

## Mass singularities in light quark correlators: The strange quark case

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The correlators of light quark currents contain mass singularities of the form  $\ln(m^2/Q^2)$ . It has been known for quite some time that these mass logarithms can be absorbed into the vacuum expectation values of other operators of appropriate dimension, provided that schemes without normal ordering are used. We discuss in detail this procedure for the case of the mass logarithms  $m^4 \ln(m^2/Q^2)$ , including also the mixing with the other dimension-4 operators to two-loop order. As an application we present an improved QCD sum rule determination of the strange quark mass. We obtain  $\bar{m}_s(1 \text{ GeV}) = 171 \pm 15 \text{ MeV}$ .

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### I. INTRODUCTION

The method of QCD sum rules, first introduced in [1], has become a popular and powerful technique to study QCD in the low-energy, nonperturbative region. The starting point is the operator product expansion (OPE) of current correlators at short distances, suitably modified to incorporate nonperturbative effects. The latter are parametrized by a set of vacuum expectation values of the quark and gluon fields entering the QCD Lagrangian. These vacuum condensates induce power corrections to asymptotic freedom, and are responsible for the rich resonance structure observed at low energies. The basic assumption here is the factorization of short and long distance effects. The former are associated with the Wilson coefficients in the OPE, and the latter with the vacuum condensates. While the Wilson coefficients are in principle calculable in perturbation theory, the vacuum condensates cannot be calculated analytically at least at present from first principles. Instead, they can be estimated in the framework of lattice QCD, or extracted from experimental data in certain channels by means of the QCD sum rules themselves. Next, making use of the analyticity properties of the relevant Green functions, and invoking the notion of QCD hadron duality, one relates the fundamental QCD parameters entering the OPE with a dispersive integral involving the hadronic spectral function. In this fashion, a relation between hadronic and QCD parameters is achieved.

An important problem which must be addressed in connection with the factorization of short and long distance effects in the OPE is the appearance of mass singularities in the coefficient functions. They are actually a long-distance effect and thus their presence in the coefficient functions spoils the desired factorization. It is possi-

ble, however, to shift them into the vacuum condensates, provided one is willing to accept the existence of perturbative vacuum expectation values of operators. This is equivalent to giving up the customary normal-ordering prescription which, by definition, sets such contributions to zero. A detailed discussion of how this can be achieved is presented in Sec. II, including the renormalization-group improvement. As a phenomenological application of these results, we address in Sec. III the problem of determining the value of the strange quark mass. Some time ago [2] a redetermination of the strange quark mass was performed in the framework of QCD sum rules, exploiting new developments in the theoretical [3] and experimental [4] understanding of the two-point function involving the strangeness-changing vector current divergence. This constituted an improvement over earlier determinations of  $m_s$  [5,6]. Of particular importance was the removal of logarithmic quark-mass singularities in two-loop quark-mass corrections of order  $O(m_q^2)$  and  $O(m_q^4)$ , achieved in [3]. However, a feature of some concern was the appearance of parametrically enhanced terms of order  $O(1/\alpha_s)$ . In this paper we remedy this problem, showing how these terms can be effectively avoided by employing a scheme without normal ordering. Working at the next-next-to-leading order in perturbative QCD, and including next-to-leading radiative corrections to the condensates, we obtain a new expression for the current correlator. This is then used in order to obtain an improved value of the strange quark mass.

### II. THE OPERATOR PRODUCT EXPANSION

We will be concerned in the following with the vacuum expectation value of the following time ordered product:

$$T(q) = i \int dx e^{iqx} T[J(x)J^\dagger(0)], \quad (1)$$

where  $J = \partial_\alpha \bar{s} \gamma^\alpha u = m_s \bar{s} i u$ , and the up- and down-quark masses are neglected. Except for a sign change, which will be given explicitly, all our results will hold also for the divergence of the strangeness-changing axial vector current  $J = \partial_\alpha \bar{s} \gamma^\alpha \gamma_5 u = m_s \bar{s} i \gamma_5 u$ . When sandwiched between vacuum states, the  $T$  product (1) becomes the corresponding two-point correlator

$$\psi(Q^2, \alpha_s, m, \mu) = \langle 0|T(q)|0 \rangle, \quad (2)$$

where  $\mu$  is the renormalization scale. Note that the polarization operator is *not* renormalization-group (RG) invariant as the function  $\langle 0|T[J(x)J^\dagger(0)]|0 \rangle$  contains non-integrable singularities in the vicinity of the point  $x = 0$ . These cannot be removed by the quark-mass and coupling constant renormalizations alone, but must be subtracted independently. We write the correlator (2) as an expansion in powers of  $1/Q$  as

$$\begin{aligned} \psi(Q^2) = m_s^2 & \left\{ \Pi_0(L, \alpha_s) Q^2 + m_s^2 \Pi_2(L, \alpha_s) \right. \\ & \pm \frac{C_u(L, \alpha_s)}{Q^2} \langle m_s \bar{u} u \rangle_0 \\ & \left. + \sum_{j=1}^3 \frac{C_j(L, \alpha_s)}{Q^2} \langle \mathcal{O}_j \rangle + \mathcal{O}(Q^{-4}) \right\}, \quad (3) \end{aligned}$$

where  $L = \ln(\mu^2/Q^2)$  and  $Q^2 = -q^2$ . The upper sign in front of the coefficient function  $C_u$  corresponds to the scalar and the lower one to the pseudoscalar case. The operators  $\mathcal{O}_i$  are

$$Q^2 \psi''(Q^2, \alpha_s, m_s, \mu) \underset{Q^2 \rightarrow \infty}{\sim} K_0(Q^2, \alpha_s, m_s, \mu) \mathbb{1} + \sum_{n \geq 1} \frac{m_s^2}{(Q^2)^{n/2}} \sum_{\dim \mathcal{O}_i = n} K_i(Q^2, \alpha_s, m_s, \mu) \langle 0|\mathcal{O}_i(\mu)|0 \rangle. \quad (9)$$

