Three-body decays of a heavy top quark in two-Higgs-doublet models

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Three-body top quark decays $t \to H^+b(Z,\gamma,g)$ and $t \to H^+bH^i$ are calculated in two-Higgs-doublet models. We find that the branching ratio of the decay $t \to H^+b(Z,H^i)$ can reach the 10^{-4}–10^{-5} level for favorable parameter values and our calculations confirm the previous results given by another group for the channel $t \to H^+b(g,\gamma)$. Also, we discuss the feasibility of observing these top quark decays at future high energy colliders.

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I. INTRODUCTION

The existence of the top quark, with a mass of $174\pm10^{+13}_{-12}$ GeV, has recently been reported by the Collider Detector at Fermilab (CDF) Collaboration [1]. The discovery of the top quark will open experimentally a number of new and interesting issues, such as the precision measurement of the mass, width, and Yukawa couplings of the top quark. Theoretically, one expects to obtain roughly $10^7 - 10^8$ $t\bar{t}$ pairs per year [2] with the operation of the CERN Large Hadron Collider (LHC), and thus may be able to present further tests of the standard model (SM) and provide fruitful information about new physics beyond the SM. Therefore, various rare decay modes of the top quark are of interest. The two-body rare decay modes $t \to cV$ and $t \to cH$ [3] have been discussed in the SM and beyond the SM. Since the top quark is heavy, it is interesting to study three-body decays of the top quark. In the SM, the dominant three-body decays of the top quark are $t \to W^+b(Z,\gamma,g)$, and $t \to W^+bH$, which have been calculated by three theoretical groups [4]. In the two-Higgs-doublet extension of the SM (2HDM) [5], there are three neutral and two charged Higgs bosons, H, h, A, $H^\pm$, of which H and h are CP even and A is CP odd. With two Higgs doublets only two different models (model I and model II defined below) can be constructed which do not have flavor-changing neutral currents in the Higgs sector at the tree level. In model I, one Higgs doublet (ϕ_1) decouples from all the fermions while the other Higgs doublet (ϕ_2) couples to both the up-, and down-type quarks in the same manner as in the standard model. But in model II, ϕ_1 couples only to the up-type quarks, while ϕ_2 couples only to the down-type quarks. The recent results of the CLEO Collaboration place a strong constraint on the mass of the charged Higgs boson [6,7]. But in model I, this constraint will be relaxed to bounds reached at the CERN e^+e^- collider LEP ($m_{H^\pm} > 41.7$ GeV) [8,9] if $\tan\beta$ is large enough ($\tan\beta \geq 1$). So the three-body decay channels of the top quark $t \to H^+b(Z,\gamma,g)$ and $t \to H^+bH^i$ ($H^i = H,h$) are open if the charged Higgs boson mass takes its lower mass limits. In Ref. [10], $t \to H^+b(\gamma,g)$ were discussed. In this paper we calculated all these three-body decays of the top quark, keeping all the masses. The analytic expression for the matrix elements squared is presented.

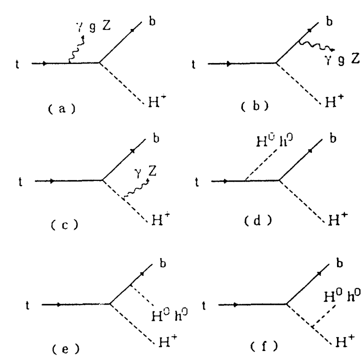

FIG. 1. Feynman diagrams for $t \to H^+b(Z,\gamma,g)$ and $t \to H^+bH^i$.

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II. CALCULATION

The corresponding Feynman diagrams for the decays $t \to H^+b(Z,\gamma,g)$ as well as $t \to H^+bH^i$ are shown in Fig. 1. The relative Feynman rules can be found in [5]. The general form of the partial width for the top decays under consideration is

$$\Gamma(t \to \text{three body}) = \frac{1}{256\pi^3 m_i^3}\int_{s_2^-}^{s_2^+} ds_2 \int_{s_1^-}^{s_1^+} ds_1 |M|^2 \ . \quad (1)$$

The matrix elements squared and the limits of the phase-space integral for all the channels are discussed in the following.

