Moments of lepton spectrum in B decays and the $m_b - m_c$ quark mass difference

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It is argued that the quark mass difference $m_b - m_c$ can be extracted with a high accuracy from experimental data on ratios of moments of the lepton energy spectrum in semileptonic decays of B mesons. Theoretical expressions for the moments are presented, which include perturbative as well as nonperturbative corrections.

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I. INTRODUCTION

Masses of quarks as well as weak mixing angles are fundamental input parameters in the standard model. Thus it is important to know them with maximal possible accuracy. Moreover, the values of the masses of the b and c quarks are correlated with the determination of the mixing parameter $|V_{bc}|$ from the data on inclusive semileptonic B meson decay rates. Therefore an independent understanding of m_b and m_c is necessary for a precision determination of $|V_{bc}|$. There is a vast literature on extracting the values of m_c and m_b from the data on charmonium, Υ resonances, charmed and B hadrons. However, there is hardly a compelling argument in the literature to invalidate the original evaluations from the QCD sum roles of the "on-shell" quark masses $m_c = 1.35 \pm 0.05$ GeV [1–3] and $m_b = 4.80 \pm 0.03$ GeV [4], which can still serve as "reference values" in discussion of dynamics of heavy hadrons. It is the purpose of this paper to point out that the accuracy of determining the difference between the quark masses $m_b - m_c$ can possibly be significantly improved by considering the ratios of the moments

$$M_n = \int E_l^n \frac{d\Gamma}{dE_l} dE_l \tag{1}$$

of the lepton energy spectrum $d\Gamma/dE_l$ in semileptonic B decays. While theoretical expressions for the moments are sensitive to combinations of m_b and m_c (and also to $|V_{bc}|$), their ratios for few first moments are to a high accuracy sensitive only to the mass difference $m_b - m_c$. Also it is natural to expect that experimentally the ratios of the moments are determined with better systematic accuracy, since the absolute normalization of the event rate cancels out in the ratios.

On the theoretical side, the ratios of the moments have the advantage of weak and controlled dependence on the infrared dynamics in QCD both perturbatively and nonperturbatively. For first few moments the perturbative corrections are expressed through $\alpha_s(m_b)$ and the nonperturbative ones are suppressed by m_b^{-2} and can be found by the operator produc expansion (OPE) technique [5–8] in terms of the quantities μ_π^2/m_b^2 and μ_g^2/m_b^2 with

$$\mu_\pi^2 = \langle B|(\bar{b}\boldsymbol{\pi}^2 b)|B\rangle \text{ and } \mu_g^2 = \langle B|(\bar{b}(\boldsymbol{\sigma}\cdot\mathbf{B})b)|B\rangle , \tag{2}$$

where $\mathbf{B}$ is the chromomagnetic field operator and $\boldsymbol{\pi} = \mathbf{p} - \mathbf{A}$ is the covariant momentum operator for the heavy quark. The spin-dependent chromomagnetic energy μ_g^2 is related to the mass splitting of B^* and B mesons, $\mu_g^2 = \frac{3}{4}(M_{B^*}^2 - M_B^2) \approx 0.36$ GeV^2, while for the kinetic energy μ_π^2 only a lower bound exists [9], $\mu_\pi^2 \geq \mu_g^2$, which follows from the non-negativity of the operator $(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})^2 = \boldsymbol{\pi}^2 - \boldsymbol{\sigma}\cdot\mathbf{B}$, and an estimate [10] $\mu_\pi^2 \approx 0.5 \pm 0.1$ GeV^2 from QCD sum rules.

It should be also noticed that the difference $m_b - m_c$, unlike each of the masses, is less sensitive to the infrared behavior in QCD and is a well-defined quantity in QCD in the limit where both masses are heavy as compared to $\Lambda_{\rm QCD}$. Indeed, because of confinement there is no real "mass shell" for a quark. Therefore its mass can only be determined off shell and then extrapolated to a would-be on-shell value. For a heavy quark its mass can be found at a virtuality scale λ (i.e., at $m^2 - p^2 \approx 2m\lambda$) such than on one hand $\lambda \gg \Lambda_{\rm QCD}$, which justifies a short-distance treatment, and on the other hand $\lambda \ll m$. The latter condition ensures that the evolution of $m(\lambda)$ towards the would-be mass shell does not depend on m in the leading order in $1/m$. In particular, in the leading-log approximation this evolution is described by the renormalization group (RG) equation [11,12]

