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Masses and widths of N and Δ resonances

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A simple approximation is derived to relate the complex pole in a resonant partial-wave T-matrix amplitude to the conventional Breit-Wigner parameters that describe the scattering resonance. The approximation is tested by using well-established resonance parameters for the $\Delta(1232)\frac{3}{2}^+$. This new result facilitates a comparison of both Breit-Wigner parameters and pole positions for 17 resonances described by four major analyses of πN scattering.

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I. INTRODUCTION

Resonance parameters are generally determined by analyzing the energy dependence of experimentally determined partial-wave amplitudes that describe resonant two-body scattering reactions. Typically the amplitudes are fitted with generalized Breit-Wigner formulas that contain a "mass parameter" M and a "width parameter" Γ. While the conventional Breit-Wigner parameters M and Γ are more often compared with model predictions, the pole positions are considered more fundamental and less model dependent [1–4]. This paper provides a simple approximate way to relate M and Γ to the pole position. In addition, questions concerning properties of specific resonances, such as the width of the Roper resonance, are addressed.

In the vicinity of a pole, a partial-wave T-matrix amplitude can be approximated by the Breit-Wigner form

$$T \approx \frac{f(W)}{M - W - i\Gamma(W)/2} , \tag{1}$$

where W is the total invariant energy in the center-of-mass (c.m.) frame. The energy dependence of $\Gamma(W)$ displaces the real part of the pole position from M, and the imaginary part is different from $-i\Gamma/2$, where Γ denotes $\Gamma(W)$ evaluated at $W = M$. If $W_p = M_0 - i\Gamma_0/2$ denotes the complex pole, then $D(W_p) = 0$ where $D(W) = M - W - i\Gamma(W)/2$. Newton's method for determining the root of a function indicates that

$$W_p \approx W_0 - \frac{D(W_0)}{D'(W_0)} , \tag{2}$$

where $D'(W) = dD(W)/dW$ and $W_0 = M - i\Gamma/2$ is an initial approximation for the pole position [5]. For $M \gg \Gamma/2$, I can make the Taylor series expansions $\Gamma(W_0) \approx \Gamma - i\alpha\Gamma$ and $\Gamma'(W_0) \approx \Gamma'$, where Γ' denotes $d\Gamma(W)/dW$ evaluated at $W = M$ and $\alpha = \Gamma'/2$. Upon substituting into Eq. (2) and simplifying, I obtain

$$M_0 \approx M - \frac{\Gamma}{2}\left(\frac{\alpha}{1+\alpha^2}\right) , \tag{3a}$$

$$\Gamma_0 \approx \frac{\Gamma}{1+\alpha^2} . \tag{3b}$$

This approximation is somewhat similar to that derived by Lichtenberg [4], who found by a Taylor-series expansion of $D(W)$ about the point $W = M$ that (in my notation) $M_0 \approx M - (\Gamma/2)\,\alpha$ and $\Gamma_0 \approx \Gamma\,[1 - \alpha^2 - (\Gamma/4)\,\alpha']$, with $\alpha' = \Gamma''/2$. Clearly Eq. (3) reduces to Lichtenberg's result if $\alpha^2 \ll 1$ and if $|\alpha'| \ll (4/\Gamma)\,\alpha^2$. Both conditions are approximately satisfied for the $\Delta(1232)\frac{3}{2}^+$, as discussed in the following section.

The approximations above are not directly useful for determining the residues at the pole positions, which clearly require knowledge of the explicit form of the function $f(W)$ in Eq. (1). This function is generally very model dependent and may be affected strongly by background contributions (from nonresonant terms and/or from overlapping resonances). An advantage of the approximations given in Eq. (3) is that they provide a convenient way to investigate how values of the conventional Breit-Wigner parameters will vary depending on the assumed energy dependence of $\Gamma(W)$. [Values of the pole parameters are thought to be approximately insensitive to the assumed energy dependence of $D(W)$].

