Electroweak sphaleron for effective theory in the limit of large Higgs boson mass

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Theoretical arguments suggest that the Higgs sector of the standard model is an effective theory. We parametrize the new physics by means of an effective Lagrangian technique and study its effect on the energy of the electroweak sphaleron. We find that in the presence of a certain class of higher dimension operators the sphaleron energy becomes arbitrarily large as the Higgs boson mass m_H increases. The physical meaning of this result and its implications to electroweak baryogenesis in the dynamical symmetry-breaking models are discussed.

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The energy of the sphaleron in the one-doublet Higgs model, calculated first by Manton and Klinkhamer [1], then improved in [2], is given by $E^{\rm sphal} = \frac{2M_W}{\alpha_W} B(\lambda/g^2)$, where M_W is the W boson mass, $\alpha_W = g^2/4\pi$, and g is the SU(2) coupling constant; B is a function of the Higgs boson mass $m_H^2 = 2\lambda v^2$ with v being the Higgs vacuum expectation value. Thus $E^{\rm sphal}$ will be fixed once the Higgs boson mass is known. Experimentally, data from the CERN e^+e^- collider LEP put a lower limit on m_H, which is about 60 GeV [3]. There are also theoretical arguments which give an upper bound on m_H. For instance, triviality arguments suggest that $m_H < 600$–800 GeV [4]. For this range of m_H, the factor $B(\frac{\lambda}{g^2})$ does not change very much. Actually, the numerical evaluation gives $1.5 \gg B \gg 2.7$ [1,2] when m_H varies from zero to infinity [5]. Thus the weakly interacting theory with a light Higgs boson is indistinguishable from the strongly coupled theory in the infinite Higgs boson mass limit as far as the $E^{\rm sphal}$ is concerned.

There are the theoretical arguments of "triviality" [4] and "naturalness" [6] against the elementary scalar sector of the standard model (SM), and one believes that the Higgs sector of the standard model is an effective theory. In this paper we will study the properties of the sphaleron solution in an effective theory and demonstrate that $E^{\rm sphal}$ will increase without bound as the Higgs boson mass goes to infinity.

First of all, we consider the effective Lagrangian (EL) with a linear realization of the SM gauge symmetry, where the effects of the new physics are parametrized by a set of higher-dimension operators in addition to those present in the SM. Within such an EL, we will look for the sphaleron solution and calculate its energy. In Ref. [7], it was shown that the dimension 6 operators have a very small effect on the $E^{\rm sphal}$. However, starting at dimension 8, there are operators which can make $E^{\rm sphal}$ diverge in the heavy Higgs boson mass limit. The dimension 8 operator in question is

$$\mathcal{O} \sim \frac{1}{\Lambda^4}\{(D_\mu\Phi)^\dagger D^\mu\Phi\}^2 \ . \tag{1}$$

There are other operators with dimension greater than 8, which may also give infinite contributions to the $E^{\rm sphal}$. However, we will concentrate here on the operator $\mathcal{O}$ for a detailed discussion.

First of all, let us write down explicitly the EL relevant to our discussions:

$$\mathcal{L}^{\rm eff} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \tfrac{1}{4}f_{\mu\nu}f^{\mu\nu} + (D_\mu\Phi)^\dagger D^\mu\Phi - V(\Phi) + \frac{1}{\Lambda^4}\{(D_\mu\Phi)^\dagger D^\mu\Phi\}^2 \ , \tag{2a}$$

where $F^a_{\mu\nu}$ are the $\mathrm{SU}_L(2)$ field strength, $f_{\mu\nu}$ the $\mathrm{U}_Y(1)$ field strength, and

$$V(\Phi) = \lambda(\Phi^\dagger\Phi - v^2/2)^2 \ ,$$
$$D_\mu\Phi = \partial_\mu\Phi - i\frac{g}{2}\vec{\tau}\cdot\vec{W}_\mu\Phi - i\frac{g'}{2}\vec{B}_\mu\Phi \ , \tag{2b}$$

where W_μ and B_μ are the gauge fields of $\mathrm{SU}_L(2)$ and $\mathrm{U}_Y(1)$, respectively. In (2a), we have not included the fermion fields since we consider the sphaleron solution in the bosonic sector of the effective theory.

