Squark and slepton mass relations in grand unified theories

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In the minimal supersymmetric standard model, assuming universal scalar masses at large energies, there are four intragenerational relations between the masses of the squarks and sleptons for each light generation. In this paper we study the scalar mass relations which follow only from the assumption that at large energies there is a grand unified theory which leads to a significant prediction of the weak mixing angle. Two new intragenerational mass relations for each of the light generations are derived. In addition, a third mass relation is found which relates the Higgs boson masses, the masses of the third generation scalars, and the masses of the scalars of the lighter generations. Verification of a fourth mass relation, involving only the charged slepton masses, provides a signal for SO(10) unification.

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I. INTRODUCTION

If supersymmetry is a symmetry of nature, broken only at the weak scale, then future experiments will discover many extra particles, in particular the superpartners of all the quarks and leptons. The masses of these scalar quarks and leptons will provide extra clues about a more fundamental theory at higher energies. However, whereas the quark and lepton masses provide information on how chiral and flavor symmetries are broken, the squark and slepton masses will provide a window to the structure of supersymmetry breaking.

It may be that the squark and slepton spectrum will show no clear pattern or regularities, and the origin of the spectrum will become a major puzzle, rather like the present situation with quark and lepton masses. However, much attention has been focused on a single theory, the minimal supersymmetric standard model (MSSM), in which a very clear pattern emerges in the scalar spectrum. By the MSSM we will mean the supersymmetric extension of the standard model with minimal field content, which has a boundary condition near the Planck scale that the soft supersymmetry-breaking mass parameters for the scalars are all equal. In this model, the physical masses of the 14 squarks and sleptons of the lighter two generations are given in terms of just five unknown parameters: the universal scalar masses at the Planck scale, m_0^2, the three gaugino masses, M_a, and the ratio of electroweak breaking vacuum expectation values (VEV's), $\tan\beta = v_2/v_1$. Due to effects of large Yukawa couplings, the physical squark and slepton masses of the heaviest generation depend on one further parameter, A. Although these effects are well understood and can easily be added, for simplicity, we consider only the lightest two generations. Thus the MSSM has many relations among the scalar masses. However, the question as to why all scalars are assumed degenerate at the Planck scale becomes extremely important. If experiments are done to check the validity of the scalar mass relations of the MSSM [1], what is the fundamental principle which is being tested?

Flavor-changing processes provide considerable experimental constraints on the form of the squark and slepton mass matrices [2,3]. However, these constraints are intimately connected with flavor violation and provide constraints between the masses of scalars of different generations. For a given generation there are five independent gauge-invariant squark and slepton masses: m_Q, m_{U^c}, m_{D^c}, m_L, and m_{E^c}, where Q and L represent SU(2) doublet squarks and sleptons, while U, D, and E are SU(2) singlet squarks and sleptons. Certainly the flavor-changing constraints do not constrain the ratios $m_Q{:}m_U{:}m_D{:}m_L{:}m_E$, and it is largely these ratios which will be addressed in this paper.

The assumption of a universal scalar mass at high energies originated from studies of $N = 1$ supergravity theories in which supersymmetry is broken in a hidden sector. The scalar mass was found to be universal in particular models [4,5] and also in a wide class of models [6]. However, the universal mass is not a general property of supergravity models, and involves an assumption about the form of the Kähler potential. If there are N fields in the observable sector of the theory, an SU(N) invariance of the Kähler potential guarantees the universality of the scalar masses at the Planck scale [6]. However, this symmetry is clearly broken elsewhere in the theory, and so the universality of the scalar masses can only be understood as a special property of certain supergravity theories. If the scalar mass relations of the MSSM were violated, it might simply mean that the Kähler potential does not possess this SU(N) invariance.

In this paper we study squark and slepton mass relations which follow from two assumptions, which have nothing to do with supergravity.

(1) The standard model is unified into a grand-unified theory (GUT). It is well known that a grand-unified symmetry, together with supersymmetry, has yielded a successful relation amongst the gauge couplings of the stan-

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dard model [7]. Much attention has also been given to quark and lepton mass relations which can follow from a grand-unified symmetry. It therefore seems well worthwhile studying what squark and slepton mass relations might follow purely from grand unification.

(2) The generation-changing entries in the squark and slepton masses (in a basis where the quark and lepton masses are diagonal) are sufficiently small not to affect the scalar mass eigenvalues at a level of accuracy to which the mass relations will be experimentally tested.

In fact, the latter is hardly an assumption, such large flavor-changing effects are almost certainly experimentally excluded. Since the grand-unified symmetry acts within a generation, we expect relations amongst squark and slepton masses of the same generation, we do not expect any relations between masses of particles in different generations.

We begin Sec. II by writing down the mass relations between squarks and sleptons of a given generation which occur in the MSSM. We then list the assumptions which a supersymmetric grand-unified theory (SGUT) must satisfy for a successful weak mixing angle prediction to occur at the 1% level. Finally, we show that, with these assumptions, we are able to derive two intrageneration scalar mass relations. The mass relation of the MSSM which relates the masses of the two charged sleptons within a generation may be violated. This is a particularly important mass relation since it is likely that the squarks will be much heavier than the sleptons, and this will be the first mass relation of the MSSM to be tested. In Sec. III, we study the extent to which this mass relation is expected to follow if the GUT gauge groups include SO(10). While this slepton mass relation is generically expected as a consequence of the SO(10) gauge symmetry, we find that radiative corrections and additional D-term contributions to the scalar masses, beyond those of the MSSM, may lead to its violation. In Sec. IV we show that even if the additional D-term contributions do not arise at tree level, they could be generated by radiative corrections. In Sec. V, we show that these extra D^2 interactions found in SO(10) could lead to an easing of the fine-tuning problem which has been found when the MSSM has large $\tan\beta$ and the universal scalar mass boundary condition.

II. SCALAR MASS RELATIONS IN A CLASS OF GRAND-UNIFIED THEORIES

Before studying grand-unified theories, we give the well-known predictions for the scalar masses in the MSSM, taken to have universal scalar masses m_0^2 at the Planck scale. Mass splittings arise from renormalization-group scaling from Planck to weak scales [8], and the renormalization-group equations are given by

$$\frac{d}{d\ln\mu}m_i^2(\mu) = \frac{1}{16\pi^2}\Bigg(-8C_2(R_a^i)g_a^2(\mu)M_a^2(\mu) + \tfrac{6}{5}Y_i g_1^2(\mu)S(\mu) + \sum_{j,k}|\lambda_{ijk}|^2(m_i^2+m_j^2 + m_k^2 + A_{ijk}^2)\Bigg) , \tag{2.1}$$

$$\frac{d}{d\ln\mu}S(\mu) = \frac{b_1}{2\pi}\alpha_1(\mu)S(\mu) , \tag{2.2}$$

$$S(\mu) = \sum_i Y_i m_i^2(\mu) , \tag{2.3}$$

where $a = 1,2,3$ represents $\mathrm{U}(1)_Y$, $\mathrm{SU}(2)_L$, and $\mathrm{SU}(3)_c$,[1] i represents the species of the scalar and Y_i is the corresponding hypercharge, A_{ijk}'s are the soft SUSY-breaking trilinear scalar couplings, and λ_{ijk}'s are the superpotential couplings. $C_2(R_a^i)$ is the second Casimir invariant of the gauge group a for the species i, $C_2 = (N^2-1)/2N$ for the fundamental representation of $\mathrm{SU}(N)$, $\frac{3}{5}Y_i^2$ for $\mathrm{U}(1)_Y$. The S term is zero under the assumption of universal scalar masses and hence does not contribute. For the lightest two generations, whose superpotential coupling contributions are negligible, the mass splittings involve only contributions from the gauginos, which have masses M_{0a} at the Planck scale. Mass splittings also arise from the D^2 terms of the potential due to $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ interactions. These are proportional to $M_Z^2\cos 2\beta$. The result is

$$m_i^2(\mu) = m_0^2 + \sum_a f_{ai}M_{0a}^2 + (T_{3_i} - Q_i\sin^2\theta_W)M_Z^2\cos 2\beta , \tag{2.4}$$

where i runs over the seven types of squark and slepton: U, D, U^c, D^c, E, N, and E^c, and it is understood that the two light generations have identical scalar spectra. The renormalization constants f_{ai} are

$$f_{ai}(\mu) = \frac{2}{b_a}C_2(R_a^i)\left(\frac{\alpha_a^2(\mu)}{\alpha_a^2(M_p)} - 1\right) , \tag{2.5}$$

where b_a is the one-loop β-function coefficient, and μ should be taken equal to the scalar mass, m_i.

