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CP violation in the heavy neutrino production process $e^+e^- \to N_1 N_2$

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The problem of CP conservation and CP violation for two heavy neutrino production in e^+e^- interactions is considered. A very convenient way of parametrizing the neutrino mass matrix, from which necessary and sufficient conditions for CP conservation easily follow, is presented. Contrary to the Kobayashi-Maskawa mechanism, the effects of CP violation in the lepton sector with Majorana neutrinos can be very large. The change of the total cross section caused by CP violation can be much larger than the cross section itself.

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I. INTRODUCTION

The origin of CP violation is one of the most important open problems in particle physics. In the standard model (SM) CP violation is explained by the Kobayashi-Maskawa (KM) mechanism [1]. In this mechanism CP violation depends on the mixing between flavor eigenstates and mass eigenstates. For the mixing to take place, the fermions with given charges must have distinguishable masses. That is why CP violation is visible in the quark sector (quark masses are distinguishable) and not visible in the lepton sector (light neutrino masses are still consistent with zero). The CP violation effect has been observed until now only in the K^0-$\bar{K}^0$ sector [2] and is small. This is because the only quantity which describes CP violation in the KM mechanism is the parameter $\delta_{\rm KM}$ given by

$$\delta_{\rm KM} = \mathrm{Im}\,(V_{cd} V_{ub} V^*_{cb} V^*_{ud})\,. \tag{1}$$

As the KM mixing matrix parameters V_{ik} are small the $\delta_{\rm KM}$ is also small:

$$\delta_{\rm KM} < 10^{-4}. \tag{2}$$

The CP violation problem is very interesting in the lepton sector if the neutrinos are Majorana particles. First of all, in contrast to Dirac particles, physical Majorana fields are not rephasing invariant. Then not so many phases can be eliminated and CP is violated already for two generations of leptons [3]. The greater number of noneliminated phase parameters is also the reason why CP violation is not mass suppressed [4] so the effect could be potentially visible even for very light neutrinos.

In this paper we consider the problem of CP violation in the case of heavy Majorana neutrinos. Such particles with masses greater then 100 GeV can be produced in future e^+e^- colliders. All our considerations are done in the framework of the left-right $(L-R)$ symmetric model which predicts the existence of the Majorana neutrino in a natural way. In the next section we find the most convenient parametrization of the mass matrix for the study of CP violation. A necessary and sufficient condition guaranteeing CP invariance on the level of weak lepton states is studied. The numerical analysis of CP violation in the $e^+e^- \to N_1N_2$ process is done in Sec. III and some conclusions are presented at the end.

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II. PARAMETRIZATION OF THE MASS AND MIXING MATRICES

We consider the $L-R$ model [5] described in detail in Ref. [6]. The relevant parts of the model's Lagrangian for studying the CP properties are the charged-current interaction and the lepton mass Lagrangian. They are given by

$$L_{\rm CC} = \frac{g}{\sqrt{2}} \left(\bar{\nu}_L \gamma^\mu l_L W^+_{L\mu} + \bar{\nu}_R \gamma^\mu l_R W^+_{R\mu} \right) + \text{H.c.} \tag{3}$$

and

$$L_{\rm mass} = -\frac{1}{2} \left(\bar{n}^c_L M_\nu n_R + \bar{n}_R M^*_\nu n^c_L \right) - \left(\bar{l}_L M_l l_R + \bar{l}_R M_l^+ l_L \right), \tag{4}$$

where n_R is a six-dimensional vector of the neutrino fields

$$n_R = \begin{pmatrix} \nu^c_R \\ \nu_R \end{pmatrix}, \quad \nu^c_R = i\gamma^2 \nu^*_L,$$
$$n_L = \begin{pmatrix} \nu_L \\ \nu^c_L \end{pmatrix}, \quad \nu^c_L = i\gamma^2 \nu^*_R. \tag{5}$$

M_ν and M_l are 6×6 and 3×3 mass matrices for neutrinos and charged leptons respectively. We consider the model with explicit left-right symmetry where the left-handed neutral Higgs triplet does not condense $(v_L = 0)$. Then the mass matrix M_ν is given by