$$\frac{dm}{d\lambda} = -c\alpha_s(\lambda) , \tag{3}$$

where the constant c depends on the specific definition of the off-shell mass. The infrared singularity of α_s (infrared renormalon) prevents us from integrating an equation such as (3) down to $\lambda = 0$ and thus really extrapolating the mass to the mass shell. However one can integrate Eq. (3) in any finite order in α_s and thus define the "on-shell" mass of a heavy quark to a finite order. In this sense the "on-shell" masses m_b and m_c quoted above are the result of such an extrapolation in the first order and are thus appropriate for using in other calculations in the first order in α_s. Naturally this definition of quark mass changes with the order in α_s. However, increasing the order in α_s does not converge at a certain value of m because of the factorial divergence of the series in α_s caused by the infrared renormalon. A minimal residual error in the "on-shell" mass in this procedure is of the order of $\Lambda_{\rm QCD}$ [12]. On the other hand this uncertainty in a heavy quark mass does not depend on m in the limit of large m. Thus this uncertainty cancels in the difference of masses of two heavy quarks. As to the preasymptotic in the heavy quark mass limit corrections to the evolution equation (3), their contribution to the residual uncertainty is of the order of $\Lambda^3_{\rm QCD}/m^2$, which is quite small even for the charmed quark.

The difference $m_b - m_c$ can be estimated from the experimental values of the masses of D and B mesons:

$$M_B - M_D = m_b - m_c + \frac{\mu_\pi^2 - \mu_g^2}{2m_b} - \frac{\mu_\pi^2 - \mu_g^2}{2m_c} + O(m_c^{-2}, m_b^{-2}) \ . \quad (4)$$

Neglecting the terms, smaller than m_c^{-1} or m_b^{-1}, and taking into account the inequality $\mu_\pi^2 \geq \mu_g^2$ one finds a lower bound for the difference of the quark masses:

$$m_b - m_c \geq M_B - M_D = 3.41 \text{ GeV} \ . \quad (5)$$

Varying μ_π^2 in the range from $\mu_\pi^2 = \mu_g^2 \approx 0.36$ GeV^2 up to $\mu_\pi^2 = 0.6$ GeV^2 one finds $m_b - m_c = 3.44 \pm 0.03$ GeV, which is perfectly compatible with the quoted above estimates of each of the quark masses from QCD sum rules.

Naturally, an independent measurement of this quark mass difference with a comparable or better accuracy would provide an additional consistency check for the heavy quark theory and, possibly, would enable a better quantitative understanding of the parameter μ_π^2. As is discussed in the rest of this paper, a measurement of the ratios of few first moments (1) provides an excellent opportunity to independently determine the difference $m_b - m_c$.

II. MOMENTS OF THE LEPTON SPECTRUM

In the simplest approximation, where the QCD effects are neglected altogether the spectrum of charged lepton l in the decay $b \to c l \nu$ is given by the well-known muon decay formula

$$\frac{d\Gamma}{dx} = \Gamma_0 w_0(x, \mu) \quad (6)$$

with $\Gamma_0 = G_F^2 |V_{cb}|^2 m_b^5/(192\pi^3)$ and

$$w_0(x, \mu) = \frac{2x^2(1-\mu^2-x)^2}{(1-x)^3} \times \left[(1-x)(3-2x) + \mu^2(3-x)\right] \ , \quad (7)$$

where $\mu = m_c/m_b$ and $x = 2E_l/m_b$, so that the physical range of x goes from $x_m = 0$ to $x_M = 1 = \mu^2$.

