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A dependence of hadron production in inelastic muon scattering and dimuon production by protons

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The A dependence of the production of hadrons in inelastic muon scattering and of the production of dimuons in high Q^2 proton interactions are simply related. Feynman x distributions and z scaling distributions in nuclei are compared with energy loss models. Suggestions for new data analyses are presented.

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I. INTRODUCTION

We start with remarks on the relative role of "soft" and "hard" interactions in nuclear collisions at very high energy: When a projectile proton strikes another proton at rest it imparts energy to the struck nucleon and therefore must lose energy. Thus energy loss must affect the cross sections for protons striking additional nucleons embedded in nuclei.

Consider first a free p-p collision: As the result of the collision, the proton is excited to a spectrum of states of differing invariant mass. The subsequent deexcitation of these states into pions, kaons, etc., shows up in the minimum bias measurements of the rapidity, multiplicity, and p_t distribution of these final on-shell products. These experimental p-p data are summarized in the well-known ISAJET minimum bias codes. They reveal the properties of the "soft" nonperturbative collisions that dominate these hadronic interactions. From this ISAJET code one can determine the invariant mass spectra and the energy loss distribution as a function of bombarding energy and hence their mean values. Such plots appeared in our earlier work on minimum bias interactions in nuclei [1,2]. To apply these p-p results to p-A reactions, i.e., to determine the effects of multiple scatters of this type as the proton traverses a nucleus, required some further but plausible assumptions. The assumptions we made in the case of minimum bias interactions in nuclei was that the cross section of the excited projectile nucleon in subsequent nucleon collisions in the path through the nucleus was close to the minimum-bias p-p measured cross section. It was also convenient to assume that the energy loss in each collision, as the nucleon energy was degraded in passing through the nucleus, followed the functional form of the energy loss experienced in a free p-p collision. With these two assumptions we found that we could reproduce the nuclear data for a variety of measurements in p-A and nucleus-nucleus collisions.

The finding that the nuclear data could be reproduced from the p-p data without invoking the underlying quark-gluon structure was fortunate since perturbative QCD is not useful in predicting the cross sections for soft collisions. There are however many codes that use interesting models incorporating quarks and gluons to understand the nuclear data.

As we have previously demonstrated it is also possible to use the same approach to study the effect of soft collisions on reactions that involve high Q^2 interactions. In particular we have previously shown [3,4] that the details of the rich data on Feynman x distributions in J/ψ production in nuclei can be accounted for by the same decoupling of hard and soft processes. Recall that when hard processes are involved, as in the cases examined in this paper, the hard process involves perturbative QCD and the elementary muon-proton cross section or the $p + p \to J/\psi + X$ cross section come directly from experiment. They can be reproduced using QCD and a knowledge of the measured empirical nucleon structure functions.

How these cross sections for multipion production by muons or dimuon production by protons are modified by the soft energy loss processes in nuclei is the subject of this paper. Figures 1–3 should be helpful in understand-

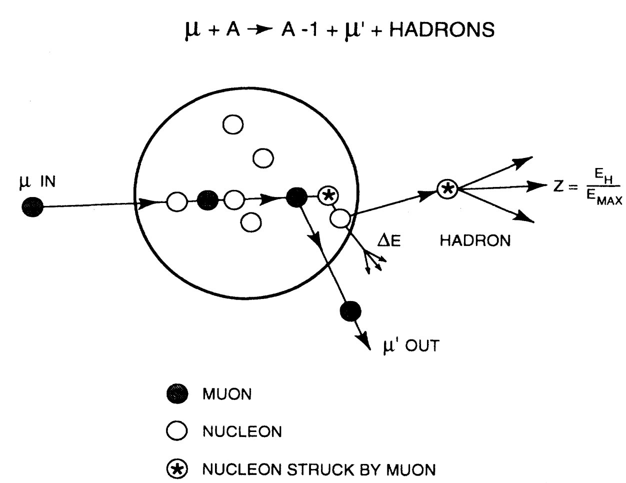

FIG. 1. Hadron production by muons: An incident muon interacts with a nucleon which rescatters off another nucleon in the nucleus. The nucleon struck by the muon hadronizes outside the nucleus.

