed form S^μ_T and an exact form S^μ_L, because there is no harmonic one-form:

$$S^\mu = S^\mu_T + S^\mu_L, \tag{27}$$

where the co-closed form S^μ_T satisfies $\nabla_\mu S^\mu_T = 0$, and the exact form S^μ_L can be written as $S^\mu_L = \partial^\mu\chi$ for a function χ. The above decomposition is orthogonal:

$$\int d^4x\sqrt{g}S^\mu_T S^\nu_L = 0. \tag{28}$$

Choosing the gauge-fixing term

$$\triangle I_2 = \frac{1}{2}\alpha\int d^4x\sqrt{g}[\nabla_\mu\tilde{S}^\mu_L + \alpha^{-1}(1+6\xi)\phi_b\tilde{\phi}]^2, \tag{29}$$

and adding the gauge-fixing term to the quadratic action Eq. (25) whose independent quantum fields are $\{\tilde{S}^\mu_L, \tilde{S}^\mu_T, \tilde{\phi}\}$, we have the following gauge-fixed quadratic action of the quantum fluctuations:

$$I_2 + \triangle I_2 = \frac{1}{2}\int d^4x\sqrt{g}\left\{\tilde{S}^\mu_T[-\Box_{\mu\nu} + R_{\mu\nu} + (1+6\xi)\phi_b^2 g_{\mu\nu}]\tilde{S}^\nu_T \right.$$
$$\left. + \alpha\tilde{S}^\mu_L[-\nabla_\mu\nabla_\nu + \alpha^{-1}(1+6\xi)\phi_b^2 g_{\mu\nu}]\tilde{S}^\nu_L + \tilde{\phi}[-\Box - \tilde{V}''(\phi_b) + \alpha^{-1}(1+6\xi)^2\phi_b^2]\tilde{\phi}\right\}. \tag{30}$$

The one-loop generating functional in the Landau gauge in which α goes to the infinity ($\alpha \to \infty$) is

$$Z_1 = \left[\frac{\det\{\mu^{-2}Q\}}{\det\{\mu^{-2}[W + (1+6\xi)\phi_b^2]\}\ \det\{\mu^{-2}[Q - \tilde{V}''(\phi_b)]\}}\right]^{1/2}, \tag{31}$$

where $Q = -\Box$ without a zero mode, $W = -\Box + \frac{\bar{R}}{4}$, μ is a parameter with mass dimension, and $\bar{R}$ is the constant scalar curvature in de Sitter space. We have dropped the spurious zero mode integrations in the path integral because the zero mode of the conformal factor can be absorbed into the fixed constant background of the dilaton field. It can be easily shown that the result Eq. (31) is consistent with the gauge-independent one-loop generating functional in case of $\phi_b = 0$.

The one-loop effective potential for the quantum fluctuations of the torsion vector and the scalar field in the Coleman-Weinberg sector [13] [we assume that λ is of order $(1+6\xi)^2$] can be obtained using ζ-function regularization [12,14]:

$$\tilde{V}_1(\phi_b) = \tilde{V}(\phi_b) + \frac{1}{2\Omega}\ln\det\{\mu^{-2}[W + (1+6\xi)\phi_b^2]\}, \tag{32}$$

where $\Omega = \frac{8\pi^2a^4}{3}$ is the volume of de Sitter space with a radius a.

In the large radius limit $(1+6\xi)a^2\phi_b^2 \gg 1$, the above effective potential becomes

$$\tilde{V}_1(\phi) = \tilde{V}(\phi) + \frac{3(1+6\xi)^2}{64\pi^2}\phi^4\left(\ln\frac{(1+6\xi)\phi^2}{\mu^2} - \frac{3}{2}\right) + \frac{(1+6\xi)}{64\pi^2}\bar{R}\phi^2\left(\ln\frac{(1+6\xi)\phi^2}{\mu^2} - 1\right), \tag{33}$$

where we have dropped the subscript ϕ_b for the sake of convenience. The fact that $\sqrt{g}\tilde{V}_1$ is not invariant under the global scale transformation $\sqrt{g} \to \lambda\sqrt{g}, \phi \to \lambda^{-1/4}\phi$ is a manifestation of the quantum anomaly of scale symmetry. Sticking, nevertheless, to the gauge fixing (29) with $\alpha \to \infty$, we will study the effect of $\tilde{V}_1$ on the inflation scenario.

