CP violation in the two-Higgs-doublet model: An example

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In a general two-scalar-doublet model without fermions, there is a unique source of CP violation, J_1, in the gauge interactions of the scalars. It arises in the mixing of the three neutral physical scalars X_1, X_2, and X_3. CP violation may be observed via different decay rates for $X_1 \to H^+W^-$ and $X_1 \to H^-W^+$ (or, alternatively, for $H^+ \to X_1W^+$ and $H^- \to X_1W^-$, depending on which decays are kinematically allowed). I compute the part of those CP-violating decay-rate differences which is proportional to J_1. The CP-invariant phase is provided by the absorptive parts of the one-loop diagrams. I check the gauge invariance of the whole calculation.

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I. INTRODUCTION

There are general reasons for the interest in the possibility of CP violation in the scalar sector. CP violation is a necessary ingredient for the generation of the baryon asymmetry of the Universe [1]. It is believed that CP violation in the Kobayashi-Maskawa matrix is not large enough to explain that asymmetry [2]. It has been speculated [3] that the scalar sector might provide the missing CP violation.

There have been studies of possible signatures of CP violation in the scalar sector. It has been remarked [4] that the simultaneous presence of the three couplings $Z^0S_1S_2$, $Z^0S_1S_3$, and $Z^0S_2S_3$, where S_1, S_2, and S_3 are three neutral scalar fields in any model, implies CP violation. Similarly, the simultaneous presence of the three couplings $S_1Z^0Z^0$, $S_2Z^0Z^0$, and $S_1S_2Z^0$, represents CP violation. This is because the C quantum number of the Z^0 is -1. Another work [5] has considered various CP-violating Lagrangians including scalars, fermions, and vector bosons, and has suggested looking for CP violation in the decay mode $S \to Z^0W^+W^-$, occurring when, in the rest frame of the decaying neutral scalar S, the momentum distribution of the W^+ is not the same as the momentum distribution of the W^-, or, in a similar fashion, in $S \to Z^0H^+H^-$. The first of these CP-violating asymmetries has later been computed [6] in the context of the two-Higgs-doublet model. However, the decay mode $S \to Z^0W^+W^-$ is phase-space disfavored as compared to the simpler decay modes $S \to W^+W^-$ and $S \to Z^0Z^0$. Other studies [7] have concentrated on CP-violating phenomena originating in the interplay of scalars and fermions, in particular the effects of top-quark physics.

The aim of this work is the computation of a CP-violating asymmetry in the two-Higgs-doublet model without any fermions. The model has gauge symmetry SU(2)⊗U(1), which is spontaneously broken to the U(1) of electromagnetism by the vacuum expectation values (VEV's) of the two Higgs doublets. I look for CP violation involving solely the gauge interactions of the scalars. For simplicity, I do not consider the presence of fermions, which would lead to extra sources of CP violation, both in the fermion sector, and in the interplay of the fermion and the scalar sectors. I also omit possible sources of CP violation in the cubic and quartic interactions of the physical scalars. Those scalars are two charged particles $H^\pm$, with mass m_H, and three neutral particles X_1, X_2, and X_3, with masses m_1, m_2, and m_3, respectively. In addition, the spectrum of the model includes the massive intermediate vector bosons $W^\pm$ and Z^0, with masses $m_W = 80$ GeV, and $m_Z = 91$ GeV, respectively, and the massless photon. For a fairly large range of the masses of the scalars, either the two decays $X_1 \to H^+W^-$ and $X_1 \to H^-W^+$, or the two decays $H^+ \to X_1W^+$ and $H^- \to X_1W^-$, are kinematically allowed (the neutral scalars may be numbered so that X_1 is the scalar for which one of these couples of decays is allowed). Then, the possibility of a CP-violating difference between the rate of one decay and the rate of its CP-conjugated decay exists. It is my purpose to calculate that difference.