A. $t \to H^+b(Z,\gamma,g)$

The matrix element squared for $t \to H^+bZ$ is given by

$$|M|^2 = \sum_{i,j=1}^{3}(T_{ij} + Y_{ij}) \ . \quad (2)$$

The analytical expressions of T_{ij}, Y_{ij} can be found in Appendix A. For $t \to H^+b\gamma$ the matrix element squared is obtained by

$$|M|^2 = \sum_{i,j=1}^{3} T_{ij}(a_t \to 0, a_b \to 0, v_t \to \tfrac{2}{3}, v_b \to -\tfrac{1}{3}, \cos 2\theta_W \to \sin 2\theta_W, m_Z \to 0) \quad (3)$$

and for $t \to H^+bg$ we have

$$|M|^2 = \sum_{i,j=1}^{2} T_{ij}(a_t \to 0, a_b \to 0, v_t \to -\lambda^a g_s/e, v_b \to -\lambda^a g_s/e, m_Z \to 0) \ , \quad (4)$$

where a_f, v_f, λ_a, θ_W, and g_s are axial vector coupling constant, vector coupling constant, Gell-Mann matrix of color SU(3), Weinberg angle, and strong-coupling constant, respectively. The kinematical variables entering (1) are defined as

$$(p_t - p_V)^2 = s_1 \ , \quad (p_b + p_{H^+})^2 = s_2 \ , \quad (5)$$

in which the phase-space boundaries are readily found to be

$$s_1^{\pm} = m_V^2 + m_b^2 - \frac{1}{2s_2}[(s_2 - m_t^2 + m_V^2)(s_2 + m_b^2 - m_{H^+}^2) \mp \lambda^{1/2}(s_2, m_t, m_V)\lambda^{1/2}(s_2, m_b, m_{H^+})] \quad (6a)$$

and

$$s_2^- = (m_b + m_{H^+})^2 \ ,$$

$$s_2^+ = \begin{cases} (m_t - m_Z)^2 & \text{for } t \to H^+bZ \ , \\ m_t^2 - 2E_{\text{cut}}\sqrt{E_{\text{cut}}^2 + m_t^2} & \text{for } t \to H^+b(\gamma,g) \ , \end{cases} \quad (6b)$$

where

$$\lambda(x,y,z) = (x - y^2 - z^2)^2 - 4y^2z^2 \ . \quad (7)$$

To avoid infrared divergences we impose a cut on the photon (gluon) energy. Since we keep $m_b \neq 0$ there are no collinear singularities.

B. $t \to H^+bH^i$

Summing the three diagrams for the decay $t \to H^+bH^i$ (see Fig. 1), the matrix element squared can be conveniently written as

$$|M|^2 = \sum_{i,j=1}^{2} T'_{ij} \ , \quad (8)$$

where the explicit expression of the T'_{ij} can be found in Appendix B.

The phase-space boundaries to be used in the numerical evaluation of (1) are given by

$$s_1^{\pm} = m_{H^i}^2 + m_b^2 - \frac{1}{2s_2}[(s_2 - m_t^2 + m_{H^i}^2)(s_2 + m_b^2 - m_{H^+}^2) \mp \lambda^{1/2}(s_2, m_t, m_V)\lambda^{1/2}(s_2, m_b, m_{H^+})] \quad (9a)$$

and

$$s_2^- = (m_b + m_{H^+})^2 \ ,$$

$$s_2^+ = (m_t - m_{H^i})^2 \ . \quad (9b)$$

III. CONCLUSIONS AND DISCUSSION

Throughout the numerical calculation of all the decay channels, we take the set of parameters

$$\alpha_{\text{em}}(M_W^2) = \tfrac{1}{128} \ , \quad \alpha_s(M_Z^2) = 0.108 \ ,$$

$$M_W = 80.6 \text{ GeV} \ , \quad M_Z = 91.175 \text{ GeV} \ , \quad m_b = 4.5 \text{ GeV} \ .$$

The branching ratios referred to later are defined as

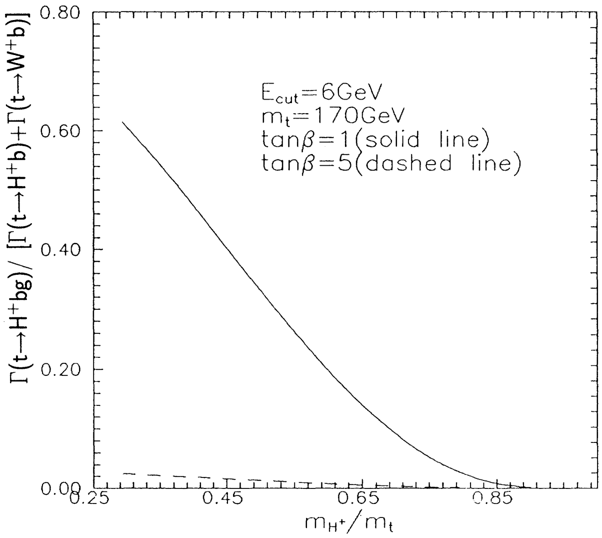

FIG. 2. Plots of the branching ratio vs m_{H^+}/m_t for $t \rightarrow H^+bg$.

