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Spacetime quantization of the Braaten-Pisarski-Frenkel-Taylor-Wong action: Spacelike plasmon cut and new phase of the thermal vacuum

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The properties of quark propagation through a hot medium are summarized by the Braaten-Pisarski-Frenkel-Taylor-Wong effective action. The fermion thermal propagator shows a pseudo-Lorentz-invariant particle pole as well as a spacelike cut. In an earlier paper, we performed the explicit quantization of the action in momentum space, and showed how a canonical Dirac field of mass T' arises. In this paper, we perform the quantization in coordinate space. In the process, we relate the spacelike plasmon cut in the propagator to the homogeneous solutions of the local equation of motion for auxiliary fields. Our quantization shows how the spacelike cut produces a 90° phase factor in the thermal vacuum at high T. This phase factor is responsible for the vanishing of $\langle\psi\bar{\psi}\rangle$ at high T.

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I. INTRODUCTION

In an earlier paper [1], we studied the canonical quantization of the Braaten-Pisarski-Frenkel-Taylor-Wong (BPFTW) [2] action for the propagation of a massless fermion through the hot environ [3]. The propagator $\langle T(\psi(\mathbf{x},t)\bar{\psi}(\mathbf{y},t'))\rangle_\beta$, despite its apparent chiral symmetry, shows a pseudo-Lorentz-invariant particle pole [4] at $p_0 = \omega \equiv \sqrt{\mathbf{p}^2+T'^2}$. This pole is described by the canonical Dirac field

$$\Psi(\mathbf{x},t) = \frac{1}{\sqrt{V}}\sum_{p,s} e^{i\mathbf{p}\cdot\mathbf{x}}\left\{U_{p,s}A_{p,s}e^{-i\omega t} + V_{-p,s}B^\dagger_{-p,s}e^{+i\omega t}\right\}, \tag{1}$$

where $A_{p,s}$, $B_{p,s}$ are the annihilation operators for a particle of mass T', and U_{ps}, $V_{-p,s}$ are the massive Dirac spinors [5]. In addition, the thermal fermion propagator shows a pair of parallel spacelike cuts, just above and below the real axis and running from $p_0 = -|\mathbf{p}|$ to $p_0 = |\mathbf{p}|$.

This pair of spacelike cuts is attributed to the hot plasma state. But how exactly is it related to the thermal vacuum in the plasma state? In this paper we investigate the origin of the spacelike cut in terms of the spacetime quantization of the BPFTW action.

II. BPFTW ACTION

The BPFTW action [2] for the propagation of the fermion through a hot environ takes the form

$$\mathcal{L}_{\text{eff}} = -\bar{\psi}\gamma_\mu\partial^\mu\psi - \frac{T'^2}{2}\bar{\psi}\left\langle\frac{\gamma_0-\boldsymbol{\gamma}\cdot\hat{\mathbf{n}}}{D_0+\hat{\mathbf{n}}\cdot\mathbf{D}}\right\rangle\psi, \tag{2}$$

where

$$T'^2 = \frac{g_r^2 C_f}{4}T^2, \tag{3}$$

and C_f is the Casimir invariant equal to $(N^2-1)/(2N)$ for the SU(N) group. The angular brackets denote an average over the orientation $\hat{\mathbf{n}}$.

Equation (2) is a nonlocal action in spacetime. If we suppress the gluon fields and concentrate on the fermion sector of the effective action, then the nonlocality is a lightlike separation between the two fermion fields:

$$\mathcal{L}_{\text{eff}} = -\bar{\psi}\gamma_\mu\partial^\mu\psi + \frac{T'^2}{8}\int dt'\epsilon(t-t') \times\{\langle\bar{\psi}(x)(\boldsymbol{\gamma}\cdot\hat{\mathbf{n}}-\gamma_0)\psi(x_P)\rangle - \langle\bar{\psi}(x)(\boldsymbol{\gamma}\cdot\hat{\mathbf{n}}+\gamma_0)\psi(x_{P'})\rangle\}, \tag{4}$$

where x_P and $x_{P'}$ refer to the pair of conjugate points lightlike separated from $(\mathbf{x},t)$:

$$x_P = (\mathbf{x}-\hat{\mathbf{n}}(t-t'),t'), \quad x_{P'} = (\mathbf{x}+\hat{\mathbf{n}}(t-t'),t'). \tag{5}$$