It has recently been observed [8] that the two-Higgs-doublet model has one and only one source of CP violation in the gauge interactions of the scalars. I describe it briefly. Because the U(1) of electromagnetism is preserved in the symmetry breaking, we can, without loss of generality, choose a basis for the two scalar doublets in which only one of them, H_1, has a VEV v, while the second one, H_2, does not have a VEV. The two doublets in that basis can be written

$$H_1 = \begin{pmatrix} G^+ \\ v + (H + iG^0)/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ (R + iI)/\sqrt{2} \end{pmatrix}. \tag{1}$$

G^+ and G^0 are the Goldstone bosons, which become the longitudinal components of the W^+ and Z^0, respectively.

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H, R, and I are linear combinations of the three neutral scalar fields X_1, X_2, and X_3, which are the eigenstates of mass. Those linear combinations are given by an orthogonal matrix T,

$$\begin{pmatrix} H \\ R \\ I \end{pmatrix} = T \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} . \quad (2)$$

Without loss of generality, I set T to have determinant $+1$. There is CP violation in the gauge interactions of the scalars [8] if and only if m_1, m_2, and m_3 are all different, and

$$J_1 \equiv T_{11}T_{12}T_{13} \quad (3)$$

is nonzero. The quantity J_1 has in the two-Higgs-doublet model a role analogous to the one of Jarlskog's [9] J in the three-generation standard model. Notice however that, here, there are in principle other sources of CP violation, in the cubic and quartic interactions of the scalars. I will neglect those extra sources of CP violation throughout this work.

It is important to remark that, though J_1 represents CP violation in the mixing of the three neutral scalars, this source of CP violation has nothing to do with the fermions and with the identification of, say, H and R as being scalars, and I as being a pseudoscalar. That identification can only be done when a specific Yukawa Lagrangian, coupling the two scalar doublets to the fermion sector, is considered, which I do not do here. Specifically, as is clear from Eq. (3), R and I play a completely equivalent role in J_1 —indeed, as long as there are no Yukawa couplings, R and I may rotate into each other by a simple U(1) rephasing of H_2. Also, J_1 cannot be the source of, say, CP violation in the kaon system. If fermions are introduced in the model, the mixing of the neutral scalars will in principle lead to more CP violation than simply J_1, because of the Yukawa interactions of the scalars with the fermions [10].

II. GENERAL FEATURES OF THE CALCULATION

Let us consider how CP violation proportional to J_1 arises in the decay modes that I consider here. The tree-level diagram with incoming particles W^-, H^+, and X_1 is proportional to $i(T_{21} - iT_{31})$, while the diagram with incoming particles W^+, H^-, and X_1 is proportional to $-i(T_{21} + iT_{31})$. Now take a look at Fig. 1, in which all the one-loop diagrams which lead to CP violation when interfering with the tree-level diagram are collected. Consider for instance the first diagram, with a loop of W^+W^-, and then X_2, as an intermediate state. That diagram is equal to $i^6T_{11}T_{12}(T_{22} - iT_{32})$, times a certain momentum integral iI_k. The seven factors of i come, three from the vertices, three from the propagators, and one from the Wick rotation in the momentum integral. Therefore, the interference of this diagram with the tree-level one is proportional to the real part of $(-i)(T_{21} + iT_{31})i^6T_{11}T_{12}(T_{22} - iT_{32})iI_k = T_{11}T_{12}(T_{11}T_{12} + iT_{13})I_k$. The T-matrix factor has an imaginary part equal to J_1. Therefore, if the momentum integral has an absorptive (i.e., imaginary) part, then the interference term will include J_1 times that absorptive part. This is CP violating. The absorptive part of the integral plays in the calculation the role of a CP-invariant final-state-interaction phase, which allows J_1 to manifest itself.

We find in a similar fashion that the absorptive parts of all other nine one-loop diagrams in Fig. 1 lead, when one considers the interference of those diagrams with the tree-level one, to CP violation. Indeed, a careful study of the model and all its vertices shows that the ten diagrams in Fig. 1 are the only ones which lead to CP violation proportional to J_1 in this process.[1] The CP violation manifests itself in a difference of the decay rates of $X_1 \to W^+H^-$ and $X_1 \to W^-H^+$, or of $H^+ \to X_1W^+$ and $H^- \to X_1W^-$, whichever pair of decays is kinematically allowed.