II. DISCUSSION

As a specific example and a test of Eq. (3), consider the $\Delta(1232)\frac{3}{2}^+$, the first resonance in elastic pion-nucleon scattering. If I write

$$\Gamma(W) = \Gamma\,\frac{\rho(W)}{\rho(M)} , \tag{4}$$

where $\rho(W)$ is a phase-space factor, then it follows that

$$\alpha = \frac{\Gamma}{2}\,\frac{\rho'(M)}{\rho(M)} . \tag{5}$$

For the $\Delta(1232)\frac{3}{2}^+$, the phase-space factor may be parametrized as [6]

$$\rho(W) = \frac{q}{W}\,B_\ell^2(qR) , \tag{6}$$

where B_ℓ is a Blatt-Weisskopf barrier-penetration fac-

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tor [7], $\ell = 1$ is the orbital angular momentum between the pion and nucleon, and q is the relative momentum between the pair in the c.m. frame; i.e., $W = \sqrt{q^2 + m_\pi^2} + \sqrt{q^2 + m_N^2}$, where m_π and m_N are the masses of the pion and nucleon, respectively. For the $\Delta(1232)$, I take $M = 1231$ MeV and $\Gamma = 118$ MeV [6]. Then using $m_\pi = 140$ MeV and $m_N = 938$ MeV, I find $q = 226$ MeV at $W = M$. Using $B_1(x) = x/\sqrt{1+x^2}$ [8] and the value $R = 1.0$ fm from Ref. [6], it is straightforward to calculate that $\alpha = 0.40$. With this value substituted into Eq. (3), I find $M_0 = 1211$ MeV and $\Gamma_0 = 102$ MeV, in excellent agreement with accepted values for the pole position [9,10]. By comparison, if Lichtenberg's approximation [4] is used, then the values obtained are $M_0 = 1207$ MeV and $\Gamma_0 = 102$ MeV, where, for the width, I have used the calculated value, $\alpha' = -8.0 \times 10^{-4}$ MeV^{-1}. As Lichtenberg has noted, his approximation effectively provides a lower bound for the mass and width.

From Eq. (3), it follows that values of M, M_0, and Γ_0 can be used to determine values of α according to the expression, $\alpha = 2(M - M_0)/\Gamma_0$. Table I lists calculated values of α for several N and Δ resonances observed in πN elastic scattering. These values were determined using the pole positions and mass parameters from the partial-wave analysis of Cutkosky *et al.* [9]. Note that in each case, $0 < \alpha < 1$, which is reflected in the observation that $M > M_0$ and $\Gamma > \Gamma_0$ [4]. This result is easily understood from Eq. (6), which implies that $\rho(W)$ is a monotonically increasing function for real values of W [$\rho(W) \propto q^{2\ell+1}$ for small q and $\rho(W) \to$ const for large q]. I next calculated the corresponding width parameters according to Eq. (3), viz., $\Gamma_{\rm calc} = (1 + \alpha^2)\Gamma_0$. For each resonance in Table I, the difference between $\Gamma_{\rm calc}$ and the value of Γ determined by Cutkosky *et al.* is generally much smaller than the quoted uncertainty in Γ. [Note that Table I lists $\alpha = 0.44$ for the $\Delta(1232)\frac{3}{2}^+$. This value corresponds to an interaction radius of $R = 0.85$ fm in Eq. (6).]

Conventional Breit-Wigner parameters M and Γ were then calculated from the pole positions (solution SM90) determined in a recent partial-wave analysis of elastic πN scattering by Arndt *et al.* [10]. (An even more recent set of partial-wave analyses employing rigorous constraints from simultaneous forward and fixed-t dispersion relations has been performed, although new pole positions are not available [11].) Results of the calculations, which used Eq. (3) with values of α from Table I, are listed in Tables II and III under the heading VPI91. Tables II and III compare these parameters with those published for three other analyses: namely, the 1992 Kent State University analysis (KSU92) [6], the 1980 Carnegie-Mellon–Berkeley analysis (CMB80) [9], and the 1980 Karlsruhe–Helsinki analysis (KH80) [12]. The tables also give unweighted means with sample standard deviations. In most cases, the different analyses agree surprisingly well for the parameter values; however, there are six cases that warrant further comment.

