Quark droplet formation in a neutron star core in the presence of a strong magnetic field

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The effect of a strong magnetic field on the thermal nucleation of quark droplets at the core of a neutron star is investigated. The surface energy of the quark phase diverges logarithmically and as a consequence there cannot be any thermal nucleation of the quark droplet. The effect of a strong magnetic field on the curvature term is also studied. The curvature term also diverges.

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The possibility of an infinite cluster of quark matter at the core of a neutron star is an important interesting problem. Although the gross properties of such a self-gravitating object (known as a hybrid star) [1] do not differ significantly from a pure neutron star, the presence of a quark phase at the core may cause rapid cooling of the star and can also produce significant γ bursts [2]. The other important aspect is the superconducting nature of the quark matter core. Since quarks carry electric charges, unlike neutron matter which exhibits superfluidity in a neutron star, this particular phase may as a whole behave like a superconductor (and possibly type I; of course, if there are very few protons present in neutron matter, they form Cooper pairs and give rise to superconducting zones). As a consequence, the magnetic field lines present will be pushed out from the superconducting quark core to the normal crust region (which is mainly superfluid neutron matter). As a result, in such objects the magnetic field lines mainly pass through the crust region, the consequence of which is the rapid Ohmic decay of the magnetic field. Therefore the surface magnetic field for such objects will be low enough compared to the corresponding pure neutron star [3].

The other interesting part of such objects is the time of quark matter nucleation. It is still unknown whether it takes place immediately after a supernova explosion or much later. This point was discussed in detail by Horvath *et al.* [4] and also by Olesen and Madsen [5] in their recent publications.

Our aim in this paper is somewhat different. We shall investigate quark matter nucleation at the core of a neutron star in the presence of a strong magnetic field. We are not at all interested in the origin of such a strong magnetic field; instead, we would like to understand its role in the formation of quark matter droplets.

From the observed features in the spectra of pulsating accreting neutron stars in binary systems, the strength of the surface magnetic field of a neutron star is found to be $\sim 10^{12}$ G. In the interior it probably reaches $\sim 10^{18}$ G. Therefore, if a phase transition from neutron (with a small admixture of protons) matter to quark matter takes place at the core through the nucleation of quark droplets, then, under such circumstances as mentioned above, it is advisable to study the effect of a strong magnetic field on the nucleation of quark droplets. In particular one should try to see how the surface energy of the quark phase, which plays a crucial role in droplet formation, changes with the magnetic field. One should also check the effect of a strong magnetic field on the curvature energy of the quark phase. The curvature term is also equally important as the surface one for thermal nucleation.

Now the rate of stable quark droplet formation per unit volume from a metastable neutron matter phase due to fluctuation is given by [4,6]

$$I = I_0 \exp(-W_m/T) \approx T^4 \exp(-W_m/T) , \tag{1}$$

where W_m is the minimum thermodynamic work to be done to create a critical quark droplet and is given by

$$W_m = \frac{4}{3}\pi \frac{\sigma^3}{(\Delta P)^2}[2+2(1+b)^{3/2}+3b] , \tag{2}$$

where $\sigma = \sigma_q + \sigma_n$ the surface tension and $\Delta P = P_q - P_n$ is the pressure difference. Here $\Delta P = P_q - P_n$ is a positive quantity and both σ_q and σ_n are also positive, $b = 2\gamma(\Delta P)/\sigma^2$, and $\gamma = \gamma_q - \gamma_n$ stands for curvature energy density [5]. Here the phase q is a droplet of quark matter and the phase n stands for the metastable neutron matter. In Eq. (1), the preexponential factor is chosen to be T^4, where T is the temperature of the metastable medium (for an exact expression see Ref. [7]). In Eq. (2) we have assumed that the two phases are in chemical equilibrium ($\Delta\mu = 0$). Since the bubble nucleation time $\tau_{\rm bubble} \approx 10^{-23}$ s $\approx \tau_{\rm strong}$, the strong interaction time scale, the creation of strange quarks through weak processes within the quark droplets is neglected. Therefore quark droplets are consisting of only u and d quarks. Since the temperature ($\sim$4–5 MeV) is much less than the quark chemical potential ($\sim$300 MeV), the presence of antiquarks can also be ignored. On the other hand, if we assume the presence of hyperons at the core of the neutron star (which is believed to be true), then as a consequence of phase transition to quark matter, s quarks will also be produced at the core. However, the quantum mechanical effect of a strong magnetic field on the s quark part of a strangelet can be ignored as long as the magnetic field strength is $\leq 10^{20}$ G, which is too large to achieve at the

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core of a neutron star. Therefore in the surface energy term of a strangelet, a finite contribution will come from the s quark part.

