

## Normalization of QCD effects in $O(m_t^2)$ electroweak corrections

Brian H. Smith

*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455*

M. B. Voloshin

*Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455  
and Institute of Theoretical and Experimental Physics, Moscow 117259*

(Received 7 February 1994)

We point out that, contrary to some recent claims, there is no intrinsic long-distance uncertainty in the perturbative calculation of the QCD effects in the  $t\bar{t}$  and  $b\bar{b}$  loops giving the electroweak corrections proportional to  $m_t^2$ . If these corrections are expressed in terms of the “on-shell” mass  $m_t$ , the only ambiguity arising is that associated with the definition of the “on-shell” mass of a quark. The latter is entirely eliminated if the result is expressed in terms of  $m_t$  defined at short distances. Applying the Brodsky-Lepage-Mackenzie criterion for determining the natural scale for normalization of  $\alpha_s$ , we find that using the “on-shell” mass makes this scale numerically small in units of  $m_t$ . Specifically, we find that by this criterion the first QCD correction to the  $O(m_t^2)$  terms is determined by  $\alpha_s^{\overline{\text{MS}}}$  ( $0.15m_t$ ). Naturally, a full calculation of three-loop graphs is needed to completely quantify the scale.

PACS number(s): 12.15.Lk, 12.38.Bx, 14.65.Ha, 14.70.-e

The present remarkable statistics and accuracy of the data from the CERN  $e^+e^-$  collider LEP at the energy of the  $Z$  peak calls for a significant theoretical precision of calculation of the electroweak loop effects with the goal of sensing the effects of Higgs boson and/or new physics once the top quark mass is known (for a recent review see, e.g., Ref. [1]). Because of the large mass of the top quark, the effects of the  $t\bar{t}$  and  $b\bar{b}$  loops, proportional to  $m_t^2$ , have to be calculated including the QCD corrections. Specifically, the leading in the limit  $m_t^2/m_Z^2 \gg 1$  electroweak corrections, related to the  $W$  and  $Z$  propagators, are universally determined [1] by the finite difference of the longitudinal parts of the  $Z$  boson and the  $W$  boson vacuum polarization insertions at  $q^2 \rightarrow 0$ . When expressed for definiteness in terms of the correction to the electroweak  $\rho$  parameter the effect of the heavy quark loop is given by

$$\Delta\rho = \frac{3G_\mu}{8\pi^2\sqrt{2}} m_t^2 \left( 1 - \frac{2\alpha_s}{9\pi} (\pi^2 + 3) \right) \quad (1)$$

in the lowest [2] and the first [3] orders in  $\alpha_s$ . The present accuracy of the data already makes necessary a quantitative understanding of the magnitude of the  $\alpha_s$  term as well as of the higher QCD corrections.

Recently doubt was cast [4–6] on the calculability of the higher QCD effects in expression (1) in terms of the QCD coupling, normalized at distances of order  $m_t^{-1}$ :  $\alpha_s(m_t)$ . The reason for this doubt arises in a calculation of the vacuum polarization at  $q^2 \approx 0$ , i.e., far below the  $t\bar{t}$  and  $b\bar{b}$  thresholds, using the dispersion relations, which involve integrals over the spectral densities of the physical states, containing  $t\bar{t}$  and  $b\bar{b}$  near and above the corresponding thresholds. The resonances and the continuum states near the threshold are governed by the perturbative and nonperturbative dynamics at long dis-

tances, which are much larger than  $m_t^{-1}$ , hence arises the doubt about the calculability of the vacuum polarization at  $q^2 \rightarrow 0$  in terms of  $\alpha_s(m_t)$ . Here we point out that this doubt is ungrounded and that the solution of this problem was known long ago, at least since the development of the QCD sum rules for charmonium [7]. Moreover, this is exactly the central point of the QCD sum rules, that though each individual hadronic state is governed by *long-distance* dynamics, the dispersion integrals over these states, which give the vacuum polarization far below the threshold, are determined by the *short-distance* QCD dynamics. This point, which was also emphasized in a recent paper [8], is further discussed below in the text.

