Perturbative QCD fragmentation functions as a model for heavy-quark fragmentation

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The perturbative QCD fragmentation functions for a heavy quark to fragment into heavy-light mesons are studied in the heavy-quark limit. The fragmentation functions for S-wave pseudoscalar and vector mesons are calculated to next-to-leading order in the heavy-quark mass expansion using the methods of heavy-quark effective theory. The results agree with the $m_b \to \infty$ limit of the perturbative QCD fragmentation functions for $\bar{b}$ into B_c and B_c^*. We discuss the application of the perturbative QCD fragmentation functions as a model for the fragmentation of heavy quarks into heavy-light mesons. Using this model we predict the fraction P_V of heavy-light mesons that are produced in the vector meson state as functions of the longitudinal momentum fraction z and the transverse momentum relative to the jet axis. The fraction P_V is predicted to vary from about $\frac{1}{2}$ at small z to almost $\frac{3}{4}$ near $z = 1$.

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I. INTRODUCTION

Heavy-quark spin-flavor symmetries are very useful for understanding the properties of hadrons containing a single heavy quark in kinematic regimes where nonperturbative aspects of the strong interaction are dominant. These symmetries arise from the fact that the charm, bottom, and top quarks are much heavier than $\Lambda_{\rm QCD}$. The symmetry is exact in the limit of infinite quark mass, and corrections can be systematically organized into an expansion in powers of $\Lambda_{\rm QCD}/m_Q$ using heavy-quark effective theory (HQET). There has been much progress on the applications of heavy-quark symmetries and HQET to the spectroscopy and to both exclusive and inclusive decays of charm and bottom hadrons [1].

It has recently been pointed out by Jaffe and Randall [2] that HQET can also be applied to the fragmentation of a heavy quark into hadrons containing a single heavy quark. They showed that when the fragmentation function is expressed in terms of an appropriate scaling variable, it has a well-defined heavy-quark mass expansion. Specifically, they showed that the fragmentation function $D_{Q\to H}(z)$ at the heavy-quark mass scale has a systematic expansion in inverse powers of m_Q when expressed as a function of the scaling variable

$$y = \frac{1-(1-r)z}{rz} , \tag{1}$$

where $r = (m_H - m_Q)/m_H$, m_H is the mass of the heavy hadron, and z is its longitudinal momentum fraction relative to the fragmenting heavy quark. In the case of a heavy-light meson, r can be interpreted as the ratio of the constituent mass of the light quark to the meson mass. For the pseudoscalar meson P and vector meson V of the same S-wave multiplet ($^1S_0$, $^3S_1$), the fragmentation functions at the scale m_Q have heavy-quark mass expansions of the form

$$D_{Q\to P}(z) = \frac{a(y)}{r} + b(y) + O(r) , \tag{2}$$

$$D_{Q\to V}(z) = \frac{a^*(y)}{r} + b^*(y) + O(r) , \tag{3}$$

where $a^*(y) = 3a(y)$. By heavy-quark spin symmetry, the leading terms differ by a spin factor of 3, while spin splittings first appear at next-to-leading order in the functions $b(y)$ and $b^*(y)$.

It was also realized recently that the fragmentation functions for mesons containing a heavy quark and a heavy antiquark can be computed using perturbative quantum chromodynamics (PQCD) [3–5]. The fragmentation functions for a $\bar{b}$ to split into the S-wave $\bar{b}c$ mesons B_c and B_c^* were calculated to leading order in α_s in Ref. [6]. These fragmentation functions have been used to predict the production rates of the B_c meson at the CERN e^+e^- collider LEP and at the Fermilab Tevatron [7,8]. The general analysis of Jaffe and Randall must certainly apply to perturbative QCD fragmentation functions in the limit where the mass of the heavier quark is taken to infinity. It was verified explic-

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itly in Ref. [6] that the PQCD fragmentation functions $D_{\bar{b}\to B_c}(z)$ and $D_{\bar{b}\to B_c^*}(z)$ reduce to the forms (2) and (3) with $r = m_c/(m_b + m_c)$ in the limit $m_b \to \infty$.

Since the PQCD fragmentation functions are consistent with heavy-quark symmetry, they can be used as models for the fragmentation of heavy quarks into heavy-light mesons. In this paper, we show how the leading and next-to-leading terms in the $1/m_Q$ expansions can be calculated directly from HQET. We then discuss the use of the PQCD fragmentation functions as a phenomenological model for the fragmentation of charm and bottom quarks into heavy-light mesons. As an application of this model, we consider the fraction P_V of heavy-light mesons that are produced in the vector-meson state.

II. PQCD FRAGMENTATION FUNCTIONS FROM HQET

The HQET Lagrangian, including the leading and $1/m_Q$ terms, is given by [1]

$$\mathcal{L} = \bar{h}_v \left\{ iv\cdot D + \frac{1}{2m_Q}\left(C_1(iD)^2 - C_2(v\cdot iD)^2 - \frac{C_3}{2} g_s \sigma^{\mu\nu} G_{\mu\nu} \right) \right\} h_v \ , \tag{4}$$

where

$$C_1 = 1 \ ,$$

$$C_2 = 3\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-8/(33-2n_f)} - 2 \ , \tag{5}$$

$$C_3 = \left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-9/(33-2n_f)}$$

These coefficients are all equal to 1 at the heavy-quark mass scale $\mu = m_Q$. The term proportional to C_2 can be omitted in calculating physical quantities, because it can be eliminated using a field redefinition involving the equation of motion $(v\cdot D)h_v = 0$ from the leading term in the Lagrangian. Our method for calculating the fragmentation function involves a heavy quark which is off shell by an amount at least of order $m_Q m_q$. To demonstrate that the C_2 term can still be omitted in this case, we keep it in our calculation throughout and show that it cancels between the vertex and propagator corrections.

In our calculation, we need the Feynman rules derived from the HQET Lagrangian for (i) the heavy-quark propagator, including $1/m_Q$ corrections, (ii) the heavy-quark-gluon vertex, including $1/m_Q$ corrections, and (iii) the propagator for the small component of the Dirac field of the heavy quark. This last Feynman rule is needed in our calculation because the fragmenting heavy quark is off its mass shell. The Feynman rule for a heavy-quark propagator is

$$\frac{i}{v\cdot k + \frac{C_1}{2m_Q}k^2 - \frac{C_2}{2m_Q}(v\cdot k)^2}\,\frac{1+\not{v}}{2} \ , \tag{6}$$

where k is the residual four-momentum of the heavy quark. The QQg vertex is

$$-ig_s T^a \left(v^\mu + \frac{C_1}{2m_Q}(k_1+k_2)^\mu - \frac{C_2}{2m_Q} v\cdot(k_1+k_2)v^\mu + i\frac{C_3}{2m_Q}\sigma^{\mu\nu}q_\nu \right) \ , \tag{7}$$

where k_1 and k_2 are the residual four-momenta of the incoming and outgoing quarks and $q = k_2 - k_1$ is the momentum of the gluon. The Feynman rule for the propagator of the small component of the Dirac field of the heavy quark is

$$\frac{i}{v\cdot k}\,\frac{1+\not{v}}{2}\left(\frac{1}{2m_Q}\sigma^{\mu\nu}q_\nu\right)\frac{1-\not{v}}{2} \ . \tag{8}$$