IV. SEMICLASSICAL EQUATION OF MOTION

In this section we will analyze the semiclassical equation of motion for the scalar field and the metric considering the effective one-loop potential which has been obtained in the previous section. We have found the effective Lagrangian density as follows:

$$\sqrt{g}L_{\text{eff}} = \sqrt{g}[-\tfrac{1}{2}\xi R(\{\})\phi^2 + \tfrac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V_{\text{eff}}(\phi)], \tag{34}$$

where

$$V_{\text{eff}}(\phi) = \frac{\lambda}{4!}\phi^4 + \frac{3(1+6\xi)^2\phi^4}{64\pi^2}\left(\ln\frac{(1+6\xi)\phi^2}{\mu^2} - \frac{3}{2}\right) + \frac{(1+6\xi)}{64\pi^2}\bar{R}\phi^2\left(\ln\frac{(1+6\xi)\phi^2}{\mu^2} - 1\right) + \rho_v. \tag{35}$$

Here we have shifted the vacuum energy by ρ_v which might be attributed to quantum corrections of other fields we have not considered. By varying the action Eq. (34), we get two equations of motion for the scalar field and the metric:

$$\Box\phi + \xi R(\{\})\phi = -\frac{\partial V_{\text{eff}}}{\partial\phi}, \tag{36}$$

$$\xi\phi^2(R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R) = \partial_\mu\phi\partial_\nu\phi - \tfrac{1}{2}g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi - \xi(g_{\mu\nu}\Box\phi^2 - \nabla_\mu\partial_\nu\phi^2) + g_{\mu\nu}V_{\text{eff}}(\phi), \tag{37}$$

where we have not considered the backward contribution of the curvature dependence of the effective potential into the Einstein equation (37).

To investigate the symmetry-breaking equation in this model, let us look for the solution of these equations with $\phi = \sigma = \text{const}$. The scalar equation of motion, Eq. (36), is reduced to

$$\xi R(\{\}) = -\frac{1}{\sigma}\frac{\partial V_{\text{eff}}}{\partial\phi}\bigg|_{\phi=\sigma}. \tag{38}$$

The trace of Einstein, Eq. (37), is

$$-\xi R(\{\})\phi^2 + \partial_\alpha\phi\partial^\alpha\phi - 4V_{\text{eff}} + 3\xi\Box\phi^2 = 0, \tag{39}$$

which implies, for constant $\phi = \sigma$,

$$\xi R(\{\}) = -\frac{4}{\sigma^2}V_{\text{eff}}(\sigma). \tag{40}$$

With the help of Eqs. (38) and (40), we have the symmetry-breaking equation

$$\left(\frac{\partial V_{\text{eff}}}{\partial\phi} - \frac{4V_{\text{eff}}(\phi)}{\phi}\right)\bigg|_{\phi=\sigma} = 0. \tag{41}$$

Therefore, the symmetry-breaking equation for the induced gravity is different from the usual $\frac{\partial V_{\text{eff}}}{\partial\phi}|_{\phi=\sigma} = 0$ in scalar theory with the Einstein-Hilbert action.