The long-distance QCD effects however produce a certain effect on expression (1) through the convention associated with it. Namely, the result in Eq. (1) is written in terms of the on-shell mass  $m_t$ , which, as discussed in this paper, effectively lowers the appropriate normalization point for  $\alpha_s$  in Eq. (1) through the contribution of the near-mass-shell region to the evolution of the quark mass from the mass shell to distances of order  $m_t^{-1}$ , which are relevant in the loops. In connection with this observation it should be emphasized that this effect by no means makes the perturbative calculation uncontrollable: the notion of the on-shell quark mass is consistent in any finite order of perturbation theory in QCD and the discussed effect manifests itself in a numerical, rather than parametrical reduction of the appropriate normalization momentum scale for  $\alpha_s$  in units of  $m_t$ . Equivalently, this implies that if the QCD corrections to the  $O(m_t^2)$  electroweak corrections are expressed in terms of  $\alpha_s(m_t)$  the coefficients of higher terms should be unnaturally large. Beyond the perturbation theory there is the known problem of defining the on-shell quark mass, which amounts to an uncertainty of the order of  $\Lambda_{\text{QCD}}$ , which effectively places the limit on the accuracy of definition of the quark

mass. This is the well-known case for the  $b$  quark and this particular uncertainty should be the same for the  $t$  quark. For the latter the situation is somewhat additionally complicated by the large width of the decay  $t \rightarrow Wb$ . Nevertheless, one can, quite probably, get to the accuracy of defining the “on-shell” top mass better than  $\sim 1$  GeV.

One can notice however that the problem of properly defining  $m_t$  is somewhat artificial for the electroweak corrections at the  $Z$  peak. This problem can be completely eliminated by expressing those corrections through the top mass, measured at short distances in some other measurable quantity, determined by the short-distance dynamics. As examples of such quantities one can pick the  $m_t$  entering the electroweak corrections to the  $Z \rightarrow b\bar{b}$  decay rate through the  $t\bar{t}W$  triangle or the total width of the decay  $t \rightarrow Wb$ , either of which is determined by distances of order  $m_t^{-1}$ , or any other measurable  $m_t$ -dependent quantity of the same nature. In other words, the relation between the electroweak corrections of the type in Eq. (1) and the quantities such as the total decay rate of the top should not contain unnaturally large or small numerical coefficients in units of  $m_t$  in the scale of normalization of  $\alpha_s$ .

Naturally, to completely fix the normalization point for  $\alpha_s$  in Eq. (1) one needs a full three-loop calculation in the order  $\alpha_s^2$ , which has not been done yet. However as pointed out some time ago by Brodsky, Lepage, and

Mackenzie [9] (BLM) one gets an appropriate estimate of the normalization point and thereby makes the coefficient of the higher-order term reasonable by evaluating the lower-order graphs with an explicitly running coupling constant. Formally, this corresponds to tagging the dependence of the higher loop term on the number of light quark flavors  $n_f$  and then shifting this dependence in the combination  $b_0 = 11 - \frac{2}{3}n_f$  into the definition of the normalization point of  $\alpha_s$  in the lower term. Here we apply this procedure to the calculation of the electroweak correction in Eq. (1), and find that the BLM criterion gives the normalization point of  $\alpha_s$  in the modified minimal subtraction scheme ( $\overline{\text{MS}}$ ) scheme as low as  $0.15m_t$ . The appearance of the small coefficient 0.15 is mainly due to the usage of the on-shell top quark mass.

For the practical calculation we use the simple fact [10] that when calculating the correction in Eq. (1) in the limit  $m_Z^2/m_t^2 \ll 1$  one can neglect the masses of gauge bosons altogether, which is equivalent to setting the electroweak gauge couplings to zero [notice that the expression in Eq. (1) contains as the overall factor only the top quark Yukawa coupling  $h_t$ ]. Therefore the quantity of interest can be expressed in terms of the dynamics of only the scalar sector coupled to the  $t$  and  $b$  quarks. In these terms the correction  $\Delta\rho$  is expressed through the vacuum polarization by the quark density operators coupled to the Goldstone bosons  $\chi^0$  and  $\chi^\pm$ :

$$\begin{aligned} P_0(q^2) &= -2i \int \langle 0 | T \{ [\bar{t}(x)\gamma_5 t(x)] [\bar{t}(x)\gamma_5 t(x)] \} | 0 \rangle e^{iqx} d^4x, \\ P_\pm(q^2) &= i \int \langle 0 | T \{ [t(x)(1 - \gamma_5)b(x)] [b(x)(1 + \gamma_5)t(x)] \} | 0 \rangle e^{iqx} d^4x \end{aligned} \quad (2)$$

(the mass of the  $b$  quark is entirely neglected throughout this paper). The electroweak correction  $\Delta\rho$  is found as [10]

$$\Delta\rho = \frac{G_\mu}{\sqrt{2}} m_t^2 [P'_\pm(0) - P'_0(0)], \quad (3)$$

where  $P'(q^2) = dP(q^2)/dq^2$ .