The nonlocal Euler-Lagrange equation of motion for the action is

$$\gamma_\mu\partial^\mu\psi(\mathbf{x},t) = \frac{T'^2}{8}\int dt'\epsilon(t-t')\langle(\boldsymbol{\gamma}\cdot\hat{\mathbf{n}}-\gamma_0)\psi(\mathbf{x}-\hat{\mathbf{n}}(t-t'),t')\rangle - \frac{T'^2}{8}\int dt'\epsilon(t-t')\langle(\boldsymbol{\gamma}\cdot\hat{\mathbf{n}}+\gamma_0)\psi(\mathbf{x}+\hat{\mathbf{n}}(t-t'),t')\rangle. \tag{6}$$

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Following Weldon [6], we may replace the nonlocal action by a local one involving auxiliary fields [7]:

$$\gamma_\mu \partial^\mu \psi(\mathbf{x},t) = \frac{T'}{2}\langle \gamma\cdot n_+ \chi_+\rangle - \frac{T'}{2}\langle \gamma\cdot n_- \chi_-\rangle, \tag{7}$$

$$n_{+,\mu}\partial^\mu \chi_+ = \frac{T'}{2}\psi(\mathbf{x},t), \tag{8}$$

$$n_{-,\mu}\partial^\mu \chi_- = \frac{T'}{2}\psi(\mathbf{x},t), \tag{9}$$

where we have introduced the two conjugate lightlike four-vectors

$$n_{+,\mu} \equiv (\hat{\mathbf{n}},1), \tag{10}$$

$$n_{-,\mu} \equiv (\hat{\mathbf{n}},-1), \tag{11}$$

so that

$$\gamma\cdot n_+ = (\boldsymbol{\gamma}\cdot\hat{\mathbf{n}} - \gamma_0), \tag{12}$$

$$\gamma\cdot n_- = (\boldsymbol{\gamma}\cdot\hat{\mathbf{n}} + \gamma_0), \tag{13}$$

and the new local effective Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{\text{eff}} = {}& -\bar\psi\gamma_\mu\partial^\mu\psi + \frac{T'}{2}\bar\psi\langle\gamma\cdot n_+\chi_+\rangle - \frac{T'}{2}\bar\psi\langle\gamma\cdot n_-\chi_-\rangle \\ & -\frac{T'}{2}\langle\bar\chi_+\gamma\cdot n_+\psi\rangle + \frac{T'}{2}\langle\bar\chi_-\gamma\cdot n_-\psi\rangle \\ & +\langle\bar\chi_+\gamma\cdot n_+ n_+\cdot\partial\chi_+\rangle - \langle\bar\chi_-\gamma\cdot n_- n_-\cdot\partial\chi_-\rangle .\end{aligned} \tag{14}$$

Note that apart from the equations of motion, the auxiliary fields $\chi_\pm$ do not satisfy any other constraints. This is unlike in the case of Weldon [6], where his auxiliary field ϕ^{Weldon} satisfies the constraint

$$\gamma\cdot n_+\gamma_0\phi^{\text{Weldon}} = 2\phi^{\text{Weldon}} \tag{15}$$

beyond the equation of motion

$$in_{+\mu}\partial^\mu\phi^{\text{Weldon}}(x) = \frac{T'}{2}\gamma\cdot n_+\psi(x). \tag{16}$$

In addition, Weldon does not distinguish between the two light cones $n_{+\mu}$ and $n_{-\mu}$. The relation between our auxiliary field χ_+ and that of Weldon is

$$\phi^{\text{Weldon}}(x) = -i\gamma\cdot n_+\chi_+(x). \tag{17}$$

Written in this form, the reason for the constraint on Weldon's field becomes clear. It comes about because of the nilpotent operator $(\boldsymbol{\gamma}\cdot\hat{\mathbf{n}} - \gamma_0)$ acting on χ_+:

$$(\gamma\cdot n_+)^2 = 0. \tag{18}$$

The advantage of the coupled set of local equations of motion (7)–(9) is that it gives us an insight into the role of the homogeneous solutions to Eqs. (8) and (9), which otherwise would not be transparent with the nonlocal equation of motion (6). These homogeneous solutions turn out to be related to the spacelike cuts of the thermal fermion propagator.