The summation on the right-hand side (RHS) extends over all Lorentz scalar composite operators  $\mathcal{O}_i$  of mass dimension  $n \geq 1$ . We have explicitly separated the contribution of the unit operator from that of the operators with a nontrivial dependence on the field variables. The coefficient functions (CF's)  $K_0$  and  $K_i$  depend upon the details of the renormalization prescription for the composite operators  $\mathcal{O}_i$ . The usual procedure of normal ordering for the composite operators appearing on the RHS of the OPE (9) becomes physically unacceptable if quark mass corrections are to be included. This is already obvious for the unit operator, representing the usual perturbative contributions if normal ordering is used, because in general it contains mass and momentum logarithms of the form

$$m_s^2 \left( \frac{m_s^2}{Q^2} \right)^n \left( \ln \frac{\mu^2}{Q^2} \right)^\alpha \left( \ln \frac{\mu^2}{m_s^2} \right)^\beta, \quad (10)$$

$$\mathcal{O}_1 = \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu}, \quad \mathcal{O}_2 = m_s \bar{s} s, \quad \mathcal{O}_3 = m_s^4, \quad (4)$$

and the explicit expressions of the functions  $\Pi_0$  and  $\Pi_2$  will be given later. The only terms in  $\psi$  which are not RG invariant are the coefficient functions  $m_s^2 \Pi_0$  and  $m_s^4 \Pi_2$ . They satisfy the inhomogeneous RG equations

$$\mu \frac{d}{d\mu} (m_s^2 \Pi_0) = m_s^2 \gamma_q, \quad \mu \frac{d}{d\mu} (m_s^4 \Pi_2) = m_s^4 \gamma_m, \quad (5)$$

with [7,8]

$$\gamma_q = \frac{1}{8\pi^2} \left[ -6 - 10 \frac{\alpha_s}{\pi} + \left( 3\zeta(3) - \frac{383}{12} \right) \frac{\alpha_s^2}{\pi^2} \right], \quad (6)$$

$$\gamma_m = \frac{1}{8\pi^2} \left( -12 - 16 \frac{\alpha_s}{\pi} \right). \quad (7)$$

The anomalous dimension  $\gamma_q$  has been given to three-loop order because the corresponding correction to  $\Pi_0$  is *a priori* not negligible, and it will be taken into account in the next section. The absorptive part of  $\psi(Q^2)$ , being an observable quantity, is invariant under the RG transformations. Without any loss of generality we will work with the second derivative  $\psi''(Q^2) \equiv d^2 \psi(Q^2)/d(Q^2)^2$ , which can be seen from (3) and (5) to satisfy a homogeneous RG equation

$$\mu \frac{d}{d\mu} \psi''(Q^2) = 0. \quad (8)$$

The high-energy behavior of  $\psi''(Q^2)$  in the deep Euclidean region may be reliably evaluated in QCD by employing the operator product expansion, i.e.,

with  $n$ ,  $\alpha$ , and  $\beta$  being non-negative integers. More specifically, one may write [9]

$$\begin{aligned} & K_0^{\text{NO}}(Q^2, \alpha_s, m_s, \mu) \\ & \underset{Q^2 \rightarrow \infty}{\sim} m_s^2 \sum_{n \geq 0, l \geq 0} \left( \frac{m_s^2}{Q^2} \right)^{n/2} \left( \frac{\alpha_s}{\pi} \right)^{l-1} F_{il}(L, M), \quad (11) \end{aligned}$$

where  $L = \ln(\mu^2/Q^2)$ ,  $M = \ln(\mu^2/m_s^2)$ , and the superscript NO is a reminder of the normal-ordering prescription being used. The function  $F_{il}(L, M)$  corresponds to the contribution of the  $l$ -loop diagrams, and is a polynomial of degree not higher than  $l$ , in both  $L$  and  $M$ . Now it is obvious that one may not choose the normalization scale  $\mu$  in such a way that for  $Q \gg m_s$  both  $M$  and  $L$  would be small. The mass logarithms signal that

even in the framework of perturbation theory there are effects coming from large distances of order  $1/m$ . Fortunately, it has been realized long ago [3,9–11] that all the mass logarithms may be neatly shifted into the vacuum expectation values (VEV's) of nontrivial composite operators appearing on the RHS of (9), provided the latter are minimally subtracted [12].

To give a simple example, let us consider the correlator (2) in the lowest order one-loop approximation. First, we use the normal-ordering prescription for the composite operators which appear into an OPE of the time ordered product in (1). To determine the coefficients of the various operators, one possible method is to sandwich both sides of the OPE between appropriate external states. By choosing them to be the vacuum, only the unit operator  $\mathbf{1}$  will contribute on the RHS, if the normal-ordering prescription is used. This means that the bare loop of Fig. 1 contributes entirely to the coefficient  $K_0$  in (9). A simple calculation gives [8] (in the following we neglect all terms of order  $1/Q^6$  and higher)

$$\begin{aligned} K_0^{\text{NO}}(Q^2, m_s, \mu) &= Q^2 \psi''(Q^2, m_s, \alpha_s, \mu)|_{\alpha_s=0} \\ &= \frac{3}{8\pi^2} m_s^2 \left[ 1 - 2 \frac{m_s^2}{Q^2} - \frac{2m_s^4}{Q^4} (L - M) \right]. \end{aligned} \quad (12)$$

This coefficient function contains mass singularities (the  $M$  term). On the other hand, if one does not follow the normal-ordering prescription, then the operator  $m_s \bar{s}s$  develops a nontrivial vacuum expectation value even if the quark gluon interaction is turned off by setting  $\alpha_s = 0$ . Indeed, after minimally removing its pole singularity, the one-loop diagram of Fig. 2 leads to [8]

$$\langle 0 | \bar{s}s | 0 \rangle^{\text{pt}} = \frac{m_s^3}{16\pi^2} 4N_c \left( \ln \frac{\mu^2}{m_s^2} + 1 \right). \quad (13)$$

By inserting this into (9), the new coefficient function  $K_0$  can be extracted, with the result

$$K_0 = \frac{3m_s^2}{8\pi^2} \left[ 1 - 2 \frac{m_s^2}{Q^2} - \frac{2m_s^4}{Q^4} (1 + L) \right]. \quad (14)$$

The mass logarithms are now completely transferred from the CF  $K_0$  to the VEV of the quark operator (13). The same phenomenon continues to hold even after the  $\alpha_s$  corrections are taken into account for (pseudo)scalar and (pseudo)vector correlators, independently of their flavor structure [3,14]. The coefficient functions of the nontriv-

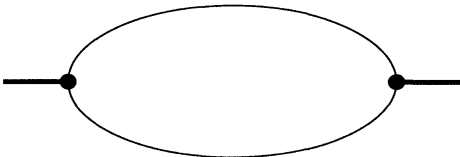


FIG. 1. Lowest-order contribution to the correlator  $\psi(Q^2)$ .

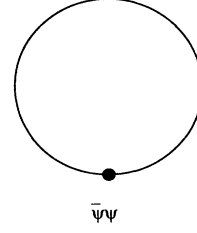


FIG. 2. Vacuum diagram contributing to the perturbative VEV of the operator  $\bar{s}s$ .

ial operators will also depend on whether or not normal ordering is employed.