Klinkhamer and Manton [1] have shown that in the SM there is a saddle-point solution, which is the sphaleron. Since the higher-dimension operator $\mathcal{O}$ does not affect the symmetry-breaking pattern of the minimal model, nor change the topology of the field configuration space, the sphaleron solution should exist in $\mathcal{L}^{\rm eff}$ (2a).

Following the usual procedure we consider the static solution by setting the time components of the gauge fields to zero. Also, as is generally done, we will set the Weinberg angle to zero, so that the $\mathrm{U}_Y(1)$ gauge field is decoupled and may be consistently set to zero. Following Ref. [1], we use the spherical symmetric ansatz for the gauge field $\vec{W}_\mu$ and the Higgs field Φ:

$$W^a_i\sigma^a dx^i = -\frac{2i}{g}f(\xi)dU^\infty(U^\infty)^{-1} \ , \tag{3a}$$

$$\Phi = \frac{v}{\sqrt{2}}h(\xi)U^\infty\begin{pmatrix}0\\1\end{pmatrix} \ , \tag{3b}$$

where $\xi = gvr$ and

$$U^\infty = \frac{1}{r}\begin{pmatrix} z & x+iy \\ -x+iy & z\end{pmatrix} \ .$$

The energy functional is given by

$$E^{\rm sphal}=\frac{4\pi v}{g}\int_0^\infty d\xi\left\{4\left(\frac{df}{d\xi}\right)^2+\frac{8}{\xi^2}f^2(1-f)^2+\tfrac{1}{2}\xi^2\left(\frac{dh}{d\xi}\right)^2+h^2(1-f)^2+\frac{1}{4}\frac{\lambda}{g^2}\xi^2(h^2-1)^2\right.$$
$$\left.+c\left[\xi^2\left(\frac{dh}{d\xi}\right)^4+4h^2\left(\frac{dh}{d\xi}\right)^2(1-f)^2+4\frac{h^4}{\xi^2}(1-f)^4\right]\right\}, \tag{4}$$

where $c\equiv(\frac{g^2}{4})\frac{v^4}{\Lambda^4}$. It is straightforward to derive the Euler-Lagrange equation for this energy functional. They read

$$\xi^2\frac{d^2}{d\xi^2}=2f(1-f)(1-2f)-\frac{\xi^2}{4}h^2(1-f)-c\left\{\xi^2h^4\left(\frac{dh}{d\xi}\right)^2(1-f)+2h^4(1-f)^3\right\}, \tag{5a}$$

$$\frac{d}{d\xi}\left(\xi^2\frac{dh}{d\xi}\right)=2h(1-f)^2+\left(\frac{\lambda}{g^2}\right)\xi^2(h^2-1)h+8c\left[h\left(\frac{dh}{d\xi}\right)^2(1-f)^2+2\frac{h^3}{\xi^2}(1-f)^4\right]$$
$$-4c\left[\frac{d}{d\xi}\left(\xi^2\left(\frac{dh}{d\xi}\right)^3\right)+2\frac{d}{d\xi}\left(\frac{dh}{d\xi}h^2(1-f)^2\right)\right]. \tag{5b}$$