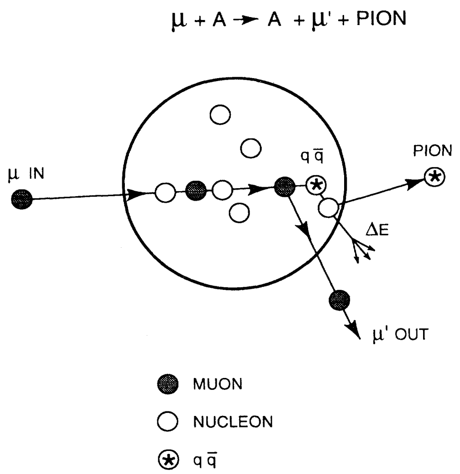

FIG. 2. Pion production by muons: An incident muon interacts with a nucleon producing an off-shell pion which rescatters off nucleons in the nucleus.

ing the model and the important role that both the time evolution to on-shell states and time dilation play in the nuclear process.

Consider first the case of hadron production by muons where the muon, because of the small cross section, may strike a proton randomly along its path, making a high-energy scatter. The struck excited nucleon recoils and strikes another nucleon in its path losing an energy δE to it in a soft collision. (The spectator nucleon struck in this soft process eventually hadronizes and produces particles mainly at negative rapidity in the cm frame. They are not detected in the geometry of the experiments we analyze.)

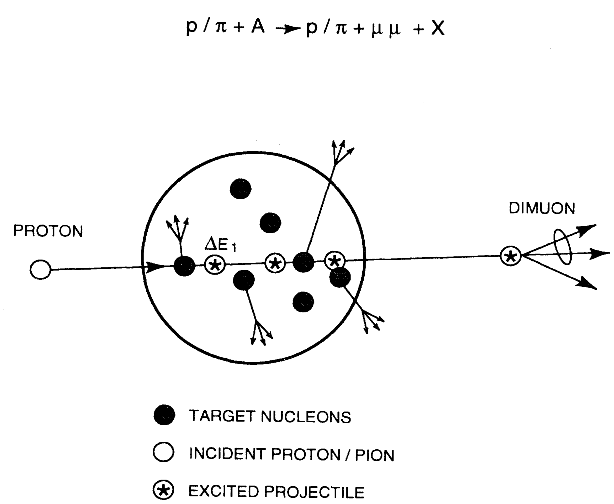

FIG. 3. Dimuon production by hadrons: An incident proton makes soft nonperturbative scatters before making a high Q^2 collision with another nucleon producing an excited proton which hadronizes outside the nucleus.

The nucleon struck by the muon eventually hadronizes into the typical exponential pion spectrum found in the present data and which we have verified is reproduced by ISAJET. (Since the nuclear effects are small, the nuclear pion spectra show approximately the same exponential dependence.) This energy loss by the excited nucleon reduces the maximum energy available to the hadrons (mainly pions) it produces and will thus affect the value of $z = E_h/E_{\max}$, shifting nuclear yields to lower z. Because the muons are weakly interacting, they only interact at their full energy with a single target nucleon.

It is important to understand that, as shown in Fig. 1, the hadronization can take place outside the nucleus because of the time dilation and after the struck nucleon has slowed down because of its soft collisions with other target nucleons. That is the model used in our previous stopping power, minimum bias, and J/ψ studies.

To show the sensitivity to the time dilation one can also carry out a similar calculation to see what the cross section would be if the q-$\bar{q}$ pairs resulting from the radiation of the struck but confined quark appeared immediately at the surface of the struck nucleon and passed through the remainder of the nucleus. Therefore we have also calculated the nuclear cross section in this scenario. To do this we must know the pair-nucleon cross section and the energy loss in such a collision. Once again we take the on-shell cross section and on-shell energy loss as our parameters, using the measured pion-nucleon cross section and the same energy loss parametrization as used in the previous model. We actually take the pion spectrum from the deuterium data since the spectrum $R(z)$ is not exactly exponential. This reaction is illustrated in Fig. 2.