At tree level, the amplitude for the decay $X_1 \to W^+H^-$, or for the decay $H^+ \to X_1W^+$, is $(\epsilon_\nu P_H^\nu)ig(T_{21} - iT_{31})$, from the tree-level vertex. Here, ϵ_ν is the polarization vector of the outgoing W^+, and P_H^ν is the incoming momentum of the $H^\pm$. At the the one-loop level, each diagram in Fig. 1 contributes $M_k = (\epsilon_\nu P_H^\nu)g^3C_kiI_k$. Here, C_k are the various i factors and T-matrix factors from the vertices and propagators in the diagram, and iI_k is the momentum integral, with the i coming from the Wick rotation. The amplitudes for the CP-conjugated decays are, at the tree level, $(\epsilon_\nu P_H^\nu)(-i)g(T_{21} + iT_{31})$, and, at the one-loop level from Fig. 1, $(\epsilon_\nu P_H^\nu)g^3(-C_k^*)iI_k$. Notice that, while the momentum integral is the same, the vertex factors are complex conjugated. Then, the CP-violating asymmetries are

$$\frac{B(X_1 \to W^+H^-) - B(X_1 \to W^-H^+)}{B(X_1 \to W^+H^-) + B(X_1 \to W^-H^+)}$$

or

$$\frac{B(H^+ \to X_1W^+) - B(H^- \to X_1W^-)}{B(H^+ \to X_1W^+) + B(H^- \to X_1W^-)}$$

$$\approx 2g^2 \sum_{k=1}^{10} \frac{\mathrm{Im}[(T_{21} + iT_{31})C_k]\,\mathrm{Re}(iI_k)}{1 - T_{11}^2} , \quad (4)$$

where I used the orthogonality of T to write $T_{21}^2 + T_{31}^2 = 1 - T_{11}^2$. The imaginary part of $(T_{21} + iT_{31})C_k$ is simply

[1] There are other diagrams which may also lead to CP violation in this process, but which include other sources of CP violation, in the cubic scalar interactions. I neglect those extra sources of CP violation, just as I neglect fermionic sources of CP violation.

J_1 times a number, typically ± 1 or ± 2. The momentum integral iI_k (the i is from the Wick rotation) has a real part if cuts in the corresponding diagram lead to absorptive parts. Notice that in Eq. (4) I have used the approximation of taking, in the denominator, only the square of the modulus of the tree-level contribution to the amplitude.

It is clear from Eq. 4 that the asymmetry will be of the form

$$g^2\, \frac{2T_{11}T_{12}T_{13}}{1-T_{11}^2}\, A\,, \tag{5}$$

where A represents the sum of the absorptive parts of all the diagrams in Fig. 1, weighted by appropriate numbers ± 1 or ± 2 (see the preceding paragraph). As one is interested in how large the asymmetry can be, I now consider the mixing-matrix factor in Eq. (5). Because one is constrained by the orthogonality condition $T_{21}^2 + T_{31}^2 = 1 - T_{11}^2$, it is obvious that that factor will be maximal when $|T_{21}| = |T_{31}|$, and we will then have