III. ANALYTIC PROPERTIES OF THE THERMAL FERMION PROPAGATOR

The BPFTW action leads to the thermal fermion propagator [8]

$$\langle T(\psi(x)\bar\psi(y))\rangle_\beta = \frac{1}{i}\int\frac{d^4p}{(2\pi)^4}\frac{-i\boldsymbol{\gamma}\cdot\mathbf{p}(1-\frac{T'^2}{2}a) + i\gamma_0 p_0(1-\frac{T'^2}{2}b)}{p^2 - p_0^2 + T'^2 - i\epsilon + \frac{T'^4}{4}(p^2a^2 - p_0^2b^2)}, \tag{19}$$

where a, b are the functions

$$a = \frac{p_0}{2p^3}\ln\left|\frac{p_0+p}{p_0-p}\right| - \frac{1}{p^2}, \tag{20}$$

$$b = \frac{1}{2p_0 p}\ln\left|\frac{p_0+p}{p_0-p}\right|, \tag{21}$$

chosen to be real along the entire real p_0 axis, and p denotes the magnitude of $\mathbf{p}$.

For $t>0$, we have

$$\langle T(\psi(x)\bar\psi(0))\rangle_\beta = \langle\psi(x)\bar\psi(0)\rangle \tag{22}$$

$$= \int\frac{d^3p}{(2\pi)^3}e^{i\mathbf{p}\cdot\mathbf{x}}\left\{ Z_p\frac{-i\boldsymbol{\gamma}\cdot\mathbf{p} + i\gamma_0\omega}{2\omega}e^{-i\omega t}\right. \tag{23}$$

$$\left. -\frac{T'^2}{8}\int_{-p}^{p}\frac{dp_0'}{p^3}\frac{i\boldsymbol{\gamma}\cdot\mathbf{p}p_0' - i\gamma_0 p^2}{p^2 - p_0'^2 + T'^2}e^{-ip_0't} + \frac{T'^4}{8}\int_{-p}^{p}\frac{dp_0'}{p^3}\frac{p_0'(i\boldsymbol{\gamma}\cdot\mathbf{p} - i\gamma_0 p_0')}{(p^2 - p_0'^2 + T'^2)^2}e^{-ip_0't}\right\} + O(T'^4). \tag{24}$$

Here, we have performed the contour integration in the p_0 plane and isolated the pole as well as the cut contributions for $t>0$. In doing so, we have dropped other $O(T'^4)$ terms along the spacelike cut that do not affect our final $O(T'^2)$ result.

Note that the spinor structure of the massive particle pole term has the (unusual) feature of being manifestly chiral invariant. The wave function renormalization constant Z_p at the pole is given by

$$Z_p = 1 - \frac{T'^2}{4p^2}\left(\ln\frac{4p^2}{T'^2} - 1\right). \tag{25}$$

IV. RELATION BETWEEN ψ AND Ψ

Based on the earlier work [1], the field ψ is related to the canonical Ψ by the expansion

$$\psi(x) = \Psi(x) + \frac{T'}{8}\int dt'\epsilon(t-t')(\langle\gamma\cdot n_+\Psi(x_P)\rangle - \langle\gamma\cdot n_-\Psi(x_{P'})\rangle) + \cdots . \tag{26}$$

This expansion correctly describes the thermal fermion propagator accurately to order T'.

