# Vacuum oscillation solution to the solar neutrino problem in standard and nonstandard pictures

Zurab G. Berezhiani*
*Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara, I-44100 Ferrara, Italy and Institute of Physics, Georgian Academy of Sciences, Tbilisi 380077, Georgia*

Anna Rossi†
*Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara, I-44100 Ferrara, Italy*
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The neutrino long wavelength (just-so) oscillation is reexamined as a solution to the solar neutrino problem. We consider the just-so scenario in various cases: in the framework of the solar models with a relaxed prediction of the boron neutrino flux, as well as in the presence of the nonstandard weak range interactions between neutrino and matter constituents. We show that the fit of the experimental data in the just-so scenario is not very good for any reasonable value of the $^8$B neutrino flux, but it substantially improves if the nonstandard $\tau$-neutrino–electron interaction is included. These new interactions could also remove the conflict of the just-so picture with the shape of the SN 1987A neutrino spectrum. Special attention is devoted to the potential of the future real-time solar neutrino detectors such as Super-Kamiokande, SNO, and BOREXINO, which could provide the model-independent tests for the just-so scenario. In particular, these imply a specific deformation of the original solar neutrino energy spectra and time variation of the intermediate energy monochromatic neutrino ($^7$Be and pep) signals.

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## I. INTRODUCTION

The deficit of the solar neutrinos, dubbed the solar neutrino problem (SNP), was observed more than 20 years ago in the Homestake Cl-Ar experiment. The 1970–1993 average of the chlorine experiment result reads [1]

$$R_{\rm Cl} = 2.32 \pm 0.26 \text{ solar neutrino units (SNU)}, \qquad (1)$$

whereas the standard solar model (SSM) of Bahcall and Pinsonneault (BP) [2] implies $R_{\rm Cl} = 8$ SNU, where 6.2 SNU comes from $^8$B neutrinos, 1.2 SNU from $^7$Be neutrinos, and the remaining 0.6 SNU from the other sources. The predictions of the other SSM's [3–5] do not differ strongly. However, the chlorine result alone does not seem sufficient to pose the problem, since the predicted flux of the boron neutrinos has rather large uncertainties. These are mainly due to the poorly known nuclear cross sections $\sigma_{17}, \sigma_{34}$ at low energies, some other astrophysical uncertainties which could change the solar central temperature, the plasma effects, etc. (see, e.g., [6] and references therein). All these, working coherently, may decrease $\phi^{\rm B}$ by more than a factor of 2 compared to the SSM prediction. Also the $^7$Be neutrino flux can have uncertainties up to 20%. Therefore, for a comprehensive analysis, it is suggestive to consider these fluxes as free parameters: $\phi^{\rm B} = f_{\rm B}\phi_0^{\rm B}$, $\phi^{\rm Be} = f_{\rm Be}\phi_0^{\rm Be}$, where $\phi_0$ are the BP model fluxes and the factors $f$ reflect the uncertainties.

However, the direct observation of solar $^8$B neutrinos by the Kamiokande detector [7] brings further evidence to the SNP. The Kamiokande signal is less than what is expected from the SSM by BP, unless $f_{\rm B} \leq 0.6$. However, more important is that the signal/prediction ratio

$$Z_K = \frac{R_K^{\rm expt}}{R_K^{\rm pred}} = \frac{1}{f_{\rm B}}\,(0.51 \pm 0.07) \qquad (2)$$

for any $f_{\rm B}$ is incompatible to the one of the chlorine experiment

$$Z_{\rm Cl} = \frac{R_{\rm Cl}^{\rm expt}}{R_{\rm Cl}^{\rm pred}} = \frac{1}{0.78 f_{\rm B} + 0.22 f_{\rm Be}}\,(0.29 \pm 0.03) \qquad (3)$$

unless $f_{\rm Be} \ll f_{\rm B}$ (for the simplicity, we have extended the factor $f_{\rm Be}$ also to other sources contributing the Cl-Ar signal). However, such a situation is absolutely improbable from the astrophysical viewpoint: Whatever effect (e.g., diminishing the central temperature) destroys $^7$Be neutrinos should destroy the $^8$B ones even more.[1]

One could even assume that the uncalibrated Homestake experiment has some uncontrollable systematical error and the true value of $\phi^{\rm B}$ is measured by the

*Electronic address: 31801::berezhiani, berezhiani@ferrara.infn.it
†Electronic address: 31801::rossi, rossi@ferrara.infn.it

[1]In fact, $f_{\rm Be}/f_{\rm B}$ could be diminished down to 0.75 if there exists a very low energy resonance in the $^3$He+$^3$He cross section [8]. This, however, cannot reconcile the solar neutrino data.

 

Kamiokande detector (i.e., $f_B \approx 0.5$). However, the data of the Ga-Ge experiment show that in doing so the SNP will not disappear. Indeed, the weighted average of the GALLEX [9] and SAGE [10] results is

$$R_{\rm Ga} = 78 \pm 10 \text{ SNU} \tag{4}$$

as compared with the BP prediction of 131 SNU. The bulk of this signal (71 SNU) comes from the *pp* source. The latter is essentially determined by the solar luminosity and, therefore, cannot be seriously altered by astrophysical uncertainties. On the other hand, the contribution of about 7 SNU is granted by the $^8$B neutrinos as measured by the Kamiokande detector. Therefore, there is not much room left for the $^7$Be neutrinos which, according to the BP model, have to provide 36 SNU: $\phi^{\rm Be}$ should be suppressed much stronger than $\phi^{\rm B}$ ($f_{\rm Be} < 0.25$). Thus, the SNP which arose initially as the boron neutrino problem now has become the problem of the beryllium neutrinos.

All these arguments are strong enough to believe that the astrophysical solutions to the SNP are excluded (for detailed discussions see Ref. [6]). It is more conceivable that on the way to Earth the solar $\nu_e$'s are partially converted into the other neutrino flavors. Moreover, the experimental data require the conversion mechanism capable to suppress differently neutrinos of different energies. According to a general paradigm, following from the experimental results, it should lead to a moderate reduction of the *pp* and $^8$B neutrino fluxes and to a strong depletion of the intermediate energy $^7$Be flux.

The neutrino oscillation picture can provide the necessary energy dependence in two regimes, which are known as the Mikheyev-Smirnov-Wolfenstein (MSW) [11] and the just-so [12, 13] scenarios.[2] The MSW resonant conversion in matter is the most attractive and elegant solution, requiring $\delta m^2$ of about $10^{-5}$ eV$^2$ and a small mixing angle $\sin^2 2\theta \sim 10^{-2}$. It provides a very good fit of the experimental data, due to the selective strong reduction of the $^7$Be neutrinos [18, 19].

Another attractive possibility is offered by the just-so oscillation, i.e., vacuum oscillation $\nu_e \to \nu_x$ ($\nu_x = \nu_\mu, \nu_\tau$) with the wavelength comparable to the Sun-Earth distance [12, 13]. This solution needs a $\delta m^2$ of about $10^{-10}\,$eV$^2$ and large mixing angles [20], a parameter range which can be naturally generated by nonperturbative quantum gravitational effects [14, 15]. The just-so scenario, because of the energy dependence of the survival probability, can provide an acceptable fit of the solar neutrino data. Its recent experimental status in the SSM framework was studied in [21], and it was shown that the data fit is much worse than in the MSW picture.

In addition, as was pointed out in Ref. [22], this scenario faces the difficulty being confronted with the SN 1987A neutrino burst [23]. The original $\bar\nu_{\mu,\tau}$ energy spectrum from the supernova has a larger average energy (about 25 MeV) than the spectrum for $\bar\nu_e$ (about 12 MeV), due to the smaller opacities of $\bar\nu_{\mu,\tau}$. The neutrino conversion $\bar\nu_e \to \bar\nu_x$ induced by the neutrino mixing results in a partial permutation of the original $\bar\nu_e$ and $\bar\nu_x$ spectra. If the permutation is strong, it would significantly alter the energy spectrum of the supernova $\bar\nu_e$ signal. The analysis [22], derived by using the SN 1987A data and different models of the neutrino burst, shows that for $\delta m^2 \sim 10^{-10}$–$10^{-11}$ eV$^2$ the range of mixing excluded at 99% C.L. is $\sin^2 2\theta \geq 0.7$, which covers the range required by the just-so scenario, $\sin^2 2\theta \geq 0.7$. Nevertheless, we do not consider the SN 1987A argument as clear evidence against large neutrino mixing. Moreover, as we will discuss below, this constraint can be removed by assuming some nonstandard neutrino interactions which could increase the $\bar\nu_x$ opacity in the supernova core, reducing thereby its average energy.