Suppose that β is known, for example, from a Higgs boson mass measurement, then the seven values of m_i^2 depend only on four unknown parameters, m_0 and M_{0a} yielding three intragenerational mass relations for the MSSM [9]. Two further relations follow if M_{0a} is independent of a. In the following the scalar masses are scaled to the same renormalization point so that these mass relations can be displayed in simpler forms.

Two of these relations have only to do with SU(2) breaking and are

$$m_U^2 - m_D^2 = m_N^2 - m_E^2 = M_Z^2\cos 2\beta\cos^2\theta_W . \tag{2.6}$$

[1]The SU(5) GUT normalization, $g_1^2 = \frac{5}{3}g'^2$, is used for the U(1) coupling.

These splittings arise because of the differing T_3 quantum numbers of the upper and lower components of the doublets $Q = (U, D)$ and $L = (N, E)$. It is convenient to define m_Q^2 and m_L^2 as the average squared mass of the doublet representation, thus $m_Q^2 = \frac{1}{2}(m_U^2 + m_D^2)$ and $m_L^2 = \frac{1}{2}(m_N^2 + m_E^2)$. In the rest of this paper it is the masses m_I^2, $I = 1, \ldots, 5$ of the five types of multiplet Q, U^c, D^c, L, and E^c which will interest us. In the MSSM, these are

$$m_I^2 = m_0^2 + \sum_a f_{aI} M_{0a}^2 - Y_I \sin^2\theta_W M_Z^2 \cos 2\beta \ , \quad (2.7)$$

where Y_I is the hypercharge of multiplet I ($Q = T_3 + Y$).

The mass predictions of (2.7) are based on several strong assumptions. The universal scalar mass is a speculative assumption about the form of the interactions in supergravity, and has been questioned, particularly by those working on string-inspired models [10]. The mass formula of Eq. (2.4) assumes the minimal particle content beneath the Planck scale. If there are extra gauge interactions then the index $a = 1, 2, 3, 4, \ldots$, yields extra terms. If there are extra chiral fields with gauge quantum number then the b_a of Eq. (2.5) will change. Furthermore, if these extra chiral fields allow further superpotential interactions of strength λ involving quark and lepton fields, then additional terms proportional to λ^2 will contribute to $m_i^2(\mu)$.

In this paper we study the scalar mass relations which follow from certain assumptions about grand unification. The assumptions appear to us to be better motivated than those listed above for the MSSM, since they are based on the successful supersymmetric GUT prediction for $\sin^2\theta_W$ [7], the weak mixing angle, which we briefly summarize. The combined fit to the precision data from the CERN e^+e^- collider LEP gives $\sin^2\theta_W(\text{LEP}) = 0.2321 \pm 0.0005$, which corresponds to $m_t = 176 \pm 15$ GeV (these results, and the results of other fits to experimental data given below, are taken from [11]). The left-right asymmetry measurement at SLAC gives $\sin^2\theta_W(\text{SLAC}) = 0.2292 \pm 0.0010$ and $m_t = 255^{+22}_{-24}$ GeV. The W mass measurements from the Collider Detector at Fermilab (CDF), D0, and UA2 Collaborations correspond to $\sin^2\theta_W = 0.2326 \pm 0.0008$. These experimental numbers should be compared with the supersymmetric GUT central prediction of $\sin^2\theta_W(\text{SGUT}) = 0.2342 \pm 0.0014$, where the only uncertainty shown is that due to $\alpha_s(M_Z) = 0.120 \mp 0.005$. In addition, simple models could have uncertainties of 0.0030 from threshold corrections at the GUT and weak scales. The weak mixing angle therefore provides the only successful theoretical prediction at the 1% level of any parameter of the standard model. This suggests that we take the assumptions which are *sufficient* to get this prediction and use them to make predictions for the squark and slepton masses. These assumptions are (1) at some scale M_G the gauge group is $\text{SU}(5) \times G$, where SU(5) contains the entire standard model gauge group; (2) at mass scales below M_G the gauge group is $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \times G'$; (3) at mass scales below M_G the only particles coupling to the standard model gauge interactions are those of the MSSM.[2]

These assumptions are not a necessary requirement for an acceptable value of $\sin^2\theta_W$. Acceptable values can be obtained in very many ways, for example, in nonsupersymmetric SU(5) theories with extra multiplets which are not SU(5) degenerate [12]. However, it is these assumptions which uniquely produce a significant prediction. All the other schemes have a free parameter which can be chosen to fit $\sin^2\theta_W$.[3]

What scalar mass relations follow from these assumptions? The first assumption imposes the boundary condition (which is taken to be at M_G now) on scalar masses within the same generation:

$$m_{Q_0} = m_{E_0^c} = m_{U_0^c} = m_{10} \ , \quad (2.8)$$

$$m_{L_0} = m_{D_0^c} = m_{\bar{5}} \ , \quad (2.9)$$

because Q, E^c, and U^c all lie in a 10-dimensional representation, and L and D^c lie in the $\bar{\mathbf{5}}$. There is no boundary condition relating masses of particles in different generations, and hence no such mass relations will result.

Let us study a particular generation, and suppose that in the $\text{SU}(5) \times G$ theory it lies in representation $(\mathbf{10}, R_1) + (\bar{\mathbf{5}}, R_2)$. If R_1 and R_2 are nontrivial and if G breaks to G' which is nontrivial, then the G' gauginos can renormalize the squark and slepton masses. However, since all members of the $\mathbf{10}$ have the same G' quantum numbers, this renormalization is common, and can simply be absorbed into the unknown parameter m_{10}. An identical situation applies to the $\bar{\mathbf{5}}$. Hence the common mass m_0^2 in formula (2.7) should be replaced by $m_0^2 \to m_{I_0}^2$ which take on the two possible values shown in (2.8) and (2.9) according to whether I lies in a $\mathbf{10}$ or $\bar{\mathbf{5}}$ representation. In addition, the S term, which vanishes under the universal boundary condition assumption, is now given by

$$\begin{aligned} S(M_G) &= \sum_i Y_i m_i^2(M_G) \\ &= m_{H_2}^2(M_G) - m_{H_1}^2(M_G) \ . \end{aligned} \quad (2.10)$$

Since H_2 and H_1 lie in different representations of SU(5), $m_{H_2}^2(M_G)$ and $m_{H_1}^2(M_G)$ are not necessarily equal. From (2.2) it follows that S scales as α_1,

$$\frac{S(\mu)}{S(\mu_0)} = \frac{\alpha_1(\mu)}{\alpha_1(\mu_0)} \ . \quad (2.11)$$

The contributions of the S term can be written as

$$\delta_S m_i^2(\mu) = Y_i T \ , \quad (2.12)$$

[2]In fact the prediction of $\sin^2\theta_W$ is not altered if extra complete, degenerate SU(5) multiplets occur beneath M_G. We assume these to be absent; it could be worth studying the extent to which such representations affect the scalar mass relations.