Consider now a calculation of the difference of the derivatives of the vacuum polarization operators in Eq. (3) by the conventional Feynman diagram technique. The one-loop graphs (no additional gluons) give logarithmic divergence for each of the derivatives, but their difference is finite and the result is the leading term in Eq. (1). Originally the two-loop graphs, which led to the  $O(\alpha_s)$  correction in Eq. (1), were calculated [3] by using the dispersion relations. Here we concentrate on the direct calculation of these graphs by Feynman's technique, which transparently reveals the structure of Euclidean distances contributing to the first QCD correction. For this purpose we first consider the integration over the quark loop, which allows one to represent the result as an integral over the gluon momentum. One can notice that starting with the first-order QCD correction the integration over the quark loop is finite for each of the derivative

terms in Eq. (3). Also at  $q^2 \approx 0$  there is no obstruction to the Wick rotation. Thus we find in terms of integrals over the Euclidean momentum of the gluon the expressions for the first QCD corrections to the derivatives of the vacuum polarization operators (2):

$$\delta^{(1)} P'_\pm(0) = \frac{1}{4\pi^3} \int w_\pm \left( \frac{k}{m_t} \right) \alpha_s d(k^2) k^2 \frac{dk^2}{m_t^2}, \quad (4)$$

$$\delta^{(1)} P'_0(0) = \frac{1}{4\pi^3} \int w_0 \left( \frac{k}{m_t} \right) \alpha_s d(k^2) k^2 \frac{dk^2}{m_t^2},$$

with  $d(k^2) = 1/k^2$  being the transverse gluon propagator<sup>1</sup> and the weight functions  $w_\pm$  and  $w_0$ , determined from the quark loop integration, are given by

$$\begin{aligned} w_\pm(x) &= 2 \frac{12 + 16x^2 + 8x^4 + x^6}{x\sqrt{4+x^2}} \operatorname{arctanh} \left( \frac{x}{\sqrt{4+x^2}} \right) \\ &\quad - 2 \frac{(1+x^2)^3}{x^2} \ln(1+x^2) + x^4 \ln x^2 - 4 \end{aligned} \quad (5)$$

<sup>1</sup>Expressions (4) are gauge invariant, hence it is only the transverse gluon propagator that is contributing.

and

$$w_0(x) = 12 \frac{2 + x^2}{x(4 + x^2)^{3/2}} \operatorname{arctanh} \left( \frac{x}{\sqrt{4 + x^2}} \right) - \frac{6}{4 + x^2}. \quad (6)$$

Correspondingly the difference, entering Eq. (3), can be written in the form

$$P'_\pm(0) - P'_0(0) = \frac{1}{4\pi^3} \int w \left( \frac{k}{m_t} \right) \alpha_s d(k^2) k^2 \frac{dk^2}{m_t^2} \quad (7)$$

with the resultant weight function

$$\begin{aligned} w(x) &= w_\pm(x) - w_0(x) \\ &= 2 \frac{36 + 70x^2 + 48x^4 + 12x^6 + x^8}{x(4 + x^2)^{3/2}} \operatorname{arctanh} \left( \frac{x}{\sqrt{4 + x^2}} \right) \\ &\quad - 2 \frac{(1 + x^2)^3}{x^2} \ln(1 + x^2) + x^4 \ln x^2 - 2 \frac{5 + 2x^2}{4 + x^2}. \end{aligned} \quad (8)$$