Three analyses of resonance formation in πN scattering experiments indicate that the Roper resonance [$N(1440)\frac{1}{2}^+$] is broad, in disagreement with the KH80 result (see Table II). It should be noted that the entry in Table II under the column VPI91 was determined from the P_{11} pole at $(1360 - i\,126)$ MeV; this pole is on the sheet of the Riemann surface most directly reached by analytic continuation from the real axis. An auxil-

TABLE I. Comparison of the calculated width parameters $\Gamma_{\rm calc}$ with values of Γ from the πN partial-wave analysis of Cutkosky *et al.* [9]. All masses and widths are in MeV. The values of α were determined as discussed in the text. Note that $0 < \alpha < 1$ for all cases.

Resonance	M_0	Γ_0	M	Γ	$\Gamma_{\rm calc}$	α
$N(1440)\frac{1}{2}^+$	1375 ± 30	180 ± 40	1440 ± 30	340 ± 70	273	0.72
$N(1520)\frac{3}{2}^-$	1510 ± 5	114 ± 10	1525 ± 10	120 ± 15	122	0.26
$N(1535)\frac{1}{2}^-$	1510 ± 50	260 ± 80	1550 ± 40	240 ± 80	285	0.31
$N(1650)\frac{1}{2}^-$	1640 ± 20	150 ± 30	1650 ± 30	150 ± 40	153	0.13
$N(1675)\frac{5}{2}^-$	1660 ± 10	140 ± 10	1675 ± 10	160 ± 20	146	0.21
$N(1680)\frac{5}{2}^+$	1667 ± 5	110 ± 10	1680 ± 10	120 ± 10	116	0.24
$N(1710)\frac{1}{2}^+$	1690 ± 20	80 ± 20	1700 ± 50	90 ± 30	85	0.25
$N(1720)\frac{3}{2}^+$	1680 ± 30	120 ± 40	1700 ± 50	125 ± 70	133	0.33
$N(2190)\frac{7}{2}^-$	2100 ± 50	400 ± 160	2200 ± 70	500 ± 150	500	0.50
$N(2220)\frac{9}{2}^+$	2160 ± 80	480 ± 100	2230 ± 80	500 ± 150	521	0.29
$N(2250)\frac{9}{2}^-$	2150 ± 50	360 ± 100	2250 ± 80	480 ± 120	471	0.56
$\Delta(1232)\frac{3}{2}^+$	1210 ± 1	100 ± 2	1232 ± 3	120 ± 5	119	0.44
$\Delta(1600)\frac{3}{2}^+$	1550 ± 40	200 ± 60	1600 ± 50	300 ± 100	250	0.50
$\Delta(1620)\frac{1}{2}^-$	1600 ± 15	120 ± 20	1620 ± 20	140 ± 20	133	0.33
$\Delta(1700)\frac{3}{2}^-$	1675 ± 25	220 ± 40	1710 ± 30	280 ± 80	242	0.32
$\Delta(1905)\frac{5}{2}^+$	1830 ± 40	280 ± 60	1910 ± 30	400 ± 100	371	0.57
$\Delta(1910)\frac{1}{2}^+$	1880 ± 30	200 ± 40	1910 ± 40	225 ± 50	218	0.30
$\Delta(1930)\frac{5}{2}^-$	1890 ± 50	260 ± 60	1940 ± 30	320 ± 60	298	0.38
$\Delta(1950)\frac{7}{2}^+$	1890 ± 15	260 ± 40	1950 ± 15	340 ± 50	315	0.46