(Because this is a complicated subject it is worthwhile to make some explanatory remarks: It is sometimes said that in a high-energy collision the quarks and gluons move independently through the nucleus and that one can talk of a quark-nucleus cross section. However, colored constituents must also conform with our views about confinement. A colored constituent in a high Q^2 scatter does move independently inside the nucleon but only until it reaches the edge of the nucleon where the strong confining forces, produced by all the nucleon constituents, prevent its escape. One can think of the quark at the "bag edge" dragging the other constituents with it. Because of the time dilation the soft subsequent rescatters from the spectator nucleons in the nucleus take place before the final hadronization into on-shell pions. Even if the struck quark radiated q-$\bar{q}$ pairs at $t = 0$ they would pass through the nuclear medium changing over to on-shell pions over time. This poorly understood time evolution also enters in what is called "nuclear transparency.")

In a parallel study, also involving muons, but now in the final state, we examine the production of dimuons by protons: The incoming proton interacts with spectator nucleons and makes soft minimum bias collisions before making the high Q^2 dimuon pair. Thus the incoming proton's energy can be decreased, thereby reducing the energy available for producing the rare dimuon pair. This shifts events to lower Feynman x. $x =$

$E(\text{dimuon})/E_{\max}(\text{dimuon})$. (The evolution for this reaction is shown in Fig. 3.)

In muon production of hadrons the A dependence is often revealed by examining $R(z) = N_A(z)/N_{\rm D}(z)$ while in dimuon production by protons the quantity is $R(x) = N_A(x)/N_{\rm D}(x)$. N is the number of events at either x or z. The comparison is usually made with the deuteron D to include possible p-n differences.

A most interesting value of R is the limiting value at z or x equal to unity. In the hadron production case, if the muon interacts with the last nucleon in its path, there will be no energy loss. Only such events will appear at $z = 1$. In the dimuon case, the events at $x = 1$ arise from those interactions where the incident proton made the dimuon on the first nucleon in its path, the weakly interacting dimuon pair passing unscathed through the nucleus. Let a_n be the normalized, $\sum a_n = 1$, (Glauber) probability of making n collisions in a proton-nucleon minimum bias interaction in a nucleus. Then, na_n is the probability of making a rare high Q^2 collision such as a muon scatter or the production of a dimuon pair or vector meson such as a J/ψ, since there will be n chances to make the rare event on the n nucleons in the path. Since the probability of making one collision is then $na_n/n = a_n$ the fraction of times the collision takes place in a first or last collision is just $\sum a_n/\sum na_n = 1/\langle n\rangle$. Thus *these end points depend only on Glauber probabilities and not on the detailed mechanisms.* This general argument shows that R cannot be a constant, independent of z or x. It is not difficult to calculate these end-point values using a Woods-Saxon spatial distribution for the nucleons and the total inelastic cross section for the nucleon-nucleon scattering. That end point does not depend on the energy loss function or its magnitude.

Actually, we really need to know the inelastic cross section for an off-shell hadron on a ground state nucleon if we wish to calculate R at any other value than $R(1)$.

II. HADRON PRODUCTION IN INELASTIC MUON INTERACTIONS

Figure 4 shows a plot of $R(z)$ taken from the data of Ref. [5]. The authors have chosen to separate their data sample into a low-Q^2–low-$x_{\rm Bj}$ bin and a high-Q^2–high-$x_{\rm Bj}$ bin, where Q^2 is the four-momentum transfer to the nucleon and $x_{\rm Bj}$ is Bjorken x. As these authors have shown, and because it can be seen that the measured points are almost identical in the two samples, there is apparently no nuclear dependence in the data on these variables. As a result, and to improve the statistical accuracy of our comparison, we have suitably averaged the two results which are shown plotted in Fig. 5.

To examine the relative merits of fits of the data to the conclusion in Ref. [5] that there is no A dependence, we have examined the relative χ^2 for a linear fit hypothesis to the data shown in Fig. 5. For the fit to $R = 1$, the χ^2 for the five measured points is 7.2. It is 3.0 for the best linear energy loss fit shown in the figure. (Eliminating the low statistics high z point hardly changes the slope of the falloff but improves the χ^2 by 2 rather than 1 unit.) The