Presently, it is assumed that we are in the broken symmetry phase with $\phi = \sigma$. If $V_{\text{eff}}(\sigma) \neq 0$, this uniform background energy density acts like a cosmological constant in the Einstein equation. By the requirement of the vanishing of the cosmological constant in the true vacuum of flat space, the constant part of $V_{\text{eff}}(\sigma, a)$ can be determined:

$$\rho_v = \frac{3(1+6\xi)^2\sigma^4}{128\pi^2}. \tag{42}$$

From Eq. (41), we can express the parameter μ in terms of σ as

$$\ln\frac{(1+6\xi)\sigma^2}{\mu^2} = 1 - \frac{8\pi^2\lambda}{9(1+6\xi)^2}. \tag{43}$$

In de Sitter space, the metric can be written as

$$ds^2 = dt^2 - e^{2Ht}d\vec{x}^2, \tag{44}$$

and the scalar curvature is $\bar{R} = -12H^2$. Using Eqs. (26), (42), and (43) the effective potential Eq. (35) for the dilaton field in de Sitter becomes

$$\tilde{V}_{\text{eff}}(\phi_s) = \frac{3}{64\pi^2}\phi_s^4\left(\ln\phi_s^2 - \frac{1}{2}\right) + \frac{3}{128\pi^2} - \frac{3}{2}H^2\phi_s^2\left(\frac{1}{8\pi^2}\ln\phi_s^2 - \frac{\lambda_s}{9}\right) - \frac{6\xi}{(1+6\xi)}H^2\phi_s^2\,, \tag{45}$$

where, for the sake of convenience, we have defined

$$\phi_s \equiv \sqrt{(1+6\xi)}\phi, \quad \sigma_s \equiv \sqrt{(1+6\xi)}\sigma, \quad \lambda_s \equiv \frac{\lambda}{(1+6\xi)^2}, \tag{46}$$

and chosen the unit $\sigma_s = 1$. This effective potential governs the evolution of the dilaton field in de Sitter space through the equation of motion

$$\Box\phi = -\frac{\partial\tilde{V}_{\text{eff}}(\phi)}{\partial\phi}. \tag{47}$$

It is found that the effective potential (45) shows a phase transition which is sensible for an inflationary scenario. The critical radius $1/H$ of the phase transition has been obtained at $\frac{1}{H} \cong 19$ in the plotting of the one-loop effective potential (45) varying the radius $\frac{1}{H}$ with fixed $\lambda_s = 1.0$ and $(1+6\xi) = 0.1$.

The combination of Eqs. (36) and (39) gives

$$\phi \Box \phi + \partial_\alpha \phi \partial^\alpha \phi + \phi \frac{\partial V_{\text{eff}}}{\partial \phi} - 4V_{\text{eff}} + 3\xi \Box \phi^2 = 0. \quad (48)$$

From the assumption that ϕ is spatially homogeneous, the above Eq. (48) is reduced to

$$(1+6\xi)\left(\ddot{\phi} + 3H\dot{\phi} + \frac{\dot{\phi}^2}{\phi}\right) + \left(V'_{\text{eff}}(\phi) - \frac{4}{\phi}V_{\text{eff}}(\phi)\right) = 0. \quad (49)$$

When $\xi = -\frac{1}{6}$, the induced gravity Lagrangian is consistent only if the form of the effective potential $V_{\text{eff}}(\phi)$ is quartic. Therefore, homogeneous spontaneous symmetry breaking is impossible for $\xi = -\frac{1}{6}$ in de Sitter space. The trace of Einstein, Eq. (39), becomes

$$12\xi H^2\phi^2 + \dot{\phi}^2 + 6\xi(\phi\ddot{\phi} + \dot{\phi}^2 + 3H\dot{\phi}\phi) - 4V_{\text{eff}}(\phi) = 0. \quad (50)$$

We are interested in the inflationary solutions of Eqs. (49) and (50), where the expansion rate H is very large in comparison with other quantities, and the scalar field changes slowly (slow rollover) [15–20]:

$$\left|\frac{\dot{\phi}}{\phi}\right| \ll H, \quad |\ \ddot{\phi}\ | \ll 3H\ |\ \dot{\phi}\ |, \quad \dot{\phi}^2 \ll |\ V_{\text{eff}}(\phi)|\ . \quad (51)$$

