Renormalons in effective field theories

Michael Luke
Department of Physics, University of Toronto, Toronto, Canada M5S 1A7

Aneesh V. Manohar
Department of Physics, University of California at San Diego, La Jolla, California 92093

Martin J. Savage
Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
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We investigate the high-order behavior of perturbative matching conditions in effective field theories. These series are typically badly divergent, and are not Borel summable due to infrared and ultraviolet renormalons which introduce ambiguities in defining the sum of the series. We argue that, when treated consistently, there is no physical significance to these ambiguities. Although nonperturbative matrix elements and matching conditions are in general ambiguous, the ambiguity in any physical observable is always higher order in $1/M$ than the theory has been defined. We discuss the implications for the recently noticed infrared renormalon in the pole mass of a heavy quark. We show that a ratio of form factors in exclusive Λ_b decays (which is related to the pole mass) is free from renormalon ambiguities regardless of the mass used as the expansion parameter of heavy quark effective theory. The renormalon ambiguities also cancel in inclusive heavy hadron decays. Finally, we demonstrate the cancellation of renormalons in a four-Fermi effective theory obtained by integrating out a heavy colored scalar.

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I. INTRODUCTION

In physical problems involving several distinct scales, it is often convenient to describe the physics using an effective field theory. Typically, one is interested in physics at energies much less than the mass of a heavy particle, in which case the physics is most easily described by an effective Lagrangian in which virtual heavy particle exchange is accounted for through a series of nonrenormalizable operators. The coefficients of these operators are perturbatively calculable as a power series in $\alpha_s(M)$, where M is the mass of the heavy particle which is integrated out. However, since the resulting perturbation series is only asymptotic, and since it is well known that perturbative QCD is not Borel summable, the perturbatively calculated coefficient functions are at some level ambiguous. One source of this ambiguity is infrared renormalons [1–4], which arise in QCD from graphs of the form shown in Fig. 1. They contribute to the factorial growth of the coefficients of the perturbation series, and introduce uncertainties in the sum of the perturbation series proportional to powers of $\Lambda_{\rm QCD}/M$.

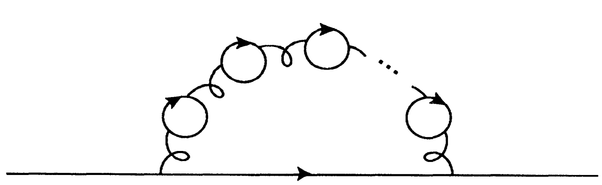

FIG. 1. The bubble chain diagrams which are the leading contribution to the renormalon as the number of light colored fermions is large, $N_f \to \infty$.

There has been much recent discussion in the literature on the effects of infrared renormalons in the context of one particular effective field theory, the heavy quark effective theory (HQET) [5–7]. In particular, the use of the pole mass as an expansion parameter in HQET has been criticized, because it has been shown to suffer from an ambiguity which prevents its definition to an accuracy better than $O(\Lambda_{\rm QCD})$ [5,6], and the authors of Ref. [5] advocate formulating HQET in terms of a short-distance mass $m(\mu)$ which does not suffer from a renormalon ambiguity at $O(\Lambda_{\rm QCD})$. The results of [5,6] clarify the formal status of the pole mass (and of nonperturbative matrix elements in an effective theory in general).

Given this ambiguity, it is important to show that physical predictions of HQET, which are usually expressed using the pole mass as an expansion parameter, are unambiguous. In this paper, we investigate the effects of renormalons on matching conditions in effective field theories. We argue that, while perturbatively calculated coefficient functions suffer from renormalons, any ambiguity in a physical observable is always higher order in $1/M$ than the theory has been defined and is consequently irrelevant. Therefore, as long as one works consistently, it does not matter that unobservable parameters such as the heavy quark mass or the matrix elements of higher dimension operators are not unambiguously defined; relations between physical quantities are unambiguous. We also argue that, while formulating HQET in terms of some short-distance mass $m(\mu)$ is certainly possible, use of an expansion parameter other than the pole mass [or some mass which differs from $m_{\rm pole}$ by $O(\Lambda_{\rm QCD})$] is both inconvenient and uneccessary.

The paper is arranged as follows: In Sec. II, we review the physics of infrared renormalons, and discuss the basic method of calculation that will be used in this paper. In Sec. III we discuss HQET and the renormalon ambiguity in the quark pole mass. We show explicitly that there are no renormalons in the ratio of form factors for $\Lambda_b \to \Lambda_c$ semileptonic decay from which the perturbatively defined pole mass may be extracted, and we comment on the renormalon cancellation for inclusive semileptonic B meson decay. In Sec. IV, we discuss renormalon ambiguities in a four-Fermi effective theory. We show that this effective theory has new renormalon ambiguities not present in the full theory which cancel corresponding ambiguities in matching conditions, and also show that the cancellation of renormalon ambiguities is not specific to HQET, but occurs in other effective field theories. This section can be omitted by readers only interested in HQET. The conclusions are presented in Sec. V.

II. RENORMALONS

QCD perturbation theory is used to express some quantity f as a power series in α_s,

$$f(\alpha_s) = f(0) + \sum_{n=0}^{\infty} f_n \; \alpha_s^{n+1}. \tag{2.1}$$

Typically, this perturbation series for f is only asymptotically convergent. The convergence can be improved by defining the Borel transform of f,

$$B[f](t) = f(0)\; \delta(t) + \sum_{n=0}^{\infty} \frac{f_n}{n!}\; t^n, \tag{2.2}$$

which is more convergent than the original expansion Eq. (2.1). The original expression $f(\alpha_s)$ can be recovered from the Borel transform $B[f](t)$ by the inverse Borel transform

$$f(\alpha_s) = \int_0^{\infty} dt\; e^{-t/\alpha_s}\; B[f](t). \tag{2.3}$$

If the integral in Eq. (2.3) exists, the perturbation series $f(\alpha_s)$ is Borel summable, and is unambiguously defined. However, if there are singularities in $B[f](t)$ along the path of integration, the function f is ambiguous. The inverse Borel transform must be defined by deforming the contour of integration away from the singularity, and the inverse Borel transform in general depends on the deformation used.