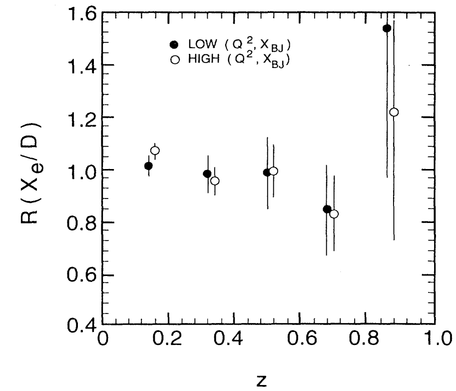

FIG. 4. The z dependence of the ratio of hadron production in xenon and deuterium, $R(\text{Xe/D})$, for high (open circles) and low (solid circles) Q^2 and $x_{\rm Bj}$. The data are taken from Ref. [1] Table XXXI.

authors of Ref. [5] remark that there are large systematic errors in the lowest z point. Without that point we find that the χ^2 is 0.53 for the linear fit and 3.2 for a flat fit with no A dependence, i.e., $R = 1$. Thus an A dependent effect seems to provide a better fit to the data, suggesting that there is a nuclear effect to be understood.

In several previous papers we have examined the effects of energy loss in nuclear interactions [3,4]. The calculation of $R(z)$ is simple if one knows the form of the energy loss per collision. Various *models* [6–8] have been proposed recently, giving different assumptions for the $\sqrt{s}$ dependence of the energy loss. We make use of the model we used in 1987 [1] since it was demonstrated to be in agreement with the data observed in low p_t production and was the functional form for the energy loss per nucleon obtained by examining the energy loss found in the ISAJET minimum bias model of Paige [9], widely used by particle physicists. That energy loss is approximately given by $d\sqrt{s}/dn$ =const= β, where n is the number of collisions. Thus in our model the energy loss varies as the square root of the laboratory energy, $\sqrt{E}$. This energy dependence lies between the values E^0 and E^1 of

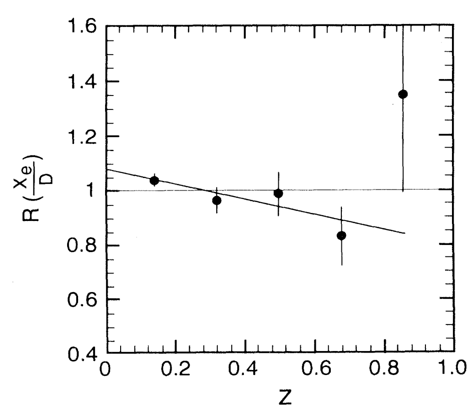

FIG. 5. R vs z for the combined sample of high and low Q^2 data. The solid curve is a least-square fit to a straight line fit to the data.

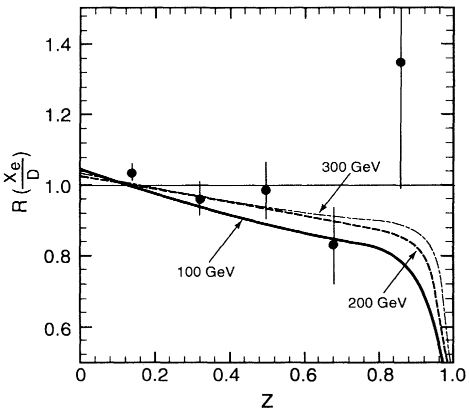

FIG. 6. Hadron energy loss: R vs z for the combined sample of high and low Q^2 data. Theoretical curves are shown for three values of the energy transfer, ν. The mean value of ν for the data is 170 GeV.

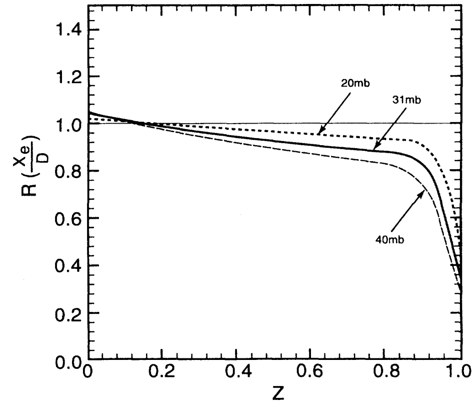

FIG. 8. Hadron energy loss: Calculations of R for different inelastic cross sections. The value of $d\sqrt{s}/dn = 0.2$ GeV. Note the different asymptotes.

Refs. [6] and [7], respectively.