With the first perturbative QCD correction and the first nonperturbative corrections, proportional to μ_π^2/m^2 and μ_g^2/m^2, taken into account the formula for the differential decay rate can be written as

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx} = w_0(x, \mu) - \frac{2}{3}\frac{\alpha_s}{\pi} w_1(x, \mu) + \frac{\mu_\pi^2}{m_b^2} w_\pi(x, \mu) + \frac{\mu_g^2}{m_b^2} w_g(x, \mu), \quad (8)$$

where the explicit expression for the perturbative correction function $w_1(x, \mu)$ is extremely lengthy and can be found in the original papers[1] [13,14] (see also [15,16]). The nonperturbative correction functions $w_\pi(x, \mu)$ and $w_g(x, \mu)$ are given by [7,8]

$$w_\pi(x, \mu) = \frac{2x^3}{3(1-x)^5}(-5 - 15\mu^4 + 20\mu^6 + 25x + 21\mu^4 x - 10\mu^6 x - 50x^2 - 6\mu^4 x^2 + 2\mu^6 x^2 + 50x^3 - 25x^4 + 5x^5) \ , \quad (9)$$

$$w_g(x, \mu) = \frac{2x^2(1-\mu^2-x)}{3(1-x)^4}(6 - 12\mu^2 - 30\mu^4 - 13x + 23\mu^2 x + 20\mu^4 x + 3x^2 - 16\mu^2 x^2 - 5\mu^4 x^2 + 9x^3 + 5\mu^2 x^3 - 5x^4) \ . \quad (10)$$

The relative correction $w_1(x, \mu)/w_0(x, \mu)$ has a logarithmic singularity at the upper end point $x = x_M$ of the spectrum [15], which is a usual consequence of the Sudakov form factor [17]. The relative nonperturbative corrections are still more singular: the ratio $w_g(x, \mu)/w_0(x, \mu)$ has a pole at $x = x_M$ and the ratio $w_\pi(x, \mu)/w_0(x, \mu)$ has a double pole at the upper end point, which in particular reflects the difference in the kinematics of decay of a free heavy quark and of a heavy quark bound in hadron [7,18]. This implies that the spectrum close to the end point is sensitive to the infrared hadron dynamics, while in integral quantities, like the moments of the lepton spectrum, the effects of this dynamics are integrated over and are present only in the form of small corrections. Naturally, this conclusion is valid only if the number n of the moment is not parametrically large, since high moments measure the spectrum near the upper end point, and all the infrared effects come back. In the expressions for the moments this growth of sensitivity to large distances reveals itself in the growth with n of the relative magnitude of the nonperturbative corrections. Therefore, one can consider as

[1] I am thankful to M. Jeżabek, for pointing out to me the papers [13] and [14], where the calculation of the function $w_1(x, \mu)$ has been finalized, and for sending me his and Czarnecki's FORTRAN code for numerical calculation of this function.

"safe" the moments with such n, for which the nonperturbative correction is still small. As will be discussed, if one chooses to keep individual terms in the corrections at a level below $10-15\%$, this would limit the range of n to $n \leq 5$. Thus in what follows explicit results are presented for the moments in this range of n.

According to Eq. (8) the ratio of the nth moment to M_0 (the total rate) $r_n = M_n/M_0$ can be written as

$$r_n = r_n^{(0)}\left(1 - \frac{2}{3}\frac{\alpha_s}{\pi}\delta_n^{(1)} + \frac{\mu_\pi^2}{m_b^2}\delta_n^{(\pi)} + \frac{\mu_g^2}{m_b^2}\delta_n^{(g)}\right), \quad (11)$$

where $r_n^{(0)}$ is the same ratio in the lowest approximation,

$$r_n^{(0)} = \left(\frac{m_b}{2}\right)^n \frac{\int_0^{1-\mu^2} w_0(x,\mu)x^n dx}{\int_0^{1-\mu^2} w_0(x,\mu)dx} \quad (12)$$

and the corrections $\delta_n^{(1)}, \delta_n^{(\pi)}$, and $\delta_n^{(g)}$ each being a function of μ are obtained from integrals with the corresponding correction function $w(x,\mu)$ in Eq. (8) as

$$\delta_n = \frac{\int_0^{1-\mu^2} w(x,\mu)x^n dx}{\int_0^{1-\mu^2} w_0(x,\mu)x^n dx} - \frac{\int_0^{1-\mu^2} w(x,\mu)dx}{\int_0^{1-\mu^2} w_0(x,\mu)dx}. \quad (13)$$