One source of singularities in $B[f]$ in QCD is infrared renormalons [1–4]. These arise from graphs of the form in Fig. 1. Physically, these graphs correspond to the running of α_s, and infrared renormalons are ambiguities in perturbation theory arising from the fact that the gluon coupling gets strong for soft gluons in the one-loop diagram in Fig. 1. The infrared renormalons produce a factorial growth in the coefficients f_n, which gives rise to poles in the Borel transform $B[f]$. The renormalon ambiguities have a power law dependence on the momentum transfer Q^2. For example, a simple pole at $t = t_0$ in $B[f]$ introduces an ambiguity in f depending on whether the integration contour is deformed to pass above or below the renormalon pole. The difference between the two choices is proportional to

$$\delta f \sim \oint_C e^{-t_0/\alpha_s(Q)}\; B[f](t) \sim \left(\frac{\Lambda_{\rm QCD}}{Q}\right)^{2(-b_0)t}, \tag{2.4}$$

where $b_0 = -(11 - 2n_f/3)/4\pi$ is the leading term in the QCD β function

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = b_0 \alpha_s^2 + O\left(\alpha_s^3\right), \tag{2.5}$$

which governs the high energy behavior of the QCD coupling constant

$$\alpha_s(Q) = \frac{1}{(-b_0)\ln\left(Q^2/\Lambda_{\rm QCD}^2\right)}, \tag{2.6}$$

and the contour C encloses t_0. It is useful to write the Borel transform $B[f](t)$ in terms of the variable $u = -b_0 t$.[1] The form of the renormalon singularity in Eq. (2.4) then implies that a renormalon at u_0 produces an ambiguity in f that is of order $(\Lambda_{\rm QCD}/Q)^{2u_0}$. This ambiguity is canceled by a corresponding ambiguity in nonperturbative effects such as in the matrix elements of higher dimension operators. The sum of the perturbation series plus nonperturbative corrections is expected to be well defined.

A. The calculational method

Clearly, one cannot sum the entire QCD perturbation series to determine the renormalon singularities. Typically, one sums bubble chains of the form given in Fig. 1 [1,9]. Beneke [8] considered a limiting case of QCD in which the bubble chain sum is the leading contribution to the renormalon. Take QCD with N_f flavors in the limit $N_f \to \infty$ with $a = N_f \alpha_s$ held fixed. Feynman diagrams are computed to leading order in α_s, but to all orders in a. Terms in the bubble sum of Fig. 1 with any number of bubbles are equally important in this limit, since each additional fermion loop contributes a factor $\alpha_s N_f$, which is not small. QCD is not an asymptotically free theory in the $N_f \to \infty$ limit, so the procedure used by Beneke is to write the Borel transform as a function of $u = -b_0 t$ but

[1] This is the definition used in [6], and is the negative of the definition used in [8].

still study renormalons for positive u. The singularities in u are taken to be the renormalons for asymptotically free QCD. This procedure is a formal way of doing the bubble chain sum, while neglecting other diagrams.

The Borel transform of the sum of Feynman graphs containing a single bubble chain can be readily obtained by performing the Borel transform before doing the loop integral [8,6]. The bubble chain sum is

$$G(\alpha_s, k) = \sum_{n=0}^{\infty} \left(\frac{k_\mu k_\nu}{k^2} - g_{\mu\nu}\right) (b_0 \alpha_s N_f)^n \left[\ln(-k^2/\mu^2) + C\right]^n , \tag{2.7}$$

where k is the momentum flowing through the gauge boson propagator, C is a constant that depends on the particular subtractions scheme, and $b_0 = 1/6\pi$ is the contribution of a single fermion to the β function. In the modified minimal subtraction ($\overline{\text{MS}}$) scheme, $C = -5/3$. The Borel transform of Eq. (2.7) with respect to $\alpha_s N_f$ is ($u = -b_0 t$)

$$\begin{aligned} B[G](u,k) &= \frac{1}{\alpha_s N_f} \sum_{n=0}^{\infty} \frac{1}{k^2} \left(\frac{k_\mu k_\nu}{k^2} - g_{\mu\nu}\right) \frac{(-u)^n}{n!} \left[\ln(-k^2/\mu^2) + C\right]^n \\ &= \frac{1}{\alpha_s N_f} \frac{1}{k^2} \left(\frac{k_\mu k_\nu}{k^2} - g_{\mu\nu}\right) \exp\left[-u \ln(-k^2 e^C/\mu^2)\right] \\ &= \frac{1}{\alpha_s N_f} \left(\frac{\mu^2}{e^C}\right)^u \frac{1}{(-k^2)^{2+u}} \left(k_\mu k_\nu - k^2 g_{\mu\nu}\right) . \end{aligned} \tag{2.8}$$

The Borel transformed loop graphs can be computed by using the propagator in Eq. (2.8) instead of the usual gauge boson propagator

$$\left(k_\mu k_\nu - k^2 g_{\mu\nu}\right) \frac{1}{(k^2)^2} . \tag{2.9}$$

III. RENORMALONS IN THE HEAVY QUARK EFFECTIVE THEORY

A. Matching conditions

An effective field theory Lagrangian is an expansion in an operator series in inverse powers of some mass scale M. By construction, the effective field theory has the same infrared physics as the full theory. However, because the ultraviolet physics (above the scale at which the theories are matched) differs dramatically in the two theories, the coefficients of operators in the effective theory must be modified at each order in $\alpha_s(M)$ to ensure that physical predictions are the same in the two theories.

Since the two theories coincide in the infrared, these matching conditions depend in general only on ultraviolet physics and should be independent of any infrared physics, including infrared renormalons. However, in a mass-independent renormalization scheme such as dimensional regularization with $\overline{\text{MS}}$, such a sharp separation of scales cannot be achieved. It is easy to see how infrared renormalons creep into matching conditions. Consider the familiar case of integrating out a W boson and matching onto a four-Fermi interaction (we will discuss a variant of this example in detail in Sec. IV). The matching conditions at one loop involve subtracting one-loop scattering amplitudes calculated in the full and effective theories, as indicated in Fig. 2. For simplicity, neglect all external momenta and particle masses, and consider the region of loop integration when the gluon is soft. When $k = 0$, the two theories are identical and the graphs in the two theories are identical. This is the well-known statement that infrared divergences cancel in matching conditions. However, for finite (but small) k, the two theories differ at $O(k^2/M_W^2)$ when one retains only the lowest dimension operators in the effective theory. Therefore, the matching conditions are sensitive to soft gluons at this order, and it is not surprising that (as we shall show) the resulting perturbation series is not Borel summable and has renormalon ambiguities starting at $O(\Lambda_{\text{QCD}}^2/M_W^2)$.