To calculate the fragmentation functions, we follow the method introduced in Refs. [3] and [5] and applied in Ref. [6] to the fragmentation processes $\bar{b} \to B_c$ and $\bar{b} \to B_c^*$. We denote the pseudoscalar and vector $Q\bar{q}$ mesons by P and V, respectively. Here Q is the heavy quark and $\bar{q}$ is the light antiquark. We calculate the cross section for producing a $Q\bar{q}$ meson plus a light quark q with total four-momentum K^μ, divide it by the cross section for producing an on-shell Q with the same three-momentum $\vec{K}$, and take the limit $K_0 \to \infty$. The fragmentation function is

$$D(z) = \frac{1}{16\pi^2}\int ds\,\theta\left(s - \frac{M^2}{z} - \frac{m_q^2}{1-z}\right) \times \lim_{K_0\to\infty}\frac{\sum|\mathcal{M}|^2}{\sum|\mathcal{M}_0|^2} \ , \tag{9}$$

where $M = m_Q + m_q$ is the mass of the meson in the nonrelativistic approximation, $s = K^2$, $\mathcal{M}$ is the matrix element for producing $P + q$ or $V + q$, and $\mathcal{M}_0$ is the matrix element for producing an on-shell Q. The calculation can be greatly simplified by using the axial gauge with the gauge parameter $n^\mu = (1, 0, 0, -1)$ in the frame where $K^\mu = (K_0, 0, 0, \sqrt{K_0^2 - s})$. In this gauge, we need only consider the production of the $Q\bar{q}$ meson plus q through a virtual Q of momentum K^μ. The part of the matrix element $\mathcal{M}$ that involves production of the virtual Q can be treated as an unknown Dirac spinor Γ. In the limit $K_0 \to \infty$, the same spinor factor Γ appears in the matrix element $\mathcal{M}_0 = \Gamma u(K)$ for an on-shell Q. The Feynman diagram for $Q^* \to Q\bar{q} + q$ is shown in Fig. 1. The usual projection of the $Q\bar{q}$ onto a nonrelativistic $^1S_0$ bound state reduces in the heavy-quark limit to the Feynman rule

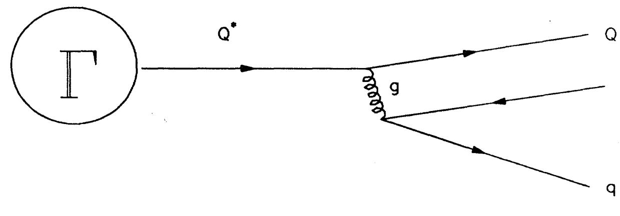

FIG. 1. Feynman diagram used to calculate the PQCD fragmentation functions in the axial gauge.

$$Q\bar{q} \to \frac{\delta^{ij}}{\sqrt{3}} \frac{R(0)\sqrt{M}}{\sqrt{4\pi}} \gamma^5 \frac{1+\not{v}}{2} , \tag{10}$$

where $R(0)$ is the radial wave function at the origin for the meson and v^μ is its four-velocity. For the ${}^3S_1$ state, the projection is the same except that γ^5 is replaced by $\not{\epsilon}$, where ϵ^μ is the polarization four-vector for the vector meson V. The rest of the amplitude corresponding to Fig. 1 is obtained by using the ordinary QCD Feynman rules for the light-quark spinor and the $q\bar{q}g$ vertex and HQET Feynman rules for the heavy-quark propagator and the QQg vertex.

The amplitude $\mathcal{M}$ for producing the ${}^1S_0$ state, including $1/m_Q$ corrections in the heavy-quark propagator and vertex, is

$$\begin{aligned} i\mathcal{M} = &-\frac{8\sqrt{\pi}\alpha_s R(0)}{3} \frac{M^2\sqrt{M}}{m_q} \frac{1}{(s-m_Q^2)^2} \frac{1}{1+\frac{C_1 m_q}{m_Q} - \frac{C_2}{2m_Q} v\cdot k} \left(g_{\mu\nu} - \frac{n_\mu k_\nu + k_\mu n_\nu}{n\cdot k} \right) \\ &\times \bar{u}(p')\gamma^\mu\gamma^5(1+\not{v}) \left(v^\nu + \frac{C_1}{2m_Q} k^\nu - \frac{C_2}{2m_Q}(v\cdot k)v^\nu + \frac{C_3}{4m_Q}(\gamma^\nu \not{k} - \not{k}\gamma^\nu) \right) \frac{1+\not{v}}{2} \Gamma , \end{aligned} \tag{11}$$

where $k = m_q v + p'$ is the momentum of the virtual gluon and also the residual momentum of the fragmenting heavy quark: $K = m_Q v + k$. Note that the term proportional to n_ν in the numerator of the axial-gauge propagator for the gluon vanishes after contracting with the Dirac factor. For the vector-meson state, the γ^5 in the above equation is replaced by $\not{\epsilon}$.

We are interested only in the sum of the first two terms $a(y)/r + b(y)$ in the heavy-quark mass expansion, where $r = m_q/(m_Q + m_q)$. We calculate separately the contributions to the fragmentation functions from the leading terms in the HQET Feynman rules from the $1/m_Q$ corrections from the propagator and from the $1/m_Q$ corrections from the vertex. For the following, we will detail the derivation for the ${}^1S_0$ state, but only quote the results for the ${}^3S_1$ state.

We first derive the fragmentation function $D_{Q\to P}(z)$ with the leading terms in the HQET propagator and vertex only. The amplitude reduces to

$$i\mathcal{M}_1 = \frac{8\sqrt{\pi}\alpha_s R(0)}{3} \frac{M^2\sqrt{M}}{m_q} \frac{1}{(s-m_Q^2)^2} \bar{u}(p') \left(1 + \frac{v\cdot k}{n\cdot k} \not{n} \right) \gamma^5 (1+\not{v})\Gamma . \tag{12}$$

Squaring and summing over spins and colors of the light quark, we get

$$\begin{aligned} \sum |\mathcal{M}_1|^2 = &\frac{64\pi\alpha_s^2 |R(0)|^2}{9} \frac{M^5}{m_q^2} \mathrm{Tr}[\Gamma\bar{\Gamma}(1+\not{v})] \left[\frac{z(1-z)}{M^3[1-(1-r)z]^2 (s-m_Q^2)^2} \right. \\ &\left. + \frac{-1+z+3rz}{M[1-(1-r)z](s-m_Q^2)^3} - \frac{4rM}{(s-m_Q^2)^4} \right] . \end{aligned} \tag{13}$$

The corresponding amplitude squared for producing an on-shell heavy quark is

$$\sum |\mathcal{M}_0|^2 = \frac{3M}{z} \mathrm{Tr}[\Gamma\bar{\Gamma}(1+\not{v})] . \tag{14}$$