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}, \tag{6}$$

where 3×3 matrices M_D (and also M_l) are Hermitian and M_R is symmetric. The most general CP transformation which leaves the gauge interactions (3) invariant is [7]

$$\begin{aligned} l_L &\to V_L C l_L^*, \quad \nu_L \to V_L C \nu_L^*, \\ l_R &\to V_R C l_R^*, \quad \nu_R \to V_R C \nu_R^*, \end{aligned} \tag{7}$$

where $V_{L,R}$ are 3×3 unitary matrices acting in lepton flavor space and C is the Dirac charge conjugation matrix. For the full Lagrangian to be invariant under (7) the lepton mass matrices M_D, M_R, and M_l have to satisfy the conditions

$$\begin{aligned} V_L^\dagger M_D V_R &= M_D^*, \\ V_R^T M_R V_R &= M_R^*, \end{aligned} \tag{8}$$

and

$$V_L^\dagger M_l V_R = M_l^*. \tag{9}$$

The relations expressed by Eqs. (8) and (9) are weak-basis independent and constitute necessary and sufficient conditions for CP invariance. This means that if for given matrices M_D, M_R, and M_l there exist two unitary matrices V_L and V_R such that relations (8) and (9) hold then our model is CP invariant and, on the other hand, if CP is the symmetry of our model then such matrices V_L and V_R exist. The most convenient basis for studying CP symmetry is the weak basis in which charged lepton mass matrix M_l is real, positive, and diagonal:

$$M_l = \text{diag}[m_e, m_\mu, m_\tau]. \tag{10}$$

Then for nondegenerate, nonvanishing $m_e \neq m_\mu \neq m_\tau$ Eqs. (8) and (9) imply that matrices $V_{L,R}$ are diagonal and equal:

$$V_L = V_R = \text{diag}[e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}]. \tag{11}$$

From Eqs. (8) and (9) it follows that the model has CP symmetry if and only if the matrices M_D and M_R have the elements

$$\begin{aligned} (M_D)_{ij} &= |\,(M_D)_{ij}\,|\, e^{+\frac{i}{2}(\delta_i - \delta_j)}, \\ (M_R)_{ij} &= |\,(M_R)_{ij}\,|\, e^{-\frac{i}{2}(\delta_i + \delta_j)} \end{aligned} \tag{12}$$

in the basis where M_l is diagonal. The number of reduced phases ($\frac{n(n+1)}{2}$ for symmetric M_R and $\frac{n(n-1)}{2}$ for Hermitian M_D give totally n^2 phases)

$$n^2 - n \ \ (= 6) \tag{13}$$

is the lepton sector number of independent CP-violating phases in the considered model (with explicit $L-R$ symmetry and $v_L = 0$).

It is easy to understand why relations (12) are necessary and sufficient conditions for CP invariance. From Eqs. (12) it follows that the neutrino mass matrix M_ν [Eq. (6)] is diagonalized by the orthogonal transformation

$$U^T M_\nu U = \text{diag}[|\, m_1' \,|, ..., |\, m_6' \,|] \tag{14}$$

and the $(2n \times 2n)$ unitary matrix U can be expressed in the form

$$U = \begin{pmatrix} V^* & 0 \\ 0 & V \end{pmatrix} O \eta \tag{15}$$

where

$$V = \text{diag}[e^{i\delta_1/2}, e^{i\delta_2/2}, e^{i\delta_3/2}], \tag{16}$$

O is a real orthogonal $2n \times 2n$ matrix ($O^T = O^{-1}$) that diagonalizes the real part of the M_ν matrix after removing the phases $e^{i\delta_i/2}$, and η is a diagonal $(2n\times 2n)$ matrix that ensures that the neutrino masses are positive numbers ($m_i =|\, m_i' \,|\geq 0$):

$$\eta_{ij} = \delta_{ij} e^{i\frac{\pi}{4}(\text{sgn}[m_i'] - 1)}. \tag{17}$$

The CP symmetry is then satisfied if we define the CP parity of Majorana neutrinos [8]:

$$\eta_{CP}(i) = i\,\text{sgn}[m_i']. \tag{18}$$

To find the mixing matrices $K_{L,R}$ for the left (right) charged current and the neutral currents $\Omega_{L,R}$ (see Ref. [6] for precise definition) we define

$$U \equiv \begin{pmatrix} U_L^* \\ U_R \end{pmatrix} = \begin{pmatrix} V^* O_L \eta \\ V O_R \eta \end{pmatrix}. \tag{19}$$