In the present paper we address certain issues in the context of the long-wavelength neutrino oscillation as a possible solution to the SNP. Namely, in Sec. II, we study how this scenario fits the experimental data in nonstandard cases: (i) NSSM+SM, in the context of models with relaxed prediction of $\phi^{\rm B,Be}$ [which we conventionally refer to as nonstandard solar models (NSSM's)], while the neutrinos are supposed to have only the standard electroweak interactions; (ii) SSM+NSM, in the SSM framework, assuming, however, that neutrinos have some additional nonstandard model (NSM) couplings with matter constituents. As we show below, the presence of NSM interactions can also reconcile the just-so scenario with the SN 1987A neutrino signal.

Section III is devoted to the model-independent analysis of the just-so scenario. This essentially implies the modification of the solar neutrino spectrum due to the energy and time dependence of the survival probability. We focus our attention on the advantages inherent in the future real-time neutrino detectors such as Super-Kamiokande [24], SNO [25], and BOREXINO [26]. All these experiments can measure the recoil electron spectrum, which could provide specific signatures allowing us to discriminate the just-so scenario, in particular from the MSW one.

At the end, we give a brief summary of our conclusions.

## II. DATA FIT IN STANDARD AND NONSTANDARD PICTURES

For simplicity, we consider the vacuum oscillations in the case of two neutrino flavors: $\nu_e \to \nu_x$, where $\nu_x$ can

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[2] According to a *cliché*, the neutrino oscillation is regarded as a nonstandard property. However, from the viewpoint of modern particle physics, the existence of the neutrino mass and mixing should be considered as a rather standard feature. In the framework of the standard model (SM) the neutrino mass can arise through the higher order operator $\frac{1}{M}(lCl)HH$, where $l$ and $H$ are, respectively, the lepton and Higgs doublets and $M$ is some regulator scale, so that $m_\nu = \langle H\rangle^2/M \approx 3\times10^{-6}(M_{\rm Pl}/M)\,$eV [14, 15]. In particular, superstring or Planck scales ($M \sim 10^{18}$–$10^{19}\,$GeV) imply the neutrino mass range needed for the just-so scenario, whereas the MSW scenario requires $M$ to be of the order of the supersymmetric grand unification scale, $M \sim 10^{16}$ GeV. As for the adjective "nonstandard," it should be rather reserved for the really nonstandard neutrino properties, implied by the SNP solutions based on the magnetic moment transition [16] or on the fast neutrino decay [17].

be $\nu_\mu$ or $\nu_\tau$.[3] The survival probability for solar $\nu_e$'s with energy $E$ is given by

$$P(L_t, E) = 1 - \sin^2 2\theta \sin^2\left(\pi\,\frac{L_t}{l}\right), \tag{5}$$

where $l = \frac{4\pi E}{\delta m^2} = \frac{E[\,\mathrm{MeV}]}{\delta m^2[10^{-10}\,\mathrm{eV}^2]} \times 2.47 \times 10^{10}$ m is the oscillation wavelength. The Sun-Earth distance $L$ depends on time as $L_t = \overline{L}[1-\varepsilon\cos(2\pi t/T)]$, where $\overline{L} = 1.5\times10^{11}$ m, $T = 365$ days, and $\varepsilon = 0.0167$ is the ellipticity of the orbit.

The time-averaged signals predicted in the radiochemical experiments are given by

$$R = \int dE\sigma(E)\sum_i \langle P(E)\phi^i\rangle_T \lambda_i(E). \tag{6}$$

Here $\sigma(E)$ is the detection cross section, $\phi^i$ are the fluxes of the relevant components of the solar neutrinos ($i =$ B, Be, etc.), $\lambda_i(E)$ are their energy spectra normalized to 1, and $\langle\cdots\rangle_T$ stands for the average over the whole time period $T$. In this way, the time dependence of the original flux $[\phi(t) \propto L_t^{-2}]$ is also taken into the account.

For the Kamiokande detector, since we consider the $\nu_e$ conversion into an active neutrino, the expression for the signal becomes

$$\begin{aligned} R_K = \int_{E_{\mathrm{th}}} & dE\lambda_B(E)\{\langle P(E)\phi^B\rangle_T\sigma_{\nu_e}(E) \\ & + \left[\langle\phi^B\rangle_T - \langle P(E)\phi^B\rangle_T\right]\sigma_{\nu_x}(E)\}. \end{aligned} \tag{7}$$

Here $\sigma_{\nu_y}$ ($y = e, x$) is the $\nu_y e^-$ scattering cross section and $E_{\mathrm{th}} = \frac{1}{2}[T_e + \sqrt{T_e(T_e + 2m_e)}]$, where $T_e = 7.5$ MeV is the recoil electron kinetic energy threshold.

Below we examine the just-so scenario in view of the recent status of the SNP. We accept the hypothesis that the solar neutrino luminocities are constant in time, and use the averaged data of the chlorine, gallium, and Kamiokande experiments to perform the standard $\chi^2$ analysis for various cases. We use as reference SSM the BP model, without taking into account the underlying theoretical uncertainties. The case of the other SSM's will be effectively recovered by relaxing $\phi^{\mathrm{B}}$ and $\phi^{\mathrm{Be}}$. Such an approach was recently used in Ref. [27] for the analysis of the MSW scenario.

### A. NSSM + SM

We consider $\phi^{\mathrm{B}}$ and, to a less extent, also $\phi^{\mathrm{Be}}$ as free parameters. So we describe the $^8$B neutrino flux as $\phi^{\mathrm{B}} = f_{\mathrm{B}}\cdot\phi_0^{\mathrm{B}}$, where evidently $\phi_0^{\mathrm{B}}$ is the prediction of the BP SSM, and the factor $f_{\mathrm{B}}$ accounting for the uncertainty is varied in the range 0.4–1.6 (the lower limit $f_{\mathrm{B}} = 0.4$ is actually set by the Kamiokande measurement of the boron neutrino flux). The analogous factor $f_{\mathrm{Be}}$ for $^7$Be neutrinos can vary within 0.7–1.3. For example, by taking $f_{\mathrm{B}} = 0.8$, $f_{\mathrm{Be}} = 0.9$ the case of the Turk-Chièze and Lopez SSM [4] is reproduced.

We have performed the $\chi^2$ analysis for various values of $f_{\mathrm{B}}$ and $f_{\mathrm{Be}}$. The corresponding best fit points of minimal $\chi^2$, and "$1\sigma$" areas containing the true parameter values with 68% probability once this solution is assumed (these are given by the condition $\chi^2 \le \chi^2_{\mathrm{min}} + 2.28$), are shown in Fig. 1. For the SSM case ($f_{\mathrm{B,Be}} = 1$) the fit is not so good: The minimal $\chi^2$ obtained is 4.4, so that just-so oscillation is allowed as a SNP solution only at the