At large  $x$  the function  $w(x)$  has the asymptotic behavior  $w(x) = 3x^{-2} + O(x^{-4})$ , so that the integral in Eq. (7) is logarithmically divergent. This divergence is regularized, once the gluon propagator is regularized by any standard procedure. Here we do not need to specify the regularization procedure, since it can be easily noticed that expression (3) for the measurable quantity  $\Delta\rho$  also contains the factor  $m_t^2$  and the divergence in Eq. (7) is the same as in the renormalization of  $m_t^2$ . More specifically the  $\alpha_s$  correction to  $\Delta\rho$  in terms of the top quark on-shell mass is determined by

$$\begin{aligned} \delta(m_t^2 [P'_\pm(0) - P'_0(0)]) \\ = m_t^2 \left( \frac{2}{m_t} \Sigma(m) + \delta(P'_\pm(0) - P'_0(0)) \right), \end{aligned} \quad (9)$$

where  $\Sigma(\gamma \cdot p)$  is the one-loop top quark self-energy. The on-shell value of the self-energy can be written as an integral over the Euclidean  $k^2$  of the gluon in the loop as

$$\frac{2}{m_t} \Sigma(m) = \frac{1}{4\pi^3} \int s \left( \frac{k}{m_t} \right) \alpha_s d(k^2) k^2 \frac{dk^2}{m_t^2} \quad (10)$$

with the weight function

$$s(x) = \frac{x^4 + 2x^2 - 8}{2x\sqrt{4 + x^2}} - \frac{x^2}{2}. \quad (11)$$

Therefore the final expression for the first  $\alpha_s$  correction to  $\Delta\rho$  [Eq. (9)] in terms of the on-shell mass  $m_t$  is proportional to the finite integral

$$\int_0^\infty [w(x) + s(x)] dx^2 = -\frac{\pi^2}{3} - 1, \quad (12)$$

which reproduces the known result [3] in Eq. (1).

However, the structure of the integral in Eq. (12) deserves a closer look. Indeed, as one can see from the plots of the weight functions  $s(x)$  and  $w(x)$  shown in Fig. 1, the integral is significantly contributed by the region of small  $x$  due to the  $-2/x$  behavior of the function  $s(x)$  in that region, i.e., at  $k \ll m_t$ . This behavior is clearly a consequence of the fact that the on-shell mass  $m_t$  is chosen as the parameter in the electroweak loop. As to the weight function of the electroweak loop itself,  $w(x)$ , as is clearly seen from the plot, it is completely dominated by the region  $x \geq 1$ , i.e.,  $k \geq m_t$ , and thus it displays practically no sensitivity to long distances.

The significance of the contribution of the region of small Euclidean  $k$  can be quantified in this calculation by applying the BLM criterion [9] for the normalization point of  $\alpha_s$ . The BLM procedure amounts to replacing the bare gluon propagator  $\alpha_s d(k^2) = \alpha_s/k^2$  by the one with the running coupling constant:  $\alpha_s^V(k)/k^2$ , where  $\alpha_s^V(k)$  is the effective coupling constant in the potential between two infinitely heavy quarks. The running constant  $\alpha_s^V(k)$  in the Coulomb gauge corresponds to including the vacuum polarization insertions in the propagator of the Coulomb gluon. In particular the BLM procedure applied to the calculation in the first order in  $\alpha_s$  correctly reproduces the dependence of the coefficient of the next term  $\alpha_s^2$  on the number  $n_f$  of light quark flavors, since these enter only through the loop insertion in the gluon propagator. However, this procedure additionally combines the  $n_f$  dependence into the factor  $b_0 = 11 - \frac{2}{3}n_f$ ,

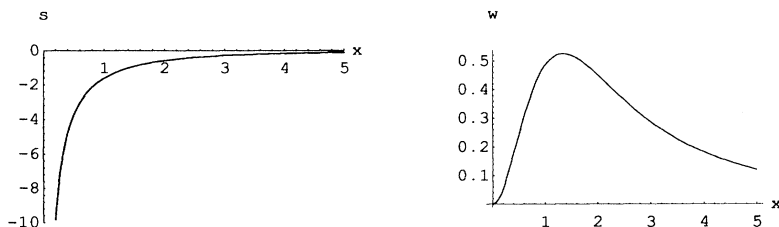


FIG. 1. The weight functions  $s(x)$  (left) and  $w(x)$  (right) vs  $x = k/m_t$ , in the integrals over the Euclidean gluon momentum in the top quark self-energy [Eq. (10)] and in the heavy quark loop [Eq. (7)]. Note the strongly different scale of the vertical axis in the plots.