To go beyond the $O(T')$ term in the expansion, we go back to Eqs. (8) and (9), and we note the homogeneous solutions

$$\chi^s_+(\mathbf{x},t) = \frac{1}{\sqrt{V}}\sum_{p,s}\{F_+(\hat{\mathbf{n}},p)U_{p,s}A_{p,s} + G_+(\hat{\mathbf{n}},p)V_{-p,s}B^\dagger_{-p,s}\}e^{i\mathbf{p}\cdot\mathbf{x}-i\hat{\mathbf{n}}\cdot\mathbf{p}t}, \tag{27}$$

$$\chi^s_-(\mathbf{x},t) = \frac{1}{\sqrt{V}}\sum_{p,s}\{F_-(\hat{\mathbf{n}},p)U_{p,s}A_{p,s} + G_-(\hat{\mathbf{n}},p)V_{-p,s}B^\dagger_{-p,s}\}e^{i\mathbf{p}\cdot\mathbf{x}+i\hat{\mathbf{n}}\cdot\mathbf{p}t}, \tag{28}$$

where $A_{p,s}$ and $B_{p,s}$ are the particle annihilation operators of the massive canonical Dirac field of Eq. (1) and the $F_\pm$ and $G_\pm$ are Dirac matrix functions of $\hat{\mathbf{n}}, \mathbf{p}, p_0$. They are to be determined so that they reproduce the location, magnitude, and phase of the spacelike cuts in the thermal fermion propagator.

These homogeneous solutions in turn contribute to the expansion for ψ through Eq. (7), so that Eq. (26) now becomes

$$\psi(x) = \Psi(x) + \frac{T'}{8}\int dt'\epsilon(t-t')[\langle\gamma\cdot n_+\Psi(x_P)\rangle - \langle\gamma\cdot n_-\Psi(x_{P'})\rangle] + \Psi^s(\mathbf{x},t) \tag{29}$$

with

$$\Psi^s(x) = i\frac{T'}{2}\int d^4y\{\langle S^{-1}(x-y)\gamma\cdot n_+\chi^s_+(y)\rangle - \langle S^{-1}(x-y)\gamma\cdot n_-\chi^s_-(y)\rangle\}, \tag{30}$$

where S^{-1} is the inverse propagator satisfying the property

$$\left(\gamma\cdot\frac{\partial}{\partial x} + T'\right)S^{-1}(x-y) = -i\delta^4(x-y). \tag{31}$$

The Dirac matrix functions $F_\pm$ and $G_\pm$ have the representation

$$F_\pm(\hat{\mathbf{n}},\mathbf{p},p_0) = (\boldsymbol{\gamma}\cdot\hat{\mathbf{n}} \pm \gamma_0)f_1 \pm (\boldsymbol{\gamma}\cdot\mathbf{p} \mp \hat{\mathbf{n}}\cdot\mathbf{p}\gamma_0)f_2, \tag{32}$$

$$G_\pm(\hat{\mathbf{n}},\mathbf{p},p_0) = (\boldsymbol{\gamma}\cdot\hat{\mathbf{n}} \pm \gamma_0)g_1 \pm (\boldsymbol{\gamma}\cdot\mathbf{p} \mp \hat{\mathbf{n}}\cdot\mathbf{p}\gamma_0)g_2, \tag{33}$$

where the scalar functions f, g are even under the exchange $\hat{\mathbf{n}} \to -\hat{\mathbf{n}}$. This representation ensures that the thermal fermion propagator is spacetime translation invariant. This arises because according to Eq. (29), the thermal fermion Green function may be written in terms of the canonical field Ψ and the new spacelike Ψ^s:

$$\begin{aligned}\langle\psi(x)\bar{\psi}(y)\rangle = {}& \langle\Psi(\mathbf{x},t)\bar{\Psi}(\mathbf{y},y_0)\rangle - \frac{T'}{4}\int dt'\epsilon(y_0-t')\langle\Psi(\mathbf{x},t)\bar{\Psi}(\mathbf{y}-\hat{\mathbf{n}}(y_0-t'),t')(\boldsymbol{\gamma}\cdot\hat{\mathbf{n}}-\gamma_0)\rangle \\ & + \frac{T'}{4}\int dt'\epsilon(t-t')\langle(\boldsymbol{\gamma}\cdot\hat{\mathbf{n}}-\gamma_0)\Psi(\mathbf{x}-\hat{\mathbf{n}}(t-t'),t')\bar{\Psi}(\mathbf{y},y_0)\rangle \\ & + \langle\Psi(\mathbf{x},t)\overline{\Psi^s}(\mathbf{y},y_0)\rangle + \langle\Psi^s(\mathbf{x},t)\bar{\Psi}(\mathbf{y},y_0)\rangle + \langle\Psi^s(\mathbf{x},t)\overline{\Psi^s}(\mathbf{y},y_0)\rangle.\end{aligned} \tag{34}$$