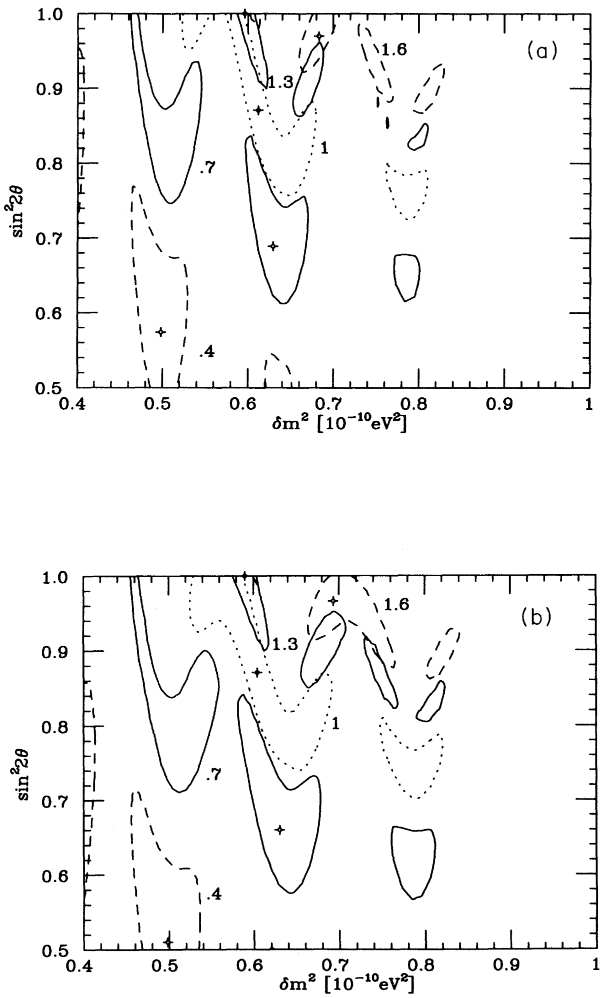


FIG. 1. The best fit points (diamonds) and the 68% C.L. regions in the case NSSM+SM for different $f_{\mathrm{B}}$, with $f_{\mathrm{Be}} = 1$ (a) or $f_{\mathrm{Be}} = 0.8$ (b). The $\chi^2_{\mathrm{min}}$ corresponding to values $f_{\mathrm{B}} =$ 0.4, 0.7, 1.0, 1.3, 1.6 are 11.7, 6.4, 4.4, 3.0, 2.8 in (a), and 10.3, 5.7, 4.3, 3.1, 2.8 in (b).

[3]Certainly, the general case of three-neutrino oscillations involves more parameters. However, in many interesting cases the three-neutrino oscillation picture effectively reduces to the case of two neutrinos. For example, in the case of the "democratic" ansatz for the gravitationally induced neutrino mass matrix [15], the oscillation picture is equivalent to the case of two neutrinos with $\sin^2 2\theta = 8/9$.

3.6% confidence level (C.L.).[4] This is in agreement with the analyses of Ref. [21], where a somewhat different way of data fitting is used. The relevant parameter range is limited by the values $\delta m^2 = (5\text{–}8)\times 10^{-11}$ eV$^2$, $\sin^2 2\theta = 0.7\text{–}1$ [see Fig. 1(a)].

By varying $f_{\rm B}$, the relevant range of $\delta m^2$ remains rather stable, while the $\sin^2 2\theta$ becomes smaller with decreasing $f_{\rm B}$.[5] The lowering (increasing) of $f_{\rm B}$ results in a weakening (strengthening) of the neutrino oscillations. Therefore, with smaller values of $f_{\rm B}$ the model could be in agreement with the SN 1987A bound $\sin^2 2\theta \le 0.7$ [22]. However, as a general tendency, by decreasing $f_{\rm B}$ the fit becomes worse, whereas it slightly improves for $f_{\rm B} > 1$. E.g., for $f_{\rm B} = 0.4$ the high value of $\chi^2_{\rm min} = 11.7$ indicates a poor fit (solution is excluded at more than 99.9% C.L.).[6] On the contrary, for $f_{\rm B} = 1.3$ we have $\chi^2_{\rm min} = 3.0$ which is acceptable at 8.3% C.L. In this case the boron neutrino flux must be depleted more strongly so that a larger mixing is required, which reconciles mutually the chlorine and the Kamiokande data. On the other hand, the large mixing contradicts the supernova bound. The variation of the beryllium neutrino flux [see Fig. 1(b)] does not alter significantly the previous results.

### B. SSM+NSM

Here we take the BP model ($f_{\rm B,Be} = 1$) as a reference SSM but assume that neutrinos have some nonstandard interactions in addition to the SM ones.[7] Namely, we suppose that the $\nu_x$ state in which the solar $\nu_e$ is converted is just $\nu_\tau$ and it has extra weak range interaction with the electron:

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}}\,\bar\nu_\tau\gamma^\mu(1-\gamma_5)\nu_\tau[\epsilon\,\bar e\gamma_\mu(1+\gamma_5)e + \epsilon'\,\bar e\gamma_\mu(1-\gamma_5)e]. \tag{8}$$

Here $\epsilon$ and $\epsilon'$ parametrize the strength of new interactions with respect to the Fermi constant $G_F$ (for $\nu_\mu$ such interactions are severely restricted by accelerator limits—see [29, 30] and references therein). The first term in this Lagrangian, with positive $\epsilon$, can be effectively obtained (after the Fierz transformation) from the exchange of some additional electroweak doublet scalar $\varphi$ (the relevant Yukawa coupling is $\bar l_{\tau L} e_R \varphi$, where $l_{\tau L}$ is the lepton doublet including $\tau$ and $e_R$ is the right-handed component of the electron). The second term could be due to the exchange of some charged Higgs singlet $\eta$. However, the same exchange of the charged singlet unavoidably contributes the $\tau \to e\nu_\tau\bar\nu_e$ decay width, which sets the strong bound $\epsilon' < 0.05$. As for the strength of the first interaction $\epsilon$, its value is not seriously constrained by any laboratory limit, while the astrophysical bounds on stellar evolution in the most conservative case imply $\epsilon \le 1$ [30].

The extra neutral current (NC) interaction of $\nu_\tau$ with the electron contributes to $\nu_\tau$-$e$ elastic scattering together with the standard neutral current and, as far as $\epsilon > 0$, it increases the $\sigma_{\nu_\tau}$ cross section (see below, Fig. 9), and thus the signal in the Kamiokande detector. This implies a stronger suppression of the boron neutrino flux, which leads to a better agreement between the Kamiokande and Homestake data.

In order to study the impact of these extra NC couplings on the just-so scenario, we have repeated the $\chi^2$ analysis for the interval $\epsilon = 0\text{–}1$. The results of the fitting are shown in Fig. 2. One can observe that the allowed region of the parameters $\delta m^2$ and $\sin^2 2\theta$ is rather stable against the variation of $\epsilon$. However, as expected, the data fit improves by increasing $\epsilon$, since now the Kamiokande signal requires larger mixing angles. E.g., for $\epsilon = 1$ we achieve $\chi^2_{\rm min} = 1.8$, which implies that in this case the just-so oscillations can be regarded as the SNP solution at the 18% C.L.

Certainly, along with the interactions (8) one can consider also the analogous nonstandard interactions of $\nu_\tau$ with protons and neutrons. They could be induced due to the exchange of some scalar leptoquark with mass of about 100 GeV. These interactions do not contribute the signal in the detectors under operation. Nevertheless,

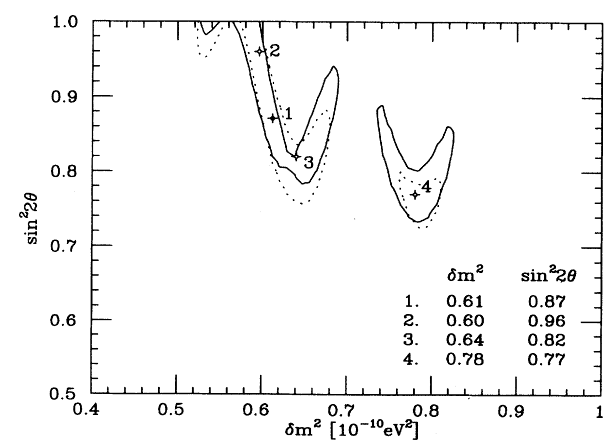


FIG. 2. The best fit point (marked as 2, $\chi^2_{\rm min} = 1.8$) and the 68% C.L. regions in the case SSM+NSSM, for $\epsilon = 1$ (solid curves) tested with use of the case SSM+SM, $\epsilon = 0$ (dotted curves, best fit point marked as 1). In the following these points, as well as the other typical points 3 and 4, will be used for demonstrating the effects of spectral distortion.

[4] We find it instructive to separate the experimental and theoretical (SSM) uncertainties, and do not include the latter in $\chi^2$ analysis (in fact, these are simulated by varying $f_{\rm B,Be}$). Thus, here and in the following the quantitative statements on the C.L. are somewhat informal, and reflect only the ideal situation with no uncertainties in neutrino fluxes, exactly fixed by choice of $f_{\rm B}$ and $f_{\rm Be}$.