$$\frac{2T_{11}T_{12}T_{13}}{1-T_{11}^2} = T_{11}\,. \tag{6}$$

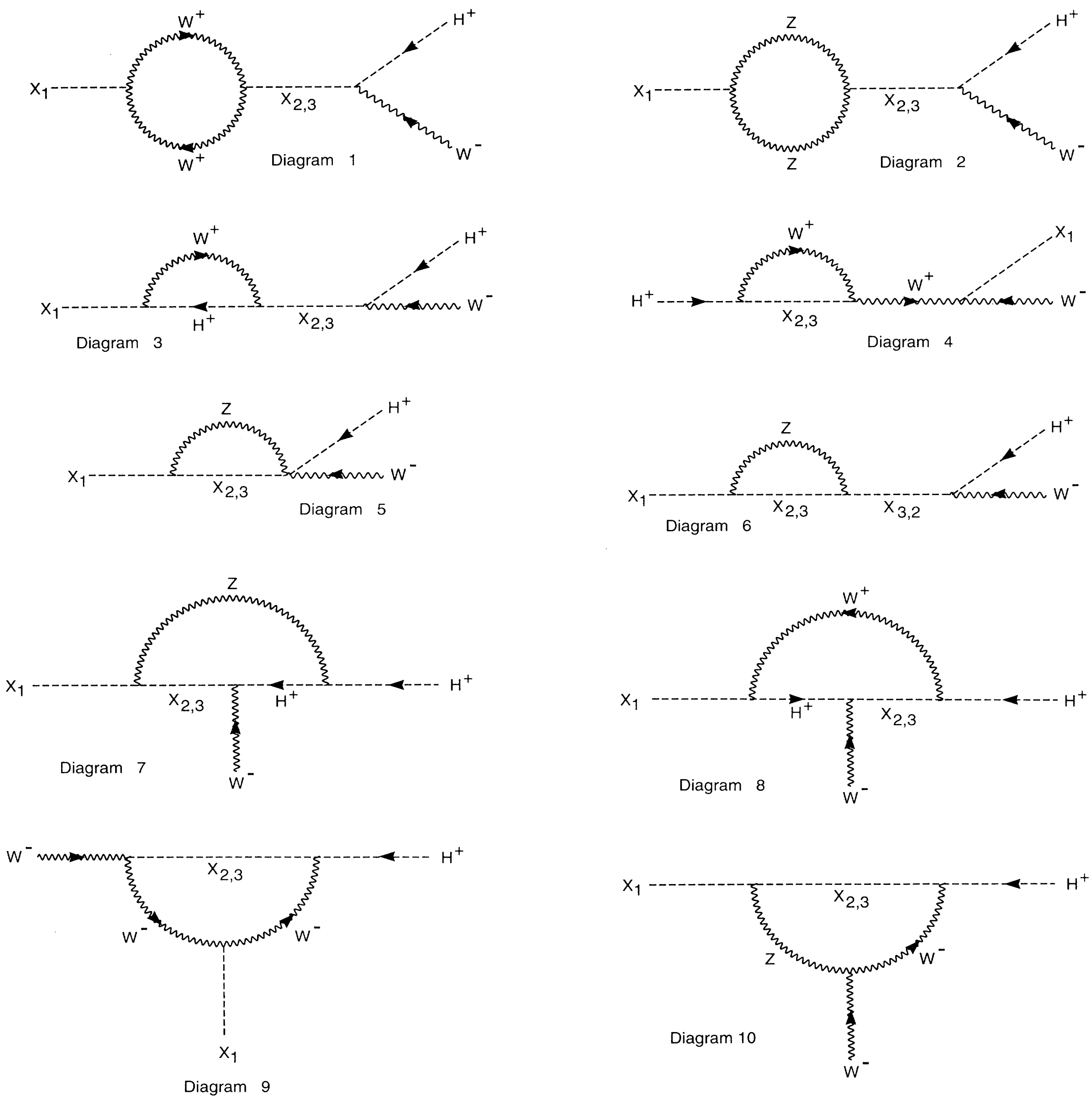

FIG. 1. The one-loop diagrams with external incoming particles W^-, H^+, and X_1, which lead to CP violation upon interfering with the tree-level diagram, if the integrals have absorptive parts. In each case, the $W^\pm$ and the Z^0 in internal lines may be substituted by the corresponding Goldstone bosons $G^\pm$ and G^0, respectively; in the first two diagrams, they may be substituted by ghost loops as well.

Clearly, the order of magnitude of this quantity T_{11} is 1. It is at this point important to remark that, in this specific example, the CP-violating asymmetry approaches its maximum when the decay rate decreases. Indeed, as $T_{11} \to 1$, the asymmetry becomes potentially larger (as long as $|T_{21}|$ remains equal to $|T_{31}|$), but the decay rate, which is proportional to $1 - T_{11}^2$, approaches zero. Similarly, the decay rate becomes larger if $T_{11} \to 0$, but then $J_1 \to 0$ and the CP asymmetry also vanishes. This situation is reminiscent of the case of CP violation in decay modes of the B^0 mesons, which is generally predicted to be larger when the branching ratios are smaller, and vice versa.