However, this ambiguity is completely spurious, and does *not* mean that the effective field theory is not well defined. Since the theory has only been defined to a fixed order, an ambiguity at higher order in $1/M_W$ is irrelevant. The renormalon ambiguity corresponded to the fact that the two theories differed in the infrared at $O(k^2/M_W^2)$. When operators suppressed by an additional power of $1/M_W^2$ in the effective theory are consistently taken into account, the two theories will coincide in the infrared up to $O(k^4/M_W^4)$, and any ambiguity is then pushed up to $O(\Lambda_{\text{QCD}}^4/M_W^4)$. Consistently including $1/M_W^4$ suppressed operators pushes the renormalon to $O(\Lambda_{\text{QCD}}^6/M_W^6)$, and so on. In general, a renormalon at $u = u_0$ in the coefficient function of a dimension D operator is canceled exactly by a corresponding ambiguity

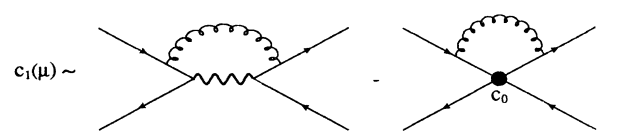

FIG. 2. The one-loop contribution to the matching of a higher dimension operator. The coefficient of the operator in the effective Lagrangian is $c_0 + c_1\alpha_s + c_2\alpha_s^2 + \cdots$. This series will have infrared renormalons due to the incomplete cancellation of the soft gluons in the two graphs.

in matrix elements of operators of dimension $D-4+2u_0$, so that physical quantities are unambiguous. This cancellation is a generic feature of all effective field theories, and also occurs in HQET.

B. HQET and the quark mass

The HQET Lagrangian has an expansion in inverse powers of the heavy quark mass,

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{2m_0}\mathcal{L}_1 + \frac{1}{(2m_0)^2}\mathcal{L}_2 + \cdots + \mathcal{L}_{\rm light},$$
$$\mathcal{L}_0 = \bar{h}_v\,(iD\cdot v)\,h_v - \delta m\;\bar{h}_v h_v, \qquad (3.1)$$
$$\mathcal{L}_1 = \cdots .$$

Here $\mathcal{L}_{\rm light}$ is the QCD Lagrangian for the light quarks and gluons, h_v is the heavy quark field, and $\mathcal{L}_k$ are terms in the effective Lagrangian for the heavy quark that are of order $1/m_0^k$. There are two mass parameters for the heavy quark in Eq. (3.1), the expansion parameter of HQET m_0, and the residual mass term δm. The two parameters are not independent; one can make the redefinition $m_0 \to m_0 + \Delta m$, $\delta m \to \delta m - \Delta m$. A particularly convenient choice is to adjust m_0 so that the residual mass term δm vanishes. Most HQET calculations have been done with this choice of m_0, but it is easy to show that the same results are obtained with a different choice of m_0 [10]. The HQET mass when $\delta m = 0$ is often referred to in the literature as the pole mass, and we will follow this practice here.

Like all effective Lagrangians, the HQET Lagrangian is nonrenormalizable, so a specific regularization prescription must be included as part of the definition of the effective theory. An effective field theory is used to compute physical quantities in a systematic expansion in a small parameter, and the effective Lagrangian is expanded in this small parameter. The expansion parameter of the HQET is $\Lambda_{\rm QCD}/m_0$. One can then use "power counting" to determine what terms in the effective theory are relevant to a given order in the $1/m_0$ expansion. For example, to second order in $1/m_0$, one needs to study processes to first order in $\mathcal{L}_2$, and to second order in $\mathcal{L}_1$. It is important that the renormalization procedure preserves the the power counting for the effective field theory to make sense. In order to preserve power counting, we must choose a mass-independent subtraction scheme; in our case, we choose to use dimensional regularization and $\overline{\rm MS}$. A mass-dependent subtraction scheme (such as a momentum space cutoff) mixes different orders in the $1/m_0$ expansion. Thus to compute a quantity to first order in $1/m_0$, one would have to retain the effective Lagrangian to all orders in $1/m_0$, which is not particularly useful. Hence, HQET is defined using a mass-independent subtraction scheme, and nonperturbative matrix elements must be interpreted in this scheme.

It has recently been shown [5,6] that there is a renormalon in the relation between the renormalized mass at short distances (such as the $\overline{\rm MS}$ mass $\overline{m}$) and the pole mass of the heavy quark at $u = 1/2$, which produces an ambiguity in the relation between the pole mass and the $\overline{\rm MS}$ mass of order $\Lambda_{\rm QCD}$. This implies that there is an ambiguity in the residual mass term δm of order $\Lambda_{\rm QCD}$ due to renormalon effects [5,6].

The quark mass in HQET and the $\overline{\rm MS}$ mass at short distances are parameters in the Lagrangian that must be determined from experiment. Any scheme can be used to compute physical processes, though one scheme might be more advantageous for a particular computation. The $\overline{\rm MS}$ mass at short distances is useful in computing high energy processes. However, there is no advantage to using the "short-distance" mass (such as the running $\overline{\rm MS}$ mass) in HQET, as advocated by [5]. In fact, from the point of view of HQET, this is extremely inconvenient. The effective Lagrangian Eq. (3.1) is an expansion in inverse powers of m_0. Power counting in $1/m_0$ in the effective theory is only valid if δm is of order one (or smaller) in m_0, i.e., *only* if δm remains finite in the infinite mass limit $m_0 \to \infty$. When m_0 is chosen to be the $\overline{\rm MS}$ mass the residual mass term δm is of order m_0 (up to logarithms), which spoils the $1/m_0$ power counting of HQET, mixes the α_s and $1/m_0$ expansions, and breaks the heavy flavor symmetry. For example, using m_0 to be the $\overline{\rm MS}$ mass at $\mu = m_0$, one finds at one loop that

$$\delta m = \frac{4}{3\pi}\alpha_s m_0. \qquad (3.2)$$

In $b \to c$ decays, including this residual mass term in the heavy c-quark Lagrangian, causes $1/m_c$ operators such as $\bar{h}_c(-i\overleftarrow{\not{D}})\Gamma h_b/m_c$ to produce effects that are of the same order in $1/m_c$ as lower dimension operators of the form $\bar{h}_c\Gamma h_b$. While physical quantities calculated in this way must be the same as those calculated using the pole mass, it unnecessarily complicates calculations to use a definition for m_0 that leaves a residual mass term that is not finite in the $m_0 \to \infty$ limit. Better choices of the expansion parameter of HQET are the heavy meson mass (with δm of order $\Lambda_{\rm QCD}$), and the pole mass (with $\delta m = 0$).