[5] The analysis of Ref. [27] shows that the MSW scenario reacts in the same way by varying $f_{\rm B}$.

[6] It is interesting to note that for $f_{\rm B} \simeq 0.6$ even the averaged short-wavelength oscillation picture with only one parameter (mixing angle) fit provides somewhat smaller $\chi^2_{\rm min}$ at $\sin^2 2\theta = 1$.

[7] The effects of such nonstandard interactions (flavor diagonal as well as flavor changing) for the MSW picture were studied in Refs. [28–30]. However, the altering of the neutrino propagation in the solar interior has no importance in the case of just-so oscillation. This interaction is relevant only for the detection cross section in the $\nu$-$e$ scattering experiment.

they can be relevant for the signal in the future real-time detectors, expecially SNO and BOREX. It is worth mentioning that nonstandard neutrino interactions with electrons as well as nucleons can naturally appear in the context of supersymmetric models with broken $R$ parity (see, e.g., [29] and references therein).

Let us conclude this section with the following remarks. As we have seen, the just-so picture can be relevant for SNP only for the following mass and mixing range:

$$\delta m^2 = (0.5\text{–}0.8) \times 10^{-10}\ \mathrm{eV}^2\,, \quad \sin^2 2\theta = 0.7\text{–}1, \tag{9}$$

for any reasonable values $f_{\mathrm{B,Be}}$ and $\epsilon$ (see Figs. 1 and 2). Moreover, for the plausible interval $f_{\mathrm{B}} = 0.7$–1.3 the best fit area is essentially located in the very narrow band around $\delta m^2 \approx 0.6 \times 10^{-10}$ eV$^2$, rather independently on the concrete values of $f_{\mathrm{B,Be}}$ and $\epsilon$, while $\sin^2 2\theta$ varies from 0.7 to 1 depending on the concrete values of these parameters. The data fit for certain cases of the simultaneous variation of $f_{\mathrm{B}}$ and $\epsilon$ is shown in the Table I.

## III. PREDICTIONS FOR FUTURE SOLAR NEUTRINO EXPERIMENTS

Although the data fit in the just-so scenario is somewhat worse than in the MSW picture, it cannot be ruled out as a SNP solution. On the other hand, these solutions cannot be discriminated by the recent experiments. However, the next generation of solar neutrino detectors will shed more light on the situation. The novel detec-

TABLE I. The expected signals in different detectors, for the best fit points corresponding to different values of $f_{\mathrm{B}}$ for the cases $\epsilon$ = (a) 0 and (b) 1, respectively. $Z$ is the ratio of the calculated signal to the one expected in the solar model with the given $f_{\mathrm{B}}$ (clearly, $Z$ does not depend on $f_{\mathrm{B}}$). Within round brackets the percentage seasonal variation of the signal, compared to the time-averaged value $Z$, is reported, where the upper sign refers to June and the lower one to December. For the $\nu$-$e$ scattering experiments the individual contributions from the survived $\nu_e$ and emerged $\nu_x$ are also shown (within the square brackets). $R$ are the annual average signals predicted for each detector. For the radiochemical experiments $R$ is given in SNU, whereas for BOREXINO in the number of events per day, for the recoil electron energy intervals indicated. For (Super) Kamiokande and SNO $R$ is given in units of the BP SSM prediction: $R = f_{\mathrm{B}} Z$. The quantity $\delta(T)$ stands for the variations of the recoil electron average energy with respect to the one predicted in the SSM.

| | (a) $\epsilon = 0$ | | | | | |
|---|---|---|---|---|---|---|
| | $f_{\mathrm{B}} = 1$ ($\chi^2_{\min} = 4.4$) | | $f_{\mathrm{B}} = 0.7$ ($\chi^2_{\min} = 6.4$) | | $f_{\mathrm{B}} = 1.3$ ($\chi^2_{\min} = 3.0$) | |
| | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ |
| Cl-Ar | 0.32 ($\mp$ 10%) | 2.55 | 0.43 ($\mp$4.7%) | 2.63 | 0.27 ($\mp$12%) | 2.65 |
| Ga-Ge | 0.50 ($\mp$10%) | 66 | 0.55 ($\mp$4.3%) | 70 | 0.51 ($\mp$10%) | 69 |
| Kamiokande ($T_{\mathrm{th}}$ = 7.5 MeV) | 0.41 ($\mp$2.4%) [0.31+0.1] | 0.41 | 0.52 ($\mp$2.0%) [0.44+0.08] | 0.36 | 0.34 ($\mp$3.5%) (0.23+0.11] | 0.44 |
| SK ($T_{\mathrm{th}}$ = 5.5 MeV) | 0.37 ($\mp$ 2.0%) [0.26+0.11] | 0.37 | 0.49 ($\mp$1.1%) [0.40+0.09] | 0.34 | 0.28 ($\mp$3.2%) [0.16+0.12] | 0.36 |
| SNO | 0.26 ($\mp$3.7%) | 0.26 | 0.41 ($\mp$1.5%) | 0.29 | 0.17 ($\mp$6.5%) | 0.22 |
| BOREXINO: $^7$Be ($T$ = 0.25–0.7 MeV) | 0.61 ($\mp$22%) [0.45+0.16] | 32 | 0.55 ($\mp$11%) [0.39+0.16] | 29 | 0.79 ($\mp$ 18%) [0.67+0.12] | 41 |
| BOREXINO: pep ($T$ = 0.7–1.3 MeV) | 0.41 ($\pm$4.7%) [0.28+0.13] | 1.0 | 0.58 ($\pm$6.2%) [0.48+0.10] | 1.5 | 0.32 ($\mp$3.2%) [0.16+0.16] | 0.8 |
| $\delta(T)_{\mathrm{SK}}$ | 2.6% | | 1.4% | | 4.0% | |
| $\delta(T)_{\mathrm{SNO}}$ | 8.0% | | 3.8% | | 14.4% | |
| | (b) $\epsilon = 1$ | | | | | |
| | $f_{\mathrm{B}} = 1$ ($\chi^2_{\min} = 1.8$) | | $f_{\mathrm{B}} = 0.7$ ($\chi^2_{\min} = 4.2$) | | $f_{\mathrm{B}} = 1.3$ ($\chi^2_{\min} = 1.0$) | |
| | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ |
| Cl-Ar | 0.31 ($\mp$13%) | 2.47 | 0.41 ($\mp$5.5%) | 2.51 | 0.25 ($\pm$8.0%) | 2.45 |
| Ga-Ge | 0.54 ($\mp$10%) | 71 | 0.55 ($\mp$7%) | 70 | 0.55 ($\pm$10%) | 75 |
| Kamiokande ($T_{\mathrm{th}}$ = 7.5 MeV) | 0.44 ($\mp$2.0%) [0.26+0.18] | 0.44 | 0.56 ($\mp$1.3%) [0.42+0.14] | 0.39 | 0.35 ($\mp$2.8%) [0.14+0.21] | 0.46 |
| SK ($T_{\mathrm{th}}$ = 5.5 MeV) | 0.46 ($\mp$1.3%) [0.20+0.26] | 0.46 | 0.58 ($\mp$0.7%) [0.37+0.21] | 0.41 | 0.41 ($\mp$0.2%) [0.11+0.30] | 0.53 |
| SNO | 0.21 ($\mp$4.5%) | 0.21 | 0.38 ($\mp$1.5%) | 0.27 | 0.11 ($\mp$5.4%) | 0.14 |
| BOREXINO: $^7$Be ($T$ = 0.25–0.7 MeV) | 1.02 ($\pm$2.0%) [0.68+0.34] | 53 | 1.04 ($\pm$1.0%) [0.42+0.62] | 54 | 1.02 ($\mp$1.5%) [0.80+0.21] | 53 |
| BOREXINO: pep ($T$ = 0.7–1.3 MeV) | 0.72 ($\mp$0.7%) [0.20+0.52] | 1.8 | 0.80 ($\pm$1.3%) [0.42+0.38] | 2.1 | 0.92 ($\pm$ 2.7%) [0.77+0.15] | 2.4 |
| $\delta(T)_{\mathrm{SK}}$ | −0.6% | | −0.6% | | −2.6% | |
| $\delta(T)_{\mathrm{SNO}}$ | 11.5% | | 4.5% | | 13.0% | |

tors such as the Super-Kamiokande [24], SNO [25], and BOREXINO or BOREX [26] could provide tests, almost independent of the SSM details. In particular, these real-time detectors will be able to observe the seasonal time variations of the various neutrino components, due to the ellipticity of Earth's orbit and sufficiently strong (but not *very* strong to be averaged) oscillation effects in the just-so regime. On the contrary, the MSW mechanism can exhibit only the standard 7% *simultaneous* variation of all signals from June to December, since in this case all neutrino conversions take place in the Sun's interior and the small oscillation effects on the way from the Sun to Earth are negligible.