Now to study the stable quark phase we have considered two different confinement models: (i) the *conventional MIT* bag model and (ii) the *dynamical density-dependent* quark mass (D3QM) model of confinement. In the MIT bag model, for the sake of simplicity we are assuming that quarks are moving freely within the droplets and as usual the current masses of both u and d quarks are extremely small ($\sim 5-10$ MeV). In the D3QM model, assuming the quark matter to be a degenerate Fermi gas, the masses of both u and d quarks are parametrized in the following manner [8]:

$$m_{u,d}=\frac{B}{n_q}\ , \tag{3}$$

where n_q is the total quark number density and B is the vacuum ($n_q \to 0$) energy density. According to this model the mass of a quark is extremely low inside a hadron and is infinitely large outside. Physically, this model is equivalent to the trapping of particles in an infinite potential well. Therefore the boundary condition to be satisfied here is the so-called *Dirichlet boundary condition* (quark wave function vanishes at the boundary).

As is well known, the energy of a charged particle changes significantly in the quantum limit if the magnetic field strength is of the order of or exceeds some critical values $B_m^{(c)}=m^2c^3/q\hbar$ (in gauss) $=4.4\times 10^{13}$ G ($B_m^{(c)(e)}$) for electrons. Therefore, for a quark of mass 5 MeV the critical field $B_m^{(c)(q)}=10^2\times B_m^{(c)(e)}$, where m and q are the particle mass and charge, respectively; c and $\hbar$ are, respectively, the velocity of light and reduced Planck constant (which along with the Boltzmann constant k_B are assumed to be unity in our choice of units). This is the typical strength of magnetic field at which the cyclotron lines begin to occur and in this limit the cyclotron quantum is of the order of or greater than its rest energy. This is equivalent to the requirement that the de Broglie wavelength is of the order of or greater than the Larmor radius of the particle in the magnetic field. In a recent publication [9], we have studied the stability of bulk strange quark matter (SQM) in the presence of strong magnetic field ($\geq$ critical value). We have seen that the stability of SQM increases with the increase of magnetic field strength.

For a constant magnetic field along the z axis ($\mathbf{A}=0$, $\mathbf{B}_m=B_{m,z}=B_m=\text{constant}$), the single-particle energy eigenvalue is given by [10]

$$\epsilon_{k,n,s}=[k^2+m^2+qB_m(2n+s+1)]^{1/2}\ , \tag{4}$$

where $n=0,1,2,\ldots$ are the principal quantum numbers for the allowed Landau levels, $s=\pm 1$ refers to spin-up (positive) and -down (negative) cases. Setting $2\nu=2n+s+1$, where $\nu=0,1,2,\ldots$, we find $\nu=0$ state is singly degenerate, whereas all other states with $\nu\neq 0$ are doubly degenerate. Here k is the component of particle momentum along the field direction. Then we can rewrite the single particle energy eigenvalue in the form

$$\epsilon_\nu=[k^2+m^2+2\nu qB_m]^{1/2}\ . \tag{5}$$

Now following Balian and Bloch [11] and also Berger and Jaffe [12] (see also Ref. [13]), we have the surface energy per unit area of surface tension of the quark droplet:

$$\sigma=\frac{T}{64\pi^2}\sum_{i=u,d} g_i\int\frac{d^3k}{k}\ln\left[1+\exp\left[-\frac{\epsilon_i-\mu_i}{T}\right]\right]G\ , \tag{6}$$

where

$$G=\left[1-\frac{2}{\pi}\arctan\left[\frac{k}{m_i}\right]\right] \tag{6a}$$

for the bag model and $G=1$ in the case of the D3QM model. Here T is the temperature of the system, g_i the degeneracy of the ith flavor (= 6 for both u and d quarks) and m_i is the corresponding mass.