which is the first coefficient of the QCD  $\beta$  function. In numerous examples [9,11] applying this criterion to the choice of the normalization point for the coupling constant removes large coefficients in the subsequent term.<sup>2</sup>

To apply the BLM procedure to the calculation of  $\Delta\rho$  we use in the integrals in Eqs. (7) and (10) the one-loop effective coupling

$$\alpha_s d(k^2) \rightarrow \frac{1}{k^2} \alpha_s^V(m_t) \left( 1 - \frac{\alpha_s^V(m_t) b_0}{2\pi} \ln(k/m_t) \right). \quad (13)$$

Then  $\alpha_s$  times the integral in Eq. (12) is replaced by

$$\begin{aligned} \alpha_s^V(m_t) \int_0^\infty \left( 1 - \frac{\alpha_s^V(m_t) b_0}{2\pi} \ln(x) \right) [w(x) + s(x)] dx^2 \\ \approx -\alpha_s^V(m_t) \left( \frac{\pi^2}{3} + 1 \right) \left( 1 + 1.034 \frac{\alpha_s^V(m_t) b_0}{2\pi} \right) \\ \approx -\alpha_s^V(0.355m_t) \left( \frac{\pi^2}{3} + 1 \right), \end{aligned} \quad (14)$$

which fixes the normalization point in terms of  $\alpha_s^V$  [the integral with the factor  $\ln(x)$  was calculated numerically]. Furthermore, the normalization of the effective coupling  $\alpha_s^V(k)$  is simply related [9] to that of the  $\alpha_s^{\overline{\text{MS}}}$ :

$$\alpha_s^V(k) = \alpha_s^{\overline{\text{MS}}}(e^{-5/6}k) \approx \alpha_s^{\overline{\text{MS}}}(0.435k).$$

Therefore from Eq. (14) we find that within the BLM scheme the effective coupling, entering the first QCD correction in Eq. (1) is  $\alpha_s^{\overline{\text{MS}}}(0.154m_t)$ . It is clear, however, that to completely quantify the magnitude of the QCD correction to  $\Delta\rho$  a full three-loop calculation of the terms  $\alpha_s^2$  is needed.

It is clear that a similar calculation, operating only with Euclidean-space integrations over the momenta of the gluons, can in principle be performed in higher orders of the QCD perturbation theory, thus making it free from the long-distance uncertainties. Exactly this point was discussed at length in connection with the QCD sum rules in the papers [7] and also in a later review [12]. Here we would like to point out a specific loophole in the reasoning of the recent papers [4–6], which state that through the dispersion relations the contribution of the near-threshold region to  $P'(0)$  makes the latter quantity sensitive to the long-distance dynamics. We also disagree with the argumentation of [13], where it is argued that the threshold effects are *numerically* small. We insist here that the long-distance effects are small *parametrically*, i.e., suppressed by powers of  $\Lambda_{\text{QCD}}/m_t$  at the non-

perturbative level, and are absent altogether in any finite order of perturbation theory in QCD.

The reasoning in those papers is as follows. The difference of the derivatives of the vacuum polarization operators in Eq. (3) can be written in the form of the dispersion integral

$$P'_\pm(0) - P'_0(0) = \frac{1}{\pi} \int \frac{\rho_\pm(s) - \rho_0(s)}{s^2} ds, \quad (15)$$

where  $\rho_\pm(s)$  and  $\rho_0(s)$  are, respectively, the spectral densities of the operators  $[t(1 - \gamma_5)b]$  and  $i\sqrt{2}(\bar{t}\gamma_5 t)$ , and the integral is running over all values of  $s$ , where the spectral densities are nonzero. Consider now the region of  $s$  near the  $t\bar{t}$  threshold, where the integral is contributed by the  $t\bar{t}$  resonances and the very beginning of the continuum, strongly distorted by long-distance interactions. Within the perturbation theory the exchange of Coulomb gluons between the quark and the antiquark with a small velocity  $v = \sqrt{1 - 4m_t^2/s}$  makes the QCD effects depend on the parameter  $\alpha_s/v$  rather than  $\alpha_s$ . Therefore at  $v$  of the order of or less than  $\alpha_s$  these effects should be summed up. The summation amounts to using the well-known solution of the Coulomb problem, and the net effect reduces to multiplying the bare spectral density  $\rho_0^{(0)}$  by the Coulomb factor