As we have seen, the just-so picture can be relevant for the SNP only in a narrow $\delta m^2$ interval (9), rather independently of the values $f_{\mathrm{B,Be}}$ and $\epsilon$. Moreover, for the moderate values $f_{\mathrm{B}} = 0.7$–$1.3$ the best fit area is essentially located at $\delta m^2 \approx 0.6 \times 10^{-10}$ eV$^2$. Then it is easy to see that for the $\delta m^2$ in the range (9) the monochromatic $^7$Be neutrinos ($E = 0.861$ MeV) oscillate along the distance $\overline{L} = 1.5 \times 10^{11}$ m about 3–5 times, pep neutrinos ($E = 1.442$ MeV) about 2–3 times, and the boron neutrinos (with typical energy of about 10 MeV) do not undergo even one full oscillation. Therefore, since the value $\varepsilon\pi\overline{L}/l$ is a small parameter (e.g., for $^7$Be neutrinos it is about 0.2), from Eq. (5) we obtain, for the $\nu_e$ survival probabilities at June and December $[L_\pm = \overline{L}(1 \pm \varepsilon)]$,

$$P_\pm(E) \approx \overline{P}(E) \mp [1 - \overline{P}(E)] \frac{2\varepsilon\pi\overline{L}/l_E}{\tan(\pi\overline{L}/l_E)}, \tag{10}$$

where the quantity $\overline{P}(E) = P(\overline{L}, E)$ essentially is the average survival probability of the $\nu_e$ with energy $E$. This formula demonstrates that the seasonal variations should be stronger for neutrinos with smaller energies, and it can be dramatic for the monochromatic neutrino lines [12, 13]. Namely, in the best fit region ($\delta m^2 \approx 0.6 \times 10^{-10}$ eV$^2$) we have $\overline{P}_{\mathrm{Be}} \sim 0.5$ for the $^7$Be neutrinos, while the phase factor $\tan(\pi\overline{L}/l_{\mathrm{Be}}) \sim 1$. This could provide about 10% seasonal variations of signal in the gallium detectors (see, Table I), which may be observable after the increasing of statistics in GALLEX and its calibration. The larger effect is expected in the BOREXINO detector: up to 50% seasonal variations of signal in the beryllium neutrino "window." The standard 7% variations are negligible in this case. At the same time, for this range of $\delta m^2$ the variation of the pep signal is expected to be smaller, less than 10%, essentially due to large $\tan(\pi\overline{L}/l_{\mathrm{pep}})$. However, for the wider range of parameters (9) also the pep neutrino signal variation could be significant. As for the $^8$B neutrinos, one cannot expect strong time variations (up to 10%), due to large oscillation length as well as smoothing effects due to continuous spectrum.[8]

Another possibility to discriminate the just-so scenario is related to the spectral distortion of the various solar neutrino components. The original energy spectra $\lambda_i(E)$ ($i$ = B, Be, etc.) are independent of the details of the solar models. They are determined only by the nuclear reactions producing the neutrinos. The energy-dependent conversion mechanisms for the SNP solution can strongly modify the initial neutrino spectra, offering thereby specific signatures for their discrimination. Below we consider the "just-so" spectral predictions for the planned experiments. We do not take into account the energy resolution functions of various detectors, so that sensitivities for the spectrum shape are somewhat overestimated. Therefore, our estimations are rather for the purpose of demonstration, and if the characteristic "just-so" signals will be really observed in future detectors, a more detailed spectral analysis has to be performed.

### A. Super-Kamiokande

This detector is expected to measure the spectrum of the high energy $^8$B neutrinos. The original neutrino distribution can be reproduced from the recoil electron spectrum due to $\nu$-$e$ scattering, though it is somewhat smeared due to the integration over the neutrino energy:

$$F(T) = \int dE \lambda_{\mathrm{B}}(E) \left[ \langle P(E)\phi^{\mathrm{B}}\rangle_T \frac{d\sigma_{\nu_e}}{dT}(E,T) + [\langle\phi^{\mathrm{B}}\rangle_T - \langle P(E)\phi^{\mathrm{B}}\rangle_T] \frac{d\sigma_{\nu_x}}{dT}(E,T) \right],$$

$$\frac{d\sigma_{\nu_y}}{dT}(E,T) = \frac{2G_F^2 m_e}{\pi} [g_{yL}^2 + g_{yR}^2 (1 - T/E)^2 - g_{yL}g_{yR}\, m_e\, T/E^2], \quad y = e, \mu, \tau, \tag{11}$$

where $T$ is the recoil electron energy. For the $\nu_e$-$e$ scattering we adopt the standard model values for the NC coupling constants $g_{eL} = \frac{1}{2} + \sin^2\theta_W$ and $g_{eR} = \sin^2\theta_W$, whereas for the $\nu_x$ state we also account for the possible nonstandard couplings given by Eq. (8): $g_{xL} = -\frac{1}{2} + \sin^2\theta_W + \epsilon'$ and $g_{xR} = \sin^2\theta_W + \epsilon$.

We calculate the ratio of the distorted spectrum $F(T)$ to that is predicted by the SSM $F_0(T)$. For the definiteness we normalize $\xi(T) = F(T)/F_0(T)$ to 1 at $T = 10$ MeV. Clearly, this ratio does not depend on the SSM details, as far as $F_0(T)$ is essentially determined by the boron $\beta$-decay spectrum $\lambda_{\mathrm{B}}(E)$.

The shape of $\xi(T)$ for various couples of the parameters $\delta m^2$ and $\sin^2 2\theta$ from the allowed area is given in Fig. 3(a) for $\epsilon = 0$ and Fig. 3(b) for $\epsilon = 1$. The present sensitivity of the Kamiokande detector (long error bars) is not enough to discriminate the just-so solution, whereas the Super-Kamiokande detector (short error bars) could distinguish it from the MSW picture, especially due to the characteristic distortion in the lower energy part of the spectrum (for comparison, see Ref. [27] for the recoil electron spectrum in the MSW case). The deformation of the energy spectrum can alter the average energy $\overline{T}$

[8]The feasibility of the Super-Kamiokande and SNO detectors for the observation of the boron neutrino signal variations was studied in details in the recent paper by Krastev and Petcov [33].

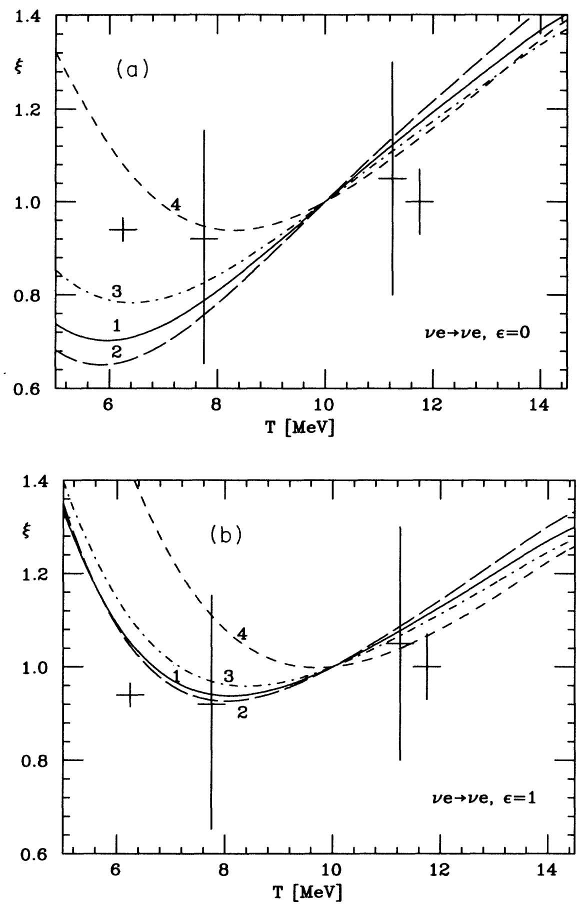


FIG. 3. Super-Kamiokande: the ratio $\xi(T)$ of the recoil electron energy spectrum, distorted due to the just-so oscillation, to the undistorted one (normalized to 1 at 10 MeV), given for the points shown in Fig. 2. (a) and (b) refer to the cases $\epsilon = 0$ and $\epsilon = 1$, respectively. The longer error bars indicate the present sensitivity of the Kamiokande detector and the shorter ones represent the expected sensitivity in the Super-Kamiokande detector.