In the slow-rolling inflationary regime, Eqs. (49) and (50) are reduced to

$$3H\dot{\phi}_s = \frac{4}{\phi_s}V_{\text{eff}}(\phi_s) - V'_{\text{eff}}(\phi_s)\ , \quad (52)$$

$$H^2 = \frac{(1+6\xi)}{3\xi\phi_s^2}V_{\text{eff}}(\phi_s)\ , \quad (53)$$

where

$$V_{\text{eff}}(\phi_s) = \frac{3}{64\pi^2}\phi_s^4\left(\ln\phi_s^2 - \frac{1}{2}\right) + \frac{3}{128\pi^2} - \frac{3}{2}H^2\phi_s^2\left(\frac{1}{8\pi^2}\ln\phi_s^2 - \frac{\lambda_s}{9}\right)\ . \quad (54)$$

From Eq. (53), it turns out that our space-time is not exactly de Sitter space, but Robertson-Walker space with the metric $ds^2 = dt^2 - a^2(t)d\vec{x}^2$. However, as long as $|\ \dot{H}/H^2\ | \ll 1$, we can use the effective potential (54), which is evaluated in the de Sitter background space-time, as a good approximation. Actually, in the slowly rolling regime, the ratio satisfies

$$\left|\frac{\dot{H}}{H^2}\right| = \sqrt{\frac{3\xi\dot{\phi}^2}{|\ V_{\text{eff}}(\phi)\ |}}\left|\left(\frac{\phi V'_{\text{eff}}(\phi)}{2V_{\text{eff}}(\phi)} - 1\right)\right| \ll 1 \quad (55)$$

due to the condition $|\ \frac{\dot{\phi}^2}{V_{\text{eff}}(\phi)}\ | \ll 1$ in Eq. (51).

In the slow-rolling phase, the contribution from the $\frac{4V_{\text{eff}}(\phi_s)}{\phi_s}$ part of the driving term on the right-hand side of Eq. (52) should be nearly equal to the contribution of the $V'_{\text{eff}}(\phi_s)$ term so that the dilaton field could roll down slowly compared with the expansion rate H. This slow-rolling inflationary phase surely cannot happen at the very center of the potential, but near the origin such that

$$\ln\phi_s^2 \cong 8\pi^2\left(\frac{\lambda_s}{9} - \frac{2\xi}{(1+6\xi)}\right). \quad (56)$$

When the scalar field ϕ_s reaches $\phi_s \cong 1$, it is expected that the dilaton field oscillates about the true vacuum with damping because the dilaton field can be coupled to other matter fields through Yukawa couplings Tr $\bar{\psi}\Gamma(\phi\psi)$). Through this dissipation process, the vacuum energy density of the symmetric phase, $\frac{3\sigma_s^4}{128\pi^2}$, is eventually converted into radiation and matter.

V. CONCLUSION

We have considered that Newton's gravitational constant G_N is generated through spontaneous symmetry breaking of a conformal symmetry. It is possible to formulate the conformally induced gravity in Riemann space. However, spontaneous symmetry breaking via a radiative correction does not work for a scalar field with $\xi = -\frac{1}{6}$. We have extended minimally Riemann space to Riemann-Cartan space to incorporate the torsion vector which is effectively playing the role of a conformal gauge field; then the dimensionless coupling constant ξ is arbitrary. With the introduction of the conformal gauge field, the mechanism of spontaneous symmetry breaking via a radiative correction does work as in the case of the massless scalar electrodynamics. The computation of the one-loop effective potential is performed by ζ-function regularization in de Sitter space. Considering this effective potential, we have analyzed the semiclassical equation of motion of the dilaton field. We will consider the case of a nonvanishing torsion background and will provide a detailed analysis of the effective potential within the context of the inflation scenario later.

ACKNOWLEDGMENTS

This work was supported in part by the Ministry of Education through Grant No. BSRI-93-206/94-2441, the Korea Science and Engineering Foundation (through Grant No. 94-1400-04-01-3 and the Center for Theoretical Physics at SNU), and Hanyang University.

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