We first consider the model of the struck proton hadronizing outside the nucleus into pions. In Fig. 6 we have plotted our results for several values of $\nu = E_\mu - E_{\mu'}$. After these calculations were made we obtained the actual ν distributions obtained in the experiment. That average value is $\nu = 170$ GeV which one can see from Fig. 6 would fall well on the experimental results. All the theoretical curves predict a rise above $R = 1$ at low z due to the sliding back of events to lower z due to energy loss. While this appears in the data, the authors of Ref. [1] caution that there are large systematic errors in the lowest z point. Note that the point at $z = 0.68$ falls well below $R = 1$. For this calculation we used $d\sqrt{s}/dn = 0.2$ GeV, which we had found in earlier work fit the available data on dimuon and J/ψ production [4]. The theoretical curves have been corrected to take into account the bin size used in the data. The $N(z)$ data for both Xe and D were fitted to a sum of two exponentials of the form $e^{-\alpha z}$ to get the best representation of the input to the energy loss calculation. To show the sensitivity of the calculations to both the inelastic cross section σ_{inel} and $d\sqrt{s}/dn$, we show in Figs. 7 and 8 how variation in these parameters affects the results. These plots also show the asymptotic limits. There is roughly a trade-off of $d\sqrt{s}/dn$ of 0.1 GeV for a change in the inelastic cross section of 10 mb.

We now turn to the second calculation, namely the model in which the $q\bar{q}$ pairs are formed at the collision and lose energy on the way out of the nucleus. Figure 9 shows our results for an inelastic pion-nucleon cross section of 20 mb and an energy loss $d\sqrt{s}/dn = 0.2$ for different values of the energy of the pion. The dependence on energy is small and all curves show a depression below $R = 1$. There is a good fit to the data if we again omit the lowest z point as suggested in Ref. [5]. Figure 10 shows the effect of varying the energy loss when $\nu = 170$

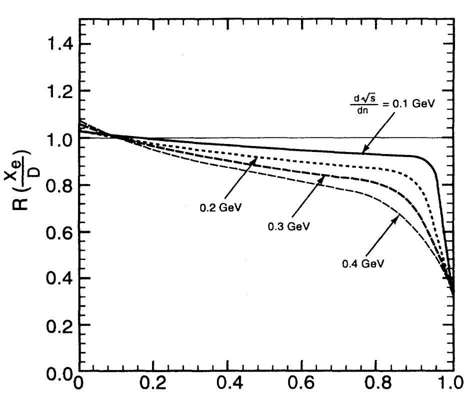

FIG. 7. Hadron energy loss: Theoretical calculations of R for different values of $d\sqrt{s}/dn$. The inelastic p-p cross section is set at 31 mb. The value of R at $z = 1$ is set by this cross section and the Woods-Saxon nucleon spatial distribution.

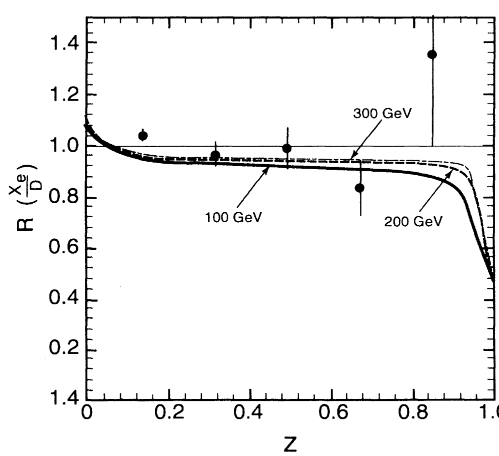

FIG. 9. "Pion energy loss": R vs z for the combined sample of high and low Q^2 data. Theoretical curves are shown for three values of the energy transfer, ν. The pion-nucleon cross section is taken as 20 mb. $d\sqrt{s}/dn = 0.2$. The mean value of ν for the data is 170 GeV.