The $\overline{\rm MS}$ mass at short distances can be determined (in principle) from experiment without any renormalon ambiguities proportional to $\Lambda_{\rm QCD}/m_Q$ (i.e., at $u = 1/2$). As an example, consider the $\overline{\rm MS}$ mass of the b quark at the grand unified theory (GUT) scale in an SU(5) unified field theory. The b-quark mass at the GUT scale is proportional to the b-quark Yukawa coupling at the GUT scale, which in turn is equal to the τ-lepton Yukawa coupling at the same scale.[2] There are no QCD renormalons at $u = 1/2$ in the relation between the τ-lepton mass at short distances and the pole mass of the τ (neglecting QED effects). Thus one could determine the b-quark

[2]There are corrections to this relation from matching conditions at the GUT scale, which will have renormalon ambiguities proportional to powers of $\Lambda_{\rm QCD}/m_{\rm GUT}$.

mass at short distances by measuring the τ-lepton mass, without any renormalon ambiguities at $u = 1/2$.

The $\overline{\text{MS}}$ quark mass can be related to other definitions of the quark mass using QCD perturbation theory. The connection between the Borel transformed pole mass and a short-distance mass (such as the $\overline{\text{MS}}$ mass) has been worked out in [6]. The relation between the two (for the c quark) is

$$B[m_c^{\text{pole}}](u) = m_c\delta(u) + \frac{m_c}{3\pi N_f}\left[\left(\frac{\mu^2}{m_c^2}\right)^u e^{-uC} 6(1-u)\frac{\Gamma(u)\Gamma(1-2u)}{\Gamma(3-u)} - \frac{3}{u} + R_{\Sigma_1}(u)\right], \tag{3.3}$$

where m_c is the renormalized (*not* the pole) mass at short distances, such as the $\overline{\text{MS}}$ mass, μ is the renormalization scale, and the constant C and the function $R_{\Sigma_1}(u)$ depend on the renormalization scheme. Equation (3.3) has a renormalon singularity at $u = 1/2$ which is the leading infrared renormalon in the pole mass. Writing $u = 1/2 + \Delta u$, we have

$$B[m_c^{\text{pole}}](u = 1/2 + \Delta u) = -\frac{2\mu e^{-C/2}}{3\pi N_f m_c\,\Delta u} + \cdots, \tag{3.4}$$

where the ellipsis denotes terms regular at $\Delta u = 0$. In the next two sections, we will only work to $O(1/m_Q)$, so poles to the right of $u = 1/2$, which are related to ambiguities at higher order in $1/m_Q$, are irrelevant at this stage.

Although m_c^{pole} is formally ambiguous at $O(1/m_c)$, we will argue in this paper that physical quantities which depend on m_c^{pole} are unambiguously predicted in HQET. We demonstrate this explicitly for a ratio of form factors in Λ_b semileptonic decay. We then comment on the cancellation of renormalon ambiguities in the expression for the inclusive semileptonic width of the B meson. Both results will make use of Eq. (3.4) and its analog for the b quark.

C. $\bar{\Lambda}$ from exclusive decays

The matrix element of the vector current for the semileptonic decay $\Lambda_b \to \Lambda_c e^- \nu_e$ decay is parametrized by the three decay form factors:

$$\langle \Lambda_c(v')|\bar{c}\gamma^\mu b|\Lambda_b(v)\rangle = \bar{u}(v')\left[F_1(v\cdot v')\gamma^\mu + F_2(v\cdot v')v^\mu + F_3(v\cdot v')v'^\mu\right]u(v). \tag{3.5}$$

In the limit m_b, $m_c \to \infty$, and at lowest order in α_s, the form factors F_2 and F_3 vanish. We will consider α_s and $1/m_c$ corrections, but work in the $m_b = \infty$ limit. Consider the ratio $r_F = F_2/F_1$, which vanishes at lowest order in α_s and $1/m_c$. The corrections to r_F can be written in the form [15]

$$r_F(\alpha_s, v\cdot v') \equiv \frac{F_2(v\cdot v')}{F_1(v\cdot v')} = \frac{\bar{\Lambda}}{m_c}\frac{1}{(1+v\cdot v')} + f_r(\alpha_s, v\cdot v'), \tag{3.6}$$

where the function $f_v(\alpha_s, v\cdot v')$ is a perturbatively calculable matching condition from the theory above $\mu = m_c$ to the effective theory below $\mu = m_c$, and the $\bar{\Lambda}$ term arises from $1/m_c$ suppressed operators in HQET. At one loop [11],

$$f_r(\alpha_s, v\cdot v') = -\frac{2\alpha_s}{3\pi}\frac{1}{\sqrt{(v\cdot v')^2 - 1}}\ln\left(v\cdot v' + \sqrt{(v\cdot v')^2 - 1}\right). \tag{3.7}$$

The ratio $r_F = F_2/F_1$ is an experimentally measured quantity, and does not have a renormalon ambiguity. The standard form for r_F in Eq. (3.6) is obtained by using HQET with the pole mass as the expansion parameter. The HQET parameter $\bar{\Lambda}$ is the meson mass in the effective theory, i.e., it is the meson mass m_D minus the pole mass of the c quark. The pole mass has the leading renormalon ambiguity [6,5] at $u = 1/2$ given in Eq. (3.4), which produces an ambiguity in the $1/m_c$ contribution to F_2/F_1 given by the first term in Eq. (3.6). There must therefore also be a renormalon at $u = 1/2$ in the radiative correction to F_2/F_1 given by the second term in Eq. (3.6). It is straightforward to show, using the techniques of Sec. II, that this is indeed the case.