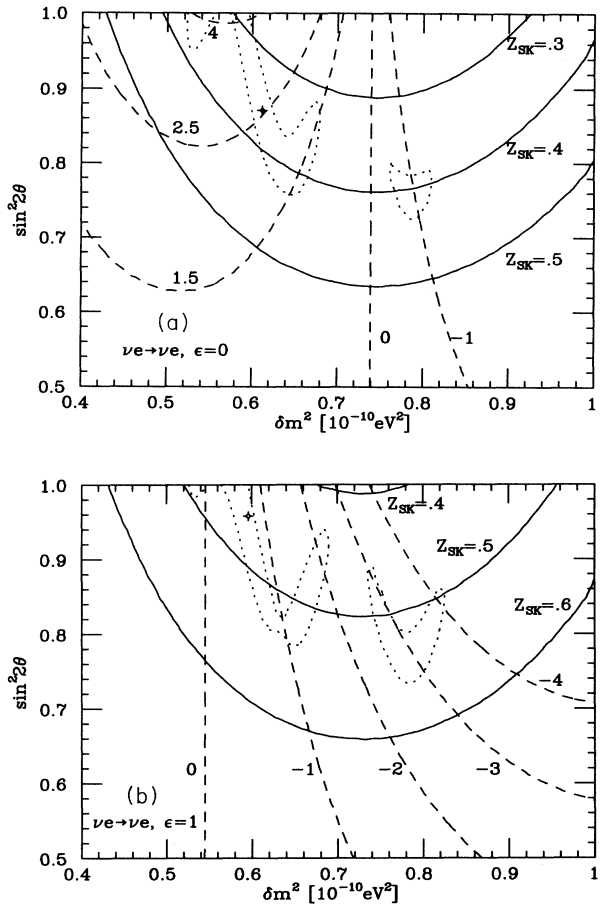


FIG. 4. The isosignal curves for $Z_{\rm SK}$ expected at the Super-Kamiokande detector, with 5.5 MeV threshold (solid line). The curves for the isopercentage variations of the average electron energy compared with the SSM value are also shown (dashed). (a) refers to the case $\epsilon = 0$ and (b) to that $\epsilon = 1$. The corresponding 68% C.L. regions are also shown (dotted curves).

of the recoil electrons as compared to the standard prediction ($\overline{T}_0 = 7.44$ MeV for an electron energy threshold $T_{\rm th} = 5.5$ MeV). In Fig. 4 the isocurves for the variation of $\overline{T}$ as compared to $\overline{T}_0$ (in percent) are plotted in the $(\delta m^2, \sin^2 2\theta)$ plane. As we see, $\overline{T}$ can change up to 4%. In the case $\epsilon = 0$ [Fig. 4(a)] the variation is rather positive than negative, whereas for $\epsilon = 1$ [Fig. 4(b)] it is dominantly negative. In particular, for the best fit solutions the variation is 2.6% for $\epsilon = 0$ and $-0.6\%$ for $\epsilon = 1$.

### B. SNO

This heavy-water real-time detector will measure the $^8$B neutrino flux through the charged-current (CC) and neutral-current (NC) processes:

$$\begin{aligned} &\text{CC}: \quad \nu_e d \to e^- p\ p; \\ &\text{NC}: \quad \nu_y d \to \nu_y p\ n, \quad y = e, \mu, \tau. \end{aligned} \tag{12}$$

The ratio $\eta = R_{\rm CC}/R_{\rm NC}$ in the SSM (i.e., when no neutrino conversion takes place) is independent of the value of $f_{\rm B}$. If neutrino conversion occurs, the flux of the survived solar $\nu_e$ is directly measured by the CC signal:

$$R_{\rm CC} = \int_{E_{\rm th}} dE \sigma_{\rm CC}(E) \lambda_{\rm B}(E) \langle P(E) \phi_{\rm B} \rangle_T, \tag{13}$$

where $E_{\rm th} = 7$ MeV and for the cross section $\sigma_{\rm CC}$ we use the data presented in [32].

If the solar $\nu_e$'s are converted into active neutrinos $\nu_x = \nu_\mu, \nu_\tau$ having only the SM neutral current couplings to nucleons ($Z$-boson exchange), then the probability conservation guarantees that the NC signal is the same as in the reference SSM: $R_{\rm NC}$ directly measures the original $\phi^{\rm B}$ flux. Therefore, if the measured ratio $\eta = R_{\rm CC}/R_{\rm NC}$ is less than that is predicted by the SSM ($\eta_0 = 1.8$ for $E_{\rm th} = 7\,$MeV, independently of $f_{\rm B}$), this would unambiguously indicate the deficit of the boron $\nu_e$, caused by neutrino conversion. In Fig. 6 below the isosignal curves are given for the ratio $Z_{\rm SNO} = R_{\rm CC}/R_{\rm CC}^{\rm pred} = \eta/\eta_0$. As we see, in the parameter region relevant for the just-so scenario this ratio varies in the range 0.2–0.35.

The CC signal will allow one to clearly discriminate the just-so picture by measuring the recoil electron spectrum $F(T)$. In fact, the ratio of the distorted spectrum to the SSM predicted one does not depend on $f_{\rm B}$ and it characterizes the energy dependence of the survival probability. In a rough approximation, the function $F(T)$ reproduces the energy spectrum of the $\nu_e$'s survived the conversion, i.e., $\lambda(E) = \langle P(E)\phi_{\rm B}\rangle_T \lambda_{\rm B}(E)$, shifted by an amount equal to the recoil energy left to nuclei: $T = E - 1.44$ MeV. However, for a given $\lambda(E)$ the correct calculation of $F(T)$ requires accounting for the effects smearing the recoil electron spectrum, as are the final state Coulomb interactions, the Fermi motion of nucleons in the deuteron, and final state three-body kinematics. In addition, for correct evaluation of the CC signal the SNO energy resolution function has to be included in Eq. (13). Having all these in mind, we analyze a shape of the survived boron $\nu_e$ spectrum $\lambda(E)$ as compared to initial spectrum $\lambda_B(E)$.

In Fig. 5 the ratio $\xi(E) = \lambda(E)/\lambda_{\rm B}(E)$, normalized to 1 at $E = 10$ MeV, is plotted for the same parameters as in Fig. 3. The presence of the pronounced minimum discriminates the just-so solution from the MSW one, which instead provides the characteristic monotonic shape of this ratio [27]. This effect should be reflected on the recoil electron spectrum $F(T)$ somewhat more strongly than in the case of the Super-Kamiokande detector, since now the spectral distortion is less smoothed by the integration over the neutrino energy. In Fig. 6 we show the isocurves of the recoil electron average energy deviation from the SSM prediction ($\overline{T}_0 = 8.42$ MeV with the electron energy threshold of 5.5 MeV). It ranges up to 12%, stronger than in the Super-Kamiokande detector. For the best fit points it is 8% for $\epsilon = 0$ and 11.5% for $\epsilon = 1$ (see Table I). The energy variation in the MSW picture has the same sign [19], but it is considerably smaller.