The moments of the lowest order function $w_0(x,\mu)$ for $n \leq 5$ are listed in the Appendix. The correction coefficients $\delta_n^{(\pi)}$ can in fact be found in a simple analytical form. This is possible due to the fact that the function $w_\pi(x,\mu)$ is related to a modification by a small boost with $\langle v^2\rangle = \mu_\pi^2/m_b^2$ of the lepton spectrum described by the function $w_0(x,\mu)$ [7,18]:

$$w_1(x,\mu) = \frac{x^2}{2}\frac{\partial}{\partial x}\left(\frac{w_0(x,\mu)}{x}\right) + \frac{x^3}{6}\frac{\partial^2}{\partial x^2}\left(\frac{w_0(x,\mu)}{x}\right) - \frac{w_0(x,\mu)}{2}. \quad (14)$$

Integrating by parts one readily finds that $\delta_n^{(\pi)}$ does not depend on μ and is given by

$$\delta_n^{(\pi)} = n(n+2)/6 . \quad (15)$$

The expression for the coefficients $\delta_n^{(g)}$ can be found in a somewhat lengthy analytical form. The integrals in Eq. (13) with the function $w_g(x,\mu)$ and $n \leq 5$ are listed in the Appendix. Similar integrals for the perturbative coefficients $\delta_n^{(1)}$ can also, perhaps, be done analytically as a function of the mass ratio μ. However, judging by the expression [13,14] for the function $w_1(x,\mu)$ and by the analytical expression for the dependence of the $O(\alpha_s)$ correction to the total rate [19], the resulting formulas should be prohibitively lengthy. For the practical purpose of analyzing experimental data it is sufficient, however, to have a table of these coefficients for values of μ around the approximate actual value $\mu \approx 0.3$. The numerical values of the coefficients $\delta_n^{(1)}$ and $\delta_n^{(g)}$ for $n \leq 5$ are given in Tables I and II. Since for each n these coefficients are slowly varying functions of μ, the tabular values can be used for an interpolation.

TABLE I. Numerical values of the perturbative correction coefficients $\delta_n^{(1)}$ in Eq. (11) for $n \leq 5$ and $\mu = m_c/m_b$ in the range from 0.25 to 0.35.

m_c/m_b	$\delta_1^{(1)}$	$\delta_2^{(1)}$	$\delta_3^{(1)}$	$\delta_4^{(1)}$	$\delta_5^{(1)}$
0.25	0.0252	0.0669	0.1193	0.1787	0.2427
0.26	0.0226	0.0619	0.1120	0.1693	0.2312
0.27	0.0201	0.0571	0.1052	0.1603	0.2202
0.28	0.0178	0.0527	0.0987	0.1519	0.2098
0.29	0.0156	0.0485	0.0925	0.1438	0.1999
0.30	0.0135	0.0446	0.0867	0.1362	0.1904
0.31	0.0116	0.0408	0.0812	0.1289	0.1813
0.32	0.0099	0.0373	0.0760	0.1219	0.1726
0.33	0.0082	0.0340	0.0711	0.1153	0.1643
0.34	0.0066	0.0309	0.0663	0.1090	0.1563
0.35	0.0052	0.0280	0.0619	0.1029	0.1486

One can see from the numerical values and from Eq. (15) that for $\alpha_s \approx 0.2, \mu_g^2/m_b^2 \approx 0.015$, and $\mu_\pi^2/m_b^2 \approx$0.015–0.025 the perturbative correction to the ratios r_n is quite small as compared to the nonperturbative terms, and that each of the later terms is within 10–15% range for $n = 5$, though the overall nonperturtative correction is significantly smaller due to a partial cancellation between the two terms.

III. DISCUSSION

The estimates presented above illustrate that both the perturbative and the nonperturbative QCD corrections are sufficiently small and controllable in a number of ratios of moments of the lepton spectrum in semileptonic B decays, which number is sufficient for a detailed experimental sutdy of the kinematical parameters of these decays. A simple numerical inspection reveals that the ratios r_n are in fact sensitive to the quark mass difference $m_b - m_c$ rather than to the individual quark masses. This is a consequence of the fact that the kinematics in the $b \to c$ transitions is not far from the so-called small velocity (sv) limit [20]. The parameter, describing the deviation from this limit, is given by