The underlying reason for this was first established in [15]. Here it was discovered that if the minimal subtraction procedure is scrupulously observed [16] then no CF may depend on mass logarithms in every order of perturbation theory, irrespective of the specific model and/or OPE at hand. This implies that all the mass logarithms  $\ln M$  in  $K_0^{\text{NO}}$  are transferred into the “condensates,” where they are hidden among various non-perturbative contributions. This remarkable property leads to the possibility of using the standard RG techniques to study mass effects in the framework of QCD sum rules without interference from unwanted mass singularities. On the other hand, the above nice features of minimal subtraction come at a price: when schemes without normal ordering are employed, then the renormalization properties of composite operators and CF's become more involved. This may already be observed in our one-loop example. Indeed, as a consequence of (8) one can immediately infer that the CF  $K_0^{\text{NO}}$  is RG invariant and hence should obey the equation

$$\mu \frac{d}{d\mu} K_0^{\text{NO}}(Q^2) = 0. \quad (15)$$

This equation is satisfied trivially for (12) but *not* for  $K_0$  as expressed by (14). The reason is that the operator  $m_s \bar{s}s$  ceases to be RG invariant in the world without normal ordering. The vacuum diagram of Fig. 2 has a divergent part which has to be removed by a new counterterm proportional to the operator  $m_s^4 \mathbf{1}$ . In other words,  $m_s \bar{s}s$  begins to mix with the “operator”  $m_s^4 \mathbf{1}$  [9].

To lowest order, the corresponding anomalous dimension matrix reads

$$\mu \frac{d}{d\mu} \begin{pmatrix} m_s \bar{s}s \\ m_s^4 \end{pmatrix} = \begin{pmatrix} 0 & \frac{3}{2\pi^2} \\ 0 & -8 \frac{\alpha_s}{\pi} \end{pmatrix} \begin{pmatrix} m_s \bar{s}s \\ m_s^4 \end{pmatrix}. \quad (16)$$

The nonvanishing off-diagonal matrix element describes the mixing of the two operators under renormalization and was obtained from the divergent part of the vacuum diagram in Fig. 2. The diagonal matrix elements are just the anomalous dimensions of the respective operators in the usual normal-ordering scheme. The lower one is equal to  $-4\gamma(\alpha_s)$ , where  $\gamma(\alpha_s)$  is the strange quark mass anomalous dimension which defines its running according to

$$\mu \frac{d}{d\mu} m_s = -\gamma(\alpha_s) m_s, \quad (17)$$

where [17]

$$\gamma(\alpha_s) = \gamma_1 \frac{\alpha_s}{\pi} + \gamma_2 \left( \frac{\alpha_s}{\pi} \right)^2 + \dots, \quad (18)$$

with  $\gamma_1 = 2$ ,

$$\gamma_2 = \frac{101}{12} - \frac{5}{18} n_f, \quad (19)$$

$$\gamma_3 = \left( 1249 - \left[ \frac{2216}{27} + \frac{160}{3} \zeta(3) \right] n_f - \frac{140}{81} n_f^2 \right) / 32, \quad (20)$$

and  $n_f$  is the number of active light quarks. The running of the coupling constant  $\alpha_s(\mu)$  is determined by

$$\mu \frac{d}{d\mu} \alpha_s = \beta(\alpha_s) \alpha_s, \quad (21)$$

where [18]

$$\beta(\alpha_s) = \beta_1 \frac{\alpha_s}{\pi} + \beta_2 \left( \frac{\alpha_s}{\pi} \right)^2 + \beta_3 \left( \frac{\alpha_s}{\pi} \right)^3 + \dots, \quad (22)$$

with

$$\beta_1 = -\frac{11}{2} + \frac{1}{3} n_f,$$

$$\beta_2 = -\frac{51}{4} + \frac{19}{12} n_f, \quad (23)$$

$$\beta_3 = \left( -\frac{2857}{2} + \frac{5033}{18} n_f - \frac{325}{54} n_f^2 \right) / 32.$$

The solutions of (17) and (21) can be written as

$$\alpha_s(\mu) = \frac{2\pi}{-\beta_1 L} \left[ 1 + \frac{2\beta_2 \ln L}{\beta_1^2 L} + \frac{4}{\beta_1^2 L^2} \left( \frac{\beta_2^2}{\beta_1^2} (\ln^2 L - \ln L - 1) + \frac{\beta_3}{\beta_1} \right) \right], \quad (24)$$

$$\begin{aligned} m_s(\mu) = & \frac{\hat{m}_s}{(\frac{1}{2}L)^{-\gamma_1/\beta_1}} \left[ 1 - \frac{2\gamma_1\beta_2 \ln L}{\beta_1^3 L} + \frac{2}{\beta_1^2} \left( \gamma_2 - \frac{\gamma_1\beta_2}{\beta_1} \right) \frac{1}{L} \right. \\ & + \frac{2}{\beta_1^6 L^2} (-\beta_1^3 \gamma_3 + \beta_1^2 \gamma_2^2 + \beta_1^2 \beta_2 \gamma_2 + \beta_1 \beta_2^2 \gamma_1 - \beta_1^2 \beta_3 \gamma_1 - 2\beta_1 \beta_2 \gamma_1 \gamma_2 + \beta_2^2 \gamma_1^2) \\ & \left. + \frac{4 \ln L}{\beta_1^6 L^2} (\beta_1^2 \beta_2 \gamma_2 - \beta_1 \beta_2 \gamma_1 \gamma_2 + \beta_2^2 \gamma_1^2) + \frac{2 \ln^2 L}{\beta_1^6 L^2} (-\beta_1 \beta_2^2 \gamma_1 + \beta_2^2 \gamma_1^2) \right]. \quad (25) \end{aligned}$$

The expressions (24) and (25) are given to three-loop order for completeness. The simplified arguments so far will only make use of the corresponding one-loop results. Now one can see that the operator  $m_s \bar{s}s$  acquires a scale dependence, which can be obtained from (16) and is given by [9]

$$\begin{aligned} m_s \bar{s}s(\mu) &= m_s \bar{s}s(\mu_0) + \frac{3}{2\pi^2} \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{dx}{x\beta(x)} \exp\left(-4 \int_{\alpha_s(\mu_0)}^x \frac{dy \gamma(y)}{y \beta(y)}\right) m_s^4(\mu_0) \\ &= m_s \bar{s}s(\mu_0) - \frac{3}{2\pi(4\gamma_1 + \beta_1)} \left( \frac{m_s^4(\mu)}{\alpha_s(\mu)} - \frac{m_s^4(\mu_0)}{\alpha_s(\mu_0)} \right). \quad (26) \end{aligned}$$

A distinctive feature of this result is the appearance of inverse powers of  $\alpha_s$  [11]. Note that in the approximation we have considered, the combination

$$I_s = (m_s \bar{s}s)(\mu) + \frac{3}{2\pi(4\gamma_1 + \beta_1)} \frac{m_s^4(\mu)}{\alpha_s(\mu)} \quad (27)$$

is a RG invariant. It corresponds (but is not generally equal) to the RG-invariant combination  $m_s \bar{s}s$  in the usual normal-ordering scheme. For simplicity, we will neglect for the moment the contributions of the dimension-4 operators  $G^2$  and  $\bar{u}u$  in the OPE (9). They will be added in

later. The lowest-order coefficients of the operators  $m_s \bar{s}s$  and  $m_s^4$  at the scale  $\mu = Q$  have the values

$$c_{m_s \bar{s}s} = \frac{1}{2}, \quad c_{m_s^4} = \frac{3}{16\pi^2}. \quad (28)$$