The Borel transformed series $B[f_r](u, v\cdot v')$ in the $1/N_f$ expansion is easily calculated from the graph in Fig. 3 using the Borel transformed propagator in Eq. (2.8). The Borel transform of the Feynman diagram is

$$B[\text{graph}] = \frac{1}{\alpha_s N_f}\frac{4}{3}g^2\left(\frac{\mu^2}{e^C}\right)^u \int \frac{d^4k}{(2\pi)^4}\frac{\gamma^\nu\left(m_c\not{v}' + \not{k} + m_c\right)\gamma^\alpha v^\mu(k_\mu k_\nu - k^2 g_{\mu\nu})}{(k^2 + 2m_c k\cdot v')(-k^2)^{2+u}k\cdot v}. \tag{3.8}$$

The radiative correction to F_2 (which determines f_r) is obtained from the terms in Eq. (3.8) which are proportional to v^α. Combining denominators using the identities

$$\frac{1}{(k^2+2m_c k\cdot v')(k^2)^{2+u}} = (2+u)\int_0^1 \frac{dx\,(1-x)^{1+u}}{[k^2+2m_c xk\cdot v']^{3+u}},$$
$$\frac{1}{(k^2+2m_c xk\cdot v')^{3+u}\,k\cdot v} = 2(3+u)\int_0^\infty \frac{d\lambda}{[k^2+2m_c xk\cdot v'+2\lambda k\cdot v]^{4+u}},$$

extracting the terms proportional to v^μ and performing the momentum integral, we obtain

$$B[f_r](u,v\cdot v') = \frac{4(u-2)}{3\pi N_f(1+u)}\left(\frac{\mu^2}{e^C}\right)^u m_c \int_0^\infty d\lambda \int_0^1 dx \frac{(1-x)^{1+u}\,x}{[\lambda^2+2\lambda m_c xv\cdot v'+m_c^2x^2]^{1+u}}. \tag{3.9}$$

Rescaling $\lambda \to xm_c\lambda$ and performing the x integral gives

$$B[f_r](u,v\cdot v') = \frac{4}{3\pi N_f}\left(\frac{\mu^2}{m_c^2 e^C}\right)^u \frac{(u-2)\Gamma(1-2u)\Gamma(1+u)}{\Gamma(3-u)}\int_0^\infty d\lambda \frac{1}{[\lambda^2+2\lambda v\cdot v'+1]^{1+u}}. \tag{3.10}$$

This expression has a pole at $u=1/2$. Expanding in $\Delta u = u-1/2$ gives

$$\begin{aligned} B[f_r](u=1/2+\Delta u, v\cdot v') &= \frac{2\mu}{3\pi m_c e^{C/2}}\frac{1}{\Delta u}\int_0^\infty d\lambda \frac{1}{[\lambda^2+2\lambda v\cdot v'+1]^{3/2}} + \cdots \\ &= \frac{2\mu}{3\pi m_c e^{C/2}}\frac{1}{\Delta u}\frac{1}{1+v\cdot v'}, \end{aligned} \tag{3.11}$$

where the ellipsis denotes terms that are regular at $u=1/2$.

The Borel singularity in Eq. (3.11) cancels the singularity in the first term of Eq. (3.6) at all values of $v\cdot v'$, so that the form factor ratio $r_F(\alpha_s, v\cdot v') = F_2(v\cdot v')/F_1(v\cdot v')$ has no renormalon ambiguities. Therefore the standard HQET computation of the $1/m_c$ correction to F_2/F_1 using the pole mass and the standard definition of $\bar\Lambda$ gives an unambiguous physical prediction for the ratio of form factors.

D. Inclusive decays

A similar situation occurs for inclusive B decays, which have been the subject of much recent interest [12–14]. The inclusive $B\to X_q e\nu$ (where $q=u$ or c) decay rate is related to the imaginary part of the forward scattering amplitude,

$$\Gamma(B\to X_q e\nu) \sim \text{Im}\,\langle B|T(J^{\mu\dagger}, J^\nu)|B\rangle, \tag{3.12}$$

where $J^\mu = \bar c\gamma^\mu(1-\gamma_5)b$. In this case the expression for the total rate as an expansion in powers of $1/m_Q$ is not the result of matching onto an effective theory, but instead is the result of performing an operator product expansion on the time ordered product of the two currents in Eq. (3.12). The final expression is

$$\begin{aligned} \Gamma(B\to X_q e\nu) = \frac{G_f^2\,|V_{bq}|^2\,m_b^5}{192\pi^3}\Big[&f_0(\alpha_s)\,\langle B(v)|\,\bar h_b h_b\,|B(v)\rangle \\ &+ \frac{5}{m_b}f_1(\alpha_s)\,\langle B(v)|\,\bar h_b\, i\,(D\cdot v)\,h_b\,|B(v)\rangle + \mathcal{O}(m_q^2/m_b^2, \Lambda_{\rm QCD}^2/m_b^2)\Big], \end{aligned} \tag{3.13}$$

where $f_0=1$ and $f_1=1$ to lowest order in α_s. Equation (3.13) is true with an arbitrary residual mass term in the HQET Lagrangian, and we have not yet applied the equations of motion to the operator $\bar h_b\,i\,(D\cdot v)\,h_b$. The total decay rate Γ is an observable, and does not have a renormalon ambiguity. It was shown in Refs. [5,7] that the total decay rate is unambiguous. It is important to note that this result does not require the use of a "short-distance" mass in Eq. (3.13). One is free to choose some definite (but arbitrary) prescription for integrating around the pole at $u=1/2$ in Eq. (3.3). The mass m_b in the leading term of Eq. (3.13) is then well defined, but there is an ambiguity at $O(\Lambda_{\rm QCD})$ in the residual mass term δm arising from the renormalon at $u=1/2$ in the pole mass. By the equations of motion,

$$i(v\cdot D)\,h_b = \delta m\,h_b + O(1/m_b), \tag{3.14}$$

so the ambiguity at $O(\Lambda_{\rm QCD})$ in the matrix element $\langle B(v)|\,\bar h_b iD\cdot v h_b\,|B(v)\rangle$ is the same as that in the pole mass, Eq. (3.4). From [7], the Borel transformed series $B[f_0(u)]$ for the α_s corrections to the leading term has a

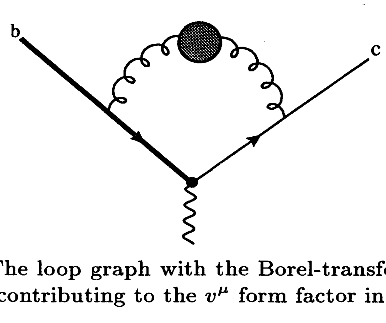

FIG. 3. The loop graph with the Borel-transformed gluon propagator contributing to the v^μ form factor in $\Lambda_b\to\Lambda_c e\nu$.

pole at $u = 1/2$,

$$B[f_0](u = 1/2 + \Delta u) = \frac{10 \mu e^{-C/2}}{3\pi N_f m_b \Delta u} + \dots \,. \tag{3.15}$$

Comparing Eq. (3.4) with Eq. (3.15) and Eq. (3.13), we see explicitly that the ambiguity in the matrix element of $iD{\cdot}v$ cancels that in f_0 to give an unambiguous prediction for the total width Γ.