Then

$$\begin{aligned} K_L &\equiv U_L^\dagger = \eta O_L^T V^\dagger, \\ K_R &\equiv U_R^\dagger = \eta^* O_R^T V^\dagger, \end{aligned} \tag{20}$$

and

$$\begin{aligned} \Omega_L &\equiv K_L K_L^\dagger = \eta O_L^T O_L \eta^*, \\ \Omega_R &\equiv K_R K_R^\dagger = \eta^* O_R^T O_R \eta, \\ \Omega_{RL} &\equiv K_R K_L^\dagger = \eta^* O_R^T O_L \eta^*. \end{aligned} \tag{21}$$

From Eqs. (20) and (21) we see that the phase factors from matrix V multiply the columns of the matrices $K_{L,R}$ and can be absorbed by rephasing of the charged lepton fields in the charged currents $l_{L,Ri} \to e^{i\delta_i/2} l_{L,Ri}$. The phase factors disappear from matrices $\Omega_{L,R}$ and Ω_{RL} which mix the physical Majorana neutrino fields for which the rephasing is not possible.[1] Then, if the CP is not spontaneously broken, the total lepton Lagrangian (gauge-gauge, gauge-leptons, Higgs-leptons, and Higgs interactions) is CP invariant. If the phases of matrices M_D and M_R differ from those that are given by Eqs. (12) the CP symmetry is broken. In the next section we investigate the effect of these CP broken phases in the production process of two heavy neutrinos.

[1]We adopt the definition of the physical Majorana fields $N(x)$ as fields that under charge conjugation stay the same without any phase factor: $N^C(x) \equiv C\bar{N}^T(x) = N(x)$. For a definition of Majorana fields where the creation phase factors are introduced, see Refs. [9]. We do not think that these definitions are useful.

III. THE CP EFFECT IN THE PROCESS $e^+e^- \to N_1N_2$; NUMERICAL ANALYSIS

The amplitude for two Majorana neutrino production process in e^+e^- interactions is given by the contributions from six diagrams with gauge boson exchange in t, u, and s channels (see Fig. 1). The contributions from Higgs boson exchange particles are negligible [10] and we do not consider them here.

Full helicity amplitudes $M(\sigma\bar{\sigma};\lambda_1\lambda_2)$ for the process

$$e^-(\sigma) + e^+(\bar{\sigma}) \to N_1(\lambda_1) + N_2(\lambda_2) \tag{22}$$

are presented in the Appendices of Refs. [6] and [10].

The CP effects are caused by phase factors that appear in the mixing matrices $K_{L,R}$ in t and u channels and $\Omega_{L,R}$ in the s channel. To observe the influence of these phases two things must happen. First, different CP phases have to contribute to various Feynman diagrams from Fig. 1, and second, the diagrams have to interfere so that at least two Feynman diagrams must contribute to the same helicity amplitude. The same mixing matrix elements give contributions to the W_1,W_2 exchange diagrams in t-u channels ($K_{L,R}$) and Z_1, Z_2 boson exchange in the s channel ($\Omega_{L,R}$). So even if these diagrams contribute to the same helicity amplitude they do not interfere (of course there are also other suppression factors as the gauge boson mixing angles are small [6]). If the energy is large compared to the masses of neutrinos N_1 and N_2 then the t channel contributes to $M(-+;-+)$ (left-handed current) and $M(+-;+-)$ (right-handed current) and the u channel gives contributions to $M(-+;+-)$ and $M(+-;-+)$ amplitudes. We can see that at high energy there is no interference between t and u channels [4]. The s-channel diagrams produce all four helicity amplitudes. So at high energy we can look for CP effects resulting from the interference between t-s and u-s channels.

For the energy just above the production threshold there is no helicity suppression mechanism and final neutrinos with all helicity states can be produced by each channel diagram. These are the best conditions for observing the CP violation effects.

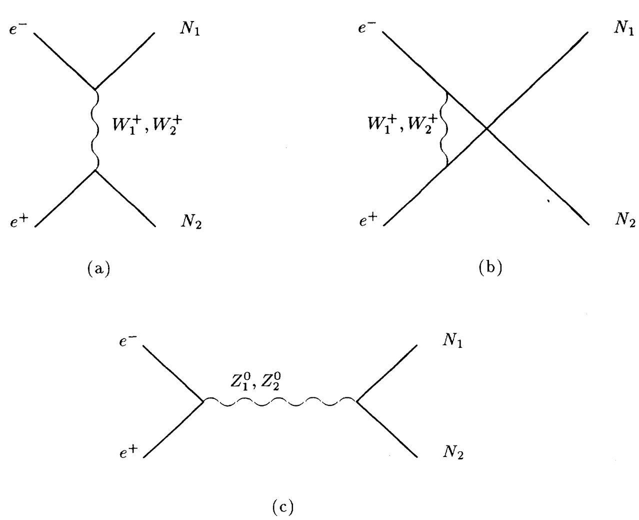

FIG. 1. Diagrams with gauge boson exchange which describe the process $e^-e^+ \to N_1N_2$ in the left-right symmetric model on the tree level.