[3]In the MSSM the scale of supersymmetry breaking is not a free parameter, it is determined to be of order the weak scale by radiative electroweak symmetry breaking.

where

$$T = -\frac{3}{5b_1}[S(M_G) - S(\mu)] = -\frac{3}{5b_1}S(M_G)\left(1 - \frac{\alpha_1(\mu)}{\alpha_1(M_G)}\right) = -\frac{3}{5b_1}S(\mu)\left(\frac{\alpha_1(M_G)}{\alpha_1(\mu)} - 1\right) . \quad (2.13)$$

Among the five masses (2.7) of each light generation, there are three combinations independent of $m_{I_0}^2$:

$$m_Q^2 - m_{U^c}^2 = (C_2 - \tfrac{15}{36}C_1)M_0^2 - \tfrac{5}{6}\sin^2\theta_W M_Z^2\cos 2\beta + \tfrac{5}{6}T , \quad (2.14a)$$

$$m_Q^2 - m_{E^c}^2 = (C_3 + C_2 - \tfrac{35}{36}C_1)M_0^2 + \tfrac{5}{6}\sin^2\theta_W M_Z^2\cos 2\beta - \tfrac{5}{6}T , \quad (2.14b)$$

$$m_{D^c}^2 - m_L^2 = (C_3 - C_2 - \tfrac{5}{36}C_1)M_0^2 - \tfrac{5}{6}\sin^2\theta_W M_Z^2\cos 2\beta + \tfrac{5}{6}T , \quad (2.14c)$$

where we have written $f_{3i} = C_3$ for a color triplet, $f_{2i} = C_2$ for a weak doublet, and $f_{1i} = Y_i^2 C_1$, and the $\alpha_a(M_p)$ in f_{ai} should be replaced by $\alpha_a(M_G)$. By rearranging the above equations, we arrive at the following two mass relations independent of T:

$$2m_Q^2 - m_{U^c}^2 - m_{E^c}^2 = (C_3 + 2C_2 - \tfrac{25}{18}C_1)M_0^2 , \quad (2.15a)$$

$$m_Q^2 + m_{D^c}^2 - m_{E^c}^2 - m_L^2 = (2C_3 - \tfrac{10}{9}C_1)M_0^2 , \quad (2.15b)$$

and also an expression for T:

$$T = \tfrac{3}{10}(m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \tfrac{10}{3}\sin^2\theta_W M_Z^2\cos 2\beta) . \quad (2.16)$$

Since $S(M_G)$ is only proportional to the difference $m_{H_2}^2(M_G) - m_{H_1}^2(M_G)$ and $b_1 = \frac{33}{5}$, we have $|T| < \frac{1}{11}|m_{H_2}^2(M_G) - m_{H_1}^2(M_G)|$. If the splitting between $m_{H_2}^2(M_G)$ and $m_{H_1}^2(M_G)$ is not too large, then T is small and these mass relations of (2.14), with $T = 0$, are approximately true. Alternatively, one can use (2.3), (2.13), and (2.16) to get

$$(m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \tfrac{10}{3}\sin^2\theta_W M_Z^2\cos 2\beta)_{\text{3rd gen}} + (m_{H_2}^2 - m_{H_1}^2) = S(\mu) - \tfrac{20}{3}T = -\left(\frac{11\alpha_1(\mu)}{\alpha_1(M_G) - \alpha_1(\mu)}\right)T - \tfrac{20}{3}T = -\left(\frac{20\alpha_1(M_G) + 13\alpha_1(\mu)}{3\alpha_1(M_G) - 3\alpha_1(\mu)}\right)T . \quad (2.17)$$

This combination does not suffer from the renormalization effects of the large third-generation Yukawa couplings. Using T from (2.16) in (2.17) gives a third (intergeneration) mass relation:

$$(m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \tfrac{10}{3}\sin^2\theta_W M_Z^2\cos 2\beta)_{\text{1st or 2nd gen}} = -\{(m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \tfrac{10}{3}\sin^2\theta_W M_Z^2\cos 2\beta)_{\text{3rd gen}} + (m_{H_2}^2 - m_{H_1}^2)\} \times \frac{10\alpha_1(M_G) - 10\alpha_1(\mu)}{20\alpha_1(M_G) + 13\alpha_1(\mu)} . \quad (2.18)$$

The MSSM provides four mass relations within each generation: those of (2.14) with $T = 0$ together with

$$m_L^2 - m_{E^c}^2 = (C_2 - \tfrac{3}{4}C_1)M_0^2 + \tfrac{3}{2}\sin^2\theta_W M_Z^2\cos 2\beta , \quad (2.19)$$

and also predicts identical spectra for each of the light generations.

In this section we have shown that two of these mass relations follow from a completely different boundary condition assumption than the one of universal scalar masses used for the MSSM. We have found that, in any GUT where the successful prediction of the weak mixing angle at the 1% accuracy level is preserved, two of the four mass relations of the MSSM for each light generation is preserved and a third one can be recovered provided that the third-generation scalar masses and Higgs boson masses are also measured.

III. AN EXTRA MASS RELATION IN SO(10)?

The mass relation (2.19) can be reformulated as a relation between the two charged slepton masses of a given generation:

$$m_E^2 - m_{E^c}^2 = (C_2 - \tfrac{3}{4}C_1)M_0^2 + (-\tfrac{1}{2} + 2\sin^2\theta_W)M_Z^2\cos 2\beta . \quad (3.1)$$

In the following we will not include the contributions from the S term, it is assumed to be small or can be obtained from (2.16) or (2.17), then be subtracted from the scalar masses. This relation is particularly important for the following reasons. (a) The supersymmetric QCD interactions tend to increase the masses of the squarks above the sleptons, hence we expect this to be the first scalar mass relation of the MSSM to be tested. (b) We have shown that this relation is precisely the one

which cannot be deduced from SU(5) unification. This is clearly because E and E^c are in different representations of SU(5).

If the gauge group is extended to include SO(10), such that a single generation lies entirely in a 16-dimensional spinor representation, then it is tempting to think that this slepton mass relation will be recovered, perhaps one can view this particular mass relation as a low energy signature of SO(10). In this section, we explore in more detail the extent to which this is true.

We will make the three assumptions, given in the last section, necessary for the GUT to yield a significant $\sin^2\theta_W$ prediction. In addition we add a fourth assumption: (4) At energy scales greater than M_{10}, which is greater than or equal to M_G, the gauge group contains a factor which includes the usual SO(10) gauge group.

This assumption provides the extra boundary condition which sets $m_L(\mu)$ and $m_{E^c}(\mu)$ equal at $\mu \geq M_{10}$. The crucial question now is: are there any additional effects which could split these masses other than those of the $SU(2)_L\times U(1)_Y$ gaugino contributions and the $SU(2)_L\times U(1)_Y$ D^2 interactions, shown in (2.19) and (3.1)?

There are four such effects, which could break the slepton mass relation in an important way [13–15]: (a) radiative contributions from the gauge couplings and gaugino masses between M_{10} and M_G, (b) radiative contributions from the superpotential couplings between M_{10} and M_G, (c) tree-level D-term contributions, and (d) radiatively generated D-term contributions.