Substituting $|\mathcal{M}|^2$ and $|\mathcal{M}_0|^2$ into (9) and integrating over s, we get

$$D_{Q\to P}(z) = \frac{2\alpha_s (2m_q)^2 |R(0)|^2}{81\pi m_q^3} \frac{rz^3(1-z)^2}{[1-(1-r)z]^6} \{3[1-(1-r)z]^2 - 8rz(1-z) + 12rz[1-(1-r)z]\} . \tag{15}$$

Expressing this in terms of y using (1) and expanding to next-to-leading order in r, we have

$$D_{Q\to P}(z) = N\frac{(y-1)^2}{ry^6}(3y^2+4y+8) - N\frac{(y-1)^3}{y^6}(3y^2+4y+8) + O(r) , \tag{16}$$

where $N = 2\alpha_s^2 |R(0)|^2/(81\pi m_q^3)$. Therefore, in terms of $a(y)$ and $b(y)$, the leading term in the HQET Lagrangian contributes

$$a(y) = N\frac{(y-1)^2}{y^6}(3y^2+4y+8) , \tag{17}$$

$$b_1(y) = N\frac{(y-1)^2}{y^6}[-(y-1)(3y^2+4y+8)] . \tag{18}$$

The corresponding calculation for the $^3S_1$ state gives $a^*(y) = 3a(y)$ and $b_1^*(y) = 3b_1(y)$. These contributions to $D_{Q\to P}(z)$ and $D_{Q\to V}(z)$ differ by a spin factor of 3, as required by heavy-quark spin symmetry.

Next we calculate the contributions from $1/m_Q$ corrections in the heavy-quark propagator and heavy-quark vertex. Expanding out the $1/m_Q$ correction to the propagator in (11) to first order, the correction to the amplitude is

$$i\mathcal{M}_2 = \frac{8\sqrt{\pi}\alpha_s R(0)}{3}\frac{M^2\sqrt{M}}{m_q}\frac{1}{(s-m_Q^2)^2}\left(-C_1\frac{m_q}{m_Q} + \frac{C_2}{2m_Q}(m_q + v\cdot p')\right)\overline{u}(p')\left(1+\frac{v\cdot k}{n\cdot k}\not{n}\right)\gamma^5(1+\not{v})\Gamma\ . \tag{19}$$

Keeping the interference terms in $|\mathcal{M}_1+\mathcal{M}_2|^2$, summing over spins and colors, and inserting into (9), we find a $1/m_Q$ correction to $D_{Q\to P}(z)$. Expressing this in terms of y, we find that the contribution to $b(y)$ is

$$b_2(y) = N\frac{(y-1)^2}{y^6}[(-2C_1 + C_2 y)(3y^2+4y+8)]\ . \tag{20}$$

A similar calculation for the $^3S_1$ state gives $b_2^*(y) = 3b_2(y)$. The $1/m_Q$ correction to the amplitude in (11) from the heavy-quark vertex is

$$\begin{aligned} i\mathcal{M}_3 = &-\frac{8\sqrt{\pi}\alpha_s R(0)}{3}\frac{M^2\sqrt{M}}{m_q}\frac{1}{(s-m_Q^2)^2}\left(g_{\mu\nu} - \frac{n_\mu k_\nu}{n\cdot k}\right)\overline{u}(p')\gamma^\mu\gamma^5(1+\not{v}) \\ &\times\left(\frac{C_1}{2m_Q}k^\nu - \frac{C_2}{2m_Q}(v\cdot k)v^\nu + \frac{C_3}{4m_Q}(\gamma^\nu\not{k} - \not{k}\gamma^\nu)\right)\frac{1+\not{v}}{2}\Gamma\ . \end{aligned} \tag{21}$$

Keeping the interference terms in $|\mathcal{M}_1+\mathcal{M}_3|^2$, we obtain after some work the contribution to $b(y)$ and to $b^*(y)$ due to the $1/m_Q$ vertex correction:

$$b_3(y) = N\frac{y-1}{y^5}[-C_2(y-1)(3y^2+4y+8) + 6C_1(y-1)(y+2) - 12C_3 y]\ , \tag{22}$$

$$b_3^*(y) = 3N\frac{y-1}{y^5}[-C_2(y-1)(3y^2+4y+8) + 6C_1(y-1)(y+2) + 4C_3 y]\ . \tag{23}$$

In (11), the $(1+\not{v})/2$ factor adjacent to Γ projects onto the *large* component of the heavy-quark spinors produced by the source Γ. There is also a contribution of order $1/m_Q$ from the *small* component of the heavy-quark spinors of the fragmenting heavy quark [2]. The corresponding amplitude is given by

$$i\mathcal{M}_4 = -\frac{8\sqrt{\pi}\alpha_s R(0)}{3}\frac{M^2\sqrt{M}}{m_q}\frac{1}{(s-m_Q^2)^2}\left(g_{\mu\nu} - \frac{n_\mu k_\nu}{n\cdot k}\right)\overline{u}(p')\gamma^\mu\gamma^5(1+\not{v})\left(\frac{1}{4m_Q}(\gamma^\nu\not{k} - \not{k}\gamma^\nu)\right)\frac{(1-\not{v})}{2}\Gamma\ . \tag{24}$$

The contributions to $b(y)$ and to $b^*(y)$ from the interference term in $|\mathcal{M}_1+\mathcal{M}_4|^2$ are

$$b_4(y) = 2N\frac{y-1}{y^5}(3y^3+5y^2+2y-4)\ , \tag{25}$$

$$b_4^*(y) = 6N\frac{y-1}{y^5}(y^3-y^2+2y-4)\ . \tag{26}$$

The complete expression for $b(y)$ is obtained by adding (18), (20), (22), and (25). Thus the fragmentation function $D_{Q\to P}(z)$ for the $^1S_0$ state, to next-to-leading order in $1/m_Q$, is given by (2) with

$$a(y) = N\frac{(y-1)^2}{y^6}(3y^2+4y+8)\ , \tag{27}$$

$$\begin{aligned} b(y) = N\frac{y-1}{y^6}[&(y-1)(3y^3+15y^2+8y-8) \\ &-12(C_3-1)y^2]\ . \end{aligned} \tag{28}$$

The complete expression for $b^*(y)$ is obtained by adding $3b_1(y)$, $3b_2(y)$, (23), and (26). The fragmentation function $D_{Q\to V}(z)$ for the $^3S_1$ state, to next-to-leading order in $1/m_Q$, is given by (3) with

$$a^*(y) = 3N\frac{(y-1)^2}{y^6}(3y^2+4y+8)\ , \tag{29}$$

$$\begin{aligned} b^*(y) = 3N\frac{y-1}{y^6}[&-(y-1)(y^3+y^2-8y+8) \\ &+4(C_3-1)y^2]\ . \end{aligned} \tag{30}$$

The terms proportional to C_2 in (28) and (30) cancel between propagator and vertex corrections. We have set $C_1 = 1$ in (28) and (30). If we further put $C_3 = 1$, we recover the next-to-leading terms in the $1/r$ expansion of the PQCD fragmentation functions given in Ref. [6].