We are now in a position to derive the RG improvement of the coefficient functions appearing in the OPE (9) when working in a scheme without normal ordering. To achieve this, one notes from Eq. (8) that the total contribution of the operators of dimension 4 is RG invariant and therefore we can choose freely the scale  $\mu$ . Setting the renormalization scale  $\mu = Q$  allows us to absorb all logarithms

$\ln(\mu^2/Q^2)$  appearing in the CF  $K_0$  into the running coupling constant and the strange quark mass. On the other hand, the matrix elements of the operators at this scale can be expressed in terms of the same matrix elements at

a lower scale  $\mu_0 \simeq 1 \text{ GeV}^2$  with the help of the RG equation (16). Our first result for the RG improvement of the OPE (9), treated entirely within the minimal subtraction prescription, reads (for  $n_f = 3$ )

$$Q^2 \psi''(Q^2, m_s, \mu) \underset{Q^2 \rightarrow \infty}{\sim} \frac{3}{8\pi^2} m_s^2(Q) \left( 1 - 2 \frac{m_s^2(Q)}{Q^2} \right) + \frac{m_s^2(Q)}{Q^4} \left( \langle 0 | (m_s \bar{s}s)(\mu_0) | 0 \rangle - \frac{3}{7\pi} \frac{m_s^4(Q)}{\alpha_s(Q)} + \frac{3}{7\pi} \frac{m_s^4(\mu_0)}{\alpha_s(\mu_0)} \right) + O(1/Q^6). \quad (29)$$

This result has been essentially obtained for the first time in [11] (to two-loop order). There are, however, a number of differences between its interpretation as given in [11] (see also [3] and [14]) and the point of view we will take in this paper. We comment briefly on these differences. In [11] the vacuum expectation value of the RG-invariant combination  $I_s$  in (27) was identified with the (RG-invariant) product  $m_s \langle 0 | \bar{s}s^{\text{NO}} | 0 \rangle$  in the usual scheme using normal ordering. Thus, the vacuum matrix element  $\langle 0 | (m_s \bar{s}s)(\mu) | 0 \rangle$  (non-normal ordered and RG noninvariant) was represented, to the order we are working, as the sum of a RG-invariant part (of a nonperturbative origin, due to the spontaneous breaking of the chiral symmetry in QCD) and of a perturbative part ( $\mu$  dependent) which represents the sum of the leading mass singularities of the form  $\alpha_s^n \ln^{n+1}(\mu^2/m^2)$ :

$$\langle 0 | (m_s \bar{s}s)(\mu) | 0 \rangle = \langle 0 | m_s \bar{s}s^{\text{NO}} | 0 \rangle - \frac{3}{7\pi} \frac{m_s^4(\mu)}{\alpha_s(\mu)}. \quad (30)$$

As can be seen from (24) and (25) in the one-loop approximation, the second term vanishes in the limit  $\mu \rightarrow \infty$  and the distinction between the VEV of the operator  $m_s \bar{s}s$  in schemes with and without normal ordering disappears. It should be stressed that the above interpretation of the relation between normal-ordered and nonordered quark condensates relies heavily on an implicit assumption which is difficult to (dis)prove. Indeed, all purely perturbative contributions to  $\langle 0 | (m_s \bar{s}s)(\mu) | 0 \rangle$  were assumed to vanish in the limit  $\mu \rightarrow \infty$ . Fortunately, even if the hypothesis fails it will only spoil the applicability of the scheme with normal ordering, but would have no effect on other renormalization schemes such as, e.g., the minimal subtraction prescription. The practical con-

sequence of this approach is a large value of the mass correction of order  $m_s^4$ , which is enhanced by the presence of one negative power of  $\alpha_s(Q)$  [the second term in the second line of Eq. (29)]. Note that in this approach there is no corresponding term containing  $1/\alpha_s(\mu_0)$  in Eq. (29), because it can be effectively combined with the contribution of the operator  $m_s \bar{s}s$  resulting in the RG-invariant VEV  $\langle 0 | I_s | 0 \rangle$ . On the other hand, considering that a typical momentum transfer for QCD sum rules is of about 1 GeV, we will work with the quark and gluon condensates normalized at this “natural” scale  $\mu_0 = 1 \text{ GeV}$  as our reference values. (For the case of the semihadronic decay rate of the  $\tau$  lepton a similar approach has been suggested in [19].) As mentioned above, this point of view is equivalent to the one taken in [11], provided the scale  $\mu_0$  were taken to infinity. Our choice of  $\mu_0$  somewhere around the characteristic momentum scale specific to the problem at hand ( $\simeq 1 \text{ GeV}$ ) helps to avoid the parametrically enhanced inverse powers of  $\alpha_s$ . Indeed, as one can see from (24) to one-loop order, one has  $\pi/\alpha_s(Q) - \pi/\alpha_s(\mu_0) = -\beta_1 \ln(Q/\mu_0)$ , which is not particularly large.

### III. DETERMINATION OF THE STRANGE QUARK MASS

We proceed now to include the contributions from the gluon operator  $G^2$  and from the light quark condensate  $\bar{u}u$ , working consistently to next-to-leading order. The coefficient functions  $\Pi_0$  and  $\Pi_2$  in (3) have, respectively, the three-loop [20] and two-loop values [for an arbitrary renormalization scale  $\mu$ , the use of the modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme is understood]

$$\begin{aligned} \Pi_0 = & \frac{1}{16\pi^2} \left[ -12 - 6 \ln \frac{\mu^2}{Q^2} + \frac{\alpha_s}{\pi} \left( -\frac{131}{2} - 34 \ln \frac{\mu^2}{Q^2} - 6 \ln^2 \frac{\mu^2}{Q^2} + 24\zeta(3) \right) \right. \\ & + \left( \frac{\alpha_s}{\pi} \right)^2 \left( -\frac{17\,645}{24} + 353\zeta(3) - 8n_f\zeta(3) + \frac{511}{18}n_f + \frac{3}{2}\zeta(4) - 50\zeta(5) \right. \\ & - 4n_f\zeta(3) \ln \frac{\mu^2}{Q^2} + n_f \frac{65}{4} \ln \frac{\mu^2}{Q^2} + 117\zeta(3) \ln \frac{\mu^2}{Q^2} - \frac{10\,801}{24} \ln \frac{\mu^2}{Q^2} \\ & \left. \left. + n_f \frac{11}{3} \ln^2 \frac{\mu^2}{Q^2} - 106 \ln^2 \frac{\mu^2}{Q^2} + n_f \frac{1}{3} \ln^3 \frac{\mu^2}{Q^2} - \frac{19}{2} \ln^3 \frac{\mu^2}{Q^2} \right) \right], \quad (31) \end{aligned}$$

$$\Pi_2 = \frac{1}{16\pi^2} \left[ -12 - 12 \ln \frac{\mu^2}{Q^2} + \frac{\alpha_s}{\pi} \left( -100 - 64 \ln \frac{\mu^2}{Q^2} - 24 \ln^2 \frac{\mu^2}{Q^2} + 48\zeta(3) \right) \right]. \quad (32)$$

The three-loop result (31) was obtained in the works [20] while the  $\alpha_s$  terms in (31) and (32) can be found in [8,21]. Note that the first evaluation of  $\Pi_0$  at three-loop level made in Ref. [22] proved to be erroneous [23]. Unfortunately, that wrong result was then used in the work [26] to find the light quark masses in the framework of the finite energy sum rules.