Another question is in what experimental observables the CP effects are visible. From the discussion presented above we can see that they can be looked for in polarized angular distribution. Unfortunately the cross sections, as we shall see, are too small to realize this possibility. And what about the unpolarized angular distribution? If CP is conserved then the helicity amplitude satisfies the relation (Θ and ϕ are c.m. scattering angles)

$$\begin{aligned} M(\sigma,\bar{\sigma};\lambda_1,\lambda_2;\Theta,\phi) &= -\eta^*_{CP}(1)\eta^*_{CP}(2) \\ &\quad\times M(-\bar{\sigma},-\sigma;-\lambda_1,-\lambda_2;\pi-\Theta,\pi+\phi), \end{aligned} \tag{23}$$

where $\eta_{CP}(i)$ are CP parities of the Majorana neutrinos. If we sum over all helicity the unpolarized angular distribution has forward-backward isotropy:

$$\frac{d\sigma}{d\Omega}(\Theta,\phi) = \frac{d\sigma}{d\Omega}(\pi-\Theta,\pi+\phi). \tag{24}$$

Does this mean that anisotropy can be observed if CP is violated? Unfortunately not, at least if we neglect the final state interaction. Without final state interactions from CPT symmetry we can prove the relation

$$\begin{aligned} M(\sigma,\bar{\sigma};\lambda_1,\lambda_2;\Theta,\phi) &= -\eta_{CP}(1)\eta_{CP}(2)e^{2i(\sigma-\bar{\sigma})(\pi+\phi)} \\ &\quad\times M^*(-\bar{\sigma},-\sigma;-\lambda_1,-\lambda_2;\pi-\Theta,\pi+\phi), \end{aligned} \tag{25}$$

from which the forward-backward isotropy also follows [11]. So the only observables where we can try to find the CP violation effect are the total cross sections. How big can the effects be? There are six phases which cause CP symmetry breaking. We do not try to find the phase for which the effect of CP breaking is maximal. We take the matrices M_D and M_R in the form

$$M_D = 10^{-3}\begin{pmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 1.0 & 0.9 \\ 0.9 & 0.9 & 0.95 \end{pmatrix},$$

and

$$M_R = \begin{pmatrix} 150e^{i\alpha} & 10 & 20 \\ 10 & 200e^{i\beta} & 10 \\ 20 & 10 & 5000e^{i\gamma} \end{pmatrix},$$

which produce a reasonable spectrum of light neutrinos. If we compare these matrices with Eq. (12) we see that if only one or more phases (α, β, or γ) are not equal to 0 or π the CP is violated. Two heavy neutrinos with masses $M_1 \simeq 150$ GeV and $M_2 \simeq 200$ GeV, almost independent of the phases α, β, and γ, result from our mass matrix. We calculate the cross section for production of these neutrinos in e^+e^- scattering

$$e^+e^- \to N_1(150)N_2(200).$$

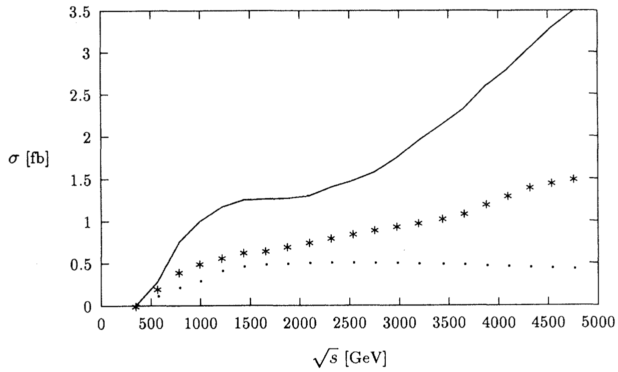

FIG. 2. CP and mixing matrix effects for the $e^- e^+ \to N_1(150) N_2(200)$ production. Solid line is for $\alpha = \beta = \gamma = 0$, dotted line is for $\alpha = \pi, \beta = \gamma = 0$, and the third line (with asterisks) is for $\alpha = 2.0, \beta = \gamma = 0$ phases. The other L-R model parameters which we used are the following: $M_{W_2} = 1500$ GeV, $\beta = \frac{M_{W_1}^2}{M_{W_2}^2}$, $M_{Z_2}^2 = \frac{2\cos^4\Theta_W M_{Z_1}^2}{\cos 2\Theta_W \beta}$, $\xi = \beta$, $\phi = -\frac{(\cos 2\Theta_W)^{3/2}}{2\cos^4\Theta_W}\beta$ (see Ref. [6]).