The heavy-quark mass expansions (2) and (3) break down in the limit $y\to\infty$, which corresponds to $z\to 0$, and also in the limit $y\to 1$, which corresponds to $z\to 1$.

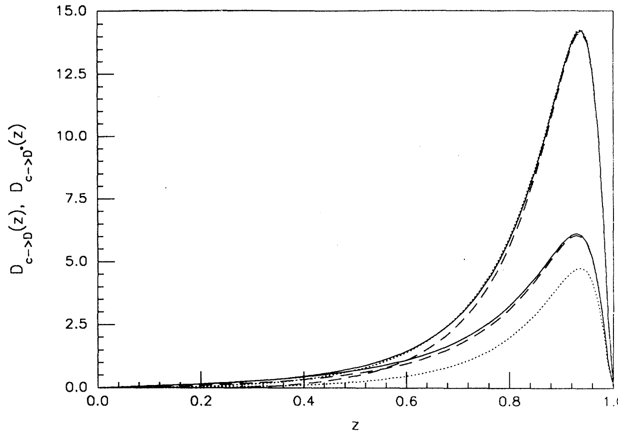

FIG. 2. Comparison of the $D_{c\to D}(z)$ (lower set of curves) and $D_{c\to D^*}(z)$ (upper set of curves) fragmentation functions. The normalization is arbitrary. Shown are the full PQCD results (solid curves), the leading terms (dotted curves) in the heavy-quark mass expansion, and the leading plus next-to-leading terms (dashed curves) in the heavy-quark mass expansion.

As $y \to \infty$, the leading terms, given by (27) and (29), scale like $1/(ry^2)$, while the next-to-leading terms in (28) and (30) scale like $1/y$. Thus the $1/m_Q$ expansion breaks down when y is of order $1/r$ or larger. In the limit $y \to 1$, the leading terms in (2) and (3) vanish like $(y-1)^2/r$, while the terms proportional to $C_3 - 1$ in the next-to-leading terms go to 0 as the first power of $y-1$. Thus, unless $C_3 = 1$, the expansion also breaks down for $y-1$ of order r or smaller.

In Fig. 2, we compare the PQCD fragmentation functions (solid curves) with the heavy-quark mass expansions (2) and (3) at leading order (dotted curves) and next-to-leading order (dashed curves) in r. We use the value $r = 0.10$, which corresponds to D mesons. The normalization is fixed by arbitrarily setting $N = 1$ in (27)–(30). Note that we have set $C_3 = 1$ in (28) and (30). For any other value of C_3, either (2) or (3) becomes negative for y very close to 1, indicating the breakdown of the $1/m_Q$ expansion when z is too close to 1. From the figure, it is clear that the next-to-leading order curves are in very good agreement with the complete PQCD fragmentation functions for both D and D^* mesons. Surprisingly, the leading order result for fragmentation into D^* mesons also agrees very well with the complete PQCD result, while the leading order result for the D meson falls about 30% low near the peak.

III. PQCD MODEL FOR HEAVY-QUARK FRAGMENTATION

It is tempting to use the heavy-quark limits of the PQCD fragmentation functions as phenomenological models for the fragmentation of a heavy quark Q into heavy-light mesons $Q\bar{q}$, where $Q = c$ or b and $q = u$, d, or s. To next-to-leading order in $1/m_Q$, these fragmentation functions are given by (2) and (3), with $a(y)$, $b(y)$, $a^*(y)$, and $b^*(y)$ given in (27)–(30). In addition to N and r, we must treat C_3 as a phenomenological parameter, since, according to (5), it depends on the low-energy scale μ where perturbation theory breaks down. These three parameters all have well-defined scaling behavior with the heavy-quark mass; namely, N is independent of m_Q, r scales like $1/m_Q$, and C_3 scales like $\alpha_s(m_Q)^{9/(33-2n_f)}$. Thus, if the parameters are determined phenomenologically from data on charm fragmentation into D and D^* mesons, then the corresponding parameters for the B and B^* mesons can be determined by scaling. The problem with this model is that unless $C_3 = 1$, either $D_{Q\to P}(z)$ or $D_{Q\to V}(z)$ becomes negative for z near 1. This unphysical behavior only arises in a region of z where the $1/m_Q$ expansion is breaking down, but it makes these fragmentation functions less attractive as a phenomenological model. If we choose $C_3 = 1$ to avoid these difficulties, we might as well avoid the $1/m_Q$ expansion altogether and use the complete PQCD fragmentation functions as our model. We therefore propose as a model of heavy-quark fragmentation the PQCD fragmentation functions calculated in Ref. [6]:

$$D_{Q\to P}(z) = N\frac{rz(1-z)^2}{[1-(1-r)z]^6}[6 - 18(1-2r)z + (21 - 74r + 68r^2)z^2 \\ -2(1-r)(6 - 19r + 18r^2)z^3 + 3(1-r)^2(1 - 2r + 2r^2)z^4] \,, \tag{31}$$

$$D_{Q\to V}(z) = 3N\frac{rz(1-z)^2}{[1-(1-r)z]^6}[2 - 2(3-2r)z + 3(3 - 2r + 4r^2)z^2 \\ -2(1-r)(4 - r + 2r^2)z^3 + (1-r)^2(3 - 2r + 2r^2)z^4] \,. \tag{32}$$

The only parameters are the normalization N, which is independent of m_Q, and r, which scales like $1/m_Q$. The parameter r, which is the PQCD calculation has the value $m_q/(m_Q + m_q)$, can be interpreted as the ratio of the constituent mass of the light quark to the mass of the meson. Integrating over z, we obtain the total fragmentation probabilities:

$$\int_0^1 dz\, D_{Q\to P}(z) = 3N\left(\frac{8 + 13r + 228r^2 - 212r^3 + 53r^4}{15(1-r)^5} + \frac{r(1 + 8r + r^2 - 6r^3 + 2r^4)\ln(r)}{(1-r)^6}\right) \,, \tag{33}$$

$$\int_0^1 dz\, D_{Q\to V}(z) = 3N\left(\frac{24+109r-126r^2+174r^3+89r^4}{15(1-r)^5} + \frac{r(7-4r+3r^2+10r^3+2r^4)\ln(r)}{(1-r)^6}\right) . \tag{34}$$