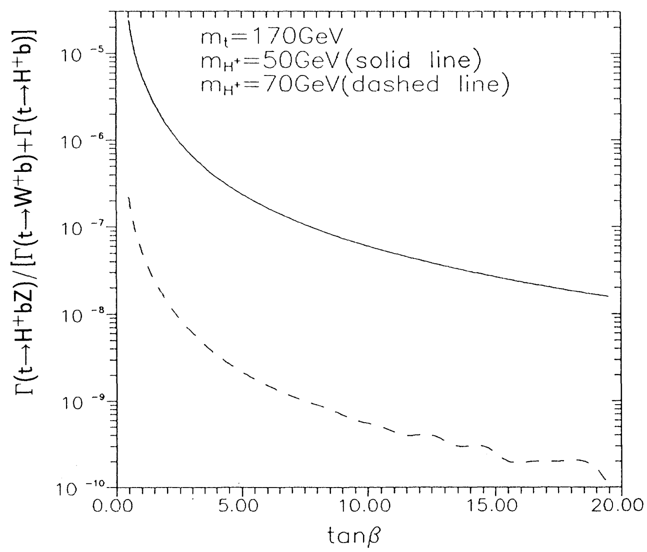

FIG. 4. Plots of the branching ratio vs tanβ for $t \rightarrow H^+bZ$ with $m_t = 190$ GeV.

$$B(t \rightarrow \text{three body}) = \frac{\Gamma(t \rightarrow \text{three body})}{\Gamma_0(t \rightarrow W^+b) + \Gamma_0(t \rightarrow H^+b)} \ . \quad (10)$$

The recent results of the CLEO Collaboration on both inclusive and exclusive radiative B decays place strong constraints on the mass of the charged Higgs boson. But in model I, since the $H^\pm$ contributions all scale as $\cot^2\beta$, enhancements to the SM decay rate of $b \rightarrow s\gamma$ only occur for small values of tanβ. If the tanβ is large enough, the contributions are negligibly small and the constraints on the mass of the charged Higgs boson from the results of the CLEO Collaboration disappear, but the LEP bound $m_{H^+} > 41.7$ GeV still remains [7]. So our discussion will be restricted within model I, in which the allowed lower mass bound of the charged Higgs boson can be less than 100 GeV if only tan$\beta \geq 1$.

A. $t \rightarrow H^+bg(\gamma)$

For these decay channels our results confirm the ones obtained in Ref. [10] in its decay widths. For complete-

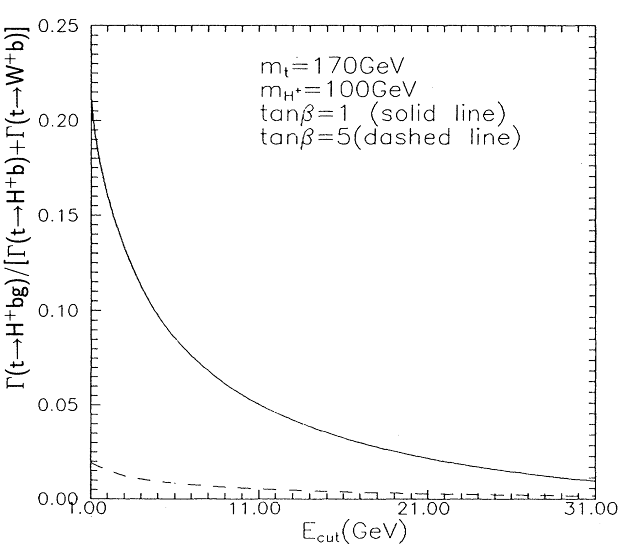

FIG. 3. Plots of the branching ratio vs $E_{\rm cut}$ for $t \rightarrow H^+bg$.

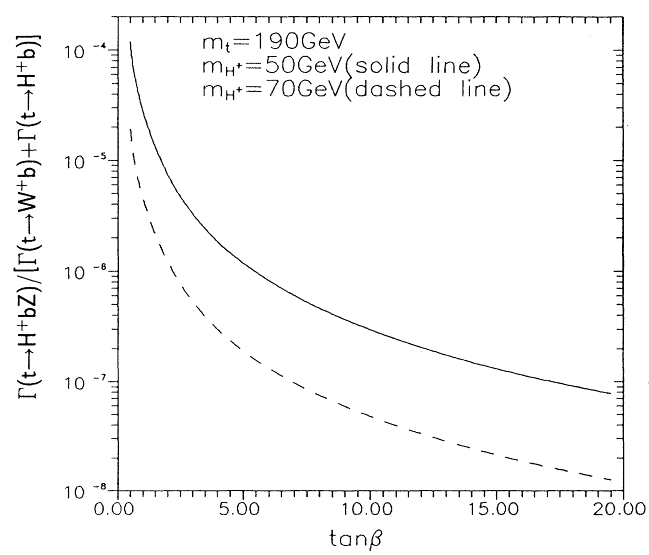

FIG. 5. Plots of the branching ratio vs tanβ for $t \rightarrow H^+bZ$ with $m_t = 170$ GeV.