iary pole, at $(1413 - i\ 128)$ MeV, was found on another sheet, and has less physical significance for the multichannel Roper resonance. Further discussion of the issue of poles in the P_{11} partial-wave amplitudes can be found in the Brief Report by Cutkosky and Wang [13]. Recently Höhler determined pole positions based on a speed-plot analysis of the KH80 amplitudes [14]. For the Roper resonance, he found $\Gamma_0 = 164$ MeV, which suggests, based on arguments presented earlier, that the KH80 width parameter ($\Gamma = 135$ MeV) is too small for this resonance. Using the mean value $M = 1441$ MeV from Table II and mean pole values (see below) of $M_0 = 1372$ MeV and $\Gamma_0 = 213$ MeV, the calculated width parameter is $\Gamma = 300$ MeV, which corresponds to $\alpha = 0.64$. This somewhat large value of α may correlate with the observation that the $\pi^- p \to \pi\pi N$ reaction is dominated near threshold by formation of the Roper resonance. [In general, $\Gamma(W)$ is expected to increase relatively fast in the vicinity of an important inelastic threshold.] Recently, Morsch *et al.* studied the Roper resonance by α-proton scattering [15]. Their production experiment led to a slightly low mass of about 1400 MeV and a rather narrow width of about 160–170 MeV. The apparently contradictory results between formation and production experiments may be understood if the parameters from the α-proton experiment can be identified with the pole parameters M_0 and Γ_0. As discussed below, estimates of M_0 and Γ_0 from the πN experiments yield the mean values, 1372 ± 11 MeV and 213 ± 48 MeV, in reasonable agreement with the results of Morsch *et al.*

From Table II, it is also clear that three analyses of resonance formation in πN scattering experiments indicate that the $N(1535)\frac{1}{2}^-$ is narrow, in disagreement with the CMB80 result. The mass and width of this resonance are nontrivial to determine because it has a large branching fraction (about 50%) to ηN, and the threshold for η production is at 1488 MeV. Recently, Clajus and Nefkens determined the mass and width of this resonance by using a Breit-Wigner term to fit available total cross-section data for the $\pi^- p \to \eta n$ reaction near threshold [16]. The mass and width obtained from their fit were $M_0 = 1483 \pm 16$ MeV and $\Gamma_0 = 204 \pm 21$ MeV, respectively. As noted by Clajus and Nefkens, this mass is well below the (Breit-Wigner) value recommended for the $N(1535)\frac{1}{2}^-$. This seemingly contradictory result is easily explained by noting that Clajus and Nefkens used a *constant* width in their Breit-Wigner fit; hence, their values *must* be interpreted as the physical mass M_0 and width Γ_0 associated with the pole position. Their values then agree with the mean pole values of $M_0 = 1502 \pm 12$ MeV and $\Gamma_0 = 169 \pm 80$ MeV (see below).

TABLE II. Conventional Breit-Wigner parameters for $I = \frac{1}{2}$ resonances as determined from four analyses: KSU92 [6]; VPI91 [10]; CMB80 [9]; and KH80 [12]. The first entry lists M and the second lists Γ.

Resonance	KSU92	VPI91[a]	CMB80	KH80	Mean
$N(1440)\frac{1}{2}^+$	1462	1451	1440	1410	1441 ± 22
	391	383	340	135	312 ± 120
$N(1520)\frac{3}{2}^-$	1524	1525	1525	1519	1523 ± 3
	124	115	120	114	118 ± 5
$N(1535)\frac{1}{2}^-$	1534	1516	1550	1526	1532 ± 14
	151	120	240	120	158 ± 57
$N(1650)\frac{1}{2}^-$	1659	1668	1650	1670	1662 ± 9
	173	163	150	180	167 ± 13
$N(1675)\frac{5}{2}^-$	1676	1668	1675	1679	1675 ± 5
	159	130	160	120	142 ± 20
$N(1680)\frac{5}{2}^+$	1684	1684	1680	1684	1683 ± 2
	139	122	120	128	127 ± 9
$N(1710)\frac{1}{2}^+$	1717	1704	1700	1723	1711 ± 11
	480	578	90	120	317 ± 248
$N(1720)\frac{3}{2}^+$	1717	1694	1700	1710	1705 ± 10
	380	127	125	190	206 ± 120
$N(2190)\frac{7}{2}^-$	2127	2176	2200	2140	2161 ± 33
	550	580	500	390	505 ± 83