The coefficient functions of the dimension-4 operators, evaluated at the scale  $\mu = Q$ , are

$$C_1 = \frac{1}{8} \left( 1 + \frac{11\alpha_s}{2\pi} \right), \quad (33)$$

$$C_2 = \frac{1}{2} \left( 1 + \frac{11\alpha_s}{3\pi} \right), \quad (34)$$

$$C_u = \left( 1 + \frac{14\alpha_s}{3\pi} \right), \quad (35)$$

$$C_3 = \frac{3}{16\pi^2} \left( 1 + \frac{\alpha_s}{\pi} [8\zeta(3) - 6] \right). \quad (36)$$

The leading order contributions to (33)–(35) were computed in [1]. The two-loop corrections to (33)–(36) were evaluated in the Refs. [27], [1,21], [1,21], and [3,14], respectively.

It is now a simple matter to derive the RG improvement of the  $\Pi_{0,2}$  terms in (3). Solving (5) with the boundary conditions (31) and (32) yields

$$\begin{aligned} (m_s^2 \Pi_0)|_\mu &= -\frac{3}{4\pi(2\gamma_1 + \beta_1)} \frac{m_s^2(Q)}{\alpha_s(Q)} \left[ 1 + (r_1 + 4 + \beta_1) \frac{\alpha_s(Q)}{\pi} + \left( r_3 + \frac{131}{6} + \frac{131\beta_1}{24} - 8\zeta(3) - 2\beta_1\zeta(3) \right) \left( \frac{\alpha_s(Q)}{\pi} \right)^2 \right] \\ &+ \frac{3}{4\pi(2\gamma_1 + \beta_1)} \frac{m_s^2(\mu)}{\alpha_s(\mu)} \left[ 1 + r_1 \frac{\alpha_s(\mu)}{\pi} + r_3 \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \right], \end{aligned} \quad (37)$$

$$(m_s^4 \Pi_2)|_\mu = -\frac{3}{2\pi(4\gamma_1 + \beta_1)} \frac{m_s^4(Q)}{\alpha_s(Q)} \left[ 1 + \left( r_2 + 4 + \frac{\beta_1}{2} \right) \frac{\alpha_s(Q)}{\pi} \right] + \frac{3}{2\pi(4\gamma_1 + \beta_1)} \frac{m_s^4(\mu)}{\alpha_s(\mu)} \left( 1 + r_2 \frac{\alpha_s(\mu)}{\pi} \right), \quad (38)$$

where

$$r_1 = \frac{5}{3} - \frac{\gamma_2}{2} + \frac{5\beta_1}{12} - \frac{\beta_2}{4} = -2 \quad (n_f = 3), \quad (39)$$

$$r_2 = \frac{4}{3} - \frac{\gamma_2}{2} + \frac{\beta_1}{6} - \frac{\beta_2}{8} = -\frac{53}{24} \quad (n_f = 3), \quad (40)$$

$$\begin{aligned} r_3 &= \frac{9[7889 - 432\zeta(3)] + 9[439 - 1824\zeta(3)]n_f - 1195n_f^2}{864(-57 + 2n_f)} \\ &= \frac{5904\zeta(3) - 8011}{4896} \quad (n_f = 3). \end{aligned} \quad (41)$$

The RG improvement of the contribution of the dimension-4 operators in (3) requires the knowledge of their mixing under renormalization. The generalization of (16) to two-loop order, by taking also into account the mixing with the gluon operator  $G^2$ , reads [9]

$$\mu \frac{d}{d\mu} \begin{pmatrix} G^2 \\ m_s \bar{s}s \\ m_s^4 \end{pmatrix} = \begin{pmatrix} -\alpha_s \frac{d\beta}{d\alpha_s} & -4\alpha_s \frac{d\gamma}{d\alpha_s} & 4\alpha_s \frac{d\gamma_0}{d\alpha_s} \\ 0 & 0 & -4\gamma_0 \\ 0 & 0 & -4\gamma \end{pmatrix} \begin{pmatrix} G^2 \\ m_s \bar{s}s \\ m_s^4 \end{pmatrix}. \quad (42)$$

Here  $\beta$  and  $\gamma$  were defined in (21) and (17), respectively, and  $\gamma_0$  is the two-loop vacuum energy anomalous dimension, given by

$$\gamma_0 = -\frac{3}{8\pi^2} \left( 1 + \frac{4}{3\pi} \alpha_s \right). \quad (43)$$

The operator  $m_s \bar{u}u$  is RG invariant. The analogous anomalous dimension matrix which describes the mixing of the dimension-5 operators in schemes without normal ordering has been calculated recently, to on-loop order, in [28]. We apply now the RG improvement of the contribution of the dimension-4 operators in (3). By taking advantage of the fact that their total contribution is RG invariant, we choose  $\mu = Q$ , where the coefficient functions are given by (33)–(36). The matrix elements of the operators  $\mathcal{O}_{1,2,3}$  can be scaled at  $\mu_0 \simeq 1$  GeV with the help of (42), where they are known. This procedure leads to

$$\begin{aligned} \sum_{j=1}^3 C_j \mathcal{O}_j &= \left( 1 + \frac{14\alpha_s(Q)}{3\pi} \right) \langle m_s \bar{u}u \rangle_0 + \frac{1}{8} \left[ 1 + \left( \frac{11}{2} - \frac{\beta_2}{\beta_1} \right) \frac{\alpha_s(Q)}{\pi} + \frac{\beta_2 \alpha_s(\mu_0)}{\beta_1 \pi} \right] \langle \mathcal{O}_1 \rangle_{|\mu_0} \\ &+ \left[ \frac{1}{2} + \left( \frac{11}{6} - \frac{\gamma_1}{2\beta_1} \right) \frac{\alpha_s(Q)}{\pi} + \frac{\gamma_1}{2\beta_1} \frac{\alpha_s(\mu_0)}{\pi} \right] \langle \mathcal{O}_2 \rangle_{|\mu_0} \\ &- \frac{3}{4\pi(4\gamma_1 + \beta_1)\alpha_s(Q)} \left( 1 + \frac{\alpha_s(Q)}{\pi} \left[ r_2 + \frac{47}{12} - \gamma_1 - \frac{\beta_1}{4} \right] \right) m_s^4(Q) \\ &+ \frac{3}{4\pi(4\gamma_1 + \beta_1)\alpha_s(\mu_0)} \left( 1 + \frac{\alpha_s(\mu_0)}{\pi} \left[ r_2 + \frac{\gamma_1}{\beta_1} + \frac{1}{4} \right] \right) m_s^4(\mu_0) + \frac{1}{4\pi^2(4\gamma_1 + \beta_1)} \frac{\alpha_s(Q)}{\alpha_s(\mu_0)} m_s^4(\mu_0) \left( 11 - \frac{3\gamma_1}{\beta_1} \right). \end{aligned} \quad (44)$$