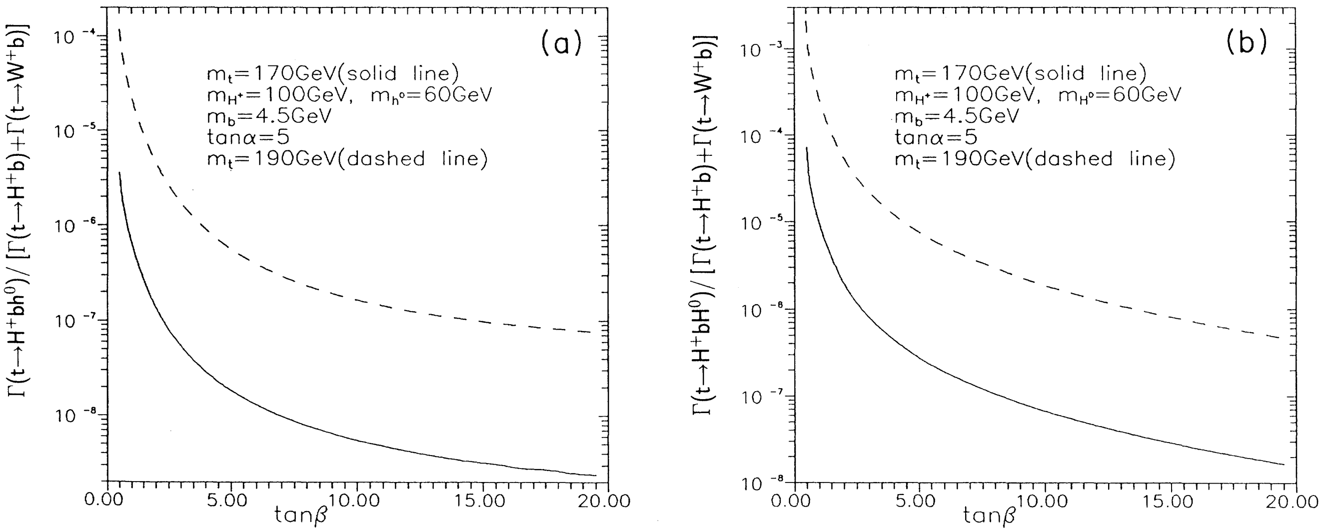

FIG. 6. Plots of the branching ratio vs tanβ for $t \to H^+ b H^i$.

ness of the paper we also plot the branching ratio for $t \to H^+ bg$ vs m_{H^+}/m_t in Fig. 2 and vs $E^g_{\rm cut}$ in Fig. 3. From which we see that the branching ratios for $t \to H^+ bg$ can reach to order 0.1 by taking $m_t = 170$ GeV obtained from the recent experimental results of CDF [1].

B. $t \to H^+ bZ$

Since the charged Higgs boson has a lower mass bound of about 100 GeV [7], this decay channel is forbidden for the typical mass of top quark $m_t = 175$ GeV in model II. If we take $m_t = 200$ GeV, the experimental upper bound of top mass, the branching ratio is

$$B(t \to H^+ bZ) \sim 10^{-6} \ .$$

The suppression due to phase space is expected. If model I, for large enough tanβ, we take $m_{H^+} = 50$ and 70 GeV and plot the branching ratios vs tanβ with $m_t = 170$ and 190 GeV in Figs. 4 and 5. Then the branching ratio can reach $\sim 10^{-5}$.

C. $t \to H^+ bH^i$

The numerical results for these decay channels are displayed in Fig. 6. Since the LEP data already restrict the neutral Higgs boson mass $m_{H^i} > 50$ GeV [11] the branching ratio is expected to be small. Indeed for $m_{H^i} = 60$ GeV, $m_{H^+} = 100$ GeV, tan$\beta = 1$, and $m_t = 190$ GeV (the upper limits allowed CDF experiment) we obtain

$$B(t \to H^+ bH^i) \sim 10^{-4} \ ,$$

where the suppression is essentially due to phase space. The dependence of the branching ratio on the free parameter tanβ is very strong. The result of $t \to H^+ bh^0$ is similar to that of $t \to H^+ bH^0$ for a small tanα.

Finally, we would like to comment on the experimental situation. The planned LHC will produce roughly 10^8 $t\bar{t}$ pairs per year. So, $t \to H^+ bZ$ and $t \to H^+ bH^i$ at its maximum level $\sim 10^{-5}$ seems to be detectable at LHC. Detailed background studies are needed for a definite conclusion. Then these decays can be used as a test of the charged Higgs boson in the mass range 50–100 GeV. As is pointed by Kane [12], a high-energy linear e^+e^- collider [next linear collider (NLC)] with $400 < \sqrt{s} < 500$ GeV and $L > 3 \times 10^{32}$ cm^{-2} sec^{-1} is a very powerful device for top quark physics and could be the ideal place to study top quark rare decays. At NLC, rare decays of the top quark could be searched for down to a very small branching ratio, and a systematic and general search for top rare decay modes may be possible. Then our results for $t \to H^+ bH^i$ might correspond to a detectable level for favorable parameter values at NLC.