[a]Calculated from the pole positions (solution SM90) in Ref. [10]. Parameters in the table for $N(1440)\frac{1}{2}^+$ were determined from the pole of most physical significance at $(1360 - i\ 126)$ MeV. An auxiliary pole was found at $(1413 - i\ 128)$ MeV (see text).

The Breit-Wigner width parameters for the $N(1710)\frac{1}{2}^+$ and the $N(1720)\frac{3}{2}^+$ are not determined consistently among the four analyses; this lack of agreement is understandable given that these two resonances are very inelastic.

From Table III, it can be noted that the calculated mass parameter of 1670 MeV for the $\Delta(1600)\frac{3}{2}^+$ (determined from the VPI91 pole position) agrees better with the KSU92 value of 1706 MeV than with the CMB80 and KH80 values of 1600 and 1522 MeV, respectively. A higher mass near 1700 MeV is in better agreement with quark-model predictions [17,18].

Finally, Table III shows that the calculated mass parameter of 2095 MeV for the $\Delta(1930)\frac{5}{2}^-$ (determined from the VPI91 pole position) is about 150 MeV higher than values of the three other analyses. The lower values have presented long-standing problems for quark-model calculations [18].

III. SUMMARY AND CONCLUSIONS

A simple approximation was derived to relate the pole position in a resonant scattering amplitude to the conventional Breit-Wigner parameters, M and Γ. The physical mass M_0 is defined as the real part of the pole position and the physical width Γ_0 is -2 times the imaginary part of the pole position [1–4]. On general grounds, it is expected that $M > M_0$ and $\Gamma > \Gamma_0$.

Tables IV and V compare the physical mass M_0 and width Γ_0 determined from the real and imaginary parts of the pole positions from four analyses, including the 1992 Kent State University analysis (KSU92) [6], the 1991 Virginia Polytechnic Institute and State University analysis (VPI91) [10], the 1980 Carnegie-Mellon–Berkeley analysis (CMB80) [9], and the 1980 Karlsruhe–Helsinki analysis (KH80) [12]. The tables also give unweighted means with sample standard deviations. The pole parameters listed under the heading KSU92 were estimated from the conventional Breit-Wigner parameters in Ref. [6] using values of α from Table I, and the pole parameters listed under the heading KH80 are from the recent speed-plot fits by Höhler [14]. Contrary to expectations, Höhler's recent determinations of pole parameters for several resonances do *not* satisfy the condition $\Gamma > \Gamma_0$ when compared with the early KH80 width parameters; consequently, the early KH80 width parameters may be underestimated in some cases (the Roper resonance, for example). This is an important point because results of that analysis have historically provided a main contribution for the estimation of standard resonance parameters, as quoted, for example, by the Particle Data Group [19].

My relationship between the pole position and Breit-Wigner parameters was tested by using results from the Carnegie-Mellon–Berkeley (CMB) partial-wave analysis of πN scattering [9]. The relationship was then used to calculate conventional Breit-Wigner parameters from the pole positions determined in a recent πN partial-wave analysis by the Virginia Tech (VPI) group [10]. For most cases, the parameters derived from the VPI pole positions agree very well with the results of a recent analysis performed at Kent State University (KSU) [6]. There

TABLE III. Conventional Breit-Wigner parameters as in Table II but for $I = \frac{3}{2}$ resonances.