III. GAUGE INVARIANCE

A way to check that the diagrams in Fig. 1 are all the relevant ones is to check whether their computation yields a gauge-invariant result. Because I just want to compute the absorptive parts of the diagrams, which are finite, it is sufficient to compute the diagrams in the unitary gauge, in which no Goldstone bosons and no ghosts are present. However, in a general 't Hooft gauge, in each of the diagrams in Fig. 1, a $G^\pm$ can be used instead of the $W^\pm$, or a G^0 can be used instead of the Z^0. In the two diagrams which have a loop only of $W^\pm$ or of Z^0 (diagrams 1 and 2), ghost loops must also be considered in a 't Hooft gauge. Now, in a 't Hooft gauge, the $W^\pm$ propagator contains an extra piece (relative to the unitary gauge) in which the $W^\pm$ has an unphysical squared mass W. Similarly, the charged Goldstone bosons $G^\pm$ and the charged ghosts $c^\pm$ have unphysical squared mass W in a 't Hooft gauge. When W is not infinite as in the unitary gauge, each diagram by itself contains a W-dependent absorptive part. However, all those unphysical absorptive parts must cancel out when one considers the whole set of diagrams. The same thing can be said about the propagators of the Z^0, which has a piece with unphysical squared mass Z, and of the Goldstone boson G^0 and ghost c^0, which have squared mass Z. (In principle, $Z \neq W$.) The sum over all diagrams of all the absorptive parts must be independent of both W and Z. I have checked that independence.

To be sure, gauge independence only applies to an observable quantity. Thus, gauge-independence in this case only occurs when (1) the three external particles are all on mass shell, that is, $P_H^2 = m_H^2$, $P_W^2 = m_W^2$, and $P_1^2 = m_1^2$, where P_H, P_W, and P_1 are the incoming momenta of the external $H^\pm$, $W^\pm$, and X_1, respectively, (2) one suppresses from the amplitude all terms proportional to P_W^ν, because the amplitude must be multiplied by the polarization vector ϵ_ν of the $W^\pm$, and $\epsilon_\nu P_W^\nu = 0$, (3) one considers, from each one-loop diagram, only the part which is proportional to J_1 upon interference with the tree-level amplitude, and (4) one only considers the absorptive part of each one-loop diagram. One has also to take into consideration that the gauge-dependent absorptive parts sometimes cancel between two similar diagrams with intermediate virtual particle X_3 instead of X_2, whenever those absorptive parts do not depend on the mass of that intermediate particle (m_3 or m_2); while other absorptive parts cancel among different-looking diagrams.

IV. CONTRIBUTION OF EACH DIAGRAM

For each diagram, I have taken separately the factors with T-matrix elements, and all the i factors from both the vertices and the propagators, multiplied that by the factor $(T_{21} + iT_{31})$ arising in the interference with the tree-level diagram, and taken the imaginary part of the result. That imaginary part is always a multiple of J_1. As a consequence, each momentum integral presented below should be looked upon as being an iI_k in the notation of Sec. II (the i arising when the Wick rotation is performed), and only the absorptive parts of each such momentum integral are meaningful in this context.

I first define the various combinations of the integration momentum k and of the incoming momenta, and of the masses, which appear in the denominators of the momentum integrals. They are

$$D_1 \equiv k^2 - m_W^2\,, \tag{7}$$

$$D_2 \equiv k^2 + 2k\cdot P_1 + m_1^2 - m_W^2\,, \tag{8}$$

$$D_3 \equiv k^2 + 2k\cdot P_1 + m_1^2 - m_H^2\,, \tag{9}$$

$$D_4 \equiv k^2 - 2k\cdot P_H + m_H^2 - m_2^2\,, \tag{10}$$

$$D_5 \equiv k^2 + 2k\cdot P_W + m_W^2 - m_Z^2\,, \tag{11}$$

$$D_6 \equiv k^2 + 2k\cdot P_W - 2k\cdot P_H - m_1^2 + 2m_W^2 + m_H^2\,. \tag{12}$$

It is convenient to define the "triangular function" λ:

$$\lambda(A,B,C) \equiv A^2 + B^2 + C^2 - 2(AB + AC + BC)\,. \tag{13}$$

This function is negative if and only if one can form a triangle with sides of length $\sqrt{A}$, $\sqrt{B}$, and $\sqrt{C}$.