In conclusion, we have presented the dominant three-body decays of a heavy top quark in 2HDM. Some of these decays, i.e., $t \to H^+ bZ$ and $t \to H^+ bH^i$, have small branching ratios. However, the enormous number of $t\bar{t}$ pairs expected at LHC should make it possible to observe even such rare decays. Also, it might be possible to observe at NLC. Others with a hard gluon (photon) have relatively large branching ratios.

ACKNOWLEDGMENT

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APPENDIX A

Below we give the expressions for different squared matrix elements. T_{ij} and Y_{ij} in Eq. (2) are given by

$$T_{11} = \frac{\alpha^2}{16\pi^2 s_W^2 \Delta_t^2(s_2)}\{4(A_tD + B_tE)[(s_2 + m_t^2 - m_Z^2) \times(s_2 + m_b^2 - m_{H^+}^2) - s_2(s_1 + s_2 - m_{H^+}^2 - m_Z^2)] -16m_t^2 C_t D(s_2 + m_b^2 - m_{H^+}^2) + 8A_tFm_tm_b(s_2 + m_t^2 - m_Z^2) -4m_t^2(B_tE - A_tD)(s_1 + s_2 - m_{H^+}^2 - m_Z^2) - 16C_tFm_tm_b(s_2 + m_t^2)\}\ , \tag{A1}$$

$$Y_{11} = \frac{\alpha^2}{16\pi^2 s_W^2 \Delta_t^2(s_2) m_Z^2}\{2(A_tD + B_tE)[(m_t^2 + m_Z^2 - s_2) \times(m_t^2 - m_Z^2 - s_2)(s_2 + m_b^2 - m_{H^+}^2) - (m_t^2 + m_Z^2 - s_2)s_2(s_1 - m_b^2 - m_Z^2) -m_Z^2(s_2 + m_t^2 - m_Z^2)(s_2 + m_b^2 - m_{H^+}^2) + m_Z^2 s_2(s_1 + s_2 - m_{H^+}^2 - m_Z^2)] +2m_t^2(A_tD - B_tE)[(m_t^2 + m_Z^2 - s_2) \times(s_1 - m_b^2 - m_Z^2) - m_Z^2(s_1 + s_2 - m_{H^+}^2 - m_Z^2)] +4m_tm_bA_tF[(m_t^2 - m_Z^2 - s_2)(m_t^2 + m_Z^2 - s_2) -m_Z^2(s_2 + m_t^2 - m_Z^2)] + 4m_t^2 m_Z^2 C_t D(s_2 + m_b^2 - m_{H^+}^2) +4m_tm_bm_Z^2 C_tF(s_2 + m_t^2)\}\ , \tag{A2}$$

$$T_{22} = \frac{\alpha^2}{16\pi^2 s_W^2 \Delta_b^2(s_1)}\{4(A_bD + B_bE)m_b^2(s_1 + s_2 - m_{H^+}^2 - m_Z^2) +4(A_bD - B_bE)[(s_1 + m_t^2 - m_{H^+}^2)(s_1 + m_b^2 - m_Z^2) -s_1(s_1 + s_2 - m_{H^+}^2 - m_Z^2)] + 8m_tm_bA_bF(s_1 + m_b^2 - m_Z^2) -16m_b^2 C_bD(s_1 + m_t^2 - m_{H^+}^2) - 16m_tm_bC_bF(s_1 + m_b^2)\}\ , \tag{A3}$$

$$Y_{22} = \frac{\alpha^2}{16\pi^2 s_W^2 \Delta_b^2(s_1) m_Z^2}\{2(A_bD - B_bE) \times[(s_1 - m_b^2 - m_Z^2)(s_1 - m_b^2 + m_Z^2)(s_1 + m_t^2 - m_{H^+}^2) -s_1(s_1 - m_b^2 - m_Z^2)(m_t^2 + m_Z^2 - s_2) - m_Z^2(s_1 + m_t^2 - m_{H^+}^2)(s_1 + m_b^2 - m_Z^2) +m_Z^2 s_1(s_1 + s_2 - m_{H^+}^2 - m_Z^2)] +4m_tm_bA_bF[(s_1 - m_b^2 + m_Z^2)(s_1 - m_b^2 - m_Z^2) -m_Z^2(s_1 + m_b^2 - m_Z^2)] + 2m_b^2(A_bD + B_bE)[(s_1 - m_b^2 - m_Z^2) \times(m_t^2 + m_Z^2 - s_2) - m_Z^2(s_1 s_2 - m_{H^+}^2 - m_Z^2)] +4m_b^2 m_Z^2 C_bD(s_1 + m_t^2 - m_{H^+}^2) + 4m_tm_bm_Z^2 C_bF(s_1 + m_b^2)\}\ , \tag{A4}$$

