

Top quark as a topological color soliton

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In this paper I propose a scenario for the top quark as a topological color soliton. I illustrate some features of the top quark as a soliton in a toy model based on a nonlinear realization of $SU_L(3) \times SU_R(3)/SU_{L+R}(3)$ with $SU_{L+R}(3)$ being the color gauge group.

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Recently the Collider Detector at Fermilab (CDF) collaboration [1] presented evidence for the top quark with a mass $m_t \sim 174$ GeV. Since m_t is of order of the Fermi scale, the top quark is expected to hold the clue to the physics of electroweak symmetry breaking [2]. The scale associated with top quark mass generation can be estimated by imposing unitarity on the processes $t\bar{t} \rightarrow V_L V_L$ [3], where V_L is a longitudinal W^\pm or Z . In the absence of the Higgs boson, such an amplitude is proportional to $G_F m_t \sqrt{s}$ and will violate unitarity at high energy Λ . The largest zeroth-partial-wave amplitude is the color-singlet, spin-zero combination of $t\bar{t}$ and the zero-isospin combination of W_L^+ , W_L^- and Z_L , Z_L , from which one has [4]

$$\Lambda \leq \frac{4\sqrt{2}\pi}{3G_F m_t} \sim 2.9 \text{ TeV} . \quad (1)$$

Since this energy scale is around $4\pi v \sim 3.1$ TeV [5,6], the scale of dynamical electroweak symmetry breaking, it suggests a possible connection between the top quark mass and the Fermi scale.

One interesting observation, based on the fact that the top quark Yukawa coupling is of order 1, is that the top quark may be compared to the constituent quarks in QCD, whose mass ratio to the pion decay constant is also a number of order unity. Since the constituent quarks get masses dynamically, one would expect logically that the top quark also gets its mass dynamically.

The above argument not only makes a heavy top quark natural, but also results in a lot of interesting phenomena associated with the top quark. As an example, one expects that the axial vector coupling constant of the top quark to the neutral gauge boson, g_A , will deviate from the standard model because a similar quantity for the constituent quark, G_A , is renormalized [6]. These predictions can be tested in future colliders. And if found experimentally to be true, it will provide a real clue to the mechanism of the breakdown in the electroweak theory. Note that in QCD chiral symmetry is broken dynamically by quark condensation. We should mention that if the top quark has different properties from the standard model prediction, it will affect experimental quantities associated with the bottom quark. Indeed, an anomalous Δg_A at the level of the order of $\Delta G_A \simeq 0.25$ for the constituent quarks can make the discrepancy of the $Z \rightarrow b\bar{b}$ width measured at the CERN e^+e^- collider LEP with the standard model expectation disappear [7].

Furthermore, the constituent quarks are argued to have a nontrivial structure [8]: i.e., consisting of a valence quark plus many quark-antiquark pairs. Kaplan has shown that the constituent quarks can be considered as color solitons (Kaplan names them "qualitons") [9]. In this picture, the constituent quark Q would look like a current quark q carrying around with it a significant deformation of the $\langle \bar{q}q \rangle$ background. The spin, color, and baryon number of the constituent quark are topologically induced and not localized at a point.

In this Brief Report, we suggest that the top quark is a soliton. For simplicity, we consider here the basic properties of the soliton, such as mass, radius, baryon number, and its color and spin representation¹ The model we use for the top quark soliton² is based on a nonlinear realization of the $SU_L(3) \times SU_R(3)/SU_{L+R}(3)$. The unbroken group $SU_{L+R}(3)$ is the color gauge group $SU_c(3)$, and so the Goldstone bosons π^a are in the adjoint representation of $SU_c(3)$. The effective action describing the soliton solution is given by

$$\begin{aligned} S = & \frac{F_\pi^2}{16} \int d^4x \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) \\ & + \frac{1}{32e^2} \int d^4x \text{Tr}([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger])^2 \\ & + n \Gamma_{WZ} \\ & + \frac{m_\pi^2 F_\pi^2}{8} \int d^4x \text{Tr}(U - 3) , \end{aligned} \quad (2)$$

where $U = \exp(2i\lambda^a \pi^a / F_\pi)$ and m_π is the Goldstone boson mass. It should be pointed out that we have neglected in (2) the operators with more powers of derivatives suppressed by powers of the symmetry-breaking scale $\sim 4\pi v \sim 3.1$ TeV. The colored Goldstone bosons π^a can decay into two gluons and the top quark pairs if $m_\pi > 2m_t$. In this paper, we will not discuss the phenomena associated with the π^a fields; instead, we take the effective action in (2) as a toy model to illustrate some features of the top quark as a soliton.