I now present the results for the first three diagrams. Diagram 1:

$$M_1 = \epsilon_\nu P_H^\nu\, g^3\, J_1\, \frac{m_3^2 - m_2^2}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)}\, \frac{-m_1^4 + 4m_1^2 m_W^2 - 12 m_W^4}{4m_W^2} \int \frac{d^4k}{(2\pi)^4}\, \frac{1}{D_1 D_2}\,. \tag{14}$$

Diagram 2:

$$M_2 = \epsilon_\nu P_H^\nu\, g^3\, J_1\, \frac{m_3^2 - m_2^2}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)}\, \frac{-m_1^4 + 4m_1^2 m_Z^2 - 12 m_Z^4}{8m_Z^2} \int \frac{d^4k}{(2\pi)^4}\, \frac{1}{D_1(m_W \to m_Z) D_2(m_W \to m_Z)}\,. \tag{15}$$

Diagram 3:

$$M_3 = \epsilon_\nu P_H^\nu\, g^3\, J_1 \, \frac{m_3^2 - m_2^2}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)} \, \frac{\lambda(m_1^2, m_W^2, m_H^2)}{2m_W^2} \int \frac{d^4k}{(2\pi)^4} \, \frac{1}{D_1 D_3} \, . \tag{16}$$

The results for each of these three diagrams are separately gauge invariant. This is because the gauge-dependent piece in each of them (the piece depending on either W or Z) is the same, for each case, in the diagram with an intermediate X_2, and in the diagram with an intermediate X_3. As the sign of the J_1 factor is opposite, the gauge-dependent piece cancels out between the diagrams with intermediate X_2 and those with intermediate X_3. The same phenomenon partially occurs in all other diagrams but, in each of them individually, some gauge dependence always remains, as is seen in the following.

Diagram 4:

$$M_4 = -\epsilon_\nu\, g^3\, J_1 \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{P_H^\nu (m_H^2 - m_2^2)}{4m_W^2 D_4 (k^2 - W)} + \frac{k^\nu (m_W^2 + m_H^2 - m_2^2)}{2D_1 D_4 (m_H^2 - m_W^2)} - P_H^\nu \, \frac{\lambda(m_2^2, m_W^2, m_H^2) + 4m_W^4}{4D_1 D_4 m_W^2 (m_H^2 - m_W^2)} - (m_2 \to m_3) \right\} . \tag{17}$$

Diagram 5:

$$M_5 = -\epsilon_\nu\, g^3\, J_1 \, \frac{m_Z^2 - m_W^2}{m_W^2} \int \frac{d^4k}{(2\pi)^4} \left\{ k^\nu \, \frac{m_2^2 - m_1^2}{4m_Z^2 D_4 [(k + P_W)^2 - Z]} + \frac{1}{D_4 D_5} \left(-\frac{P_H^\nu}{2} + k^\nu \, \frac{m_Z^2 - m_2^2 + m_1^2}{4m_Z^2} \right) - (m_2 \to m_3) \right\} . \tag{18}$$

Diagram 6:

$$M_6 = \epsilon_\nu P_H^\nu\, g^3\, J_1 \, \frac{1}{4} \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{m_2^2 - m_1^2}{m_W^2} \, \frac{1}{D_4 [(k + P_W)^2 - Z]} + \frac{\lambda(m_Z^2, m_1^2, m_2^2)}{D_4 D_5 m_W^2 (m_1^2 - m_3^2)} - (m_2 \leftrightarrow m_3) \right\} . \tag{19}$$

Diagram 7:

$$M_7 = -\epsilon_\nu\, g^3\, J_1 \, \frac{2m_W^2 - m_Z^2}{4m_W^2} \int \frac{d^4k}{(2\pi)^4} (P_H - k)^\nu \left\{ \frac{m_1^2 - m_2^2}{m_Z^2 D_4 [(k + P_W)^2 - Z]} + \frac{1}{D_4 D_6} + \frac{m_2^2 - m_1^2 - m_Z^2}{m_Z^2} \, \frac{1}{D_4 D_5} + (2m_W^2 + m_Z^2 - 2m_H^2 - m_1^2 - m_2^2) \frac{1}{D_4 D_5 D_6} - (m_2 \leftrightarrow m_3) \right\} . \tag{20}$$