TABLE II. Numerical values of the nonperturbative correction coefficients $\delta_n^{(g)}$ in Eq. (11) for $n \leq 5$ and $\mu = m_c/m_b$ in the range from 0.25 to 0.35.

m_c/m_b	$\delta_1^{(g)}$	$\delta_2^{(g)}$	$\delta_3^{(g)}$	$\delta_4^{(g)}$	$\delta_5^{(g)}$
0.25	−1.188	−2.419	−3.674	−4.94	−6.213
0.26	−1.187	−2.414	−3.664	−4.926	−6.195
0.27	−1.186	−2.411	−3.657	−4.915	−6.17
0.28	−1.185	−2.408	−3.651	−4.905	−6.165
0.29	−1.185	−2.406	−3.647	−4.898	−6.155
0.30	−1.185	−2.405	−3.644	−4.893	−6.147
0.31	−1.186	−2.406	−3.643	−4.89	−6.142
0.32	−1.187	−2.407	−3.644	−4.89	−6.14
0.33	−1.189	−2.409	−3.646	−4.891	−6.141
0.34	−1.192	−2.413	−3.65	−4.895	−6.144
0.35	−1.195	−2.417	−3.655	−4.901	−6.151

$(m_b - m_c)^2/(m_b + m_c)^2 \approx 1/4$. In this limit the recoil of the charmed quark enters as a subleading effect, and the spectrum is dominantly determined by the quark mass difference. Therefore it is quite likely that the value of the mass difference $m_b - m_c$ can be determined with high precision from experimental data, while to separate each of the masses, one will have to rely on other types of analyses, e.g., on the existing determination of m_b from the Υ sum rules, or, possibly, on one from the inclusive spectrum of photons, generated by the process $b \to s\gamma$, which may become possible in a future development of the experiment [21].

Also, as already mentioned, the moments and their ratia are only weakly sensitive to the somewhat uncertain parameter μ_π^2, which is due to the fact that it is the ratio μ_π^2/m_b^2 which enters the expressions for the moments. In othe words, the coefficient of μ_π^2 in the moments is not singular in the limit $m_c \to 0$, as opposed to the expression in Eq. (4) for the meson messes. Therefore if the quark mass difference is extracted from the discussed ratios of the moments, its value can be used to determine the parameter μ_π^2 from Eq. (4). Alternatively, one can use the mass formula (4) as a constraint in an analysis of the moments of the lepton spectra.

One last remark is in order concerning a possible experimental measurement of the moments M_n in e^+e^- annihilation at the $\Upsilon(4S)$ resonance. Since the resonance is slightly above the $B\bar{B}$ threshold, the B mesons have momentum of about 0.3 GeV, and their measured lepton spectrum is slightly distorted by the boost. However, in order to account for this boost in the integral quantities like the moments M_n there is no need to transform the measured lepton energy distribution to the B rest frame. The reason is that for a small boost the expressions for the moments in the laboratory frame remain valid after adding in quadrature the "intrinsic" average momentum squared of the b quark in meson, μ_π^2 with that of the B meson in the laboratory from $\langle p^2 \rangle$. This obviously amounts to replacing the μ_π^2 by the effective quantity

$$\bar{\mu}_\pi^2 = \mu_\pi^2 + \langle p^2 \rangle \ . \tag{16}$$

One can notice that at the energy of the $\Upsilon(4S)$ resonance the effect of the boost, $\langle p^2 \rangle \approx 0.09\ \text{GeV}^2$, is rather small in comparison with μ_π^2.

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APPENDIX

The moments of the lowest-order energy distribution function, $I_n^{(0)} = \int_0^{1-\mu^2} w_0(x,\mu) x^n dx$, entering Eq. (12), for $n \le 5$ are given by the expressions

$$I_0^{(0)} = 1 - 8\mu^2 + 8\mu^6 - \mu^8 - 12\mu^4 \ln(\mu^2), \tag{A1}$$

$$I_1^{(0)} = \frac{7}{10} - \frac{15\mu^2}{2} - 12\mu^4 + 20\mu^6 - \frac{3\mu^8}{2} + \frac{3\mu^{10}}{10} - 6\mu^4(3+\mu^2)\ln(\mu^2) \ , \tag{A2}$$