Thus in this operator product expansion the cancelation of renormalon ambiguities occurs in the same manner as in the construction of HQET: the ambiguity in the matrix element of a higher dimension operator cancels that in the perturbation series for the coefficient of the leading operator.

IV. FOUR-FERMI THEORY

As we argued in Sec. III, the cancellation of renormalon ambiguities between matrix elements and matching conditions is a general feature of an effective field theory. In this section, we illustrate this in a more familiar effective field theory, four-Fermi theory in which a heavy colored scalar is integrated out. (We choose the scalar theory as our example because it has Feynman graphs which are slightly easier to compute than in the four-Fermi theory of weak interactions.) The theory is QCD with N_f light flavors in the $N_f \to \infty$ limit, with $\alpha_s N_f$ fixed. Two of the N_f flavors (called d and s) couple via a color triplet scalar of mass $M \gg \Lambda_{\rm QCD}$ to color singlet particles (called e and τ) according to the interaction Lagrangian

$$\mathcal{L} = \lambda \left(\bar{s}\tau + \bar{d}e \right) \phi + \text{H.c.} \tag{4.1}$$

We will study the theory in the $d \to s$ sector.

The effective Lagrangian obtained from Eq. (4.1) after integrating out the heavy scalar has the form

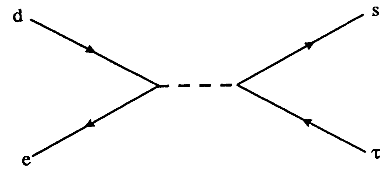

FIG. 4. The tree-level exchange of a heavy colored scalar. It only contributes to $c_S(\mu)$ and $d_S(\mu)$ and not to $c_T(\mu)$.

$$\mathcal{L}_{\rm eff} = \frac{\lambda^2}{M^2} c_S(\mu)\ \bar{s}\tau\ \bar{e}d + \frac{\lambda^2}{M^2} c_T(\mu)\ \bar{s}\sigma_{\mu\nu}\tau\ \bar{e}\ \sigma^{\mu\nu} d - \frac{\lambda^2}{M^4} d_S(\mu)\ \bar{s}\tau\ D^2 [\ \bar{e}d\ ] + \cdots \,, \tag{4.2}$$

where the ellipsis indicates additional operators of dimension 6 and higher. The scale μ is the scale at which the effective Lagrangian is computed from the full theory, and is usually chosen to be $\mu = M$. The effective Lagrangian at zeroth order in the strong interactions is computed by equating the $d\tau \to se$ scattering amplitude obtained from Eq. (4.2) with that obtained by expanding the scalar exchange graph in the full theory (Fig. 4) in a power series in $1/M^2$, to give $c_S = 1$, $c_T = 0$, and $d_S = 1$.

We now consider matching onto the effective theory at high orders in α_s, and concentrate on the coefficient of the operator $O_T \equiv \bar{s}\sigma_{\mu\nu}\tau\ \bar{e}\sigma^{\mu\nu}d$. This is a particularly convenient operator because its coefficient c_T is zero at tree level, and at one loop in the full theory it only receives a contribution from the graph in Fig. 5. The other graphs do not have the correct γ structure to contribute to c_T, and contribute only to the scalar amplitude. c_T is obtained by equating the tensor scattering amplitudes in the full and effective theories, and its Borel transform is computed using the techniques discussed in Sec. II.

The Borel transform of the tensor scattering amplitude A_T (at zero momentum transfer) in the full theory is obtained by evaluating the Feynman graph in Fig. 5,

$$B\left[A_T^{\rm full}\right] = \frac{16\pi\lambda^2}{3N_f} \left(\frac{\mu^2}{e^C}\right)^u \bar{U}_s \sigma_{\mu\alpha} U_\tau\ \bar{U}_e \sigma_{\nu\beta} U_d \int \frac{d^4k}{(2\pi)^4} \frac{k^\alpha k^\beta \left(k^\mu k^\nu - k^2 g^{\mu\nu}\right)}{\left(k^2 - m^2\right)^2 \left(k^2 - M^2\right) \left(-k^2\right)^{2+u}}, \tag{4.3}$$

using the gluon propagator Eq. (2.8). The U_i are Dirac spinors for the external fermion lines, and the quarks have been given a common mass m to regulate the infrared behavior of the diagram. Performing the k integral gives

$$B\left[A_T^{\rm full}\right] = \frac{i\lambda^2}{12\pi N_f} \left(\frac{\mu^2}{e^C}\right)^u \bar{U}_s \sigma^{\mu\nu} U_\tau\ \bar{U}_e \sigma_{\mu\nu} U_d\ \Gamma(u-1)\ \Gamma(2-u) \times \left[\frac{\left(M^2\right)^{1-u} - \left(m^2\right)^{-u} \left[(1-u) M^2 + u m^2\right]}{\left(M^2 - m^2\right)^2} \right]. \tag{4.4}$$

Despite the appearance of $\Gamma(u-1)$ in Eq. (4.3), the amplitude is finite at $u = 0$ and 1 as the term in the square brackets vanishes at these points, but it has singularities at $u = 2, 3, \ldots$. However, we stress that renormalon singularities in Eq. (4.4) are of no interest, since we are not going to attempt to calculate scattering amplitudes in perturbation theory. Any attempt to do so will of course face serious infrared problems. We are only interested in using perturbation theory to calculate the coefficient functions in Eq. (4.2), which requires us to subtract the corresponding amplitude calculated in the effective theory.