Diagram 8:

$$M_8 = \epsilon_\nu\, g^3\, J_1 \, \frac{1}{2m_W^2} \int \frac{d^4k}{(2\pi)^4} (k - P_H)^\nu \left[\frac{m_H^2 - m_2^2}{D_4 (k^2 - W)} + \frac{m_2^2 - m_H^2 - m_W^2}{D_1 D_4} + \frac{m_W^2}{D_3 D_4} + \frac{1}{D_1 D_3 D_4} (3m_W^4 - m_H^4 + m_1^2 m_H^2 - m_1^2 m_2^2 + m_H^2 m_2^2 - m_1^2 m_W^2 - m_2^2 m_W^2 - 2m_W^2 m_H^2) - (m_2 \leftrightarrow m_3) \right] . \tag{21}$$

Diagram 9:

$$M_9 = -\epsilon_\nu\, g^3\, J_1 \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{(k - P_H)^\nu}{4m_W^2} \, \frac{m_H^2 - m_2^2}{D_4 [(k + P_1)^2 - W]} + \frac{(k - P_H)^\nu}{4m_W^2} \, \frac{m_H^2 - m_2^2}{D_4 (k^2 - W)} + \frac{(k - P_H)^\nu}{4} \left(\frac{1}{D_2 D_4} - \frac{1}{D_1 D_4} \right) - \frac{(k - P_H)^\nu}{4} \, \frac{m_H^2 - m_2^2}{m_W^2} \left(\frac{1}{D_2 D_4} + \frac{1}{D_1 D_4} \right) + \frac{1}{D_1 D_2 D_4} \left[\frac{-6m_W^2 + 2m_H^2 + m_1^2}{4} P_H^\nu + \frac{4m_W^2 - 2m_2^2 - m_1^2}{4} k^\nu + \frac{m_1^2 (m_H^2 - m_2^2)}{4m_W^2} (k - P_H)^\nu \right] - (m_2 \leftrightarrow m_3) \right\} . \tag{22}$$

Diagram 10:

$$M_{10} = -\epsilon_\nu \, g^3 \, J_1 \, \frac{1}{4} \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{1}{(k^2 - W)[(k+P_W)^2 - Z]} \, k^\nu \, \frac{m_2^2}{m_W^2} \right.$$
$$+ \frac{1}{D_4[(k+P_W)^2 - Z]} \, (k - 2P_H)^\nu \, \frac{m_1^2 - m_2^2}{m_Z^2} + \frac{1}{D_4(k^2 - W)} \, (k - 2P_H)^\nu \, \frac{m_H^2 - m_2^2}{m_W^2}$$
$$-k^\nu \left(\frac{1}{D_1 D_4} + \frac{1}{D_4 D_5} \right) + k^\nu \, \frac{1}{D_1 D_4 D_5} \, (4m_2^2 - 2m_W^2 + 2m_1^2 + 2m_H^2 - m_Z^2)$$
$$+k^\nu \, \frac{(m_2^2 - m_H^2)(m_1^2 - m_2^2)}{m_W^2} \, \frac{1}{D_1 D_4 D_5} - k^\nu \, \frac{m_2^2}{m_W^2 D_1 D_5}$$
$$+(2P_H - k)^\nu \left[\frac{3m_2^2 + 2m_W^2 - m_1^2 - 2m_H^2}{D_1 D_4 D_5} + \frac{m_1^2 - m_2^2}{m_Z^2} \, \frac{1}{D_4 D_5} \right.$$
$$\left. \left. + \, \frac{m_H^2 - m_2^2}{m_W^2} \left(\frac{1}{D_1 D_4} + \frac{m_Z^2}{D_1 D_4 D_5} \right) \right] - (m_2 \to m_3) \right\} . \tag{23}$$

It is simple to check now that most terms depending on either of the unphysical squared-masses W or Z cancel among M_4 through M_{10}. The only two exceptions are the first terms in the curly brackets in the expressions for M_9 and for M_{10}. However, it is easily found that the absorptive parts of both these integrals are proportional to P_W^ν, and therefore give a vanishing contribution to the amplitude. This ensures the gauge invariance of the whole computation. In each of Eqs. (17)–(23), therefore, one only has to consider the terms independent of W and of Z. Those terms are the ones that would have been obtained had the computation been performed directly in the unitary gauge. Some of those terms, however, still yield zero absorptive contributions.