$$I_2^{(0)} = \frac{8}{15} - \frac{36\mu^2}{5} - 26\mu^4 + 32\mu^6 + \frac{4\mu^{10}}{5} - \frac{2\mu^{12}}{15} - 8\mu^4(3+2\mu^2)\ln(\mu^2) \ , \tag{A3}$$

$$I_3^{(0)} = \frac{3}{7} - 7\mu^2 - \frac{83\mu^4}{2} + \frac{85\mu^6}{2} + 5\mu^8 + \mu^{10} - \frac{\mu^{12}}{2} + \frac{\mu^{14}}{14} - 30\mu^4(1+\mu^2)\ln(\mu^2) , \tag{A4}$$

$$I_4^{(0)} = \frac{5}{14} - \frac{48\mu^2}{7} - \frac{291\mu^4}{5} + \frac{252\mu^6}{5} + 15\mu^8 - \mu^{12} + \frac{12\mu^{14}}{35} - \frac{3\mu^{16}}{70} - 12\mu^4(3+4\mu^2)\ln(\mu^2) \ , \tag{A5}$$

$$I_5^{(0)} = \frac{11}{36} - \frac{27\mu^2}{4} - \frac{759\mu^4}{10} + \frac{328\mu^6}{6} + \frac{63\mu^8}{2} - \frac{7\mu^{10}}{2} - \frac{7\mu^{12}}{6} + \frac{9\mu^{14}}{10} - \frac{\mu^{16}}{4} + \frac{\mu^{18}}{36} - 14\mu^4(3+5\mu^2)\ln(\mu^2) \ . \tag{A6}$$

The first moments of the function $w_g, I_n^{(g)} = \int_0^{1-\mu^2} w_g(x,\mu) x^n dx$, necessary for calculation of the coefficients $\delta_n^{(g)}$, are given by the expressions

$$I_0^{(g)} = -\frac{3}{2} + 4\mu^2 - 12\mu^4 + 12\mu^6 - \frac{5\mu^8}{2} - 6\mu^4 \ln(\mu^2) \ , \tag{A7}$$

$$I_1^{(g)} = -2 + \frac{5\mu^2}{3} - 4\mu^4 + 8\mu^6 - \frac{14\mu^8}{3} + \mu^{10} - 4\mu^2 \ln(\mu^2) \ , \tag{A8}$$

$$I_2^{(g)} = -\frac{104}{45} - \frac{8\mu^2}{3} + 25\mu^4 - \frac{40\mu^6}{3} - \frac{28\mu^8}{3} + \frac{16\mu^{10}}{5} - \frac{5\mu^{12}}{9} - 4\mu^2\left(2 - 3\mu^2 - \frac{10\mu^4}{3}\right)\ln(\mu^2) \ , \tag{A9}$$

$$I_3^{(g)} = -\frac{53}{21} - \frac{42\mu^2}{5} + \frac{155\mu^4}{2} - \frac{95\mu^6}{2} - 25\mu^8 + 8\mu^{10} - \frac{73\mu^{12}}{30} + \frac{5\mu^{14}}{14} - 2\mu^2(6 - 15\mu^2 - 25\mu^4)\ln(\mu^2) \ , \tag{A10}$$

$$I_4^{(g)} = -\frac{75}{28} - \frac{76\mu^2}{5} + \frac{1553\mu^4}{10} - 86\mu^6 - \frac{395\mu^8}{6} + 20\mu^{10} - \frac{73\mu^{12}}{10} + \frac{206\mu^{14}}{105} - \frac{\mu^{16}}{4} - 2\mu^2(8 - 27\mu^2 - 60\mu^4)\ln(\mu^2) , \tag{A11}$$

$$I_5^{(g)} = -\frac{151}{54} - \frac{160\mu^2}{7} + \frac{1299\mu^4}{5} - \frac{1057\mu^6}{9} - \frac{455\mu^8}{3} + 49\mu^{10} - \frac{175\mu^{12}}{9} + \frac{103\mu^{14}}{15} - \frac{23\mu^{16}}{14} + \frac{5\mu^{18}}{27} - 4\mu^2\left(5 - 21\mu^2 - \frac{175\mu^6}{3}\right)\ln(\mu^2) . \tag{A12}$$

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