In order to keep the expressions within a reasonable size, we will replace here the various constants by their numerical values corresponding to  $n_f = 3$ . At the same time, to help the reader who might want to reproduce our result, the  $n_f$  dependence which appears from other sources will be left explicit in the following. Thus, putting together (37) and (44) and taking two derivatives with respect to  $Q^2$  we obtain

$$\begin{aligned} \psi''(Q^2) &= \frac{3m_s^2(Q)}{8\pi^2 Q^2} \left[ 1 + \frac{11\alpha_s(Q)}{3\pi} + \left( \frac{5071}{144} - \frac{35}{2} \zeta(3) \right) \left( \frac{\alpha_s(Q)}{\pi} \right)^2 - 2 \frac{m_s^2(Q)}{Q^2} \left( 1 + \frac{28\alpha_s(Q)}{3\pi} \right) \right] \\ &+ \frac{m_s^2(Q)}{Q^6} \left\{ 2 \langle m_s \bar{u}u \rangle \left( 1 + \frac{23\alpha_s(Q)}{3\pi} \right) + \frac{1}{4} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{|\mu_0} \left( 1 + \frac{16\alpha_s(\mu_0)}{9\pi} + \frac{121\alpha_s(Q)}{18\pi} \right) \right. \\ &+ \langle m_s \bar{s}s \rangle_{|\mu_0} \left( 1 - \frac{4\alpha_s(\mu_0)}{9\pi} + \frac{64\alpha_s(Q)}{9\pi} \right) - \frac{3}{7\pi^2} m_s^4(Q) \left( \frac{\pi}{\alpha_s(Q)} + \frac{155}{24} \right) \\ &\left. + \frac{3}{7\pi^2} m_s^4(\mu_0) \left( \frac{\pi}{\alpha_s(\mu_0)} - \frac{173}{72} \right) + \frac{64}{21\pi^2} \frac{\alpha_s(Q)}{\alpha_s(\mu_0)} m_s^4(\mu_0) \right\}. \end{aligned} \quad (45)$$

A similar relation has been previously used in a QCD sum rule determination of the strange quark mass [2], where it was interpreted in the spirit of [11]. As explained earlier, in the approach of [11] the normal-ordered strange quark condensate (times  $m_s$ ) is identified with the VEV of the RG-invariant combination  $I_s$  defined at one-loop level in Eq. (27). At two-loop level it has the form

$$I_s = (m_s \bar{s}s)(\mu) + \frac{3}{2\pi(4\gamma_1 + \beta_1)} \frac{m_s^4(\mu)}{\alpha_s(\mu)} \left( 1 + r_2 \frac{\alpha_s(\mu)}{\pi} \right). \quad (46)$$

Besides this, our result (45) differs from the one in [11] (see also [14]) because there the mixing of the gluon condensate with the other operators of dimension 4 has been neglected.

We perform the Borel transform  $\hat{L}$  of  $\psi''(Q^2)$ , i.e.,

$$\begin{aligned} \hat{L}[\psi''(Q^2)] &= \lim_{N \rightarrow \infty} \frac{(-1)^N}{(N-1)!} (Q^2)^N \frac{\partial^N}{(\partial Q^2)^N} \psi''(Q^2) \\ &= \frac{1}{M^6} \int_0^\infty ds e^{-s/M^2} \frac{1}{\pi} \text{Im} \psi(s). \end{aligned} \quad (47)$$

A simple calculation using the methods of [29] gives

$$\begin{aligned}
\hat{L}[\psi''(Q^2)] = & \frac{3}{8\pi^2} \frac{\hat{m}_s^2}{M^2} \frac{1}{[\frac{1}{2} \ln(M^2/\Lambda^2)]^{-2\gamma_1/\beta_1}} \left( 1 + \frac{4}{9 \ln M^2/\Lambda^2} \left[ \frac{11}{3} - \gamma_1 \psi(1) + \frac{4\beta_2}{\beta_1^2} \ln \ln \frac{M^2}{\Lambda^2} - \frac{4}{\beta_1 \gamma_1} \left( \gamma_2 - \gamma_1 \frac{\beta_2}{\beta_1} \right) \right] \right. \\
& + \frac{2}{81 \ln^2(M^2/\Lambda^2)} \left\{ \frac{2510167}{6561} - \frac{1340}{9} \zeta(3) + 34\psi^2(1) - \frac{17332}{81} \psi(1) - \frac{17}{3} \pi^2 \right. \\
& \left. \left. + \ln \ln(M^2/\Lambda^2) \left( -\frac{1109248}{6561} + \frac{4352}{81} \psi(1) \right) + \ln^2 \ln(M^2/\Lambda^2) \frac{139264}{6561} \right\} \right. \\
& - 2 \frac{\hat{m}_s^2}{M^2} \frac{1}{[\frac{1}{2} \ln(M^2/\Lambda^2)]^{-2\gamma_1/\beta_1}} \left\{ 1 + \frac{4}{9 \ln M^2/\Lambda^2} \left[ \frac{28}{3} - 2\gamma_1 \psi(2) + 8 \frac{\beta_2}{\beta_1^2} \ln \ln \frac{M^2}{\Lambda^2} \right. \right. \\
& \left. \left. - \frac{8}{\beta_1 \gamma_1} \left( \gamma_2 - \gamma_1 \frac{\beta_2}{\beta_1} \right) \right] \right\} \left. \right) + \frac{1}{2M^6} \frac{\hat{m}_s^2}{[\frac{1}{2} \ln(M^2/\Lambda^2)]^{-2\gamma_1/\beta_1}} \\
& \times \left( A(\mu_0) + \frac{4}{9 \ln M^2/\Lambda^2} \left\{ B(\mu_0) + \left[ -\gamma_1 \psi(3) + 4 \frac{\beta_2}{\beta_1^2} \ln \ln \frac{M^2}{\Lambda^2} - \frac{4}{\beta_1 \gamma_1} \left( \gamma_2 - \gamma_1 \frac{\beta_2}{\beta_1} \right) \right] A(\mu_0) \right\} \right) \\
& - \frac{3}{7\pi^2} \frac{1}{2M^6} \frac{\hat{m}_s^6}{[\frac{1}{2} \ln(M^2/\Lambda^2)]^{-6\gamma_1/\beta_1}} \left\{ \frac{155}{24} - \frac{\beta_1}{2} \ln \frac{M^2}{\Lambda^2} - \frac{\beta_1}{2} \left( \frac{6\gamma_1}{\beta_1} + 1 \right) \psi(3) \right. \\
& \left. + \left( 12 \frac{\beta_2}{\beta_1^2} + \frac{\beta_2}{\beta_1} \right) \ln \ln \frac{M^2}{\Lambda^2} - \frac{12}{\beta_1 \gamma_1} \left( \gamma_2 - \gamma_1 \frac{\beta_2}{\beta_1} \right) \right\}. \tag{48}
\end{aligned}$$