The PQCD fragmentation functions (31) and (32) give the distributions in the longitudinal momentum fraction z for the mesons P and V in a heavy-quark jet. This model can be easily extended to give the distribution in their transverse momentum k_T relative to the jet momentum [21]. In Ref. [6], the fragmentation functions were obtained as integrals over the invariant mass s of the fragmenting heavy quark:

$$D_{Q\to P/V}(z) = \int_{s_{\min}(z)}^{\infty} \frac{ds}{s} d_{Q\to P/V}(z,s) , \tag{35}$$

where the lower limit of the integration is

$$s_{\min}(z) = \frac{M^2}{z} + \frac{r^2M^2}{1-z} . \tag{36}$$

The functions $d_{Q\to P/V}(z,s)$ in the integrand are given by

$$\begin{aligned} d_{Q\to P}(z,s) = 6NM^2rs\Bigg[&\frac{(1-z)(1+rz)^2}{[1-(1-r)z]^2[s-(1-r)^2M^2]^2} \\ &-\frac{[2(1-2r)-(3-4r+4r^2)z+(1-r)(1-2r)z^2]M^2}{[1-(1-r)z][s-(1-r)^2M^2]^3} - \frac{4r(1-r)M^4}{[s-(1-r)^2M^2]^4}\Bigg] , \end{aligned} \tag{37}$$

$$\begin{aligned} d_{Q\to V}(z,s) = 6NM^2rs\Bigg[&\frac{(1-z)[1+2rz+(2+r^2)z^2]}{[1-(1-r)z]^2[s-(1-r)^2M^2]^2} \\ &-\frac{[2(1+2r)-(1+12r-4r^2)z-(1-r)(1+2r)z^2]M^2}{[1-(1-r)z][s-(1-r)^2M^2]^3} - \frac{12r(1-r)M^4}{[s-(1-r)^2M^2]^4}\Bigg] . \end{aligned} \tag{38}$$

The invariant mass s is related to k_T and z by

$$s = \frac{M^2+k_T^2}{z} + \frac{m_q^2+k_T^2}{1-z} , \tag{39}$$

where $M = m_Q + m_q$ in the nonrelativistic limit. If, instead of integrating over s, we integrate over z with k_T^2 held fixed, we obtained the k_T distribution for the fragmentation process. Introducing the dimensionless variable $t = k_T/M$, we can define the k_T-dependent functions $d_{Q\to P/V}(z,t)$ and $D_{Q\to P/V}(t)$ by

$$\int_0^\infty dt\, D_{Q\to P/V}(t) = \int_0^\infty dt \int_0^1 dz\, d_{Q\to P/V}(z,t) \tag{40}$$

$$= \int_0^1 dz \int_{s_{\min}(z)}^\infty \frac{ds}{s} d_{Q\to P/V}(z,s) . \tag{41}$$

This implies

$$D_{Q\to P/V}(t) = 2M^2t\int_0^1 dz \frac{1}{z(1-z)s(z,t)} d_{Q\to P/V}(z,s(z,t)) , \tag{42}$$

with

$$s(z,t) = M^2\left(\frac{1+t^2}{z} + \frac{r^2+t^2}{1-z}\right) .$$

Carrying out the integrals over z, we find

$$\begin{aligned} D_{Q\to P}(t) = \frac{Nr}{2(1-r)^6}\frac{1}{t^6}\Bigg\{&-24rt[4r^2-(2+r+2r^2)t^2]\ln(r) \\ &-(1-r)t[30r^3-r(61-20r+28r^2)t^2-(3-48r+48r^2-12r^3)t^4] \\ &+12t[4r^3-r(2+r+2r^2)t^2+(1-r)^2t^6]\ln\left(\frac{r^2+t^2}{1+t^2}\right) \\ &+3[10r^4-3r^2(11+2r+2r^2)t^2+(3+4r+19r^2-6r^3)t^4 \\ &+(3+12r-20r^2+8r^3)t^6]\arctan\left(\frac{(1-r)t}{r+t^2}\right)\Bigg\} , \end{aligned} \tag{43}$$

$$D_{Q\to V}(t) = \frac{3Nr}{2(1-r)^6}\frac{1}{t^6}\Big\{-8rt[12r^2-(6+7r+2r^2)t^2]\ln(r)$$
$$-(1-r)t[30r^3-r(61+28r-20r^2)t^2+(5-8r+8r^2+4r^3)t^4]$$
$$+4t[12r^3-r(6+7r+2r^2)t^2+(1-r)^2t^6]\ln\left(\frac{r^2+t^2}{1+t^2}\right)$$
$$+[30r^4-3r^2(33+22r-10r^2)t^2+(9+20r+r^2+22r^3+8r^4)t^4$$
$$+(9-12r+4r^2+8r^3)t^6]\arctan\left(\frac{(1-r)t}{r+t^2}\right)\Big\}\,. \tag{44}$$

In general, fragmentation functions $D(z,\mu^2)$ depend not only on z, but also on a factorization scale μ. In a high-energy process that produces a jet with transverse momentum p_T, the scale μ should be chosen to be on the order of p_T. The functions (31) and (32) should be regarded as models for heavy-quark fragmentation functions at a scale μ of order m_Q. For values of μ much larger than m_Q, the fragmentation functions (31) and (32) should be evolved from the scale m_Q to the scale μ using the Altarelli-Parisi equation,

$$\mu^2\frac{\partial}{\partial\mu^2}D_{Q\to H}(z,\mu^2) = \int_z^1\frac{dy}{y}P_{Q\to Q}\left(\frac{z}{y},\mu\right)\times D_{Q\to H}(y,\mu^2)\,, \tag{45}$$

where $P_{Q\to Q}(x)$ is the appropriate splitting function:

$$P_{Q\to Q}(x,\mu) = \frac{2\alpha_s(\mu)}{3\pi}\left(\frac{1+x^2}{1-x}\right)_+\,. \tag{46}$$

One aspect of the initial conditions (31) and (32) and the evolution equation (45) that may cause problems in practical applications is that they do not respect the phase space constraint:

$$D_{Q\to H}(z,\mu^2) = 0 \quad \text{for } z < M^2/\mu^2\,. \tag{47}$$

This can be remedied [9] by using (47) as the initial condition on the fragmentation function equation and replacing (45) by the inhomogeneous evolution equation

$$\mu^2\frac{\partial}{\partial\mu^2}D_{Q\to H}(z,\mu^2) = \int_z^1\frac{dy}{y}P_{Q\to Q}\left(\frac{z}{y},\mu\right)\times D_{Q\to H}(y,y\mu^2) + d_{Q\to H}(z,\mu^2)\theta(\mu^2-s_{\min}(z))\,, \tag{48}$$

where $d_{Q\to H}(z,s)$ is defined by the integrand in (35) and $s_{\min}(z)$ is given in (36).

IV. COMPARISON WITH OTHER FRAGMENTATION MODELS

The model for heavy-quark fragmentation which has been used most extensively in phenomenological applications is the Peterson fragmentation function [10]

$$D_{Q\to H}(z) = N_H\frac{z(1-z)^2}{[(1-z)^2+\epsilon_H z]^2}\,, \tag{49}$$

where N_H and ϵ_H are adjustable parameters that may depend on the hadron H. This fragmentation function has the correct behavior in the heavy-quark limit if N_H scales like $1/m_Q$ and ϵ_H scales like $1/m_Q^2$. Identifying ϵ_H with r^2 and expressing (49) in terms of the Jaffe-Randall scaling variable y defined in (1), we find that it reduces in the limit $r\to 0$ to

$$D_{Q\to H}(z) \to \frac{N_H}{r^2}\frac{(y-1)^2}{[(y-1)^2+1]^2}\,. \tag{50}$$

The Peterson fragmentation function is just the square of a light-cone energy denominator multiplied by a phase space factor. It contains no spin information; the normalization parameter N_H is to be determined independently for the pseudoscalar and vector mesons of a heavy-quark spin multiplet.