Here, for simplicity, we have identified the two light cone averages and written them as a single average over the orientation $\hat{\mathbf{n}}$. The first three terms in the expansion involve only the canonical Dirac field Ψ, and, as pointed out in the earlier work [1], they *together correctly reproduce the spinor as well as the pole structure* of the thermal Green function. The chiral flip part of the canonical massive Green function $\langle\Psi\bar{\Psi}\rangle$ is canceled by the other two terms, thus agreeing with the apparent chiral symmetry of the thermal Green function $\langle\psi\bar{\psi}\rangle$.

The cross terms involving Ψ with Ψ^s would, however, give rise to Fourier integrals of the type

$$e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}e^{-ip_0 t\pm i\hat{\mathbf{n}}\cdot\mathbf{p}y_0},$$

which are manifestly not invariant under time translations. The Dirac matrix representation in Eqs. (32) and (33) ensures the absence of these terms. The last term in the expansion reproduces correctly the spacelike cuts of the thermal Green function, when

$$F_+ = \frac{\eta}{4}\left\{\frac{\gamma\cdot\hat{\mathbf{n}}+\gamma_0}{2} + i\frac{\gamma\cdot\mathbf{p}-\hat{\mathbf{n}}\cdot\mathbf{p}\gamma_0}{\sqrt{p^2-(\hat{\mathbf{n}}\cdot\mathbf{p})^2}}\right\}, \tag{35}$$

$$F_- = \frac{\eta}{4}\left\{\frac{\gamma\cdot\hat{\mathbf{n}}-\gamma_0}{2} - i\frac{\gamma\cdot\mathbf{p}+\hat{\mathbf{n}}\cdot\mathbf{p}\gamma_0}{\sqrt{p^2-(\hat{\mathbf{n}}\cdot\mathbf{p})^2}}\right\}, \tag{36}$$

$$G_+ = \frac{\eta'}{4}\left\{\frac{\gamma\cdot\hat{\mathbf{n}}+\gamma_0}{2} + i\frac{\gamma\cdot\mathbf{p}-\hat{\mathbf{n}}\cdot\mathbf{p}\gamma_0}{\sqrt{p^2-(\hat{\mathbf{n}}\cdot\mathbf{p})^2}}\right\}, \tag{37}$$

$$G_- = \frac{\eta'}{4}\left\{\frac{\gamma\cdot\hat{\mathbf{n}}-\gamma_0}{2} - i\frac{\gamma\cdot\mathbf{p}+\hat{\mathbf{n}}\cdot\mathbf{p}\gamma_0}{\sqrt{p^2-(\hat{\mathbf{n}}\cdot\mathbf{p})^2}}\right\}. \tag{38}$$

Here η, η' are overall phase factors that remain arbitrary. The requirement that the $\langle\Psi^s\overline{\Psi^s}\rangle$ reproduce the cut structure exhibited in Eq. (24) only demands an internal 90° phase difference between f_1 and f_2, and likewise between g_1 and g_2. The choice for the overall phase factors in $F_\pm$ and $G_\pm$ will become apparent only when we turn to the relation between ψ and the canonical Dirac field Ψ at time $t=0$ and study their connection with the thermal vacuum at high T.