Suppose that M_{10} is higher than M_G, and that beneath M_{10} SO(10) breaks down to SU(5) [or $SU(5)\times U(1)_X$]. The two charged sleptons of a given generation belong to $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations of SU(5), respectively, and therefore their masses receive different radiative corrections. The radiative correction contributions from the SU(5) gaugino mass is

$$\delta m^2(R) = \frac{2}{b_5}C_2(R)\left[1-\frac{\alpha_5^2(M_{10})}{\alpha_5^2(M_G)}\right]M_5^2(M_G)\ , \quad (3.2)$$

where $C_2(\bar{\mathbf{5}}) = \frac{12}{5}$ and $C_2(\mathbf{10}) = \frac{18}{5}$. Therefore we have $\delta m^2(\mathbf{10})/\delta m^2(\bar{\mathbf{5}}) = \frac{3}{2}$. If $U(1)_X$ survives beneath M_{10}, the $U(1)_X$ gaugino mass also contributes to the radiative corrections and reduces this ratio ($X_{\mathbf{10}} = -1,\ X_{\bar{\mathbf{5}}} = 3$), but in general its contributions are smaller.

If this is the only source which violates the slepton mass relation, then we have

$$1 \leq \frac{m_{10}}{m_{\bar{\mathbf{5}}}} \leq \frac{3}{2}\ , \quad (3.3)$$

and the violation should be small if gaugino mass is found to be small unless the gauge coupling increases very rapidly above M_G.

In addition to the radiative corrections from the gauge couplings, if the sleptons have some superpotential coupling of strength λ with fields which acquire masses $O(M_G)$, then there are radiative corrections to the slepton masses between M_{10} and M_G at order λ^2. In order to generate significant violations of the slepton mass relation, λ has to be large, probably $\gtrsim \frac{1}{3}$, but such a large superpotential coupling could also destroy the degeneracy of scalar masses of different generations and induce unacceptable flavor-changing effects unless there is a horizontal symmetry above M_G which keeps the scalar masses of the two lighter generations degenerate.

D-term contributions to scalar masses can arise when the rank of the gauge group is reduced. To see this, consider the following situation. Suppose the $U(1)_X$ subgroup of SO(10) [$SO(1)\supset SU(5)\times U(1)_X$] is broken by the vacuum expectation values (VEV's) of N and $\bar{N}$ fields which lie in $\mathbf{16}$ and $\overline{\mathbf{16}}$ representations of SO(10). The $U(1)_X$ gauge interaction contains a piece

$$\frac{1}{2}g_X^2\left(X_N|N|^2 - X_N|\bar{N}|^2 + \sum_i X_i|\phi_i|^2\right)^2\ , \quad (3.4)$$

where X_i is the X charge of the ϕ_i field. When the VEV's of N and $\bar{N}$ fields are not equal, it gives extra contributions to the squared masses of scalar fields of nonzero X charges. This happens if the soft SUSY-breaking masses of N and $\bar{N}$ are different [14,16,17]. The relevant part of the scalar potential for these fields we take to be

$$V(N,\bar{N}) = \tfrac{1}{2}g_X^2(X_N|N|^2 - X_N|\bar{N}^2|)^2 + m_N^2|N|^2 + m_{\bar{N}}^2|\bar{N}^2| + \lambda^2|N\bar{N}-\mu^2|^2\ , \quad (3.5)$$

where m_N^2 and $m_{\bar{N}}^2$ are the soft SUSY-breaking masses of the N and $\bar{N}$ fields, and they are of the order of the SUSY-breaking scale m_S. The last term is to give large VEV's ($\simeq \mu$) to N and $\bar{N}$ fields.[4] Defining $\Sigma \equiv |N|^2 + |\bar{N}^2|$, $\Delta \equiv |N|^2 - |\bar{N}^2|$, $m_\Sigma^2 \equiv \frac{1}{2}(m_N^2 + m_{\bar{N}}^2)$, and $m_\Delta^2 \equiv \frac{1}{2}(m_N^2 - m_{\bar{N}}^2)$, we can rewrite V as

$$V = \tfrac{1}{2}g_X^2(X_N\Delta)^2 + m_\Sigma^2\Sigma + m_\Delta^2\Delta + \lambda^2|\tfrac{1}{2}\sqrt{\Sigma^2-\Delta^2} - \mu^2|^2\ . \quad (3.6)$$

Minimizing the potential with respect to Δ we obtain

$$\Delta = -\frac{m_\Delta^2}{X_N^2 g_X^2} + O\left(\frac{m_S^3}{\mu}\right)\ . \quad (3.7)$$

This shifts the mass of the scalar particle with charge X_i by the amount

$$\delta m_i^2 = g_X^2 X_i X_N \Delta \simeq -\frac{X_i}{X_N}m_\Delta^2\ . \quad (3.8)$$

Therefore, any scalar particle which carries $U(1)_X$ charge will receive a tree-level D-term contribution which is proportional to its $U(1)_X$ charge and the difference of the soft-breaking masses m_N^2 and $m_{\bar{N}}^2$. Since N and $\bar{N}$ lie in different representations of SO(10), SO(10) allows m_N^2 to be very different from $m_{\bar{N}}^2$, and also $X_{\mathbf{10}}$ and $X_{\bar{\mathbf{5}}}$ are different ($X_{\mathbf{10}} = -1,\ X_{\bar{\mathbf{5}}} = 3$), this provides a large breaking of the slepton relation (2.19), (3.1).

[4]Different ways of stabilizing the VEV's of N and $\bar{N}$ do not change the basic result, they only give corrections to the higher-order terms in Eq. (3.7).

From the above discussion it follows that a significant violation of the slepton mass relation by the D term requires a large difference between m_N^2 and $m_{\bar{N}}^2$ (of the same order of the slepton masses). If some symmetry of the Kähler potential guarantees that m_N^2 and $m_{\bar{N}}^2$ are equal at the tree level, a large difference between them can still be generated by radiative corrections, especially if $U(1)_X$ is broken by the same radiative corrections at some much lower energy. We consider such a model in the next section.

IV. LARGE D-TERM CORRECTIONS FROM RADIATIVE BREAKING OF $U(1)_X$

If the scalar masses are universal at the Planck scale because of some symmetry of the Kähler potential, the difference between m_N^2 and $m_{\bar{N}}^2$ can still be generated by radiative corrections below the Planck scale if N and $\bar{N}$ couple to other fields differently. An interesting case is that $U(1)_X$ is also broken by the same radiative corrections which modify m_N^2 and $m_{\bar{N}}^2$, i.e., N and $\bar{N}$ fields get VEV's when $m_\Sigma^2 = \frac{1}{2}(m_N^2 + m_{\bar{N}}^2)$ is renormalized to negative. In this case, $m_\Delta^2 = \frac{1}{2}(m_N^2 - m_{\bar{N}}^2) \simeq m_N^2$ which is presumably comparable to the masses of the squarks and sleptons, then the D-term correction the particle spectrum can be quite large. In what follows we consider a simple model which will demonstrate this case.

We assume, for simplicity, $M_{\mathbf{10}} = M_G$, and beneath M_G, the particle contents are the usual ones in the MSSM with three right-handed neutrinos, the additional $U(1)_X$ gauge field, N and $\bar{N}$ fields discussed above which break the $U(1)_X$ when they get nonzero VEV's, and three gauge singlets S_k, $k = 1, 2, 3$. N and $\bar{N}$ belong to the **16** and $\overline{\mathbf{16}}$ representations of SO(10) at the GUT scale with all other components get superheavy masses and decouple below the GUT scale. This can be achieved by a **45** representation of Higgs bosons with VEV's in the hypercharge direction (see the Appendix). The two low energy Higgs doublets H_1 and H_2 are assumed to belong to the 10 representations of SO(10) and their X charges are -2 and 2, respectively. The X charges of all chiral fields are shown in Table I. Note that we only add the $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlets to the MSSM so that the successful prediction of $\sin^2\theta_W$ in the SGUT's is retained.