Now we shall investigate the magnetism arising from quantization of the orbital motion of charged particles in presence of strong magnetic field ($B_m \geq B_m^{(c)}$). We know that if the magnetic field is along the z axis, the path of the charged particle will be a regular helix whose axis lies along the z axis and whose projection on x-y plane is a circle. If the magnetic field is uniform, both the linear velocity along field direction and the angular velocity in the x-y plane will be constant, the latter arises from the constant Lorentz force experienced by the particle. Quantum mechanically the energy associated with the circular motion in the x-y plane is quantized in units of $2eB_m$. The energy associated with the linear motion along the z axis is also quantized; but in view of the smallness of the energy intervals, this may be taken as a continuous variable. We thus have Eq. (4) or (5) as the single particle energy eigenvalue. Now these magnetized energy levels are degenerate because they result from an almost continuous set of zero field levels. All these levels for which the value of the quantity $k_x^2+k_y^2$ lies between $2eB_m\nu$ and $2eB_m(\nu+1)$ now coalesce together into a single level characterized by the quantum number ν. The number of those levels is given by

$$\frac{S}{(2\pi)^2}\int\int dk_x dk_y=\frac{SeB_m}{2\pi}\ ,$$

where S is the area of the orbit in the x-y plane. This expression is independent of ν. Then in the integral of the form $\int d^3k f(k)$, we can replace $\int dk_x dk_y$ by the expression given above, where as the limit of k_z, which is a continuous variable, ranges from $-\infty$ to $+\infty$. Then we can rewrite expression (6) in the form

$$\sigma=\frac{TB_m}{16\pi}\sum_{i=u,d} g_i q_i\sum_{\nu=0}^{\infty}\int_0^{\infty}\frac{dk_z}{\sqrt{k_z^2+k_{\perp(i)}^2}}\times\ln\left[1+\exp\left[-\frac{\varepsilon_i^{(\nu)}-\mu_i}{T}\right]\right]G\ , \tag{7}$$

q_i being the magnitude of ith flavor charge, equal to $2e/3$ for u quark and $e/3$ for d quark, e being the magnitude of electronic charge, $k_{\perp,(i)}^2=2\nu q_i B_m$, and k_z is the quark momentum along the direction of magnetic field, $g_i=3$ for both u and d quarks.

The upper limit of the ν sum can be obtained from the condition

$$k^2=\mu_i^2-m_i^2-2\nu q_i B_m \geq 0 \ . \tag{8}$$

This gives

$$\nu \leq \frac{\mu_i^2-m_i^2}{2q_i B_m}(\text{nearest integer})=\nu_{\max}\ (\text{say}) \ . \tag{8a}$$

Therefore the upper limit $\nu_{\max}$ may not be the same for u and d quarks. Now to visualize the effect of magnetic field on the nucleation of quark droplets, we shall evaluate integral (6) by parts for $G=1$ and we take $T\to 0$ (which is a valid approximation for $T \ll \mu_i$); then

$$\sigma(G=1)=\frac{B_m}{16\pi}\sum_{i=u,d} g_i q_i \left[\sum_{\nu=0}^{\nu_{\max}} \int_0^{k_{F_i}} \frac{k_z dk_z}{\sqrt{k_z^2+m_i^2+k_{\perp,(i)}^2}} \ln(k_z+\sqrt{k_z^2+k_{\perp,(i)}^2}) - \sum_{\nu=0}^{\nu_{\max}} \ln(k_{\perp,(i)})(\mu_i-\sqrt{k_{\perp,(i)}^2+m_i^2}) \right]$$
$$=\sigma_{\mathrm{D3QM}} \ . \tag{9}$$