An alternative fragmentation model which does contain spin information has been proposed by Suzuki [11]. Suzuki's fragmentation functions are derived from the same Feynman diagram in Fig. 1 as the PQCD fragmentation functions, but with two essential differences. First, the diagram was calculated in the Feynman gauge. If a general covariant gauge had been used, Suzuki's fragmentation functions would have depended on the gauge parameter. The PQCD fragmentation functions that we calculated are gauge invariant. We calculated the diagram in the axial gauge only for simplicity. If we had used a covariant gauge, we would have had to also include diagrams in which both the virtual heavy quark and the virtual gluon are emitted by the source Γ in Fig. 1. Alternatively, we could have calculated the PQCD fragmentation functions for the fragmentation of a heavy quark into S-wave heavy quarkonium directly from the general gauge-invariant definition [12]. Such a calculation has been carried out for the equal-mass case of charmonium by Ma [13]. A second essential difference between the PQCD model and Suzuki's is that we integrated over the invariant mass s of the fragmenting quark [see Eq. (9)]. The invariant mass is related to the transverse momentum k_T of the meson relative to the fragmenting quark by (39). Rather than integrating over k_T^2, Suzuki chose to evaluate the integrand at a typical value $\langle k_T^2\rangle$. Suzuki's model therefore has three parameters: the overall normalization N, the mass ratio r, and $\langle k_T^2\rangle/m_Q^2$. When expressed in terms of the scaling variable y defined in (1), Suzuki's fragmentation function $D_{Q\to P}(z)$ reduces in the limit $r\to 0$ to

$$D_{Q\to P}(z) \to \frac{N}{r}(y-1)^2\frac{(y-2)^2+\kappa^2}{[y^2+\kappa^2]^4}\,, \tag{51}$$

where $\kappa^2 = \langle k_T^2\rangle/(r^2 m_Q^2)$. By heavy-quark spin symmetry, $D_{Q\to V}(z)$ differs, in this limit, only by a factor of 3.

The Peterson, Suzuki, and PQCD fragmentation functions all vanish like $(1-z)^2$ as $z\to 1$. An alternative fragmentation function which vanishes like the first power of $(1-z)$ has been proposed by Collins and Spillers [14]. This was motivated by incorrect dimensional counting rules. The correct dimensional counting rules for QCD [15] do in fact give a limiting behavior of $(1-z)^2$ for the fragmentation function. The Collins-Spillers fragmentation function can be derived in a similar way to ours, except that in the Feynman diagram in Fig. 1, the interaction mediated by the virtual gluon is replaced by a pointlike scalar Yukawa coupling between the meson, heavy quark, and light quark. Consequently, the denominator of the matrix element contains only one power of $(s-m_Q^2)$, in contrast to the two powers in (12). It is the omission of the gluon propagator that changes the behavior as $z\to 1$ from $(1-z)^2$ to $(1-z)$. Also, instead of integrating over the invariant mass of the fragmenting quark as in (9), Collins and Spillers, like Suzuki, evaluated the integral at a typical value $\langle k_T^2\rangle$. Taking the scaling limit $r\to 0$, the fragmentation function of Collins and Spillers reduces to

$$D_{Q\to P}(z) \to \frac{2N}{r}(y-1)\frac{(y-2)^2+\kappa^2}{[y^2+\kappa^2]^2}\ , \qquad (52)$$

where $\kappa^2 = \langle k_T^2\rangle/(r^2 m_Q^2)$.

The various fragmentation models in the literature have been summarized in Ref. [16] and compared with experimental data on D and D^* production. The string and parton cluster models are very different in spirit from those discussed above. One can derive analytic expressions for the heavy-quark fragmentation functions from the string models [17]. They contain a tunneling factor $\exp(-Bm_H^2/z)$, which suppresses the small-z region. In the scaling limit, the Lund symmetric fragmentation function behaves like

$$D_{Q\to H}(z) \to N r^{\beta} e^{-B(m_H^2+\langle k_T^2\rangle)}(y-1)^{\beta}\ . \qquad (53)$$

Unless N scales like $e^{Bm_Q^2} m_Q^{\beta+1}$, this is inconsistent with heavy-quark symmetry, which requires the leading term to scale like m_Q as $m_Q\to\infty$.

The PQCD model for heavy-quark fragmentation has a number of advantages over those described above. First, it is rigorously correct in the limit $m_q \gg \Lambda_{\rm QCD}$. Higher-order perturbative corrections can be systematically calculated. Relativistic corrections can also be calculated in terms of additional nonperturbative matrix elements [18]. Second, our model is consistent with heavy-quark symmetry in the limit $m_Q\to\infty$. The logarithms of m_Q that are predicted by HQET would be reproduced by the higher-order perturbative corrections. The PQCD model is also more predictive than those in Refs. [10,11,14]. It describes spin-dependent effects, like Suzuki's model, but without introducing any additional parameters. The PQCD model not only predicts the z dependence of the fragmentation functions, but also their dependence on k_T, the transverse momentum of the meson relative to the jet. The fragmentation functions (31) and (32) apply only to S-wave mesons, but the fragmentation functions for higher-orbital-angular-momentum states can also be calculated. The PQCD fragmentation functions for the P-wave mesons have been calculated to leading order in α_s in Ref. [19].

V. VECTOR-TO-PSEUDOSCALAR RATIO

In any production process for heavy-light mesons, one of the most fundamental experimental observables is the ratio

$$P_V = \frac{V}{V+P}\ , \qquad (54)$$

which measures the relative number of vector mesons V and pseudoscalar mesons P that are produced. If the mesons are produced within a heavy-quark jet, then V and P in (54) can be identified as the fragmentation probabilities for the heavy quark to fragment into vector and pseudoscalar mesons, respectively. The ratio P_V can depend on kinematic variables, such as the longitudinal momentum fraction z of the meson or its transverse momentum k_T relative to the jet. In the PQCD model for fragmentation, the normalization factor N cancels out in the ratio (54), so that P_V is determined by the parameter r only.