We consider a superpotential given by

$$W = \mathbf{Q}\boldsymbol{\lambda}_U \mathbf{U}^c H_2 + \mathbf{Q}\boldsymbol{\lambda}_D \mathbf{D}^c H_1 + \mathbf{L}\boldsymbol{\lambda}_E \mathbf{E}^c H_1 + \mathbf{L}\boldsymbol{\lambda}_\nu \boldsymbol{\nu}_R^c H_2 + \mu H_1 H_2 + \sum_{k=1}^{3} \lambda_k \nu_R{}^c{}_k S_K \bar{N} \ . \tag{4.1}$$

Other possible interactions, such as $N S_k \bar{N}$, $m S_k^2$, and S_k^3, could vanish either because S_k's are embedded in some nontrivial representations of SO(10), or because of some discrete symmetry. (For example, a parity whose lepton fields change sign and S_k and $\bar{N}$ are multiplied by i.) The scalar potential involving N and $\bar{N}$ fields is given by[5]

$$\begin{aligned} V &= \frac{1}{2} g_X^2 \left(X_N |N|^2 + X_{\bar{N}} |\bar{N}^2| + \sum_i X_i |\phi_i|^2 \right)^2 + \sum_{k=1}^{3} \lambda_k \tilde{\nu}_R{}^c{}_k \bar{N}|^2 \\ &\quad + \sum_{k=1}^{3} |\lambda_k S_k \bar{N}|^2 + m_N^2 |N|^2 + m_{\bar{N}}^2 |\bar{N}^2| + \sum_{k=1}^{3} A_k \lambda_k \tilde{\nu}_R{}^c{}_k S_k \bar{N} \\ &= \frac{1}{2} g_X^2 \left(X_N \Delta + \sum_i X_i |\phi_i|^2 \right)^2 + m_\Sigma^2 \Sigma + m_\Delta^2 \Delta \\ &\quad + \sum_{k=1}^{3} |\lambda_k \tilde{\nu}_R{}^c{}_k \bar{N}|^2 + \sum_{k=1}^{3} |\lambda_k S_k \bar{N}|^2 + \sum_{k=1}^{3} A_k \lambda_k \tilde{\nu}_R{}^c{}_k S_k \bar{N} \ , \end{aligned} \tag{4.2}$$

where Σ, Δ, m_Σ^2, and m_Δ^2 are defined as before. When m_Σ^2 is driven negative by the Yukawa interactions $\lambda_k \nu_{Rk}^c S_k \bar{N}$ at some intermediate mass scale M_I, (λ_k's are assumed to be ~ 1) N and $\bar{N}$ fields will get nonzero VEV's and break the $U(1)_X$. The difference of the squares of their VEV's Δ is given by $\Delta = -m_\Delta^2 / X_N^2 g_X^2$ by minimizing V with respect to Δ, and the sum Σ is fixed by the one-loop correction

$$\Delta V = \frac{1}{64\pi^2} \mathrm{Str} M^2 \left(\ln \frac{M^2}{\mu^2} - \frac{3}{2} \right) \tag{4.3}$$

TABLE I. The $U(1)_X$ charges of different fields.

Field:	q_L	$u_R{}^c$	$d_R{}^c$	l_L	$e_R{}^c$	$\nu_R{}^c$	H_1	H_2	N	$\bar{N}$	S
X	-1	-1	3	3	-1	-5	-2	2	-5	5	0

to the scalar potential [18], $\Sigma \sim M_I^2$ where M_I is the scale at which $m_\Sigma^2(M_I) = m_N^2(M_I) + m_{\bar{N}}^2(M_I) = 0$ [16]. Figure 1 shows the evolutions of the soft breaking masses of N, $\bar{N}$, S_k, and $\tilde{\nu}_R{}^c{}_k$ fields. For simplicity, we have assumed that the soft SUSY-breaking parameters are universal at M_G and the parameters are chosen to be $\lambda_{t0} = \lambda_{\nu_\tau 0} = 1.5$, $\lambda_{b0,\tau 0} \ll 1$, $\lambda_{k0} = 1$, $k = 1, 2, 3$, and the universal soft breaking trilinear couplings $A_0 = 3m_0$. The $m_{\tilde{\nu}_{R3}}^2$ is also driven negative at low energies because of the large λ_{ν_3} coupling. How-

[5]We use S and N to represent both the superfields and their scalar components. It should be clear which one they represent.

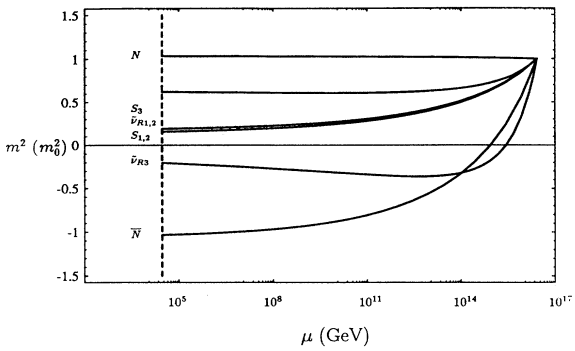

FIG. 1. The evolutions of the soft breaking masses of N, $\bar{N}$, S_k, and $\tilde{\nu}_R{}^c{}_k$ fields from the GUT scale (2.7×10^{16} GeV) to the $U(1)_X$ breaking scale (30 TeV). A universal soft breaking mass m_0 is assumed at the GUT scale and the parameters are chosen to be $\lambda_{t0} = \lambda_{\nu_\tau 0} = 1.5$, $\lambda_{b0,\tau 0} \ll 1$, $\lambda_{k0} = 1$, $k = 1,2,3$, and the universal soft breaking trilinear couplings $A_0 = 3m_0$.

ever, the terms $\sum_{k=1}^{3} |\lambda_k \tilde{\nu}_R{}^c{}_k \bar{N}|^2$ in the scalar potential V [Eq. (4.2)] prevent both $\bar{N}$ and $\tilde{\nu}_R{}^c$ get nonzero VEV's. After $U(1)_X$ is broken, the mass square of $\tilde{\nu}^c_{R_3}$ gets a large positive contribution from the $\bar{N}$ VEV and $\langle\tilde{\nu}^c_{R_3}\rangle$ remains zero.

The present bounds on the mass of the $U(1)_X$ gauge boson Z_χ are $M_{Z_\chi} > 320$ GeV (direct) and > 670 GeV (indirect) [19]. The primordial nucleosynthesis may put a more stringent limit on M_{Z_χ}, taking $N_\nu < 3.5$, M_{Z_χ} has to be greater than ~ 1 TeV [20] because of the extra massless states present in our model. Cosmological constraints also put an upper limit on M_I. The flaton (a linear combination of N and $\bar{N}$ which corresponds to the quasiflat direction) decays into light particles through the heavy intermediate states of $O(M_I)$ after the phase transition of $U(1)_X$ breaking. The decay rate must be fast enough in order not to affect the primordial nucleosynthesis or overdilute the baryon asymmetry. This gives an upper bound on M_I [21]. With these considerations, we will take M_I to be in the range of 10^3 to 10^7 GeV.