Resonance	KSU92	VPI91[a]	CMB80	KH80	Mean
$\Delta(1232)\frac{3}{2}^+$	1231	1232	1232	1233	1232 ± 1
	118	119	120	116	118 ± 2
$\Delta(1600)\frac{3}{2}^+$	1706	1670	1600	1522	1625 ± 81
	430	288	300	220	310 ± 88
$\Delta(1620)\frac{1}{2}^-$	1672	1607	1620	1610	1627 ± 30
	154	133	140	139	142 ± 9
$\Delta(1700)\frac{3}{2}^-$	1762	1679	1710	1680	1690 ± 18[b]
	600	229	280	230	246 ± 29[b]
$\Delta(1905)\frac{5}{2}^+$	1881	1860	1910	1905	1889 ± 23
	327	305	400	260	323 ± 58
$\Delta(1910)\frac{1}{2}^+$	1882	2010	1910	1888	1923 ± 60
	239	434	225	280	295 ± 96
$\Delta(1930)\frac{5}{2}^-$	1956	2095	1940	1901	1973 ± 85
	530	457	320	195	376 ± 149
$\Delta(1950)\frac{7}{2}^+$	1945	1939	1950	1913	1937 ± 16
	300	289	340	224	288 ± 48

[a]Calculated from the pole positions (solution SM90) in Ref. [10].
[b]Without KSU92.

TABLE IV. Pole parameters for $I = \frac{1}{2}$ resonances as determined from four analyses: KSU92 [6]; VPI91 [10]; CMB80 [9]; and KH80 [12,14]. The first entry lists the physical mass M_0 and the second lists the physical width Γ_0.

Resonance	KSU92[a]	VPI91	CMB80	KH80	Mean
$N(1440)\frac{1}{2}^+$	1369	1360	1375	1385	1372 ± 11
	257	252	180	164	213 ± 48
$N(1520)\frac{3}{2}^-$	1509	1511	1510	1510	1510 ± 1
	116	108	114	120	115 ± 5
$N(1535)\frac{1}{2}^-$	1513	1499	1510	1487	1502 ± 12
	138	110	260		169 ± 80
$N(1650)\frac{1}{2}^-$	1648	1657	1640	1670	1654 ± 13
	170	160	150	163	161 ± 8
$N(1675)\frac{5}{2}^-$	1660	1655	1660	1656	1658 ± 3
	152	124	140	126	136 ± 13
$N(1680)\frac{5}{2}^+$	1668	1670	1667	1673	1670 ± 3
	132	116	110	135	123 ± 12
$N(1710)\frac{1}{2}^+$	1661	1636	1690	1690	1669 ± 26
	452	544	80	200	319 ± 216
$N(1720)\frac{3}{2}^+$	1660	1675	1680	1686	1675 ± 11
	342	114	120	187	191 ± 106
$N(2190)\frac{7}{2}^-$	2017	2060	2100	2042	2055 ± 35
	440	464	400	482	447 ± 35

[a]Calculated from the Breit-Wigner parameters in Ref. [6].

TABLE V. Pole parameters as in Table IV but for $I = \frac{3}{2}$ resonances.

Resonance	KSU92[a]	VPI91	CMB80	KH80	Mean
$\Delta(1232)\frac{3}{2}^+$	1211[b]	1210	1210	1209	1210 ± 1
	102[b]	100	100	100	100 ± 1
$\Delta(1600)\frac{3}{2}^+$	1620	1612	1550	1550	1583 ± 38
	344	230	200		258 ± 76
$\Delta(1620)\frac{1}{2}^-$	1649	1587	1600	1608	1611 ± 27
	139	120	120	116	124 ± 10
$\Delta(1700)\frac{3}{2}^-$	1675	1646	1675	1651	1657 ± 16[c]
	545	208	220	159	196 ± 32[c]
$\Delta(1905)\frac{5}{2}^+$	1811	1794	1890	1829	1831 ± 42
	247	230	260	303	260 ± 31
$\Delta(1910)\frac{1}{2}^+$	1849	1950	1880	1874	1888 ± 43
	219	398	200	283	275 ± 89
$\Delta(1930)\frac{5}{2}^-$	1867	2018	1890	1850	1906 ± 76
	462	398	260	180	325 ± 128
$\Delta(1950)\frac{7}{2}^+$	1888	1884	1890	1878	1885 ± 5
	247	238	260	230	244 ± 13