$$F_c = \frac{4\pi\alpha_s/3v}{1 - \exp(-4\pi\alpha_s/3v)}. \quad (16)$$

At  $v \sim \alpha_s$  the spectral density is of order of  $\alpha_s$  and the size of this region of integration in Eq. (15) is  $\Delta s \sim 4m_t^2\Delta v^2 \sim m_t^2\alpha_s^2$ . Therefore the contribution of the Coulomb region above the threshold in Eq. (15) is of order  $\alpha_s^3$ , which is the same as that of the under-the-threshold resonances. The point of papers [4–6] is that the  $\alpha_s$  in this effect is normalized at long distances:  $\alpha_s(m_t v \sim m_t \alpha_s)$ . Because of favorable numerical factors of  $\pi$  this  $O(\alpha_s^3)$  effect is stated to be a sizable fraction of the  $\alpha_s$  term in Eq. (1). To quantify this statement the dispersion integral over the near-threshold region is calculated [5,6] with the factor  $F_c$  in Eq. (16) in which the running of the coupling constant is parametrized as  $\alpha_s(m_t v)$ .<sup>3</sup>

The loophole in this argument is that in terms of the running constant  $\alpha_s$  in Eq. (16) the normalization point is *not* given by  $m_t v$ . Rather, the proper normalization point is related to  $m_t v$  by a function  $f(\alpha_s/v)$ :  $\alpha_s[m_t v f(\alpha_s/v)]$ . This phenomenon is clearly seen in the QED calculation [14] of the excitation curve of the  $\tau^+\tau^-$  at the threshold, including the Uehling-Serber running of the QED constant  $\alpha$ . Namely, in the region  $v \sim \alpha$  the formula, obtained by the simple substitution of  $\alpha$  by  $\alpha(m_\tau v)$ , significantly deviates from the exact result. The

<sup>2</sup>The only known cases where large numerical coefficients are not removed by this procedure are those associated with the annihilation of heavy quarkonia in QCD [9,11], which parallels a similar behavior of the three-photon annihilation of orthopositronium in QED.

<sup>3</sup>More precisely, in [5] a relativistic parametrization is used, which however, does not change the main point of the argument.

contribution of the sum over resonances and of the integral over the continuum in Eq. (15) in higher orders in  $\alpha_s$  contains delicate cancellations [7,13]: starting from order  $\alpha_s^3$  the integration over the continuum partly compensates the contribution of resonances.<sup>4</sup> Moreover, in calculation of the dispersion integral the function  $F_c$  cannot be expanded in powers of  $\alpha_s$ , since this expansion does not converge at  $v < \alpha_s$  [within such expansion the integrals of individual terms would diverge at  $v = 0$  starting from the order  $(\alpha_s/v)^3$ ]. The result of the integration however can be perfectly expanded as a series in  $\alpha_s$  [7]. Thus an approximate parametrization is certainly prone to giving misleading results by destroying the correct structure of the spectral density at small  $v$ . Therefore we conclude that clarifying this point by calculating the spectral density near the threshold is a far more complicated problem than the initial one of calculating the vacuum polarization far below the threshold. On the other hand, those detailed calculations of the near-threshold region are not needed, since, as discussed before, the  $O(m_t^2)$  electroweak corrections can be calculated by the Feynman diagrams entirely in the Euclidean space, in which at no step the long-distance uncertainty of the QCD dynamics shows up at the perturbative level. In other words, though the integrand in Eq. (15) is poorly calculable near the  $t\bar{t}$  threshold, the entire integral is well calculable within the short-distance QCD.