To match onto the effective theory, we expand (4.4) in a power series in $1/M$:

$$B[A_T^{\rm full}] = \frac{i\lambda^2}{12\pi N_f}\left(\frac{\mu^2}{e^C}\right)^u \bar{U}_s\sigma^{\mu\nu}U_\tau\,\bar{U}_e\sigma_{\mu\nu}U_d\,\Gamma(u-1)\,\Gamma(2-u)\left[\frac{\left(M^2\right)^{-u}}{M^2} - (1-u)\,\frac{\left(m^2\right)^{-u}}{M^2} + \cdots\right], \tag{4.5}$$

where we have only retained terms up to order $1/M^2$, since c_T/M^2 is the coefficient of a dimension-six operator. It is perhaps useful at this point to relate this to the standard perturbation series for $A_T^{\rm full}$,

$$A_T^{\rm full} = i\frac{\lambda^2}{12\pi N_f M^2}\,\bar{U}_s\sigma^{\mu\nu}U_\tau\,\bar{U}_e\sigma_{\mu\nu}U_d\sum_{n=0}^{\infty} a_n(\alpha_s N_f)^n + O\left(\frac{1}{M^4}\right). \tag{4.6}$$

Taking the appropriate derivatives of (4.5) gives for the first few terms of the series

$$\begin{aligned} a_0 &= 0,\\ a_1 &= -1 - 2\ln(m/M),\\ a_2 &= -b_0\left[C + 2(1+C)\ln(m/M) - 2\ln(\mu/M) + 2\ln^2(m/M) - 4\ln(m/M)\ln(\mu/M)\right],\\ a_3 &= \cdots\ . \end{aligned} \tag{4.7}$$

The tensor scattering amplitude in the effective theory is computed from the loop correction to the lowest order operator c_S,

$$B\left[A_T^{\rm eff}\right] = -\frac{16\pi\lambda^2}{3N_f}\left(\frac{\mu^2}{e^C}\right)^u \bar{U}_s\sigma_{\mu\alpha}U_\tau\bar{U}_e\sigma_{\nu\beta}U_d\int\frac{d^4k}{(2\pi)^4}\,\frac{k^\alpha k^\beta\left(k^\mu k^\nu - k^2 g^{\mu\nu}\right)}{\left(k^2-m^2\right)^2 M^2\left(-k^2\right)^{2+u}}, \tag{4.8}$$

where we have used the tree-level value $c_S = 1$ in evaluating the graph. This result is the same as that obtained by setting the scalar propagator in Eq. (4.3) to $1/M^2$, since the c_S term in the effective Lagrangian reproduces this piece of the four-Fermi vertex. Evaluating the k integral gives

$$B[A_T^{\rm eff}] = \frac{i\lambda^2}{12\pi N_f}\left(\frac{\mu^2}{e^C}\right)^u \bar{U}_s\sigma^{\mu\nu}U_\tau\,\bar{U}_e\sigma_{\mu\nu}U_d\,\Gamma(u)\,\Gamma(2-u)\,\frac{\left(m^2\right)^{-u}}{M^2}. \tag{4.9}$$

This reproduces the $\left(m^2\right)^{-u}$ term in Eq. (4.5), including the entire u dependence. Comparing Eq. (4.5) with Eq. (4.9), we obtain

$$B[c_T(\mu)] = \frac{\lambda^2}{12\pi N_f}\left(\frac{\mu^2}{e^C}\right)^u\,\Gamma(u-1)\,\Gamma(2-u)\,\frac{\left(M^2\right)^{-u}}{M^2}. \tag{4.10}$$

Note that any dependence on the quark mass m has dropped out of (4.10), so that the matching condition is independent of the infrared regulator. Therefore, in terms of the original perturbation series

$$c_T(\mu) = \frac{\lambda^2}{12\pi N_f}\sum_{n=0}^{\infty} c_n(\alpha_s N_F)^n, \tag{4.11}$$

all large logarithms of m/M have dropped out of the matching conditions:[3]

$$\begin{aligned} c_0 &= 0,\\ c_1 &= C' - 2\ln(\mu/M),\\ c_2 &= -b_0\left[C'' + 2C\ln(\mu/M) - 2\ln^2(\mu/M)\right],\\ c_3 &= \cdots \end{aligned} \tag{4.12}$$

(where C' and C'' are scheme-dependent renormalization constants). However, despite the fact that the individual terms c_i in the expansion of c_T are now well defined, expression (4.10) has poles at $u = 0, 1, 2, \ldots$. The pole at $u = 0$ is removed by renormalization [6], but the renormalons at $u = 1, 2, \ldots$ correspond to ambiguities of order $(\Lambda_{\rm QCD}/M)^{2u}$ in the coefficient function $c_T(\mu)$. Note that these are different from the singularities in expression (4.4). The singularity at $u = 1$ in (4.10) is not present in (4.4), while the coefficients of the singularities at $u = 2, \ldots$ also differ. For the singularity at $u = n$ we find

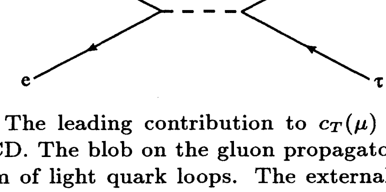

FIG. 5. The leading contribution to $c_T(\mu)$ arises at one loop in QCD. The blob on the gluon propagator denotes the bubble sum of light quark loops. The external quarks have been given a small mass m which regulates the infrared behavior of the graph.

[3]The logarithms of m/M are reproduced in the effective theory by scaling the operators from $\mu = M$ to low energies.

$$B[A_T^{\rm full}] \sim \frac{1}{M^2} \times \frac{(-1)^n (n-1)}{u-n} \left(\frac{\mu^2}{m^2}\right)^n \sum_{i=1}^{n-1} (n-i) \left(\frac{m^2}{M^2}\right)^{i-1} + \cdots ,$$
$$B[c_T(\mu)] \sim -\frac{(-1)^n}{u-n} \left(\frac{\mu^2}{M^2}\right)^n + \cdots \tag{4.13}$$

where the ellipses denote terms regular at $u = n$. Note that the singular piece of $B[A_T^{\rm full}]$ at $u = n$ has no term proportional to $(\mu^2/M^2)^n$.