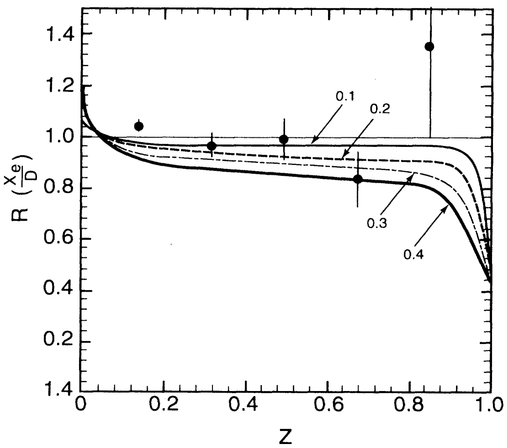

FIG. 10. "Pion" energy loss: Theoretical calculations of R for different values of $d\sqrt{s}/dn = 0.1$, 0.2, 0.3, and 0.4. The inelastic p-p cross section is set at 20 mb. The value of R at $z = 1$ is set by this cross section and the Woods-Saxon nucleon spatial distribution.

GeV and $\sigma_{\rm tot} = 20$ mb.

Figure 11 shows the effect of varying the cross section of the pion using the average value of ν of 170 GeV and the same energy loss parameter. Here we see that 20 mb appears to give the better fit than 10 or 31 mb. Once again we note different limits at $z = 1$.

Comparing Figs. 9–11 with Figs. 6–8, we note that in the case of the pion energy loss the slopes of R vs z are smaller and the main effect is a depression of R below unity.

We conclude that while the data are not very precise they do not rule out the presence of energy loss mechanisms, as suggested by Busza [11].

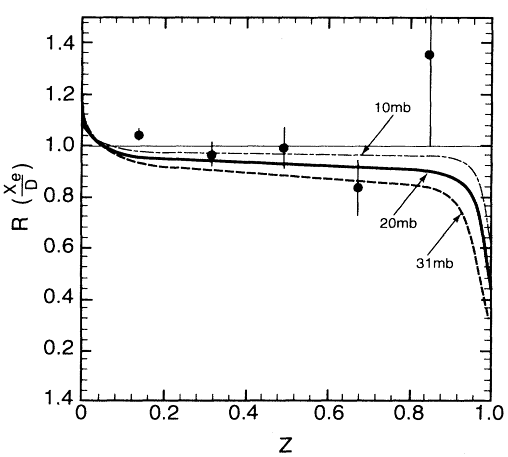

FIG. 11. "Pion" energy loss: Calculations of R for different inelastic cross sections, 10, 20, and 31 mb. The value of $d\sqrt{s}/dn = 0.2$ GeV, $\nu = 170$ GeV. Note the different asymptotes.

III. DIMUON PRODUCTION IN p-A COLLISIONS

Dimuon production in p-A collisions can be calculated on the same energy loss model, but is slightly more complicated since the dimuon production cross section is energy dependent and the shape of the cross section depends on the invariant mass of the dimuon pair.

The effect on $R(x_F) = \sigma_{h-A}(x_F)/A\sigma_{h-p}(x_F)$ of any loss in energy comes about because of the $\sqrt{s}$ dependence of the Drell-Yan cross sections which varies as $e^{\gamma M/\sqrt{s}}$ [12]. M is the invariant mass of the muon pair and $M/\sqrt{s}$ is the well known scaling variable [13]. The energy loss therefore produces two effects (a) a reduction in the dimuon yield and (b) a displacement of events to lower x_F. Before demonstrating the effects of this energy loss on $R(x_F)$ we present the result of a "back of the envelope" calculation of $R = \sigma_{p-A}/A\sigma_{p-p}$, which shows the qualitative features:

$$R = 1 - (\gamma)(M/\sqrt{s})(\beta/\sqrt{s})(\langle n\rangle - 1). \qquad (1)$$

This formula demonstrates how the various parameters enter into the dimuon "depression." [Empirically the last set of parentheses in Eq. (1) varies very roughly as $\ln A$ so, for small departures from unity, R will vary as A raised to a small constant.] While R should approach unity at large s [10], it will not be unity at laboratory energies as high as 800 GeV, nor will $R(x_F)$ be independent of x_F. It is precisely the $1/s$ dependence of R which makes this analysis consistent with perturbative QCD calculations.