As to the nonperturbative QCD effects, these can be understood by adapting the results of the discussion of the charmonium sum rules (Secs. 7.3–7.6 of the second paper in [7]). The result is that any finite distortion

<sup>4</sup>The objection [6] that these terms in the integral over the continuum cannot be negative, since the quark and the antiquark are attracting each other, is obviously erroneous: these are small negative corrections of order  $\alpha_s^3$  to the positive contribution in the orders  $\alpha_s$  and  $\alpha_s^2$ .

of the quark-antiquark interaction at a finite distance  $r_0 \gg m_t^{-1}$  produces only an effect on the  $t\bar{t}$  vacuum polarization at  $q^2 \approx 0$ , which is suppressed by  $\exp(-2m_t r_0)$ . For instance, one can cut off the Coulomb interaction at a radius  $r_0 \ll (m_t \alpha)^{-1}$  (but still  $r_0 \gg m_t^{-1}$ ), so that the Coulomb-like bound states disappear, and the actual spectral density  $\rho_0$  would look nothing like that determined by the factor  $F_c$  in Eq. (16). Still, up to the exponentially suppressed terms the vacuum polarization at  $q^2 \approx 0$  in this situation would be given by the dispersion integral in Eq. (15) with the *perturbative* spectral density, i.e., the one containing Coulomb-like poles, and the factor  $F_c$  above the threshold. Fully appreciating the nontrivial character of this phenomenon, we point out that this is a direct consequence of the analyticity of quantum amplitudes. The only way in which the long-distance effects give a contribution to the vacuum polarization at  $q^2 \approx 0$  is through the “tail” of the long-distance effects at distances of order  $m_t^{-1}$ . In the potential models with a powerlike nonperturbative potential of the form  $V(r) = ar^n$ , the effect is proportional [7] to  $am_t^{-(n+1)}$  [which is the action  $\int V(r)dt$  at distances  $r \sim m_t^{-1}$  over the time  $t \sim m_t^{-1}$ ]. To evaluate this particular effect in QCD there is however no need of invoking model potentials, and the leading effect is calculable in terms of the vacuum gluon condensate and its relative magnitude is given by  $\langle 0|\pi\alpha_s G_{\mu\nu}^a G_{\mu\nu}^a|0\rangle/m_t^4 \sim 10^{-10}$ . This would be the only nonperturbative contribution to the  $O(m_t^2)$  corrections if these corrections were expressed in terms of the top mass, normalized at short distances. However when the corrections are expressed through the on-shell mass of top, the relative nonperturbative contribution is that in the on-shell mass, i.e.,  $O(\Lambda_{\text{QCD}}/m_t)$ .

We are thankful to L. B. Okun for stimulating discussions. This work was supported, in part, by the DOE Grant No. DOE-AC02-83ER40105.

- 
- [1] V. A. Novikov, L. B. Okun, and M. I. Vysotsky, *Mod. Phys. Lett. A* **8**, 2529 (1993); **8**, 3301(E) (1993).
  - [2] M. Veltman, *Nucl. Phys.* **B123**, 89 (1977).
  - [3] A. Djouadi and C. Verzegnassi, *Phys. Lett. B* **195**, 265 (1987); A. Djouadi, *Nuovo Cimento A* **100**, 357 (1988).
  - [4] B. A. Kniehl and A. Sirlin, *Phys. Rev. D* **47**, 883 (1993).
  - [5] M. C. Gonzalez-Garcia, F. Halzen, and R. A. Vázquez, *Phys. Lett. B* **322**, 233 (1994).
  - [6] B. A. Kniehl and A. Sirlin, DESY Report No. DESY 93-194, 1993 (unpublished); NYU Report No. NYU-Th-93/12/01, 1993 (unpublished).
  - [7] V. A. Novikov *et al.*, *Phys. Rev. Lett.* **38**, 626 (1977); V. A. Novikov *et al.*, *Phys. Rep. C* **41**, 1 (1978); M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys.* **B147**, 385 (1979).
  - [8] V. A. Novikov *et al.*, *Phys. Lett. B* **331**, 433 (1994).
  - [9] S. J. Brodsky, G. P. Lepage, and P. M. Mackenzie, *Phys. Rev. D* **28**, 228 (1983).
  - [10] R. Barbieri *et al.*, *Phys. Lett. B* **288**, 95 (1992); **312**, 511(E) (1993).
  - [11] S. J. Brodsky and Hung Jung Lu, SLAC Report No. SLAC-PUB-6389, 1993 (unpublished).
  - [12] L. J. Reinders, H. Rubinstein, and S. Yazaki, *Phys. Rep.* **127**, 1 (1985).
  - [13] F. J. Ynduráin, *Phys. Lett. B* **321**, 400 (1994).
  - [14] B. H. Smith and M. B. Voloshin, *Phys. Lett. B* **324**, 117 (1994).