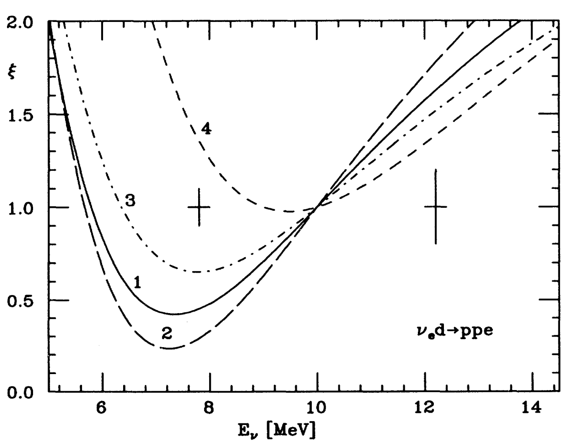


FIG. 5. SNO: the ratio $\xi(E)$ of the distorted boron neutrino energy spectrum to that expected in absence of solar neutrino conversion, normalized to 1 at 10 MeV. The curves correspond to the points marked in Fig. 2. The error bars indicate the expected sensitivity of the detector.

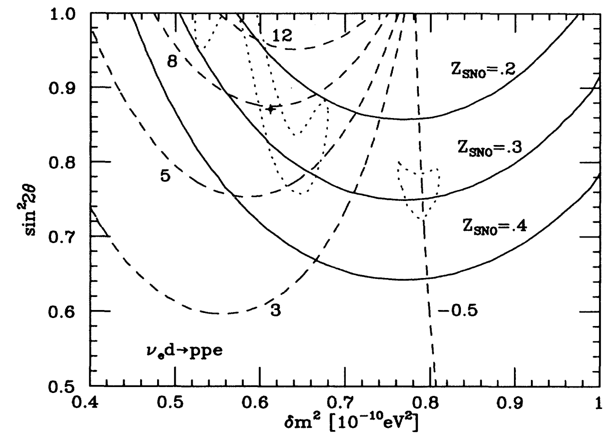


FIG. 6. The isosignal contours due to the CC reaction at the SNO detector with 5.5 MeV threshold (solid line). The dashed curves represent the isopercentage variations of the average electron energy as compared to that expected in the SSM.

The nonstandard interactions (8) of $\nu_\tau$ with electrons do not contribute the signal neither in CC nor NC channels. However, the presence of the analogous nonstandard $\nu_\tau$ interactions with quarks, violating universality of the neutrino interactions with nucleons, could be relevant. In this case the neutral current signal becomes

$$R_{\rm NC} = R_{\rm NC}^{\rm SM} + \int_{E_{\rm th}} dE \Delta\sigma_{\rm NC}^{\rm NSM}(E)\lambda_{\rm B}(E)[\langle\phi^{\rm B}\rangle_T - \langle P(E)\phi^{\rm B}\rangle_T], \tag{14}$$

where $\Delta\sigma_{\rm NC}^{\rm NSM}$ is the additional (to the SM) contribution to the $\nu_x d \to \nu_x p\ n$ cross section arising due to the nonstandard interactions. This extra contribution can differently affect the ratio $\eta$ expected, depending on the sign of $\Delta\sigma_{\rm NC}^{\rm NSM}$. In particular, in the case of sterile $\nu_x$ (i.e., when the extra contribution exactly cancels the standard one), we have $\eta \approx \eta_0$ independently of whether the conversion occurs or not [31].

### C. BOREXINO

Because of the high radiopurity of this scintillator, the detection threshold is low: $T = 0.25$ MeV. This allows us to have enough statistics to detect the $^7$Be and pep neutrino lines through $\nu$-$e$ scattering. In fact, the beryllium neutrino flux can be measured by exploring the energy window $T = 0.25$–$0.7$ MeV for the recoil electrons. In this window, according to the BP SSM, about 50 events are expected per day, versus about 10 events provided by the natural radioactivity background [26]. As for the pep neutrinos whose contribution dominates the recoil electron energy range $T = 0.7$–$1.3$ MeV, their detection is less feasible, since the predicted signal (about three events per day) is comparable with the internal background.

In the just-so picture the strong oscillations of the intermediate energy neutrinos prevent one from making some definite prediction for the time-averaged signals

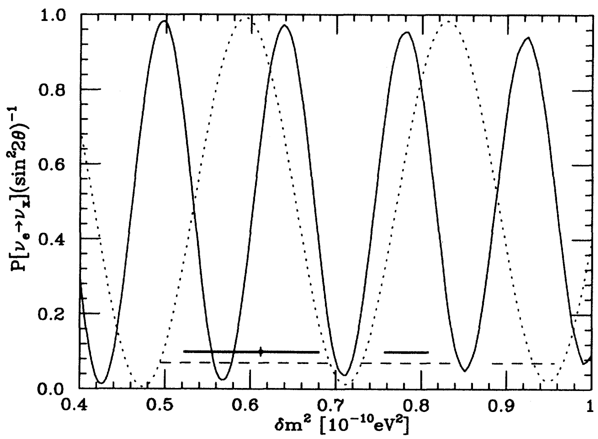


FIG. 7. Time-averaged transition probabilities (modulo $\sin^2 2\theta$) for the $^7$Be and pep neutrinos as functions of $\delta m^2$ (solid and dotted curves, respectively). Horizontal lines indicate 68% C.L. (solid line) and 95% C.L. (dashed line) regions for $\delta m^2$ in the case of the BP SSM. The diamond marks $\delta m^2$ value at the best fit point.

$R_{\text{Be}}$ and $R_{\text{pep}}$, as well as for their ratio.[9] For the sake of demonstration, in Fig. 7 we show the $\delta m^2$ dependence of the time averaged $\nu_e \to \nu_x$ transition probability for the monochromatic $^7$Be and pep neutrinos. For the best fit point these probabilities are large, in agreement with the general paradigm implying a stronger suppression for the intermediate energy neutrinos. However, as we see, in general there is no definite prediction and even the ratio of the signals behaves rather chaotically: In the relevant parameter region it can be much less or more than 1.

The high sensitivity of the BOREXINO detector will allow one to measure the recoil electron energy spectrum due to the $^7$Be neutrinos and, to some extent, also due to the pep ones. In this respect it is of interest to study how these spectra are affected in the just-so oscillation picture. Typical curves of the $\nu$-$e$ event distribution for some parameter values are plotted in Figs. 8(a) and 8(b) for the cases $\epsilon = 0$ and $\epsilon = 1$. In the former case, when $\nu_\tau$ has only SM interactions with the electron, the energy spectrum appears generally depleted throughout the relevant energy interval. However, the shape of the spectrum is not substantially changed and it essentially repeats the SSM predicted one [see Fig. 8(a)]. In the case of the NSM the rate of events is less depleted in the $^7$Be energy window: In the presence of new interactions the $\nu_\tau$ contribution becomes very effective for lower energies, which compensates the deficit of the original $\nu_e$'s. Moreover, for $\epsilon \simeq 1$ the signal can be even larger than that expected in the SSM [see Fig. 8(b)]. Also, the shape of the spectrum becomes steeper as compared to the SSM predicted one. Let us remark also that the compensating effects of the $\tau$-neutrino NSM interactions can smear the time variations of $^7$Be and pep neutrino signals [compare Tables I(a) and I(b)].

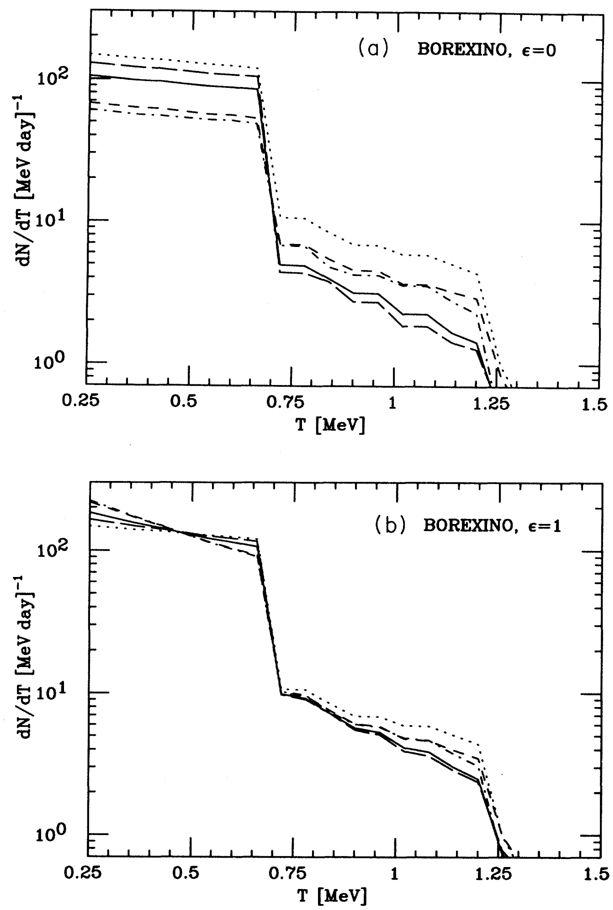


FIG. 8. Distribution of the $\nu$-$e$ scattering events expected at the BOREXINO detector as a function of the recoil electron energy $T$, for the cases $\epsilon = 0$ (a) and $\epsilon = 1$ (b). These are given for the typical points shown in Fig. 3 (solid, long dashed, dot-dashed, and short dashed curves, respectively). For comparison, the dotted curve corresponds the electron spectrum expected in the BP SSM, in the absence of neutrino conversion.