$$T_{33} = -\frac{\alpha^2}{8\pi^2 s_W^2 \Delta_{H^+}^2(s_3)}\frac{\cos^2 2\theta_W}{c_W^2} m_{H^+}^2 \times[D(s_1 + s_2 - m_{H^+}^2 - m_Z^2) + 2m_tm_bF]\ , \tag{A5}$$

$$Y_{33} = -\frac{\alpha^2}{16\pi^2 s_W^4 \Delta_{H^+}^2(s_3)}\frac{\cos^2 2\theta_W}{c_W^2}\frac{1}{2m_Z^2} \times[\Delta_t(s_2) + \Delta_b(s_1)][D(s_1 + s_2 - m_{H^+}^2 - m_Z^2) + 2m_tm_bF]\ , \tag{A6}$$

$$
\begin{aligned}
T_{12} = & -\frac{\alpha^2}{4\pi^2 s_W^2 \Delta_t(s_2)\Delta_b(s_1)}\{(GK-HL) \\
& \times(s_1+m_t^2-m_{H^+}^2)(s_2+m_b^2-m_{H^+}^2) \\
& +2(GM+HN)m_t m_b(s_1+m_t^2-m_{H^+}^2) \\
& -m_t m_b(IM+JN)(m_t^2+m_b^2-m_{H^+}^2-m_Z^2) \\
& -m_t^2(IK-JL)(s_1+m_b^2-m_Z^2)-m_b^2(GM-HN)(s_2+m_t^2-m_Z^2) \\
& -m_t m_b(GK+HL)(s_1+s_2-m_{H^+}^2-m_Z^2) \\
& +2m_t m_b(IK+JL)(s_2+m_b^2-m_{H^+}^2)+4m_t^2 m_b^2(IM-JN)\}\ , \qquad \text{(A7)}
\end{aligned}
$$

$$
\begin{aligned}
Y_{12} = & -\frac{\alpha^2}{4\pi^2 s_W^2 \Delta_t(s_2)\Delta_b(s_1)m_Z^2}((GK-HL) \\
& \times\{[\Delta_b(s_1)+m_Z^2][m_Z^2-\Delta_t(s_2)](m_t^2+m_b^2-m_{H^+}^2-m_Z^2) \\
& -[\Delta_b(s_1)+m_Z^2]^2(s_2+m_t^2-m_Z^2) \\
& -[\Delta_b(s_1)+m_Z^2][\Delta_t(s_2)+m_Z^2](s_1+m_t^2-m_{H^+}^2) \\
& -2s_1[\Delta_t^2(s_2)-m_Z^4]+2s_1 m_Z^2(s_2+m_t^2-m_Z^2) \\
& -m_Z^2(m_t^2+m_Z^2-s_2)(m_t^2+m_b^2-m_{H^+}^2-m_Z^2) \\
& +m_Z^2(s_1-m_b^2+m_Z^2)(s_2+m_t^2-m_Z^2)-m_Z^2(m_t^2-m_Z^2-s_2)(s_1+m_t^2-m_{H^+}^2)\} \\
& +2m_t m_b m_Z^2(MG+HN)(s_1+m_t^2-m_{H^+}^2) \\
& +2m_t m_b(MI+NJ)[(s_1-m_b^2+m_Z^2)(m_t^2-m_Z^2-s_2) \\
& -m_Z^2(m_t^2+m_b^2-m_{H^+}^2-m_Z^2)] \\
& +2m_t^2(IK-JL)[(s_1-m_b^2)(s_1-m_b^2+m_Z^2) \\
& -2s_1 m_Z^2]+2m_b^2(GM-HN)[(m_t^2+m_Z^2-s_2)(m_t^2-m_Z^2-s_2) \\
& -m_Z^2(s_2+m_t^2-m_Z^2)]+2m_t m_b(GK+HL)[(m_t^2+m_Z^2-s_2) \\
& \times(s_1-m_b^2-m_Z^2)-m_Z^2(s_1+s_1-m_{H^+}^2-m_Z^2)] \\
& +2m_t m_b m_Z^2(IK+JL)(s_2+m_b^2-m_{H^+}^2)+4m_t^2 m_b^2 m_Z^2(IM-JN))\ , \qquad \text{(A8)}
\end{aligned}
$$

$$
\begin{aligned}
T_{13} = & \frac{\alpha^2}{16\pi^2 s_W^3 \Delta_t(s_2)\Delta_{H^+}(s_3)}\frac{\cos 2\theta_W}{c_W} \\
& \times\{(Dv_t+Ea_t)[(s_2-m_b^2+m_{H^+}^2)(s_1+s_2-m_{H^+}^2-m_Z^2) \\
& -(s_2+m_t^2-m_Z^2)(s_2-m_b^2-m_{H^+}^2)+(s_2+m_b^2-m_{H^+}^2)(m_t^2+m_{H^+}^2-s_1)] \\
& +2m_t^2(Dv_t-Ea_t)(s_2-m_b^2-m_{H^+}^2)+2m_t m_b F v_t(s_2-m_b^2+m_{H^+}^2) \\
& +2m_t m_b F v_t(m_t^2+m_{H^+}^2-s_1)\}\ , \qquad \text{(A9)}
\end{aligned}
$$