The simplest measure of the ratio P_V comes from the total numbers of vector and pseudoscalar mesons in the jet integrated over z and k_T. Setting P and V in (54) to the fragmentation probabilities in (33) and (34), we find that the ratio P_V in the PQCD model of fragmentation is

$$P_V = \frac{(1-r)(24+109r-126r^2+174r^3+89r^4)+15r(7-4r+3r^2+10r^3+2r^4)\ln(r)}{2(1-r)(16+61r+51r^2-19r^3+71r^4)+60r(2+r+r^2+r^3+r^4)\ln(r)} \qquad (55)$$

This ratio is plotted as a function of r in Fig. 3. From the graph, it is clear that P_V is not strongly dependent on r. At $r=0$, $P_V=\frac{3}{4}$ as required by heavy-quark spin symmetry. As r increases, P_V decreases slowly to $P_V=0.51$ at $r=0.5$. Thus, at nonzero values of r, the vector state is less populated than would be given by naive spin counting. We can determine the value of r for the D and D^* system using experimental measurements of P_V. A complete compilation of the experimental data for P_V from the CERN e^+e^- collider LEP, CLEO, ARGUS, the DESY e^+e^- collider PETRA, and KEK TRISTAN can be found in Ref. [20]. The key point in obtaining consistency between these measurements is using the updated branching ratio $B(D^{+*}\to D^0\pi^+)\simeq 0.68$ instead of the

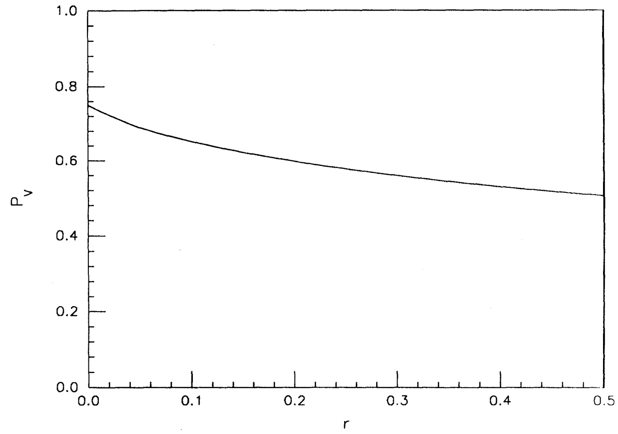

FIG. 3. Ratio P_V as a function of r.

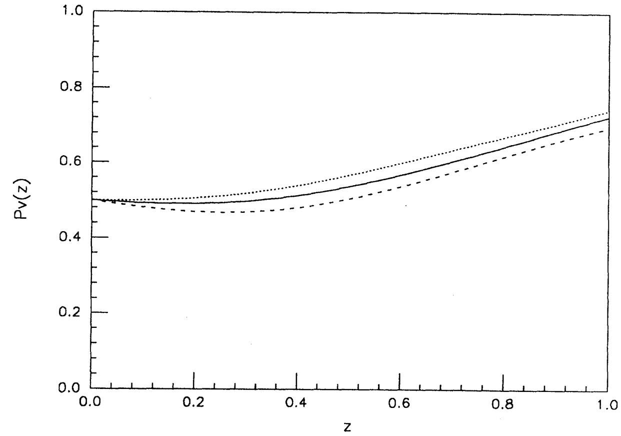

FIG. 4. Predictions for the ratio $P_V(z)$ as a function of z for $r = 0.10$ (solid curve), $r = 0.03$ (dotted curve), and $r = 0.22$ (dashed curve).

old value 0.55. The experimental value $P_V = 0.65 \pm 0.06$ determines the parameter r_D for the D-D^* system to be $r_D = 0.10^{+0.12}_{-0.07}$. If we interpret r as the ratio of the constituent mass of the light quark to the mass of the meson, then the value $r_D = 0.10$ corresponds to a constituent mass of 200 MeV. Given a value of r_D, we can determine the corresponding value for the B-B^* system by using the simple scaling behavior $r_B = (m_D/m_B)r_D$. This gives $r_B = 0.03^{+0.04}_{-0.02}$. Our determination of the parameters r_D and r_B is rather crude, with uncertainties of nearly 100%. Surprisingly, in spite of such a crude determination of r, the PQCD model still gives useful quantitative predictions for heavy-quark fragmentation.

Having determined the parameter r from data on D-D^* production, we can now predict how the vector-to-pseudoscalar ratio should vary as a function of the longitudinal momentum fraction z. The z-dependent ratio $P_V(z)$ is defined by (54), where P and V are given by the fragmentation functions (31) and (32):

$$P_V(z) = \frac{3}{4}\frac{n(z)}{d(z)} , \tag{56}$$

with

$$\begin{aligned} n(z) = {} & 2 - 2(3-2r)z + 3(3-2r+4r^2)z^2 \\ & -2(1-r)(4-r+2r^2)z^3 + (1-r)^2 \\ & \times(3-2r+2r^2)z^4 , \end{aligned} \tag{57}$$

$$\begin{aligned} d(z) = {} & 3 - 3(3-4r)z + (12-23r+26r^2)z^2 \\ & -(1-r)(9-11r+12r^2)z^3 + 3(1-r)^2 \\ & \times(1-r+r^2)z^4 . \end{aligned} \tag{58}$$

This ratio is plotted as a function of z in Fig. 4 for the values $r = 0.10$ (solid curve), $r = 0.03$ (dotted curve), and $r = 0.22$ (dashed curve). At $z = 0$, $P_V(z) = \frac{1}{2}$, regardless of the value of r. It decreases slightly for small z and then increases monotonically to a maximum value, at $z = 1$, of 0.74 for $r = 0.03$, 0.73 for $r = 0.10$, and 0.70 for $r = 0.22$. Note that, in spite of the nearly 100% uncertainty in our determination of r_D, the uncertainty in $P_V(z)$ is less than about 11%. Thus the PQCD model gives a rather unambiguous prediction that P_V should vary from around $\frac{1}{2}$ at small values of z to almost $\frac{3}{4}$ near $z = 1$.

The k_T-dependent ratio $P_V(k_T)$ is defined by (54), with P and V given by (43) and (44):

$$P_V(k_T) = \frac{3}{4}\frac{n_1 + n_2\ln(r) + n_3\ln\left(\frac{r^2+t^2}{1+t^2}\right) + n_4\arctan\left(\frac{(1-r)t}{r+t^2}\right)}{d_1 + d_2\ln(r) + d_3\ln\left(\frac{r^2+t^2}{1+t^2}\right) + d_4\arctan\left(\frac{(1-r)t}{r+t^2}\right)} , \tag{59}$$

where

$$\begin{aligned} n_1 = {} & -(1-r)t[30r^3 - r(61+28r-20r^2)t^2 \\ & +(5-8r+8r^2+4r^3)t^4] , \end{aligned} \tag{60}$$

$$n_2 = -8rt[12r^2 - (6+7r+2r^2)t^2] , \tag{61}$$

$$n_3 = 4t[12r^3 - r(6+7r+2r^2)t^2 + (1-r)^2t^6] , \tag{62}$$

$$\begin{aligned} n_4 = {} & [30r^4 - 3r^2(33+22r-10r^2)t^2 \\ & +(9+20r+r^2+22r^3+8r^4)t^4 \\ & +(9-12r+4r^2+8r^3)t^6] , \end{aligned} \tag{63}$$

and

$$\begin{aligned} d_1 = {} & -(1-r)t[30r^3 - r(61+16r-8r^2)t^2 \\ & +3(1+2r-2r^2+2r^3)t^4] , \end{aligned} \tag{64}$$

$$d_2 = -12rt[8r^2 - 2(2 + 2r + r^2)t^2] \ , \qquad (65)$$

$$d_3 = 6t[8r^3 - 2r(2 + 2r + r^2)t^2 + (1 - r)^2t^6] \ , \qquad (66)$$

$$d_4 = 30r^4 - 9r^2(11 + 6r - 2r^2)t^2 + 3(1 + r)^2(3 + 2r^2)t^4 + 3(3 - 4r^2 + 4r^3)t^6 \ . \qquad (67)$$

This ratio is plotted as a function of $t = k_T/M$ in Fig. 5 for the three values $r = 0.10$ 0.03, and 0.22. At $t = 0$, $P_V(t) = \frac{3}{4}$, regardless of the value of r. As t increases, $P_V(t)$ quickly decreases to its asymptotic value at $t = \infty$. At $t = 1$, $P_V(t)$ is within 0.1% of its asymptotic values of 0.65 for $r = 0.03$, 0.61 for $r = 0.10$, and 0.60 for $r = 0.22$. Again, we find that, in spite of the large uncertainty in r, we obtain a rather precise prediction for P_V as a function of k_T.