We now consider using the effective Lagrangian in Eq. (4.2) to calculate the cross section for the spin-flip scattering process $\tau d \to es$. This is of course not a physical process, since the d and s quarks are not physical asymptotic states, and the perturbatively calculated rate $A_T^{\rm full}$ exhibits serious infrared problems, as is clear from Eq. (4.4). However, we may use this process to demonstrate that physical predictions in the effective theory are well defined, by demonstrating that the renormalon ambiguities cancel between the coefficient function $c_T(\mu)$ and the (perturbatively computed) matrix elements of higher dimension operators. Since the graphs which cancel the ambiguity also occur for the matrix elements of physical hadrons, this cancellation will also take place for physical amplitudes such as $\tau \to K^0 e$.

The order $1/M^2$ contribution to $\tau d \to se$ process from the operator O_T is

$$\frac{\lambda^2}{12\pi N_f} \left(\frac{\mu^2}{e^C}\right)^u \Gamma(u-1)\,\Gamma(2-u)\, \frac{(M^2)^{-u}}{M^2}\, \bar{U}_s \sigma^{\mu\nu} U_\tau\, \bar{U}_e \sigma_{\mu\nu} U_d. \tag{4.14}$$

Writing $u = 1 + \Delta u$, one finds that the singularity at $u = 1$ is

$$\frac{\lambda^2}{12\pi N_f} \left(\frac{\mu^2}{e^C}\right) \frac{1}{M^4}\, \bar{U}_s \sigma^{\mu\nu} U_\tau\, \bar{U}_e \sigma_{\mu\nu} U_d\, \frac{1}{\Delta u} \tag{4.15}$$

which is of order $1/M^4$. The renormalons at $u = 2, \ldots$ produce singularities of order $1/M^6, \ldots$. Since we have only computed the effective Lagrangian to order $1/M^4$, we can ignore the renormalon singularities at $u \geq 2$, and the only singularity that is relevant to the order we are working is the one at $u = 1$. This singularity is canceled by a singularity in the Borel-transformed matrix element of the d_S operator, which is also of order $1/M^4$. The matrix element of the $\bar{s}\tau\, D^2\, [\, \bar{e}d\,]$ operator between quark states is evaluated using the graphs of Fig. 6, where only the first graph contributes to the spin-flip scattering amplitude

$$-\frac{16\pi\lambda^2}{3N_f} \left(\frac{\mu^2}{e^C}\right)^u \bar{U}_s \sigma_{\mu\alpha} U_\tau\, \bar{U}_e \sigma_{\nu\beta} U_d \int \frac{d^4k}{(2\pi)^4}\, \frac{k^\alpha k^\beta \left(k^\mu k^\nu - k^2 g^{\mu\nu}\right) k^2}{\left(k^2 - m^2\right)^2 M^4 \left(-k^2\right)^{2+u}}. \tag{4.16}$$

Evaluating the k integral gives

$$-\frac{i\lambda^2}{12\pi N_f} \left(\frac{\mu^2}{e^C}\right)^u \bar{U}_s \sigma^{\mu\nu} U_\tau\, \bar{U}_e \sigma_{\mu\nu} U_d\, \Gamma(u-1)\,\Gamma(3-u)\, \frac{m^2 \left(m^2\right)^{-u}}{M^4}. \tag{4.17}$$

Expanding around $u = 1$ gives the singular term

$$B[A_T^{(1)}] = -\frac{i\lambda^2}{12\pi N_f} \left(\frac{\mu^2}{e^C}\right) \bar{U}_s \sigma^{\mu\nu} U_\tau\, \bar{U}_e \sigma_{\mu\nu} U_d\, \frac{1}{M^4}\, \frac{1}{\Delta u}. \tag{4.18}$$

This is precisely the negative of Eq. (4.15), so that the singularity cancels in the total amplitude, which is the sum of the two terms, Eqs. (4.14) and (4.17).

Clearly, at $u = 2, 3, \ldots$ similar cancellations will take place with $1/M^6, 1/M^8, \ldots$ operators. This is simply because the singularities in $B[c_T(\mu)]$ are not found in the scattering amplitudes in the full theory, Eq. (4.13). Since the full and effective theories are, by construction, identical up to the order to which the effective theory has been defined, the singularities must cancel between matching conditions and the matrix elements of higher dimension operators in the effective theory, as they do at $u = 1$.

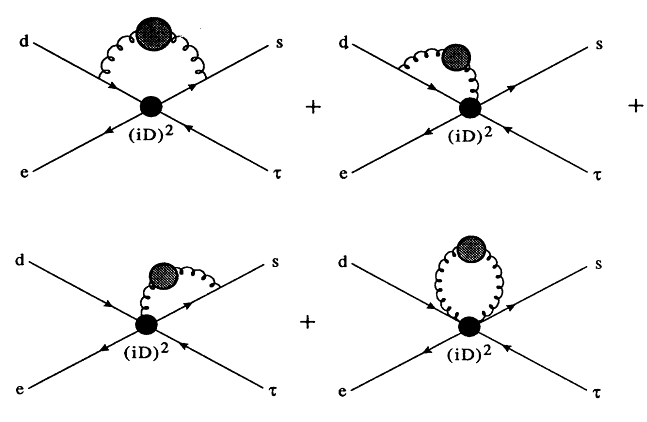

FIG. 6. The contribution to the scattering amplitude from the $(iD)^2$ operator which is $O(1/M^4)$. Only the first diagram contributes to the spin-flip amplitude at order $1/N_f$. The ambiguity at $u = 1$ in this matrix element cancels that in $c_T(\mu)$.

V. CONCLUSIONS

Perturbatively calculated matching conditions in an effective field theory suffer from renormalon ambiguities. However, we have argued that any ambiguity in a physical quantity is always higher order in $1/M$ than the effective theory has been defined and is therefore of no consequence. In practice, one calculates matching conditions to a given number of loops, and from physical measurements then determines the value of nonperturbative matrix elements. The cancellation of renormalon ambiguities in physical observables then simply means that, although the values obtained for the nonperturbative matrix elements will depend sensitively on the number of loops at which the theories are matched, relations between physical quantities will not. If the unphysical parameters are extracted from observables at a given order in α_s, then they can be used to predict other observables to the same order in α_s, as was done for example for the extraction of $|V_{bc}|$ in [16,17]. Inclusive and exclusive semileptonic decays of hadrons containing a heavy quark are free of renormalon ambiguities, regardless of the mass parameter of the $1/m_0$ expansion. In addition, we have demonstrated renormalon cancellation in an effective field theory other than HQET—the four-Fermi theory.

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