Compared with MSSM, the scalar masses contain two extra contributions: the $U(1)_X$ gaugino contribution and the $U(1)_X$ D term. For the first two generations where the Yukawa couplings are negligible, the scalar masses are given by

$$m_i^2 = m_0^2 + \sum_{a=1}^{3} f_{ai} M_0^2 + f_{X_i} M_0^2 + (T_{3i} - Q_i \sin^2\theta_W) M_Z^2 \cos 2\beta - \frac{X_i}{X_N} m_\Delta^2 , \quad (4.4)$$

where m_0 and M_0 are the scalar mass and gaugino mass at M_G, respectively, f_{ai}, $a = 1,2,3$, are the same as before and f_{X_i} is given by

$$f_{X_i} = \frac{2}{b_X}\frac{X_i^2}{40}\left(\frac{\alpha_X^2(M_I)}{\alpha_X^2(M_G)} - 1\right) . \quad (4.5)$$

In this simple model, m_Δ^2 can also be expressed in terms of m_0 and M_0,

$$m_\Delta^2 = m_N^2 = m_0^2 + f_{X_N} M_0^2 , \quad (4.6)$$

then we have

$$m_i^2 = -\left(1 - \frac{X_i}{X_N}\right) m_0^2 + \sum_{a=1}^{3} f_{ai} M_0^2 + \left(f_{X_i} - \frac{X_i}{X_N} f_{X_N}\right) M_0^2 + (T_{3i} - Q_i \sin^2\theta_W) M_Z^2 \cos 2\beta . \quad (4.7)$$

The corrections $-(X_i/X_N)m_0^2 + [f_{X_i} - (X_i/X_N) f_{X_N}]M_0^2$ to the masses of squarks and sleptons compared to the MSSM can be as large as 60% for $X_i = 3$ in the limit $m_0 \gg M_0$. Figure 2 shows the comparison of the scalar spectra with and without the $U(1)_X$ D-term corrections for a set of m_0 and M_0. We see that the corrections are more significant for the sleptons than for the squarks because of the smaller gaugino mass contributions to the sleptons than to the squarks. Now the slepton mass relation (3.1) is modified to be

$$m_E^2 - m_{E^c}^2 = (C_2 - \tfrac{3}{4}C_1) M_0^2 + 8 C_X M_0^2 + \tfrac{4}{5} m_\Delta^2 + (-\tfrac{1}{2} + 2\sin^2\theta_W) M_Z^2 \cos 2\beta , \quad (4.8)$$

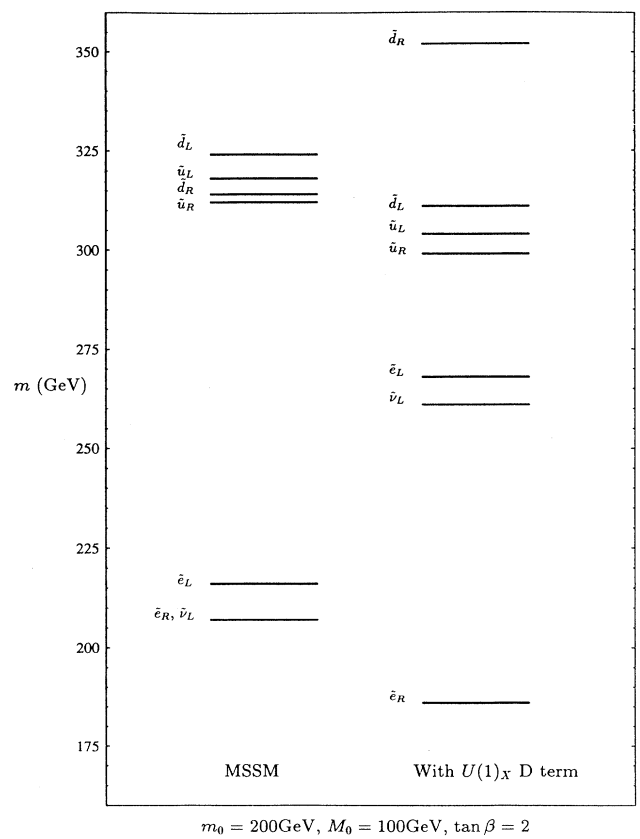

FIG. 2. Comparison of the scalar particle spectra with and without the $U(1)_X$ D-term corrections for a set of m_0, M_0, and $\tan\beta$.

where $f_{X_i} = X_i^2 C_X$. In a more general SO(10) theory there is no simple relation between m_Δ^2 and m_0^2 and m_Δ^2 has to be treated as a parameter.

Before going to the next section, we have three comments on this model.

(1) S-term contributions: When $U(1)_X$ is broken at intermediate energy M_I, $S(M_I)$ is also shifted by $\delta m_{H_2}^2 - \delta m_{H_1}^2 = \frac{4}{5} m_\Delta^2$. Then Eqs. (2.13), (2.17), and (2.18) are not valid. Therefore, if (2.15a) and (2.15b) hold but (2.18) does not, it may be a hint of a $U(1)_X$ breaking at intermediate energy scale and providing a shift of the S term.

(2) Neutrino masses: In our simplest model, there are three heavy Dirac neutrinos and three massless neutrinos because of the three singlet states we introduced [22,23]. We can see them from the mass terms of the neutrinos (for simplicity, we only consider one family here and drop the family indices)

$$m_D \nu_L \nu_R^c + M_D S \nu_R^c \ , \tag{4.9}$$

where $m_D = \lambda_\nu \langle H_2^0 \rangle = O(m_{u,c,t})$ and $M_D = \lambda \langle \bar{N} \rangle = O(M_I)$. One linear combination of ν_L and S, $\nu_L \sin\theta + S\cos\theta$, where $\tan\theta = m_D/M_D$, is married with ν_R and gets a large mass $\sqrt{m_D^2 + M_D^2} = O(M_I)$, which is consistent with experimental constraints [23], and the other combination $\nu_L \cos\theta - S\sin\theta$ is left massless. However, it is possible to give the three light neutrinos small majorana masses which are favored to solve the solar neutrino problem by just adding some extra interactions to the superpotential of the model. For example, if we add to the superpotential the nonrenormalizable interaction $(1/M_G) S^2 N \bar{N}$ which gives a small majorana mass term $m_S S^2 = (1/M_G)\langle N \rangle \langle \bar{N} \rangle S^2$ to S, then the mass matrix of the fields ν_L, ν_R^c, and S becomes

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_D \\ 0 & M_D & m_S \end{pmatrix} \ . \tag{4.10}$$

The product of the three mass eigenvalues is given by $\det\mathcal{M} = -m_D^2 m_S$, and the two larger mass are approximately equal to M_D, so the mass of the light neutrino is approximate

$$m_{\text{light}} \simeq \frac{m_D^2 m_S}{M_D^2} \sim \frac{m_D^2 M_I^2}{M_D^2 M_G} \sim \frac{m_D^2}{M_G} \tag{4.11}$$

which is similar to that generated by the usual seesaw mechanism.

(3) b-τ Yukawa unification: Because the $U(1)_X$ is broken at low energy, there are extra interactions surviving at low energies compared with the MSSM. Especially the τ-neutrino Yukawa coupling λ_{ν_τ} which should be about the same as λ_t at the GUT scale enters the renormalization-group (RG) equations of many parameters. The RG equation for the b-τ mass ratio R is modified to be

$$\frac{dR}{dt} = \frac{R}{16\pi^2}\left(-\tfrac{16}{3} g_3^2 + \tfrac{4}{3} g_1^2 + \lambda_t^2 - \lambda_{\nu_\tau}^2 + 3\lambda_b^2 - 3\lambda_\tau^2\right) \ . \tag{4.12}$$

In the small $\tan\beta$ case where λ_b and λ_τ can be neglected, the unification of b and τ Yukawa couplings in SGUT requires a large top Yukawa coupling to compensate the contribution from the SU(3) gauge coupling. In our model the contribution of λ_t is largely cancelled out by λ_{ν_τ}, making it difficult to achieve the b-τ unification for the top Yukawa coupling staying in the perturbative regime at the GUT scale. However, since the b and τ Yukawa couplings are small, they do not necessarily come from a single renormalizable interaction of the form $\mathbf{16}_3\ \mathbf{10}\ \mathbf{16}_3$ in SO(10) and therefore their unification is not mandatory. In the large $\tan\beta$ case where λ_b and λ_τ are comparable to λ_t (which we will discuss in the next section), the terms $3\lambda_b^2 - 3\lambda_\tau^2$ in the RG equation for R also contribute and make up the negative contribution from λ_{ν_τ} ($\lambda_b > \lambda_\tau$ below the GUT scale). In addition, the couplings between b and H_2 through the bottom squark-gluino loops and top squark-chargino loops [24,25] could also give a significant contribution to R if $\tan\beta$ is large. Therefore, the b-τ unification is possible in this case.