The boundary conditions for $f(\xi)$ and $h(\xi)$ are given by

$$f(\xi)\to\xi^2 \text{ and } h(\xi)\to\xi \text{ for } \xi\to 0, \tag{6a}$$

$$f(\xi) \text{ and } h(\xi)\to 1 \text{ for } \xi\to\infty. \tag{6b}$$

Note that these are the same boundary conditions as those [1] in the absence of the higher-dimension operator $\mathcal{O}$. We numerically integrate Eq. (4) by minimizing $E^{\rm sphal}$ for a given value of the parameter $\frac{\lambda}{g^2}$ in the range of zero to 10^{10} [8]. In Fig. 1 we plot the $E^{\rm sphal}$ of the present case together with that of the standard model. One can see that for small $\frac{\lambda}{g^2}$ the new physics effect on $E^{\rm sphal}$ is negligible and the two energies are practically indistinguishable. However, it becomes important for larger $\frac{\lambda}{g^2}$. For instance, for $\frac{\lambda}{g^2}>10^5$, the sphaleron energy has exceeded the maximal value of the SM sphaleron. From our numerical results we can extract the behavior of the sphaleron energy in the larger Higgs boson mass limit. Approximately, we have $E^{\rm sphal}\sim(\frac{\lambda}{g^2})^{1/4}$ for $\frac{\lambda}{g^2}>10^5$. This behavior can also be understood by using a particularly simple ansatz, considered by Manton and Klinkhamer [1], for the radial functions $f(\xi)$ and $h(\xi)$:

$$f(\xi)=\begin{cases}(\frac{\xi}{\Xi})^2, & \xi\le\Xi,\\ 1, & \xi\ge\Xi,\end{cases} \tag{7a}$$

$$h(\xi)=\begin{cases}\frac{\xi}{\Omega}, & \xi\le\Omega,\\ 1, & \xi\ge\Omega,\end{cases} \tag{7b}$$

where Ω and Ξ are two variational scale parameters. With such an ansatz, the dominant terms in $E^{\rm sphal}$ for very large $\frac{\lambda}{g^2}$ are given by

$$E^{\rm sphal}\sim\frac{4\pi v}{g}\frac{1}{210}\left\{4\left(\frac{\lambda}{g^2}\right)\Omega^3+210c\frac{97}{15}\frac{1}{\Omega}\right\}. \tag{8}$$

The scale parameter Ω that minimizes (8) goes like $(\frac{\lambda}{g^2})^{-1/4}$, which gives

$$E^{\rm sphal}\sim\left(\frac{\lambda}{g^2}\right)^{1/4}. \tag{9}$$

One remark here is that the blowup of the sphaleron energy is caused technically by the "frozen" Higgs field $h(\xi)\equiv 1$. To see this clearly, let us rewrite $E^{\rm sphal}$ in terms of Ω. We have $E^{\rm sphal}\sim 1/\Omega$. From Eq. (7b), $\Omega\to 0$ means that $h(\xi)\to 1$. Because of the triviality of the Higgs sector, however, the physical meaning of the $m_H\to\infty$ limit is questionable. In the following we will argue that our results indicate that the $E^{\rm sphal}$ is infinity in the nonlinear EL of the dynamical symmetry breaking (DSB) models.

To proceed with the discussion of the sphaleron in the

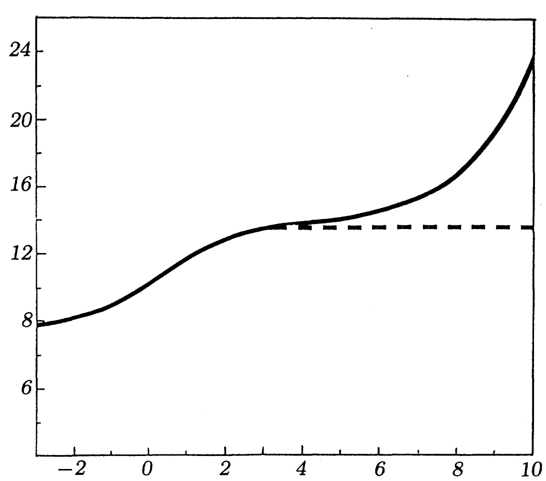

FIG. 1. Sphaleron energy as function of λ/g^2. The vertical axis is the sphaleron energy in units of TeV and the horizontal axis is $\log_{10}(\lambda/g^2)$. The solid curve is for the present case and the dash curve for the standard model. In our numerical calculation we take $\Lambda=1$ TeV. A different value of Λ will change the absolute value of the $E^{\rm sphal}$, but will not modify the behavior of the $E^{\rm sphal}$ in the large Higgs boson mass limit.