$$
\begin{aligned}
Y_{13} = & -\frac{\alpha^2}{16\pi^2 s_W^3 \Delta_t(s_2)\Delta_{H^+}(s_3)}\frac{\cos 2\theta_W}{c_W}\frac{\Delta_t(s_2)+\Delta_b(s_1)}{2m_Z^2} \\
& \times((Dv_t+Ea_t)\{[\Delta_t(s_2)+m_Z^2](s_1+s_2-m_{H^+}^2-m_Z^2) \\
& +(s_2+m_t^2-m_Z^2)[\Delta_b(s_1)-m_Z^2]+(s_2+m_b^2-m_{H^+}^2)[\Delta_t(s_2)-m_Z^2]\} \\
& -2m_t^2(Dv_t-Ea_t)[\Delta_b(s_1)-m_z^2]+4m_t m_b F v_t \Delta_t(s_2))\ , \qquad \text{(A10)}
\end{aligned}
$$

$$
\begin{aligned}
T_{23} = & \frac{\alpha^2}{16\pi^2 s_W^3 \Delta_b(s_1)\Delta_{H^+}(s_3)}\frac{\cos 2\theta_W}{c_W} \\
& \times\{(Dv_b-Ea_b)[(m_t^2-m_{H^+}^2-s_1)(s_1+s_2-m_{H^+}^2 m_Z^2) \\
& -(m_t^2+m_{H^+}^2-s_1)(s_1+m_b^2-m_Z^2)+(s_2-m_b^2-m_{H^+}^2)(s_1+m_t^2-m_{H^+}^2)] \\
& +2m_b^2(Dv_b+Ea_b)(m_t^2+m_{H^+}^2-s_1)+2m_t m_b F v_b(m_t^2-m_b^2-2m_{H^+}^2+s_2-s_1)\}\ , \qquad \text{(A11)}
\end{aligned}
$$

$$Y_{23} = \frac{\alpha^2}{16\pi^2 s_W^3 \Delta_b(s_1)\Delta_{H^+}(s_3)} \frac{\cos 2\theta_W}{c_W}$$

$$\times \frac{\Delta_t(s_2)+\Delta_b(s_1)}{2m_Z^2}\{(Dv_b - Ea_b)[(s_1 - m_b^2 + m_Z^2)(s_1 + s_2 - m_{H^+}^2 - m_Z^2)$$

$$-(m_t^2 + m_Z^2 - s_2)(s_1 + m_b^2 - m_Z^2) + (s_1 - m_b^2 - m_Z^2)(s_1 + m_t^2 - m_{H^+}^2)]$$

$$+2m_b^2(Dv_b - Ea_b)(m_t^2 + m_Z^2 - s_2) + 4m_t m_b F v_b(s_1 - m_b^2)\} \ , \quad \text{(A12)}$$

where

$$a = \frac{1}{2\sqrt{2}m_W}(-m_b \cot\beta + m_t \cot\beta) \ , \quad \text{(A13a)}$$

$$b = \frac{1}{2\sqrt{2}m_W}(-m_b \cot\beta - m_t \cot\beta) \ , \quad \text{(A13b)}$$

$$v_t = \frac{1}{2s_W c_W}(0.5 - \tfrac{4}{3}s_W s_W) \ , \quad a_t = \frac{1}{4s_W c_W} \ , \quad \text{(A14)}$$

$$v_b = \frac{1}{2s_W c_W}(-0.5 + \tfrac{2}{3}s_W s_W) \ , \quad a_b = -\frac{1}{4s_W c_W} \ , \quad \text{(A15)}$$

$$A_t = v_t^2 + a_t^2 \ , \quad B_t = 2v_t a_t \ , \quad C_t = v_t^2 - a_t^2 \ , \quad \text{(A16)}$$

$$A_b = v_b^2 + a_b^2 \ , \quad B_b = 2v_b a_b \ , \quad C_b = v_b^2 - a_b^2 \ , \quad \text{(A17)}$$

$$D = a^2 + b^2 \ , \quad E = 2ab \ , \quad F = a^2 - b^2 \ , \quad \text{(A18)}$$

$$G = v_t a + a_t b \ , \quad H = v_t b + a_t a \ , \quad I = v_t a - a_t b \ , \quad \text{(A19)}$$

$$J = v_t b - a_t a \ , \quad K = v_b a - a_b b \ , \quad L = -v_b a + a_b a \ , \quad \text{(A20)}$$

$$M = v_b a + a_b b \ , \quad N = -v_b a - a_b a \ , \quad \text{(A21)}$$

$$s_3 = m_t^2 + m_{H^+}^2 + m_b^2 + m_Z^2 - s_1 - s_2 \ , \quad \Delta_i(s) = s - m_i^2 \ . \quad \text{(A22)}$$