The action in (2) has the same form as that for an ordinary $SU(3)$ Skyrmion [11]. However, the parame-

¹The flavor quantum number of the soliton in the coset space $SU(2) \times U(1)/U(1)$ has been worked out by Arnade and Bagger [10].

²This model is similar to that considered by Kaplan [9] for qualitons.

ters are quantitatively different. In the original SU(3) Skyrminion model, $F_\pi \sim 100$ MeV and $n = N_c = 3$, but here $F_\pi \sim v \sim 100$ GeV and $n = 1$. Thus the soliton will be much heavier than the ordinary Skyrminion and its quantum numbers will also be completely different from that of the SU(3) Skyrminion. In the Skyrminion model, the baryon number of the soliton is 1, and the lowest baryonic state is in the octet representation of $SU_{L+R}(3)$. Instead, we have here that the baryon number is $\frac{1}{3}$, and the lowest baryonic state is in the fundamental representation of $SU_{L+R}(3)$ (for a detailed discussion, see Ref. [9]). Thus the soliton carries the quantum number of the top quark.³

The technique of studying the properties of the soliton is well known. First, take the ‘‘hedgehog’’ ansatz for U :

$$U_0 = \exp\{iF(r)\hat{\mathbf{r}} \cdot \boldsymbol{\tau}\}, \quad (3)$$

where the τ^i are the Pauli matrices embedden in the color $SU_{L+R}(3)$. In this ansatz,

$$F(0) = \pi, \quad F(\infty) = 0. \quad (4)$$

The energy of the solution (3) is given by

$$M = M_{\text{cl}}[F] + m_{\text{cl}}[F], \quad (5)$$

where

$$M_{\text{cl}}[F] = 4\pi \frac{F_\pi}{e} \int_0^\infty dx \left[\frac{x^2}{8} \left(\frac{dF}{dx} \right)^2 + \frac{\sin^2 F}{4} + \frac{\sin^4 F}{2x^2} + \left(\frac{dF}{dx} \right)^2 \right], \quad (6a)$$

$$m_{\text{cl}}[F] = \frac{m_\pi^2 \pi}{e^3 F_\pi} \int_0^\infty x^2 dx [1 - \cos F], \quad (6b)$$

where $x = eF_\pi r$.

The soliton state can be constructed by using the standard quantization method [13]. For our purpose, we just give the expression of the Hamiltonian for the soliton:

$$H = M + \frac{1}{2} \left(\frac{1}{I_A} - \frac{1}{I_B} \right) j(j+1) + \frac{1}{2I_B} \left(C_2 - \frac{1}{12} \right), \quad (7)$$

where j is the spin and C_2 is the color $SU_c(3)$ Casimir operator. In (7),

$$I_A = \frac{2\pi}{3e^3 F_\pi} \int_0^\infty dx \sin^2 F \left\{ x^2 + 4 \left[\sin^2 F + x^2 \left(\frac{dF}{dx} \right)^2 \right] \right\}, \quad (8a)$$

$$I_B = \frac{\pi}{e^3 F_\pi} \int_0^\infty dx \sin^2 \frac{F}{2} \left[x^2 + 2 \sin^2 F + x^2 \left(\frac{dF}{dx} \right)^2 \right]. \quad (8b)$$

For a spin- $\frac{1}{2}$, a color-triplet soliton, $j = \frac{1}{2}$ and $C_2 = \frac{4}{3}$. Then the mass of the soliton is given by

$$M_{(\text{soliton})} = M + \frac{3}{8I_A} + \frac{1}{4I_B}. \quad (9)$$

The isoscalar radius of the soliton is given by

$$r_0 = \langle r^2 \rangle_{I=0}^{1/2} = \frac{1}{eF_\pi} \left\{ \frac{-2}{\pi} \int_0^\infty dx x^2 \sin^2 F \left(\frac{dF}{dx} \right) \right\}^{1/2}. \quad (10)$$