effective theory of a DSB model, let us consider a limit $\lambda \to \infty$ in $\mathcal{L}_{\rm eff}$. In this limit, the Higgs field Φ can be parametrized as $\Phi = \frac{v}{\sqrt{2}}\Sigma\binom{0}{1}$, where Σ is a 2×2 matrix for the Goldstone boson fields. Thus $\mathcal{L}_{\rm eff}$ in (2a) becomes

$$\mathcal{L}_\sigma = \tfrac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{v^2}{4}{\rm Tr}D_\mu\Sigma^\dagger D^\mu\Sigma + \frac{v^4}{4^2\Lambda^4}({\rm Tr}D_\mu\Sigma^\dagger D^\mu\Sigma)^2 \ , \tag{10}$$

where we have neglected the $\mathrm{U}_Y(1)$ field. The $\mathcal{L}_\sigma$ has the form of the gauged nonlinear σ models and will describe the low-energy physics of dynamical electroweak symmetry-breaking models.

The sphaleron solution of $\mathcal{L}_\sigma$ [9] has been considered by Klinkhamer and Boguta [10]. They use the following spherical symmetric ansatz, which is a gauge transformation of that in Eq. (3):

$$\Sigma = 1 \ , \quad \frac{g}{2}W_i = \frac{1-f}{r^2}(\vec{r}\times\vec{\tau})_i \ , \tag{11}$$

with the boundary conditions $f(0) = 0$ and $f(\infty) = 1$. The sphaleron energy functional is

$$E^{\rm sphal}_\sigma = \frac{4\pi v}{g}\int d\xi\left\{4\left(\frac{df}{d\xi}\right)^2 + \frac{8}{\xi^2}f^2(1-f)^2 + (1-f)^2 + (4c)\frac{(1-f)^4}{\xi^2}\right\} , \tag{12}$$

where $c = \frac{g^2}{4}\frac{v^4}{\Lambda^4}$. Klinkhamer and Boguta noticed that the $E^{\rm sphal}_\sigma$ blows up [11] because of the last term. Actually, the sphaleron energy density is singular and the boundary condition is not satisfied by the differential equation for $f(\xi)$, which minimizes $E^{\rm sphal}_\sigma$.

We realize that the singularity in $E^{\rm sphal}_\sigma$ is removable once the sphaleron solution of $\mathcal{L}_\sigma$ is understood as the sphaleron solution of $\mathcal{L}^{\rm eff}$ (2a) in the limit $\frac{\lambda}{g^2} \to \infty$. In fact, in the limit $\frac{\lambda}{g^2} \to \infty$, $h(\xi) \to 1$, then $\mathcal{L}^{\rm eff}$ goes to $\mathcal{L}_\sigma$ and $E^{\rm sphal} \to E^{\rm sphal}_\sigma$. Thus in the nonlinear EL $\mathcal{L}_\sigma$, the sphaleron energy is indeed divergent. To illustrate the physical meaning of this result, we consider a toy DSB model. This is the one family standard model, but no elementary Higgs field is introduced. In this model the SM gauge symmetry is broken by the quark condensate driven by the QCD interaction, where both the electroweak symmetry-breaking scale and the weak gauge boson masses are, of course, very low.

It is well known that the strong interaction of a DSB model can be described by an EL similar to $\mathcal{L}_\sigma$, when the heavy fermions, i.e., the quarks, are frozen out at an energy below the DSB strong interaction scale. Below the DSB strong interaction scale, there are only leptons in the fermion sector, and the lepton number current is violated by an SU(2) anomaly which involves two SU(2) currents. However, its amplitude will vanish according to our results since the sphaleron energy is infinity.