V. RELATION TO THERMAL VACUUM

Equation (29) provides the expansion of $\psi(x)$ in terms of the canonical field $\Psi(x)$. In this section, we shall use it to relate the ψ field at $t=0$ to the canonical operators. Let

$$\psi(\mathbf{x},0) = \frac{1}{\sqrt{V}}\sum_p e^{i\mathbf{p}\cdot\mathbf{x}}\left\{\begin{pmatrix}\chi_L a_{p,L}\\ \chi_R a_{p,R}\end{pmatrix} + \begin{pmatrix}\chi_R b^\dagger_{-p,R}\\ -\chi_L b^\dagger_{-p,L}\end{pmatrix}\right\}, \tag{39}$$

where $\chi_{L,R}$ is an eigenfunction of the helicity operator $\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}$ with eigenvalue ±1, respectively. A straightforward evaluation of the γ matrix algebra implied in Eq. (29) shows that in order to reproduce Eq. (24)

$$a_{p,s} = A_{p,s} + \eta' s\frac{T'}{2p}B^\dagger_{-p,s} + O(T'^2/p^2), \tag{40}$$

$$b_{p,s} = B_{p,s} - \eta^* s\frac{T'}{2p}A^\dagger_{-p,s} + O(T'^2/p^2). \tag{41}$$

At this stage, the rule that $a_{p,s}$ and $b_{p,s}$ should anticommute, places the requirement

$$\eta' = \eta^*, \tag{42}$$

leaving us with still an overall phase factor η to be determined.

In the absence of Ψ^s, the relation between $a_{p,s}, b_{p,s}$ and $A_{p,s}, B_{p,s}$ would have been simply $a_{p,s} = A_{p,s}$ and $b_{p,s} = B_{p,s}$. The additional terms of $B^\dagger_{-p,s}$ and $A^\dagger_{-p,s}$ arose entirely from the spacelike Ψ^s field in Eq. (29).

Thus the spacelike cut in the fermion propagator signals the presence of Bogoliubov pairing in the thermal vacuum.

Can we determine the precise nature of this Bogoliubov pairing in the new thermal vacuum? The answer is yes. For a study of the order parameter $\langle\bar{\psi}\psi\rangle_\beta$ will yield information on the properties of the vacuum. Since we now have the relation between ψ and Ψ at time $t=0$, we may proceed to obtain the canonical expansion for $\bar{\psi}\psi$. Let us introduce the order operator [9]

$$\frac{1}{2}\int d^3x\bar{\psi}(\mathbf{x},0)\psi(\mathbf{x},0) = -\sum_p Y_{1p}, \tag{43}$$

where

$$Y_{1p} \equiv -\sum_s \frac{s}{2}(a^\dagger_{p,s}b^\dagger_{-p,s} + a_{p,s}b_{-p,s}). \tag{44}$$

The order parameter $\langle\bar{\psi}\psi\rangle_\beta$ is the expectation value of the order operator with respect to the new thermal vacuum $|\text{vac}\rangle$.

Recall that the original Fock space vacuum satisfies the property

$$a_{p,s}|0\rangle = b_{p,s}|0\rangle = 0, \tag{45}$$

while the new thermal vacuum is the one annihilated by $A_{p,s}, B_{p,s}$:

$$A_{p,s}|\text{vac}\rangle = B_{p,s}|\text{vac}\rangle = 0. \tag{46}$$

Upon substituting the relations Eqs. (40) and (41) into the order operator Eq. (44), we find the canonical expansion

$$\int d^3x\bar{\psi}(\mathbf{x},0)\psi(\mathbf{x},0)=\sum_{p,s}\left\{s(A^\dagger_{p,s}B^\dagger_{-p,s}+B_{p,s}A_{p,s})-(\eta+\eta^*)\frac{T'}{2p}(A^\dagger_{p,s}A_{p,s}-B_{p,s}B^\dagger_{-p,s})\right\},\tag{47}$$

and we have the order parameter at $t=0$:

$$\int d^3x\langle\bar{\psi}\psi\rangle_\beta=\sum_{p,s}\langle\mathrm{vac}|\left\{s(A^\dagger_{p,s}B^\dagger_{-p,s}+B_{p,s}A_{p,s})-(\eta+\eta^*)\frac{T'}{2p}(A^\dagger_{p,s}A_{p,s}-B_{p,s}B^\dagger_{-p,s})\right\}|\mathrm{vac}\rangle$$
$$=(\eta+\eta^*)\sum_p\frac{T'}{p}.\tag{48}$$