where

$$\begin{aligned}
A(\mu_0) = & 2\langle m_s \bar{u}u \rangle_0 + \frac{1}{4} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 \left( 1 + \frac{16}{9} \frac{\alpha_s(\mu_0)}{\pi} \right) \\
& + \langle m_s \bar{s}s \rangle_0 \left( 1 - \frac{4}{9} \frac{\alpha_s(\mu_0)}{\pi} \right) + m_s^4(\mu_0) \frac{3}{7\pi^2} \frac{\pi}{\alpha_s(\mu_0)} \left( 1 - \frac{173}{72} \frac{\alpha_s(\mu_0)}{\pi} \right), \tag{49}
\end{aligned}$$

$$B(\mu_0) = \frac{46}{3} \langle m_s \bar{u}u \rangle_0 + \frac{121}{72} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 + \frac{64}{9} \langle m_s \bar{s}s \rangle_0 + m_s^4(\mu_0) \frac{64}{21\pi^2} \frac{\pi}{\alpha_s(\mu_0)} \left( 1 - \frac{519}{512} \frac{\alpha_s(\mu_0)}{\pi} \right). \tag{50}$$

The expression (48) represents the ‘‘theoretical’’ side of the QCD sum rule. The ‘‘phenomenological’’ side is given by the RHS of (47), with the (hadronic) spectral function  $\text{Im}\psi(s)$  written as

$$\text{Im}\psi(s) = \text{Im}\psi(s)|_{\text{res}}\theta(s_0 - s) + \text{Im}\psi(s)|_{\text{QCD}}\theta(s - s_0), \tag{51}$$

where the first term above describes the contributions of the resonances up to  $s = 6.8 \text{ GeV}^2$  and the second term, i.e., the hadronic continuum, is identified as usual with the perturbative QCD expression, which in this case is given by

$$\frac{1}{\pi} \text{Im}\psi(s)|_{\text{QCD}} = \frac{3}{8\pi^2} m_s^2(s) s \left( 1 + \frac{17\alpha_s(s)}{3\pi} \right). \tag{52}$$

The continuum threshold is expected to be close to the upper limit of the experimental data, i.e.,  $s_0 \simeq 6\text{--}7 \text{ GeV}^2$ . In principle, though,  $s_0$  is a free parameter. Predictions will be meaningful provided they do not depend strongly on the value of this parameter.

Chiral dynamics provides a strong constraint on the behavior of the hadronic spectral function near threshold, viz.,

$$\frac{1}{\pi} \text{Im}\psi(s) = \frac{3}{32\pi^2} |d(s_+)|^2 \sqrt{\left( 1 - \frac{s_+^{K\pi}}{s} \right) \left( 1 - \frac{s^{K\pi}}{s} \right)}, \tag{53}$$

with  $s_{\pm}^{K\pi} = (M_K \pm M_\pi)^2$  and  $|d(s_+)| \simeq 0.3 \text{ GeV}^2$ . A good fit to the experimental data [4] is obtained by using (53), which simulates nonresonant background, to normalize two Breit-Wigner forms for the  $K_0^*(1430)$  and  $K_0^*(1950)$  resonances, with masses and widths:  $M_1 = 1.40 \pm 0.01 \text{ GeV}$ ,  $\Gamma_1 = 325 \pm 30 \text{ MeV}$ ,  $M_2 = 1.94 \pm 0.03 \text{ GeV}$ , and  $\Gamma_2 = 450 \pm 100 \text{ MeV}$ .

As for the QCD parameters, we adopt the following values for the nonperturbative condensates:  $\langle \bar{u}u \rangle_{\mu_0} = -(0.25)^3 \text{ GeV}^3$  at a scale  $\mu_0 = 1 \text{ GeV}$  and  $\langle \bar{s}s \rangle_{\mu_0} / \langle \bar{u}u \rangle_{\mu_0} = 0.7\text{--}1$ . The gluon condensate has been extracted some time ago [30] from data on  $e^+e^-$  annihilation and  $\tau$  decay, with values in the range  $\langle (\alpha_s/\pi) G^2 \rangle =$



0.02–0.06 GeV<sup>4</sup>. The QCD scale for three flavors in  $\Lambda \simeq 200\text{--}400$  MeV [31,32].

The invariant strange-quark mass  $\hat{m}_s$  is determined by solving the equation resulting from inserting (51) on the RHS of (47) and using (48) on the LHS. Typical results are shown in Fig. 3 (for  $\Lambda = 200$  MeV) and Fig. 4 (for  $\Lambda = 400$  MeV), corresponding to  $\langle(\alpha_s/\pi)G^2\rangle_{\mu_0} = 0.02$  GeV<sup>4</sup>. In both these figures we used  $\langle\bar{s}s\rangle_{\mu_0}/\langle\bar{u}u\rangle_{\mu_0} = 1$ . Results are essentially unchanged if this ratio deviates from unity by some 30%. On the theoretical side of the sum rule, the most important contribution numerically comes from  $\Pi_0$  in (3) which accounts for about 90% from the total in the region of the Borel parameter relevant for the sum rule.  $\Pi_2$  contributes with an opposite sign at the 3–5% level, and the nonperturbative contributions together do not exceed a few percent. The latter are rather insensitive to the precise value of the quark condensate and depend mostly on the value of the gluonic condensate.

The  $\alpha_s^2$  corrections to the coefficient functions of the gluon and quark condensates may be substantial [27,33]. We have effectively incorporated this uncertainty by allowing rather generous error bars on the gluon condensate value. Taking into account the rather small contribution of the quark condensates (under 2%), the  $\alpha_s^2$  correction to their CF's should not influence our results noticeably.