V. RESULTS

The asymmetry is equal to $g^2 \approx 0.43$ times a mixing-matrix factor, studied in Sec. II, which should be of order 0.1 to 1, times the sum A of all absorptive parts, which itself includes a suppression factor $1/(16\pi)$.

Now, one should note that the absorptive parts of diagrams 1, 2, 3, and 6 all diverge when m_2 (or m_3) approach m_1. This is simply because in those diagrams one has a scalar X_2 (or X_3) propagating with momentum P_1 such that $P_1^2 = m_1^2$. Those divergences do not cancel in the absorptive parts, because the specific values of those absorptive parts depend on different parameters. Of course, we know that these divergences are not genuine, they would be eliminated by a proper treatment in which one would take into account the finite width of the propagating X_2 or X_3, and, in addition, from a different line of reasoning [8], one knows anyway that CP violation disappears and J_1 loses its meaning when the masses of any two of the three neutral scalars become equal. A proper treatment of these divergences at $m_1 = m_2$ or $m_1 = m_3$ would lead me far astray, and therefore I simply avoided, in general, considering the region of the parameter space in which either m_2 or m_3 are close to m_1.

TABLE I. A few examples of the sum A.

m_H (GeV)	m_1 (GeV)	m_2 (GeV)	m_3 (GeV)	A
300	150	250	60	-5.23×10^{-2}
200	100	70	450	1.08×10^{-2}
200	500	100	150	-1.90×10^{-2}
250	500	250	80	5.61×10^{-2}
300	100	200	400	5.28×10^{-3}

Avoiding those regions in which the present approximation loses its validity, I find that the sum A of all absorptive parts is typically of order of magnitude 10^{-3} or 10^{-2}. A few examples are presented in Table I.

It is worth remarking that the total absorptive part A is always the final result of substantial cancellations among the absorptive parts, with different signs, of the various individual diagrams.

VI. CONCLUSIONS

In this paper I have presented a model calculation of a CP-violating asymmetry in the two-scalar-doublet model. The asymmetry chosen has been the different decay rates for $X_1 \to H^+W^-$ and for $X_1 \to H^-W^+$. I believe this to be a quite interesting place to look for CP violation in the scalar sector, even if the present calculation turns out not to be very relevant. This might happen mainly because I have taken into account only one source of CP violation, J_1 in the gauge interactions of the scalars, while I neglected further sources of CP violation, in the cubic scalar interactions and in the Yukawa interactions with the fermions. Those further sources of CP violation will in principle lead to extra contributions to the total asymmetry.

My interest here has been to ilustrate the specific way in which J_1 arises in the computation of a CP asymmetry. I observed that there is a kind of balance between the CP asymmetry and the decay rate in this specific case, but only if the only source of CP violation is taken to be J_1, with a large asymmetry being possible only when the decay rate is small, and vice versa.

I found that the asymmetry can attain values of order 10^{-2}. These values would increase or decrease if the interference with other sources of CP violation in this mode were constructive or destructive.

Because of the presence of gauge bosons in the internal lines of the one-loop diagrams that I had to compute, I found it convenient to check the gauge invariance of the whole calculation. I checked that the fictitious masses that appear in the propagators of the $W^{\pm}$ and of the Z^0, and of the corresponding Goldstone bosons and ghosts, in a general 't Hooft gauge, lead to gauge-dependent absorptive parts for the individual diagrams, which however cancel out when all the diagrams which lead to CP violation proportional to J_1 are considered. This constitutes a good check that one did not omit any diagram.

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