It is easy to check that Eq. (9) is also a part of σ_{bag} for which $G\neq 1$. Since $\mu_i>\sqrt{k_{\perp,(i)}^2+m_i^2}$, therefore, for $\nu=0$ the second term of Eq. (9) becomes $+\infty$. Which is therefore true for σ_{bag} also. Consequently, the surface energy of quark droplets in both models diverges for $B_m \geq B_m^{(c)}$. The term $\nu=0$ corresponds to the lowest Landau level. If the magnetic field is too strong only this level will be populated, which is also obvious from Eq. (8a). Although we have assumed here $T\to 0$, this important conclusion is equally valid for any finite T. Since both P_n and P_q are finite even in the presence of strong magnetic field [14] and T is also finite, we have from Eq. (1), $I=0$, $\equiv$ the thermal nucleation rate of droplet formation becomes zero; i.e., there cannot be a single quark droplet formation by thermal nucleation at the core if the magnetic field is of the order of or greater than the corresponding critical value. Therefore, the formation of quark droplet at the neutron star core (if any) will be controlled by some other mechanism, namely, the quantum effects, triggering by strangelet capture, etc. [15].

Following Ref. [5], we shall now investigate the effect of strong magnetic field on the quark curvature term, given by

$$\gamma=\frac{T}{48\pi^3}\sum_i g_i \int \frac{d^3k}{k^2}\ln\{1+\exp[-\beta(\epsilon_i-\mu_i)]\}G \ , \tag{10}$$

where

$$G=1-\frac{3}{2}\frac{k}{m_i}\left[\frac{\pi}{2}-\arctan\left[\frac{k}{m_i}\right]\right] \tag{11}$$

for MIT bag model and = 1 for D3QM model of confinement. As discussed before, in the presence of strong magnetic field Eq. (10) can be rewritten as

$$\gamma=\frac{T}{12\pi^2}\sum_{i=u,d} g_i q_i \sum_{\nu=0}^{\nu_{\max}} \int_0^\infty \frac{dk_2}{k_z^2+k_{\perp,(i)}^2} \times \ln\{1+\exp[-\beta(\epsilon_i-\mu_i)]\}G \ . \tag{10$'$}$$

Now we shall evaluate Eq. (10′) term by term. Consider

$$I_1=\int_0^\infty \frac{dk_2}{k_z^2+k_{\perp,(i)}^2}\ln\{1+\exp[-\beta(\epsilon_i-\mu_i)]\} \ . \tag{12a}$$

Integrating by parts, we have

$$I_1=\frac{1}{Tk_{\perp,(i)}}\int_0^\infty \tan^{-1}\left[\frac{k_z}{k_{\perp,(i)}}\right]\frac{k_z dk_z}{(k_z^2+k_{\perp,(i)}^2+m_i^2)^{1/2}} \times \frac{1}{\exp[\beta(\epsilon_i-\mu_i)]+1} \ , \tag{12b}$$

which diverges for $\nu=0$, but the divergence is not logarithmic ($I_1 \sim 1/\nu$).

Next consider

$$I_2=-\frac{3\pi}{4m_i}\int_0^\infty \frac{dk_z}{(k_z^2+k_{\perp,(i)}^2)^{1/2}} \times \ln\{1+\exp[-\beta(\epsilon_i-\mu_i)]\} \ . \tag{12c}$$

Integrating by parts, we have

$$I_2=\frac{3\pi}{4m_i}\left[(\ln(k_{\perp,(i)})\ln\{1+\exp[-\beta(\epsilon_i-\mu_i)]\}) - \frac{1}{T}\int_0^\infty \ln[k_z+(k_z^2+k_{\perp,(i)}^2)^{1/2}] \times \frac{k_z dk_z}{(k_z^2+k_{\perp,(i)}^2+m_i^2)^{1/2}} \times \left[\frac{1}{\exp[\beta(\epsilon_i-\mu_i)]+1}\right]\right] \ . \tag{12d}$$

For $\nu=0$, the first term diverges logarithmically, but unlike the second term of Eq. (9), it becomes $-\infty$. The second term of Eq. (12d) and the last term of Eq. (10′) have been checked numerically and they remain finite for $\nu=0$. Now the divergences of the first two terms of Eq. (10′) cannot cancel each other. The first divergence is much faster as $\nu\to 0$ than the second one; therefore, the overall divergence remains positive as $\nu\to 0$. Now from Eq. (2) it is obvious that if somehow the quantity b becomes finite (even zero), the effect of σ then makes W_m

infinitely large [see Eqs. (1) and (2)]; physically it means that an infinite amount of work has to be done to create a critical nucleus.