The error on  $m_s$  is determined by its variation when all relevant parameters are changed within the ranges indicated above. This gives, for the two extreme choices of  $\Lambda$ ,

$$\hat{m}_s = 213\text{--}222 \text{ MeV}, \quad \bar{m}_s(1 \text{ GeV}) = 171\text{--}179 \text{ MeV} \quad (\Lambda = 200 \text{ MeV}), \quad (54)$$

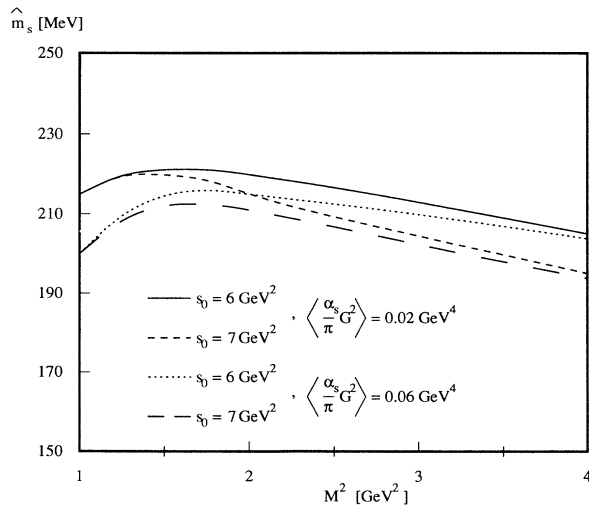


FIG. 3. Invariant strange quark mass  $\hat{m}_s$  as a function of the Borel variable  $M^2$ , for  $\Lambda = 200$  MeV. The values of the gluon condensate  $\langle(\alpha_s/\pi)G^2\rangle_{\mu_0}$  and of the continuum threshold  $s_0$  have been varied between 0.02 and 0.06 GeV<sup>4</sup> and, respectively,  $s_0 = 6$  and 7 GeV<sup>2</sup>.

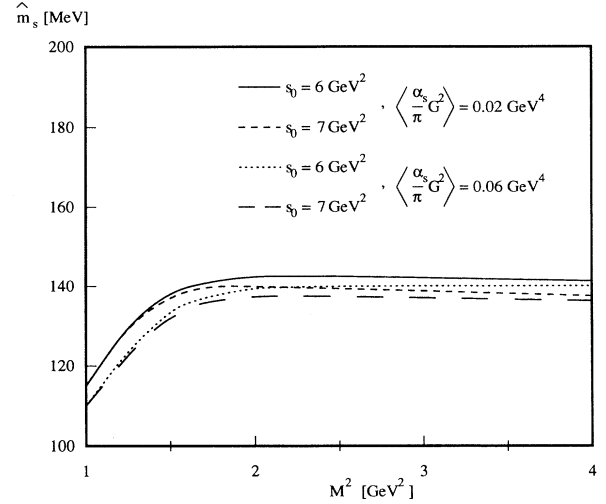


FIG. 4. Same as Fig. 3, except for  $\Lambda = 400$  MeV.

$$\hat{m}_s = 142\text{--}147 \text{ MeV}, \quad \bar{m}_s(1 \text{ GeV}) = 162\text{--}168 \text{ MeV} \quad (\Lambda = 400 \text{ MeV}), \quad (55)$$

where the variation of the strange quark mass, for a given value of  $\Lambda$ , reflects the uncertainties in the values of the gluon condensate and  $s_0$ .

The results of this determination show a welcome stability in the Borel variable  $M^2$ , as well as in the continuum threshold  $s_0$ . To estimate the error induced by the uncertainties in the hadronic spectral function, we have varied the resonance parameters within the limits shown above. This gives an additional error of about  $\pm 7$  MeV. The final uncertainty in  $m_s$  is almost exclusively due to the influence of  $\Lambda$  and the gluon condensate (see Fig. 5).

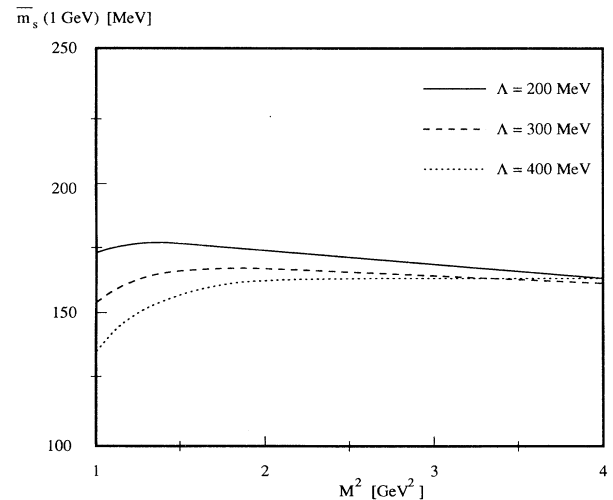


FIG. 5. Results for the running strange quark mass  $m_s(1 \text{ GeV})$  at the scale  $\mu = 1$  GeV. (—)  $\Lambda = 200$  MeV, (---)  $\Lambda = 300$  MeV, and (···)  $\Lambda = 400$  MeV. The continuum threshold  $s_0 = 6.5$  GeV<sup>2</sup>.

The effect of the three-loop radiative correction to  $\Pi_0$ , and hence to  $\psi''$ , has been to reduce the value of the invariant mass  $\hat{m}_s$  by 5–10%. Combining the results in (54) and (55) into a single prediction and including the additional error due to uncertainties in the hadronic parameters leads to

$$\hat{m}_s = 182 \pm 45 \text{ MeV} , \quad \bar{m}_s(1 \text{ GeV}) = 171 \pm 15 \text{ MeV} . \quad (56)$$

#### IV. CONCLUSIONS

In this paper we have discussed in detail how to absorb mass singularities into the vacuum expectation value of other operators of appropriate dimension, for the case of the mass logarithms  $m^4 \ln(m^2/Q^2)$ . We have also included the mixing with other dimension-4 operators to two-loop order. A comparison has been made with earlier analyses of this problem [3,9–11]. In particular, we have shown that in our approach it is possible to avoid terms involving inverse powers of  $\alpha_s$  which, being parametrically enhanced, might lead to large corrections. We have then used the QCD expression of the current correlator involving the strangeness-changing vector current, together with a fit to the experimental data on the  $I = \frac{1}{2}$ ,

$S$ -wave  $K\pi$  amplitude, to determine the strange-quark mass through a Borel QCD sum rule. Our results for  $m_s$  are in agreement, within errors, with the determination of [2], which used the same fit to the data, but employed the QCD approach of [3] to remove mass singularities. The errors we quote for the strange quark mass are larger than those in [2]. This is mostly due to the fact that in [2] the gluon condensate was fixed at the single value  $\langle(\alpha_s/\pi)G^2\rangle = 0.03 \text{ GeV}^4$ , and  $\Lambda$  was allowed to change in the narrower interval  $\Lambda = 100\text{--}200 \text{ MeV}$ .

*Note added.* After submitting our paper for publication we received a similar work [34] where the strange quark mass is calculated in a similar way. Our results agree well with those in [34].

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