V. FINE-TUNING PROBLEM IN THE YUKAWA UNIFICATION SCENARIO

Recently, the large $\tan\beta$ scenario in which the τ lepton and the bottom- and top-quark Yukawa couplings unify at the grand-unification scale has drawn considerable interest [25–27]. This happens in an SO(10) GUT if the two light Higgs doublets lie predominantly in a single **10** representation of the gauge group SO(10) and the t, b, and τ masses originate in the renormalizable Yukawa interactions of the form $\mathbf{16}_3\ \mathbf{10}\ \mathbf{16}_3$. In this case, the top quark mass can also be predicted and it was predicted to be heavy [25]. In fact, such a heavy top quark is favored by the recent CDF results, $m_t = 174^{+10+13}_{-10-12}$ GeV [28]. The problem with this scenario is that radiative electroweak symmetry breaking is hard to achieve although significant progress has already been made [27,29–31]. The masses of the up- and down-type Higgs bosons are the same at $M_{\mathbf{10}}$ because they lie in the same representation and run almost in parallel because of the boundary condition $\lambda_t(M_{\mathbf{10}}) = \lambda_b(M_{\mathbf{10}})$. Usually one relies on heavy gauginos to amplify the small hypercharge-induced difference in the running of $m_{H_1}^2$ and $m_{H_2}^2$. However, all these attempts require severe fine-tuning of the parameters which we will explain below.

The relevant part of the Higgs potential is given by

$$\mu_1^2 |H_1^0|^2 + \mu_2^2 |H_2^0|^2 + B\mu(H_1^0 H_2^0 + \text{H.c.}) + \tfrac{1}{8}(g^2 + g'^2)(|H_1^0|^2 - |H_2^0|^2)^2 \ . \tag{5.1}$$

Minimizing the Higgs potential we obtain the following conditions:

$$\frac{\mu_1^2 - \tan^2\beta\,\mu_2^2}{\tan^2\beta - 1} = \frac{M_Z^2}{2} \ , \tag{5.2}$$

$$\frac{-\mu B}{\mu_1^2 + \mu_2^2} = \frac{1}{2}\sin 2\beta \ . \tag{5.3}$$

In the case of $\lambda_t(M_{10}) = \lambda_b(M_{10})$, $\tan\beta \simeq m_t/m_b \sim 50 \gg 1$. We see that $\mu_2^2 \simeq -M_Z^2/2$ for μ_1^2 not too large, then

$$m_A^2 = \mu_1^2 + \mu_2^2 \simeq (\mu_1^2 - \mu_2^2) - M_Z^2 < \mu_1^2 - \mu_2^2 \equiv \epsilon_c m_S^2 \ , \tag{5.4}$$

where m_A is the CP-odd scalar mass, m_S^2 is the typical supersymmetric particle mass scale, $m_S \sim \max(m_0, M_0)$, and ϵ_c represents the custodial symmetry-breaking effects. Equation (5.4) tells us that both m_A^2 and M_Z^2 are smaller than $\epsilon_c m_S^2$, so there is an $O(\epsilon_c)$ fine-tuning of the Z mass. In addition, writing $m_A^2 = \epsilon m_S^2$, $\epsilon < \epsilon_c \ll 1$, we have

$$\frac{-\mu B}{\epsilon m_S^2} = \frac{1}{2}\sin 2\beta \simeq \frac{1}{\tan\beta} \Rightarrow -\mu B \simeq \frac{\epsilon}{\tan\beta} m_S^2 \ . \tag{5.5}$$

While μ is typically of the order m_S in order to satisfy $\mu_2^2 = m_{H_2}^2 + \mu^2 \simeq -M_Z^2/2$. The B parameter which receives contributions from the gaugino masses and the soft SUSY-breaking trilinear scalar coupling A and therefore is also naturally of the order m_S has to be fine-tuned to $O[(\epsilon/\tan\beta)m_S]$. The fine-tuning is at least one part in 10^3 and is much worse than the naive expectation $1/\tan\beta$.

The $\mathrm{U}(1)_X$ D term which gives the opposite contributions to $m_{H_1}^2$ and $m_{H_2}^2$ provides the desired ingredient to solve this problem [29,32]. One can either simply have $m_N^2 \neq m_{\bar{N}}^2$ at tree level [29] or have the difference m_Δ^2 generated by radiative corrections as described in the last section. However, the simple model discussed in the previous section gives a positive contribution to $m_{H_2}^2$ and a negative contribution to $m_{H_1}^2$ which is incompatible with the fact that $\mu_1^2 > \mu_2^2$. We thus modify the model so that it has interactions $\lambda_k' \nu_{R_k}^c S_k' N$, $k = 1,2,3$, instead of $\lambda_k \nu_{R_k}^c S_k \bar{N}$. The S_k''s are still $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ singlets, but carry $\mathrm{U}(1)_X$ charge $+10$ [they may belong to the $\overline{\mathbf{126}}$ of SO(10)]. We also have to add $\bar{S}_k'(X = -10)$ to the model in order to cancel the anomaly and we assume that they only have the $\mathrm{U}(1)_X$ gauge interaction. Then, the m_N^2, instead of $m_{\bar{N}}^2$, is driven negative by the Yukawa interactions. The $m_\Delta^2 = \frac{1}{2}(m_N^2 - m_{\bar{N}}^2)$ becomes negative in this case and therefore it gives the correct-sign D-term contributions to $m_{H_1}^2$ and $m_{H_2}^2$. Let δm_H^2 be the difference between $m_{H_1}^2$ and $m_{H_2}^2$ generated by the renormalization group from M_{GUT} to m_S without the D-term correction:

$$\delta m_H^2 = m_{H_1}^2 - m_{H_2}^2 = \epsilon_c m_S^2 \ll m_S^2 \ . \tag{5.6}$$

The parameter $m_A^2 = \mu_1^2 + \mu_2^2$ is now given by

$$\begin{aligned} m_A^2 &= \mu_1^2 + \mu_2^2 = (\mu_1^2 - \mu_2^2) + 2\mu_2^2 \\ &\simeq D + \delta m_H^2 - M_Z^2 \ , \end{aligned} \tag{5.7}$$

where $D = (-\frac{2}{5}m_\Delta^2) - (+\frac{2}{5}m_\Delta^2) = -\frac{4}{5}m_\Delta^2 \sim m_S^2$. For m_S larger than M_Z, m_A^2 is naturally of the order m_S^2 and the problem of a light m_A^2 can be avoided. The fine-tuning problem of μB is also relieved though not totally eliminated as we can see from Eq. (5.5) that a fine tune of $1/\tan\beta \sim \frac{1}{50}$ is still required. However, it should be generic since a large pure number $\tan\beta$ has to be generated.

VI. CONCLUSIONS

It is well known that quark and lepton mass and mixing angle relations may provide evidence for grand unification. Although squarks and sleptons have yet to be discovered, mass relations amongst scalars provide a much more reliable test of unification than do the relations involving fermion masses. This is because chiral and gauge symmetry-breaking effects mask the grand-unified symmetry relations for the fermions, but are not present for the scalars. In this paper we have derived several scalar mass relations which follow directly from the grand-unified symmetry, and we have studied the reliability of such relations as a probe of supersymmetric unification.