In conclusion, we may state that in the presence of a strong magnetic field, the surface tension as well as the curvature energy of quark droplets diverge. The surface term diverges logarithmically, whereas the divergence of curvature term is much faster. As a consequence there cannot be any thermal nucleation of quark droplet at the neutron star core. It can only occur by some nonthermal mechanisms. Therefore, in the future, if we get some direct or indirect experimental evidence for quark core and if the above conclusion is assumed to be correct, then either the magnetic field of such objects will be extremely low ($<B_m^{(c)}$) from its time of birth, if quark matter is assumed to be produced immediately after supernova explosion, or it must have been produced much later, when the neutron star magnetic field has decayed to a value $B_m < B_m^{(c)}$ or the quark droplets are produced through some nonthermal means.

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[1] C. Alcock, in *Numerical Experiments and Neutron Star Formation,* Proceedings of IAU Symposium, Nanjing, China, 1986, edited by D. J. Helfand and J.-H. Huang, IAU Symposium No. 125 (Reidel, Dordrecht, 1987), p. 413; F. Grassi, Z. Phys. C **44**, 129 (1989); A. Rosenhauer and E. F. Staubo, in *Strange Quark Matter in Physics and Astrophysics,* Proceedings of the International Workshop, Aarhus, Denmark, 1991, edited by J. Madsen and P. Haensel [Nucl. Phys. B (Proc. Suppl.) **24B**, 156 (1991)].

[2] O. G. Benvenuto, H. Vucetich, and J. E. Horvath, in *Strange Quark Matter in Physics and Astrophysics* [1], p. 160; P. Haensel, B. Paczynski, and P. Amsterdamski, Astrophys. J. **375**, 209 (1991).

[3] S. Chakrabarty, Phys. Rev. D **43**, 627 (1991).

[4] J. E. Horvath, O. G. Benvenuto, and H. Vucetich, Phys. Rev. D **45**, 3865 (1992); Univ. de Sao Paulo report, 1994 (unpublished).

[5] M. L. Olesen and J. Madsen, Phys. Rev. D **49**, 2698 (1994).

[6] S. Chakrabarty, J. Phys. G **20**, 469 (1994).

[7] L. P. Csernai and J. I. Kapusta, Phys. Rev. D **46**, 1379 (1992); Phys. Rev. Lett. **69**, 737 (1992).

[8] S. Chakrabarty, S. Raha, and B. Sinha, Phys. Lett. B **229**, 112 (1989).

[9] S. Chakrabarty, Astron. Space Sci. **213**, 121 (1994).

[10] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, Oxford, 1965).

[11] R. Balian and C. Bloch, Ann. Phys. (N.Y.) **60**, 401 (1970).

[12] M. S. Berger, Phys. Rev. D **40**, 2128 (1989); **43**, 4150(E) (1991); R. L. Jaffe, in *Strange Quark Matter in Physics and Astrophysics* [1], p. 8.

[13] S. Chakrabarty, Phys. Rev. D **48**, 1409 (1993); and (unpublished).

[14] S. Chakrabarty and A. K. Goyal (unpublished).

[15] We can evaluate $\sigma_{\rm D3QM}$ or equivalently $\sigma(G=1)$ for some unphysical situations, namely, for a $(d+1)$-dimensional flat space-time, when d is an integer spatial dimension >3. Then

$$\sigma_{\rm D3QM}=\sigma(G=1)$$
$$\propto B_m \sum_{i=u,d} g_i q_i \sum_{\nu=0}^{\nu_{\max}} \int_0^\infty \frac{k^{d-3}dk}{\sqrt{k^2+k_{\perp,(i)}^2}} \times \ln\left[1+\exp\left[-\frac{\epsilon_i-\mu_i}{T}\right]\right] .$$

Now we can perform the same exercise as discussed before Eq. (9) for a number of d values ($d>3$). Then it is just a matter of simple algebraic manipulation to see that σ does not diverge in those cases for $\nu=0$, which lead to a finite droplet formation rate. That means there can be quark droplet formation even in the presence of a strong magnetic field ($B_m \geq B_m^{(c)}$), provided the spatial dimension is greater than three, which is physically impossible.