[a]Calculated from the Breit-Wigner parameters in Ref. [6].
[b]Calculated with $\alpha = 0.40$, as discussed in the text.
[c]Without KSU92.

were, however, three notable exceptions: the $N(1720)\frac{3}{2}^+$ was found to have a significantly narrower width than determined in the KSU analysis, and the $\Delta(1910)\frac{1}{2}^+$ and $\Delta(1930)\frac{5}{2}^-$ were found to have masses over 100 MeV higher than in the KSU analysis. These differences possibly arise because these resonances couple weakly to the πN channel; therefore, they are not excited strongly in elastic πN scattering.

My relationship was also used to estimate pole parameters from the KSU Breit-Wigner parameters [6]. These were compared with pole parameters from the CMB analysis [9], the VPI analysis [10], and from the recent work by Höhler [14], who fitted speed plots of amplitudes from the KH80 analysis [12]. Mean values for M_0 and Γ_0 were determined from these four sets of pole parameters. The standard deviations among the pole values for a given resonance are generally smaller than the corresponding standard deviations among the Breit-Wigner parameters. The four sets agree particularly well for the pole mass and width of the $\Delta(1232)\frac{3}{2}^+$, $N(1520)\frac{3}{2}^-$, $N(1650)\frac{1}{2}^-$, $N(1675)\frac{5}{2}^-$, $N(1680)\frac{5}{2}^+$, and $\Delta(1950)\frac{7}{2}^+$. This stability in the determination of the pole parameters for resonances with large πN couplings provides further evidence that these parameters are less model dependent than the corresponding Breit-Wigner parameters, as noted in the Introduction.

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APPENDIX: MODEL DEPENDENCY OF BREIT-WIGNER PARAMETERS

If one assumes a "relativistic" Breit-Wigner form, rather than Eq. (1), the partial-wave T-matrix amplitude in the vicinity of a pole becomes

$$T \approx \frac{g(W)}{M^2 - W^2 - iM\Gamma(W)} \; . \tag{A1}$$

For $M \gg \Gamma/2$, a similar derivation to that presented in Sec. I yields the relationship

$$M_0 \approx M - \frac{\Gamma}{2}\left(\frac{\alpha}{1+\alpha^2}\right) - \frac{\Gamma^2/(8M)}{1+\alpha^2} \; , \tag{A2a}$$

$$\Gamma_0 \approx \frac{\Gamma}{1+\alpha^2} - \frac{\Gamma^2}{4M}\left(\frac{\alpha}{1+\alpha^2}\right) \; , \tag{A2b}$$

where here $\alpha = \Gamma'/2 - \Gamma/(2M)$. The last terms on the right-hand side (RHS) of the expressions in Eq. (A2) are generally very small. Clearly these expressions for M_0 and Γ_0 reduce to the results in Sec. I when $\Gamma/M \ll \Gamma'$. In that situation, the differences in fitted Breit-Wigner parameters arising from the use of either relativistic or nonrelativistic models are expected to be small.

Finally, it is noteworthy that other versions of the Breit-Wigner formula are in common use. For example, in fitting the e^+e^- cross section near the Z-boson resonance, it is common to represent the Z-boson propagator by a term proportional to [3]

$$T \approx \frac{g(W)}{M^2 - W^2 - iW^2\Gamma/M} \; , \tag{A3}$$

where the conventional Breit-Wigner parameters M and Γ are constants. This equation may be related to Eq. (A1) by making the identification, $\Gamma(W) = (W/M)^2\Gamma$, which implies that in Eq. (A2), $\alpha = \Gamma/(2M)$.

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