The small size of flavor-changing processes suggests that in models with weak-scale supersymmetry the squarks of a given charge should be approximately degenerate. This has led to the speculation that squarks and sleptons of different charge might also be degenerate. Although only a speculation, such a boundary condition of universal scalar masses has become a ubiquitous feature of supersymmetric models and is incorporated in the minimal supersymmetric standard model. Since there are five types of quark and leptons, the quark and lepton weak doublets Q and L and the weak singlets U^c, D^c, and E^c, such a boundary condition leads to four relations between the scalar masses. However, the origin of these relations is more a matter of simplicity than of any underlying fundamental principle.

In this paper we have derived mass relations, between scalars of a given generation, which result from the most general possible boundary condition that respects a grand-unified symmetry. With SU(5) unification, the five types of quarks and leptons are unified into two irreducible representations (Q, U^c, E^c) and (L, D^c), leading to the expectation of three mass relations, which are given in Eq. (2.14). However, these three relations involve a quantity T, which depends on the mass splitting of the Higgs scalars at the unification mass. It is likely that this mass splitting is small enough that the relations (2.14) with $T = 0$ will result. However, if the mass splitting is very large there are only two mass relations between the scalar mass parameters of each of the light generations. These relations are given by eliminating T, and are given in Eq. (2.15). We believe that these relations must be correct in any grand-unified theory which incorporates the usual SU(5) group. If these relations are found to be incorrect, then it is unlikely that grand unification is correct. Although extra particles and interactions could be added to a grand-unified theory to invalidate these mass relations, such particles and interactions will lead to extra renormalizations of the weak mixing angle, upsetting the outstanding agreement between the theoretical prediction and the experimental value.

Even if the parameter T is large, a third mass relation can be derived because T can be evaluated by measuring the Higgs boson and third-generation scalar masses. This mass relation is given in Eq. (2.18).

If the quark and leptons are further unified, so that all five species of a generation are unified in a single representation, as occurs in SO(10) theories, a fourth mass relation is to be expected. This is written, ignoring T, in Eq. (3.1), as a relation between the masses of the two charged sleptons. This mass relation is likely to be the first which is subject to precise experimental test. If it were verified it would provide striking support for SO(10) unification. However, unlike the two mass relations mentioned above, it is not a necessary consequence of SO(10) unification. We have shown in this paper that it is possible to have large corrections to this mass relation from $U(1)_X$ D^2 interactions, either at tree level or by radiative corrections.

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APPENDIX

In this appendix we show that it is possible to give superheavy masses to all components but the $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet of a multiplet of SO(10). This can be achieved by a **45** of SO(10), A_Y with a VEV in the hypercharge direction. The interaction,

$$\bar{C} A_Y C \ , \tag{A1}$$

where $C = \mathbf{16}$ or $\mathbf{126}$, will give superheavy masses to the components of C and $\bar{C}$ which have nonzero hypercharges, and leave the $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlets (N, $\bar{N}$, S'_k, $\bar{S}'_k$) massless. Those singlets survive below the GUT scale and serve to break the $U(1)_X$ at low energy.

To generate a **45** VEV in the hypercharge direction, we start with the following SO(10) invariant superpotential [we denote **54** of SO(10) by S, **45** by A, and the singlet by χ]:

$$W_1 = m_1 A_1^2 + m_2 S^2 + m_3 \chi^2 + \lambda_1 A_1^2 S + g_1 A_1^2 \chi \ . \tag{A2}$$

The equations for a supersymmetric minimum are

$$\begin{aligned} 0 &= F_A = 2(m_1 + \lambda_1 S + g_1 \chi) A_1 \ , \\ 0 &= F_S = 2 m_2 S + \lambda_1 (A_1^2 - \tfrac{1}{10} \mathrm{Tr} A_1^2) \ , \\ 0 &= F_\chi = 2 m_3 \chi + g_1 \mathrm{Tr} A_1^2 \ . \end{aligned} \tag{A3}$$

Choosing

$$\langle S \rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \ \mathrm{diag}\ (s, s, s, -\tfrac{3}{2}s, -\tfrac{3}{2}s) \ ,$$
$$\langle A \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \ \mathrm{diag}\ (a, a, a, b, b) \ ,$$

and

$$\langle \chi \rangle = c \ , \tag{A4}$$

the above equations become

$$\begin{aligned} (m_1 + \lambda_1 s + g_1 c) a &= 0 \ , \\ (m_1 - \tfrac{3}{2}\lambda_1 s + g_1 c) b &= 0 \ , \\ 2 m_2 s + \tfrac{2}{5}\lambda_1 (b^2 - a^2) &= 0 \ , \\ 2 m_3 c + g_1 (6a^2 + 4b^2) &= 0 \ . \end{aligned} \tag{A5}$$

We are interested in the solution $s = 0$, $a = b = [(m_3/5g_1)c]^{1/2} \neq 0$, $c = -m_1/g_1$, in which $\langle A_1 \rangle$ is in the SU(5) singlet (X-charge) direction. We have obtained the breaking pattern $SO(10) \to SU(5) \times U(1)_X$. We next add to the theory in such a way that SU(5) breaks to the gauge groups of the standard model, and the only light states beneath the GUT scale are those of the MSSM with some standard model singlets. We add the following terms to the superpotential:

$$\begin{aligned} W_2 = {} & g_2 \chi' A_1 A_Y + \lambda_2 S' A_Y^2 + m_4 S' S_Y + m_5 S_Y^2 + \lambda_3 S_Y^3 \\ & + \lambda_4 A A_1 A_Y + m_6 A^2 \ . \end{aligned} \tag{A6}$$

We assume that χ' and S' have no VEV's so that the minimization of W_1 is not affected, and S_Y and A_Y have nonzero VEV's of the following forms:

$$\langle S_Y \rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \ \mathrm{diag}\ (s_Y, s_Y, s_Y, -\tfrac{3}{2}s_Y, -\tfrac{3}{2}s_Y) \ ,$$
$$\langle A_Y \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \ \mathrm{diag}\ (a_Y, a_Y, a_Y, b_Y, b_Y) \ . \tag{A7}$$

s_Y is determined by the equation

$$0 = F_{S_Y} = m_4 S' + 2 m_5 S_Y + 3\lambda_3 (S_Y^2 - \tfrac{1}{10} \mathrm{Tr} S_Y^2) \ . \tag{A8}$$

We obtain $s_Y = 4m_5/3\lambda_3$. The $F_{\chi'}$ equation,

$$F_{\chi'} = A_1 A_Y = 0 \ , \tag{A9}$$

forces $\langle A_Y \rangle$ to be in the direction orthogonal to $\langle A_1 \rangle$ and the $F_{S'}$ equation,

$$F_{S'} = \lambda_2 (A_Y^2 - \tfrac{1}{10} \mathrm{Tr} A_Y^2) + m_4 S_Y = 0 \ , \tag{A10}$$

forces $\langle A_Y \rangle$ to be in the hypercharge direction. We have

$$b_Y = -\tfrac{3}{2} a_Y$$

and

$$a_Y^2 = \frac{2 m_4 s_Y}{\lambda_2} \tag{A11}$$

from these equations. Finally, we need the trilin-

ear interaction $\lambda_4 A A_1 A_Y$ to make sure that there are no extra massless states which are not eaten by the gauge bosons present to destroy the successful $\sin^2\theta_W$ prediction. We have checked the mass matrices of these fields and indeed there are only 32 massless modes which are needed for the symmetry breaking $SO(10)\to SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_X$. Now we have successfully constructed a superpotential which generates a **45** VEV in the hypercharge direction. The $SU(3)_c\times SU(2)_L\times U(1)_Y$ singlet-nonsinglet splitting required in our model could be obtained from the interaction (A1) consequently.

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