APPENDIX B

The T's are given by

$$T'_{11} = \frac{2g^4 V_{H^i tt}^2}{\Delta_t^2(s_2)}((a^2+b^2)\{[(s_2 + m_t^2 - m_{H^i}^2)(s_2 + m_b^2 - m_{H^+}^2)$$

$$-s_2(s_1 + s_2 - m_{H^+}^2 - m_{H^i}^2)]$$

$$+m_t^2[2(s_2 + m_b^2 - m_{H^+}^2) + (s_1 + s_2 - m_{H^+}^2 - m_{H^i}^2)]\}$$

$$+4(a^2 - b^2)m_t m_b(2s_2 + 2m_t^2 - m_{H^i}^2)) \ , \quad \text{(B1)}$$

$$T'_{22} = \frac{2g^4 V_{H^i bb}^2}{\Delta_b^2(s_1)}((a^2+b^2)\{[(s_1 + m_b^2 - m_{H^i}^2)(s_1 + m_t^2 - m_{H^+}^2)$$

$$-s_1(s_1 + s_2 - m_{H^+}^2 - m_{H^i}^2)]$$

$$+m_b^2[2(s_1 + m_t^2 - m_{H^+}^2) + (s_1 + s_2 - m_{H^+}^2 - m_{H^i}^2)]\}$$

$$+4(a^2 - b^2)m_t m_b(2s_1 + 2m_b^2 - m_{H^i}^2)) \ , \quad \text{(B2)}$$

$$T'_{33} = \frac{2g^4\eta^2_{H^i}}{\Delta^2_{H^+}(s_3)}[(a^2+b^2)(s_1+s_2-m^2_{H^+}-m^2_{H^i})+2(a^2-b^2)m_t m_b] \;, \tag{B3}$$

$$\begin{aligned} T'_{12} = \frac{g^4 V_{H^i tt} V_{H^i bb}}{\Delta_t(s_2)\Delta_b(s_1)}\{&(a^2-b^2)[(s_2+m_t^2-m^2_{H^i})(s_1+m_b^2-m^2_{H^i}) \\ &-(m_t^2+m_b^2-m^2_{H^+}-m^2_{H^i})(s_1+s_2-m^2_{H^+}-m^2_{H^i}) \\ &+(s_1+m_t^2-m^2_{H^+})(s_2+m_b^2-m^2_{H^+}) \\ &+2m_t^2(s_1+m_b^2-m^2_{H^i})+2m_b^2(s_2+m_t^2-m^2_{H^+})+4m_t^2 m_b^2] \\ &+4(a^2+b^2)m_t m_b(s_1+s_2+m_t^2+m_b^2-2m^2_{H^+}-m^2_{H^i})\} \;, \end{aligned} \tag{B4}$$

$$\begin{aligned} T'_{13} = \frac{g^4 V_{H^i tt}\eta_{H^i}}{\Delta_t(s_2)\Delta_{H^+}(s_3)}[&2m_b(a^2-b^2)(3m_t^2+s_2-m^2_{H^i}) \\ &+2m_t(a^2+b^2)(s_1+2s_2+m_b^2-2m^2_{H^+}-m^2_{H^i})] \;, \end{aligned} \tag{B5}$$

$$\begin{aligned} T'_{23} = \frac{g^4 V_{H^i bb}\eta_{H^i}}{\Delta_b(s_1)\Delta_{H^+}(s_3)}[&2m_t(a^2-b^2)(3m_b^2+s_1-m^2_{H^i}) \\ &+2m_b(a^2+b^2)(2s_1+s_2+m_t^2-2m^2_{H^+}-m^2_{H^i})] \;, \end{aligned} \tag{B6}$$

where

$$V_{Htt} = \frac{m_t \sin\alpha}{2m_W \sin\beta} \;, \quad V_{Hbb} = \frac{m_b \cos\alpha}{2m_W \cos\beta} \;, \tag{B7}$$

$$V_{htt} = \frac{m_t \cos\alpha}{2m_W \sin\beta} \;, \quad V_{hbb} = \frac{-m_b \sin\alpha}{2m_W \cos\beta} \;, \tag{B8}$$

$$\eta_H = m_W \cos(\beta-\alpha) - \frac{m_Z}{2c_W}\cos 2\beta \cos(\beta+\alpha) \;, \tag{B9}$$

$$\eta_h = m_W \sin(\beta-\alpha) - \frac{m_Z}{2c_W}\cos 2\beta \sin(\beta+\alpha) \;. \tag{B10}$$

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