The PQCD fragmentation functions for vector mesons have been applied previously [21] as a phenomenological model to describe the fragmentation processes $c \to D^*$ and $b \to B^*$. The fragmentation functions were separated into the transverse and longitudinal polarization components. The spin alignment, which measures the ratio of transverse to longitudinal polarizations, was calculated as a function of z and k_T. In the case of production of D^* by charm fragmentation, the spin alignment predicted by the PQCD fragmentation model was shown to be consistent with CLEO measurements [21]. In addition, the predicted value of the average longitudinal momentum fraction $\langle z \rangle$ for $c \to D^*$ and for $b \to B^*$ was shown to be in excellent agreement with data from LEP, CLEO, and ARGUS [21]. The values of r used for D^* and B^* mesons in these comparisons were $r = 0.17$ and 0.058, respectively, which lie within the range determined above from measurements of P_V.

The PQCD fragmentation functions have also been applied in Ref. [22] to predict the fragmentation spectra for the B_s and B_s^* mesons based on the production rates of the B_s mesons measured at LEP. Instead of treating the normalization N as a phenomenological parameter as advocated in this paper, the authors calculated N using the PQCD expression, which involves α_s at the scale of the strange quark mass.

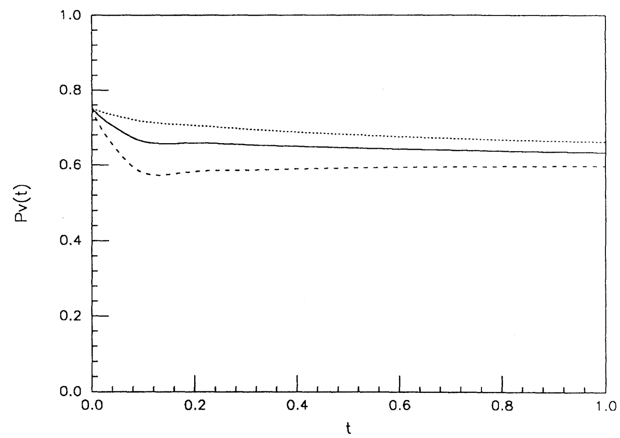

FIG. 5. Predictions for the ratio $P_V(k_T)$ as a function of k_T for $r = 0.10$ (solid curve), $r = 0.03$ (dotted curve), and $r = 0.22$ (dashed curve).

VI. SUMMARY

We have studied the heavy-quark mass limit of the PQCD fragmentation functions for producing S-wave mesons. The leading and next-to-leading terms in $1/m_Q$ were calculated directly from HQET. The PQCD fragmentation functions were proposed as a phenomenological model for fragmentation into heavy-light mesons. With only two parameters, this model describes fragmentation into the $^1S_0$ pseudoscalar meson state and the transverse and longitudinal polarization states of the $^3S_1$ vector meson. It describes not only the z dependence of the fragmentation probabilities, but also their dependence on the transverse momentum k_T of the meson relative to the jet within which it is produced. This model can easily be extended to describe heavy-quark fragmentation into P-wave states using the PQCD fragmentation functions calculated in [19]. The PQCD fragmentation functions were compared with other models for heavy-quark fragmentation in the literature. As an application, the PQCD fragmentation functions were used to predict the ratio of vector-to-pseudoscalar states as a function of z and k_T. The ratio P_V is predicted to vary from around $\frac{1}{2}$ at small values of z to almost $\frac{3}{4}$ near $z = 1$.

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[1] See, for example, B. Grinstein, Annu. Rev. Nucl. Part. Sci. **42**, 101 (1992); M. Neubert, Phys. Rep. **245**, 259 (1994).
[2] R. Jaffe and L. Randall, Nucl. Phys. **B412**, 79 (1994).
[3] E. Braaten and T. C. Yuan, Phys. Rev. Lett. **71**, 1673 (1993).
[4] C.-H. Chang and Y.-Q. Chen, Phys. Lett. B **284**, 127 (1992); Phys. Rev. D **46**, 3845 (1992).
[5] E. Braaten, K. Cheung, and T. C. Yuan, Phys. Rev. D **48**, 4230 (1993).
[6] E. Braaten, K. Cheung, and T. C. Yuan, Phys. Rev. D **48**, R5049 (1993).
[7] K. Cheung, Phys. Rev. Lett. **71**, 3413 (1993).
[8] K. Cheung and T. C. Yuan, Phys. Lett. B **325**, 481

(1994).
[9] E. Braaten, M. A. Doncheski, S. Fleming, and M. L. Mangano, Phys. Lett. B **333**, 548 (1994).
[10] C. Peterson, D. Schlatter, I. Schmidt, and P. Zerwas, Phys. Rev. D **27**, 105 (1983).
[11] M. Suzuki, Phys. Rev. D **33**, 676 (1986).
[12] G. Curci, W. Furmanski, and R. Petronzio, Nucl. Phys. **B175**, 27 (1980); J. C. Collins and D. E. Soper, *ibid.* **B194**, 445 (1982); **B193**, 381 (1981).
[13] J. P. Ma, Phys. Lett. B **332**, 398 (1994).
[14] P. D. B. Collins and T. P. Spiller, J. Phys. G **11**, 1289 (1985).
[15] F. Ezawa, Nuovo Cimento **23A**, 271 (1974); G. R. Farrar and J. D. Jackson, Phys. Rev. Lett. **35**, 1416 (1975); G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
[16] D. Bortoletto *et al.*, Phys. Rev. D **37**, 1719 (1988).
[17] B. Andersson *et al.*, Phys. Rep. **97**, 33 (1983); M. G. Bowler, Z. Phys. C **11**, 169 (1981); **22**, 155 (1984).
[18] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995).
[19] Y.-Q. Chen, Phys. Rev. D **48**, 5158 (1993); T. C. Yuan, *ibid.* **50**, 5664 (1994).
[20] A. F. Falk and M. E. Peskin, Phys. Rev. D **49**, 3320 (1994).
[21] K. Cheung and T. C. Yuan, Phys. Rev. D **50**, 3181 (1994).
[22] K. Cheung and R. J. Oakes, Phys. Lett. B **333**, 181 (1994).