We now turn to two pieces of data that illustrate the effect of energy loss. We use the Drell-Yan formalism and the Duke-Owens form factors for the calculation [14], rather than using the empirical values for the energy de-

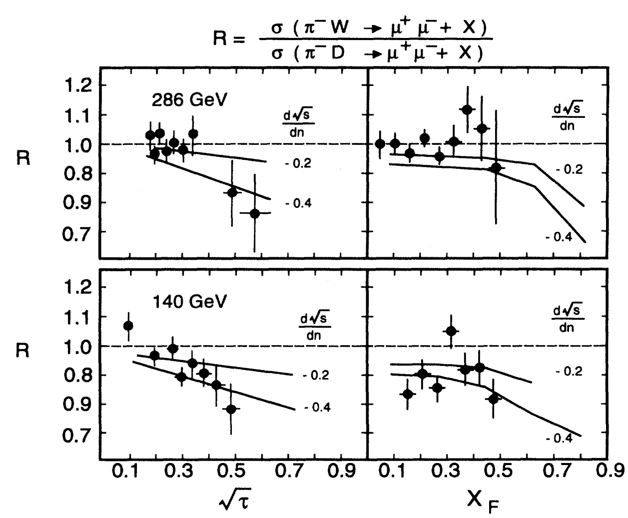

FIG. 12. Data of Ref. [15] for dimuon production by pions. The predicted falloff of R with the dimuon invariant mass M as well as with Feynman X are shown along with the theoretical energy loss predictions.

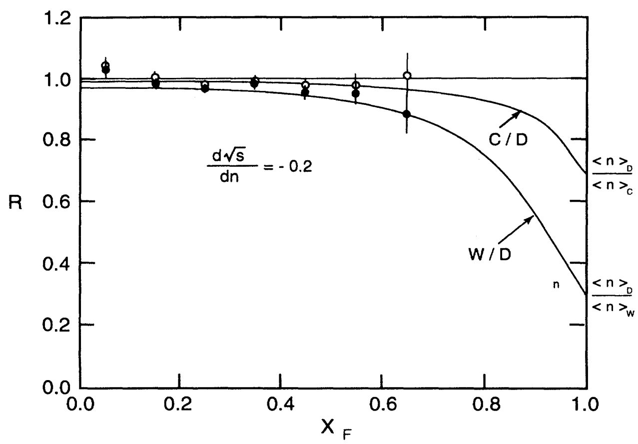

FIG. 13. Dimuon production by 800 GeV protons. R vs x_F. Data from Ref. [16].

pendence of the cross section, since we have checked that they give essentially the same results.

Figure 12 shows the data of the NA10 Collaboration on dimuon production by 140 and 286 GeV pions [15]. Superimposed on their data are our calculations for 0.2 and 0.4 GeV energy loss. There appears to be a clear M dependence as well as deviations from $R = 1$ in the x_F distributions. The data is not very precise but the general features of energy loss are borne out.

Figure 13 shows the 800 GeV data [16] of Fermilab experiment E772. We have obtained the unpublished mass distributions [17] for each x_F bin so we can make a comparison between the data for different values of $d\sqrt{s}/dn$. The published x_F distributions for their W to deuterium ratios show a deviation in R from unity at large x_F. Our calculations for $d\sqrt{s}/dn = 0.2$ are superimposed on the data.

We conclude that there are A dependent effects in dimuon production which can be accounted for using the same energy loss mechanism that can account for the A dependence in inelastic muon scattering and using the same approximate energy loss parameter.

IV. DISCUSSION

Unfortunately, none of the data extend to very large values of z or x to enable the asymptotic values at $R(1)$ to be compared with the Glauber prediction, verifying the importance of the energy loss in the most unambiguous way. There are other recent estimates for the functional form of the energy loss [6,7] but the present data shown in this paper cannot easily be used to discriminate among the various models.

However, the question of the form of the energy dependence of $d\sqrt{s}/dn$ can be more easily examined with present data on hadron production by muons. E665 can use all its data not just their published low and high Q^2 Bjorken x bins to improve the statistical accuracy for a new study: the separation of the present E665 muon inelastic data into separate ν bins to study $R(\nu)$. Newer E665 data could be analyzed to study R for a wide range of A. For example, with our parametrization $d\sqrt{s}/dn$, the laboratory energy loss dE/dn would vary as $\sqrt{E}$ so that the loss parameter would clearly change in the region covered by the E665 data (110 to 490 GeV). While it is difficult to study the energy dependence of $d\sqrt{s}/dn$ in dimuon production, since different accelerator energies are needed, it is simple to select ν from the inelastic muon data to make the same study.

ACKNOWLEDGMENTS

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