Now let us calculate $M_{(\text{soliton})}$ and r_0 numerically. We take a variational approach in Ref. [14] and assume that $F(r)$ has the form

$$F(x) = 2 \arctan \left[\left(\frac{x_0}{x} \right)^2 \right], \quad (11)$$

where x_0 , the soliton size, is the variational parameter. For simplicity, we consider first the case where $m_\pi = 0$; then

$$M = \frac{F_\pi}{e} \pi^2 \frac{3\sqrt{2}}{16} \left(4x_0 + \frac{15}{x_0} \right), \quad (12a)$$

$$I_A = \frac{1}{e^3 F_\pi} \pi^2 \frac{\sqrt{2}}{12} (6x_0^3 + 25x_0), \quad (12b)$$

$$I_B = \frac{1}{e^3 F_\pi} \pi^2 \frac{\sqrt{2}}{16} (4x_0^3 + 9x_0). \quad (12c)$$

The x_0 can be estimated by minimizing (12a) with respect to x_0 , which gives $x_0 = \sqrt{15/4}$. Then we have

$$M_{(\text{soliton})} \simeq F_\pi \left(\frac{40.544}{e} + 0.007e^3 \right) \quad (13)$$

and

$$r_0 \simeq \frac{2.19}{eF_\pi}. \quad (14)$$

Following Ref. [15], let us consider a quantity $M_{(\text{soliton})}r_0$:

$$M_{(\text{soliton})}r_0 \simeq 2.19 \left(\frac{40.544}{e^2} + 0.007e^2 \right). \quad (15)$$

Since $M_{(\text{soliton})}r_0$ depends only on one parameter e , we would be able to find out a possible minimum value of the $M_{(\text{soliton})}r_0$ by minimizing (15) with respect to e . It gives that

$$M_{(\text{soliton})}r_0 \geq 2.333, \quad (16)$$

which is comparable to 2.52 [15], Keaton’s numerical value. One can see that for a top quark $m_t \sim 174$ GeV, we have $r_0 \sim 1/v$. It should be pointed out that the minimum value of Eq. (16) corresponds to $e^2 = 76.11$.

³The soliton considered by D’Hoker and Farhi [12] in the effective action generated by integrating out the heavy quark of the standard model is a color neutral object with integer baryon number.

In such a situation the quantum and the classical mass terms in Eq. (13) are exactly equal and the collective quantization, as carried out for the soliton here, is not well justified. So a smaller e^2 should be chosen. Consequently, $M_{(\text{soliton})r_0}$ is expected to be larger than 2.333. For example, if one takes $e^2 = 50$, the quantum mass is only about half of the classical mass and now $M_{(\text{soliton})r_0}$ is about 2.542, which is slightly larger than the minimum value 2.333.

In summary, we have shown that top quark can be described by a soliton. The spin, color, and baryon number of the top quark are topologically induced and its radius is of order $\sim 1/v$. The top quark field is not localized at a point and will have color form factors. These new physics effects may show up experimentally as excessive or anomalous top quark production rates and distributions at the hadron colliders [16].

Before concluding, we would like to point out that in order to construct the color topological solitons, we have made use of the chiral color symmetry $SU_L(3) \times SU_R(3)$.

Whether it exists or not depends on the dynamics of the fundamental theory of the top quark. As an existence proof, let us consider the gauge version of the top quark condensation theory [17], where new strong physics of the top quark at the ~ 1 TeV scale is introduced [18]. In a specific model proposed by Lindner and Ross [19], the four-fermion interaction responsible for the top quark condensation is

$$\mathcal{L}_{\text{eff}} \sim \bar{T}_L \gamma_\mu T_L \bar{t}_R \gamma^\mu t_R, \quad (17)$$

where T_L stands for the left-handed top and bottom quark doublet and t_R the right-handed top quark. In (17) chiral color symmetry does exist [20].

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