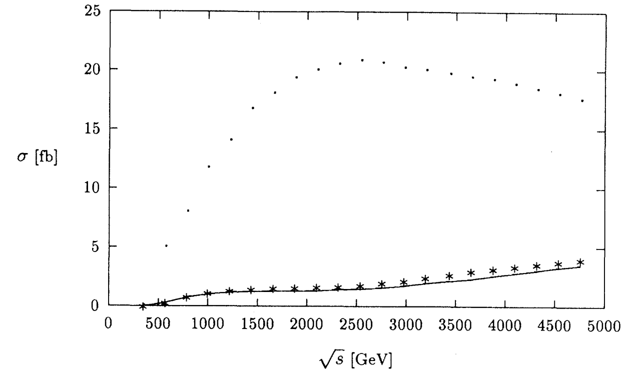

FIG. 3. The effect of CP violation only on the $e^- e^+ \to N_1(150) N_2(200)$ production. Absolute values of mixing matrix elements are the same as the ones for the solid line in Fig. 2 $[(K_L)_{1e} = 0.005\,35, (K_R)_{1e} = 0.9819, (K_L)_{2e} = 0.0058, (K_R)_{2e} = 0.189, (\Omega_L)_{12} = -(\Omega_R)_{12} = 0.000\,09]$. Dotted (solid) line is for opposite (the same) CP parity of neutrinos [Eqs. (27) and (26)]; line with asterisks is for $\alpha = 2.0, \beta = \gamma = 0$, the same as in Fig. 2.

The appropriate mixing matrix elements $(K_{L,R})_{1e}$, $(K_{L,R})_{2e}$, and $(\Omega_{L,R})_{12}$ depend on the phases α and β and are almost independent of the phase γ. For $\alpha = \beta = \gamma = 0$ two neutrinos have equal CP parity and CP is conserved

$$\eta_{CP}(N_1) = \eta_{CP}(N_2) = +i. \qquad (26)$$

For $\alpha = \pi, \beta = \gamma = 0$ CP is also conserved if we introduce the CP parities

$$-\eta_{CP}(N_1) = \eta_{CP}(N_2) = +i. \qquad (27)$$

For any other values of phases CP is violated. The production cross sections as energy functions are presented in Fig. 2. Two factors affect the behavior of the cross section. First, there is the real CP effect which causes the different interference between various diagrams. Second, for different phases different mixing matrix elements are obtained. In Fig. 2 both these effects are taken into account. To find the influence of CP interference only, we present in Fig. 3 the cross sections for the same mixing matrix elements but with all phases the same as in Fig. 2. We can see that the influence of the CP interference is very large. The cross section for production of two neutrinos with opposite CP parity can be several times bigger then the cross section for production of the same CP parity neutrinos. The cross sections for the real CP-breaking case are placed between two CP-conserving situations. We would like to stress that now the CP effect can be quite large contrary to the Kobayashi-Maskawa mechanism in the quark sector. In the lepton sector with Majorana neutrinos the changes in cross section which result from CP breaking can be several times bigger than the cross section itself. Unfortunately, the calculated cross sections are of the range of several femtobarns so the actual observation of the process for reasonable luminosity will be difficult.

IV. CONCLUSIONS

If Majorana neutrinos are present in the lepton sector the CP violation effect can be very strong. For the two heavy neutrino production process $e^+e^- \to N_1 N_2$ the CP violation signals appear as an effect of t-u channel interference just above the threshold and t-s, u-s channel interference for higher energy. The angular distribution for unpolarized e^+e^- beams and without the measurement of the final neutrino polarization has forward-backward symmetry even if CP is violated but the final state interaction may be neglected. The total cross section is the quantity which changes dramatically with various CP-violating parameters. Even if the change of total cross section is large the cross section is small, which makes the observation of this effect difficult.

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