Since $\langle\bar{\psi}\psi\rangle_\beta$ vanishes, we finally arrive at the result that[1]

$$\eta=-\eta'=i\tag{49}$$

which in turn implies that to order $O(T'/p)$, we have

$$a_{p,s}=A_{p,s}-is\frac{T'}{2p}B^\dagger_{-p,s},\tag{50}$$

$$b_{p,s}=B_{p,s}+is\frac{T'}{2p}A^\dagger_{-p,s}.\tag{51}$$

This relation spells out the precise nature of the Bogoliubov transformation that takes us from the original Fock space vacuum to the new thermal vacuum. The new vacuum is the generalized Nambu–Jona-Lasinio (NJL) vacuum [10]

$$|\mathrm{vac}\rangle=\prod_{p,s}(\cos\theta_p-is\ \sin\theta_p a^\dagger_{p,s}b^\dagger_{-p,s})|0\rangle,\tag{52}$$

where

$$\tan 2\theta_p=\frac{T'}{p}.\tag{53}$$

The interesting feature of this generalized NJL vacuum is the presence of the phase factor i in the quark-antiquark pair.

As has been pointed out elsewhere [1,11] this phase factor is responsible for the vanishing of $\langle\bar{\psi}\psi\rangle$, without however breaking up the quark-antiquark pairs. As a result, the generalized NJL vacuum is not chiral invariant. By this, we mean that under the old (zero temperature) chirality

$$a_{p,s}\to e^{is\alpha}a_{p,s},\tag{54}$$

$$b_{-p,s}\to e^{is\alpha}b_{-p,s},\tag{55}$$

the thermal vacuum goes over into a new unitarily inequivalent vacuum.

And yet there is an apparent chiral symmetry of the BPFTW action in Eq. (2). The resolution of this paradox comes when we recognize that the new chiral symmetry at high T is associated with a different Noether charge [6] than the original (zero temperature) chirality. This new Q_5^β has the expansion [1]

$$Q_5^\beta=-\frac{1}{2}\sum_{p,s}s(A^\dagger_{p,s}A_{p,s}+B^\dagger_{-p,s}B_{p,s}),\tag{56}$$

which clearly annihilates the thermal vacuum $|\mathrm{vac}\rangle$ and explains the apparent chiral symmetry of the high temperature effective action. But this high temperature chirality is not identical to the old chirality Q_5 given by

$$Q_5=-\frac{1}{2}\sum_{p,s}s(a^\dagger_{p,s}a_{p,s}+b^\dagger_{-p,s}b_{-p,s}).\tag{57}$$

[1]There is, of course, a conjugate solution with $\eta=-\eta'=-i$. To the extent that they yield the same spacelike cut contribution, it is physically equivalent to the solution we pick.

VI. CONCLUSION

In this paper, we have performed the quantization of the BPFTW action in coordinate space. By reformulating it in terms of auxiliary fields, the action may be made local. The auxiliary fields $\chi_\pm$ satisfy local equations of motion that admit homogeneous solutions, which turn out to be directly related to the spacelike cuts in the thermal propagator.

The homogeneous solutions impact on the relation between ψ and the canonical Ψ field at $t=0$. These relations spell out the connection between the vacuum of the massless free field $a_{p,s}$,$b_{-p,s}$ and the thermal vacuum of the effective action. Our results show how the presence of the spacelike cuts in the fermion propagator signals a new phase in the underlying thermal vacuum.

This thermal vacuum continues to exhibit the rich and complex structure of chiral symmetry violations as the zero temperature case. That $\langle\bar{\psi}\psi\rangle$ vanishes is no proof of chiral restoration at high T. In a separate communication [9], we introduce and discuss the $\mathrm{SU}(2N_f)_p\otimes\mathrm{SU}(2N_f)_p$ chirality algebra of order parameters that probe the state the chiral symmetry breaking.

Many questions remain to be answered. What is the underlying physics origin of the auxiliary field? How do the quark-antiquark pairing at high temperatures generate the intriguing 90° phase in the BCS-like ground state? There is clearly an interplay between the new chirality Q_5^β at high T and the old Q_5 chirality. How does it impact on the interactions of the pion at high T [12]?

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