## IV. DISCUSSION

We have confronted the just-so oscillation scenario with the recent experimental data on the solar neutrinos experiments in the context of nonstandard solar models. Namely, we studied the response of this scenario to possible changes of the boron and beryllium neutrino fluxes. In the framework of the BP SSM the data fit is not excellent, $\chi^2_{\text{min}} = 4.4$, while it becomes worse for $f_{\text{B}} < 1$ and slightly improves for $f_{\text{B}} > 1$. The better data fit can be achieved by assuming that the $\nu_x$ state, emerged from the oscillation, has some nonstandard neutral current coupling to the electron. The existing laboratory and astrophysical bounds indeed allow the $\tau$ neutrino to have such NSM interactions in the weak range, with $\epsilon \leq 1$. In this case, also with moderate increasing of $f_{\text{B}}$ (up to 1.3), one can achieve quite reasonable $\chi^2$ fit [see Table I(b)]. It is

[9]In the MSW case, no precise prediction can be obtained for $R_{\text{Be}}$ and $R_{\text{pep}}$ [19], but the ratio $R_{\text{Be}}/R_{\text{pep}}$ remains close to that is expected in the SSM.

interesting to note that the relevant mass range is rather stable against the variation of $f_{\mathrm{B,Be}}$ and $\epsilon$: For the best fit area we have $\delta m^2 \simeq 6 \times 10^{-11}$ eV$^2$. In Table I we show the average rates and their seasonal variations in the chlorine, gallium, and Kamiokande experiments, as well as in the future detectors (Super-Kamiokande, SNO, and BOREXINO), for the best fit points corresponding to different values of $f_{\mathrm{B}}$ and $\epsilon$.

The new generation of the real-time solar neutrino detectors can test the just-so scenario independently of the SSM details, and distinguish it from other candidates to the SNP solution. Even more, the possible NSM neutrino interactions can be also tested, since these detectors will be able to measure the spectra of various solar neutrino components, as well as to detect the effects of their seasonal variations. This will allow one to determine unambiguously all unknown parameters, namely, the SSM ones ($f_{\mathrm{B,Be}}$, etc.) and possible NSM ones ($\epsilon$, etc.) as well as the neutrino mass and mixing range itself.

Indeed, in the case of $\nu_e \to \nu_x$ just-so oscillation the recoil electron energy spectra appear to be specifically altered, and different from the one expected due to the MSW conversion. Imagine that the SNO and/or Super-Kamiokande spectral measurements really point to the just-so oscillation. These spectra separately cannot tell us anything about the presence of the NSM interactions of $\nu_x$ with the electron [compare the curves in Figs. 3(a) and 3(b)]. However, both the CC and NC reactions in the SNO detector provide measurements of the boron neutrino energy spectrum on Earth, which also constitutes the only contribution to the Super-Kamiokande signal. Therefore, the presence of nonstandard $\nu_x$-$e$ interactions could be determined by testing the spectra measured by the SNO and Super-Kamiokande detectors. In fact, the energy spectrum $\lambda(E)$ of the survived boron $\nu_e$'s reaching Earth, and thus the value $P(E)\phi_{\mathrm{B}}$, can be extracted from the spectral measurements of recoil electrons from the CC reaction (with uncertainties related to smearing effects listed in Sec. III). Substituting this value in Eq. (11) for the Super-Kamiokande signal, one can deduce (within inherent uncertainties) the only "unknown" quantity, the differential cross section $\frac{d\sigma_{\nu_x}}{dT}(E,T)$, and test it against the SM prediction. (As we mentioned above, the nonstandard $\nu_x$-$e$ couplings can be also tested by the spectral shape of the recoil electrons in the BOREXINO detector.) By confronting the CC and NC signals in an analogous manner, one can extract also the information on possible NSM couplings of $\nu_x$ with nucleons [see Eq. (14)]. Thus, as far as we believe that the SNP is related to some conversion mechanism of solar $\nu_e$'s into the other neutrino flavors, the Sun appears to be quite a strong and cheap source of the latter. Then measurement of the recoil electron energy spectra in the novel real-time detectors offers a test not only for possible SNP solutions, but also for the neutrino NSM interactions or, in other words, for the electroweak standard model itself.

Last but not least we wish to emphasize that the nonstandard neutrino interactions, in addition to improving the data fit in the just-so picture, could also resolve its potential conflict with the SN 1987A $\nu$ signal, pointed out in Ref. [22]. Namely, these interactions would increase the $\bar{\nu}_\tau$ opacity in the supernova core, and thereby reduce their average energy. This could occur due to the dramatic increase of the $\bar{\nu}_\tau$-$e$ cross section, as compared with the $\nu_\tau$-$e$ one, for large values of $\epsilon$ (see Fig. 9, where these cross sections are plotted versus the neutrino energy for different values of $\epsilon$). Then the interference of the original $\bar{\nu}_e$ and $\bar{\nu}_\tau$ spectra due to the neutrino mixing will less affect the expected $\bar{\nu}_e$ signal in the terrestrial detectors. According to Ref. [22], the problem will be dissolved if the average energy of $\bar{\nu}_\tau$ drops below 17–20 MeV: Then even the maximal mixing $\sin^2 2\theta = 1$ cannot be excluded. Moreover, in this case the partial permutation between the $\bar{\nu}_e$ and $\bar{\nu}_\tau$ spectra could explain the certain excess of the higher energy $\bar{\nu}_e$ events from SN 1987A, indicated by the comparison of the IMB and Kamiokande data [23]. On the other hand, the difference between $\bar{\nu}_\tau$ and $\nu_\tau$ opacities can provide a significant asymmetry in their average energies. For the terrestrial detectors this asymmetry, after oscillation into the $\nu_e$ and $\bar{\nu}_e$ states, could alter the typical ratio of $\bar{\nu}_e$ (isotropic) and $\nu_e$ (directional) events expected from the supernova burst. Obviously, for the precise evaluation of the effects from the NSM neutrino interactions it is necessary to consistently include them into a detailed computer analysis of the stellar core collapse at the beginning.

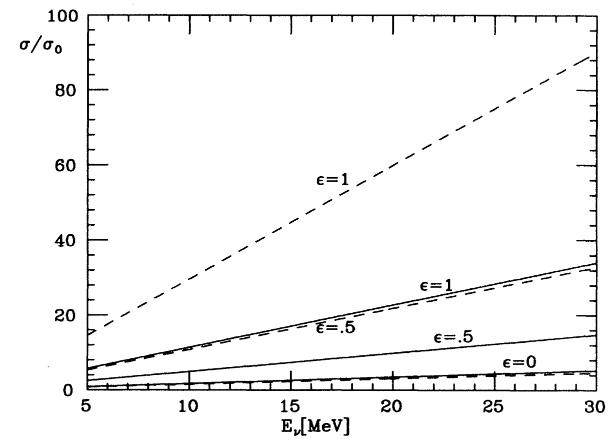


FIG. 9. The energy dependence of the $\bar{\nu}_\tau$-$e$ and $\nu_\tau$-$e$ scattering cross sections (dashed and solid lines, respectively), normalized to $\sigma_0 = 2G_F^2 m_e^2/\pi$, for different values of $\epsilon$.

*Note added.* When our paper was under preparation, we received the papers by Krastev and Petcov [33] and Bilenky and Giunti [34], devoted to the same subject. Our analysis differs from these in many aspects. In particular, we studied the case of nonstandard solar models as well as the impact of the possible nonstandard neutrino interactions.

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