Is the above reasonable? What is its significance? In the fundamental Lagrangian of quarks and leptons, there are two kind of fermion number currents: one lepton number, and the other baryon number. Both have an SU(2) anomaly; however, their difference is anomaly-free, which means that the total change of the lepton number must equal to that of the baryon number. Since baryon fields do not exist in the low-energy Lagrangian lepton number violations processes are forbidden. In other words, the sphaleron energy should be infinity.

Our results have direct implications on the electroweak baryogenesis. As argued in Ref. [12], to avoid the washout of the baryon asymmetry, the following condition is needed:

$$\frac{E^{\rm sphal}(T_t)}{T_t} > 45 \ , \tag{13}$$

where T_t denotes the electroweak phase transition temperature. In computing $E^{\rm sphal}(T_t)$, one should use the full effective action, which includes terms depending on derivatives, denoted by D_T, and terms independent of derivatives, which are generally known as the effective potential, denoted as V_T. The V_T in the standard model and its extensions has been studied in detail in recent years [13]. And the sphaleron solution with a temperature-dependent V_T has also been studied by Braibant, Brihaye, and Kunz in Ref. [14]. They concluded that for the temperature dependent V_T in the absence of a cubic term, the sphaleron energy as a function of the temperature is given by

$$E^{\rm sphal}(T) = E^{\rm sphal}(T=0)\frac{v(T)}{v} \ , \tag{14}$$

where $v(T)$ is the vacuum expectation value of the Higgs field at the temperature T. With a cubic term in the V_T, Eq. (14) has to be modified, but it remains a rather good approximation [14]. The finite temperature correction to D_T in the standard model has been considered by Dine, Huet, and Singleton in Ref. [15]. They estimated the contribution of several typical Feynman diagrams and argued that the correction to $E^{\rm sphal}(T)$ is about 20%.

In the effective theory we consider in this paper, the higher-dimension operators will contribute to V_T as well as D_T. As a result, the SM Higgs boson mass limit required by electroweak baryogenesis will be changed. Assuming the validity of Eq. (15), the higher-dimension operators can alter this upper limit for the Higgs boson mass in two ways: (1) it changes the usual relation between $v(T)$ and the Higgs boson mass; (2) it changes the dependence of the $E^{\rm sphal}(T=0)$ on the Higgs boson mass. For small Higgs boson mass, the impact of the higher-dimension operator on $E^{\rm sphal}(T=0)$ is negligible. However, the relation between $v(T)$ and m_H is changed due to a dimension 6 operator in V_T [16]. As a result, the Higgs mass limit can be relaxed to the experimental allowed region. For a large Higgs boson mass, as discussed in this paper, the presence of higher-dimensional operators will change $E^{\rm sphal}(T=0)$ significantly, and will relax the Higgs boson mass limit further. In particular, in the dynamical symmetry breaking theory, if the physics in the true vacuum can be described by an EL $\mathcal{L}_\sigma$ with temperature dependent $v(T)$, which is now the composite Goldstone boson decay constant, it will help to avoid the washout of the baryon asymmetry because of the infinity sphaleron energy [17]. Certainly, to construct a successful DSB model for the electroweak baryogenesis, one must examine in detail the phase transition and the source of CP violation.

In summary, we have taken the standard model Higgs sector as an effective theory, then studied its sphaleron solution and calculated the sphaleron energy in the presence of a dimension 8 operator. We found that the sphaleron energy diverges in the large Higgs boson mass limit. This implies that the sphaleron in $\mathcal{L}_\sigma$ of the DSB models has an infinity energy and will be helpful in preventing washout of the baryon number asymmetry.

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[9] Since this is an effective theory with cutoff $\Lambda \sim 1$ TeV, which is less than the mass of the sphaleron, one may wonder about the reliability of our results. However, in determining the region of the applicability of the effective theory, it is the size (inverse momentum of the typical fluctuation), not the mass of the sphaleron, that is relevant. The size of the sphaleron is of the order of $1/M_W$. So our approach is valid since $\Lambda \gg M_W$.

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